Appendix

Three-State Device Networks

All derivations in the appendix are based on the binomial expansion $(P+q)^n$. In the case of three-state device structures, the expansion is modified to $(P+q_o+q_s)^n$, where P is the component probability of success, q_o open failure probability, q_s short failure probability, and n is the number of independent elements.

A.1 SERIES STRUCTURE

A.1.1 Reliability Expression

Let $n=2$. Therefore

$$(P+q_o+q_s)^2=1$$

Thus

$$P^2+2Pq_o+2Pq_s+q_o^2+2q_oq_s+q_s^2=1$$

The number of state combinations for $(n=2)=3^2=9$. The state combination truth table may be represented as follows:

Table A1.

NN	*NO*	*NS*
ON	*OO*	*OS*
SN	*SO*	*SS*

where N = the normal mode of the device (success)
O = the open mode failure state
S = the short mode failure state

The reliability terms, by inspection of the truth table, Table A1, are

$$PP+Pq_s=Pq_s=P^2+2Pq_s$$

Since

$$P=1-q_s-q_o, \tag{A.1}$$

$$R=(1-q_o^2)-q_s^2 \tag{A.2}$$

Now let $n=3$:
Therefore

$$(P+q_o+q_s)^3=1$$

Thus

$$P^3+3P^2q_o+3P^2q_s+3Pq_o^2+3Pq_s^2+6Pq_oq_s+q_o^3+3q_o^2q_s+3q_s^2q_o+q_s^3=1$$

Number of state combinations for $(n=3)=3^3=27$. The new state combinations truth table is as shown in Table A2.

Table A2.

NNN	SSS	OOO
NOO	SOO	OSS
OON	OOS	SSO
NON	OSO	ONN
ONO	SOS	NNO
NSS	SNN	NSO
SSN	NNS	NOS
SNS	NSN	SNO
SON	OSN	ONS

The reliability terms by inspection of Table A2

$$PPP+Pq_sq_s+Pq_sq_s+Pq_sq_s+PPq_s+PPq_s+PPq_s$$

therefore

$$R=P^3+3Pq_s^2+3P^2q_s \tag{A.3}$$

By using (A.1) and (A.3):

$$R=(1-q_o)^3-q_s^3 \tag{A.4}$$

Obviously, from (A.2) and (A.4), the general equation for the reliability of the series structure can be written as follows. For identical components:

$$R=(1-q_o)^n-q_s^n \tag{A.5}$$

and in the case of nonidentical components:

$$R=\prod_{i=1}^{n}(1-q_{oi})-\prod_{i=1}^{n}q_{si} \tag{A.6}$$

A.1.2 Probabilities of Failure

At $n=2$. From Table A1 open mode failure terms

$$q_oP+Pq_o+q_oq_o+q_sq_o+q_oq_s$$

Since

$$P=1-q_o-q_s \tag{A.7}$$

Therefore the series system open mode failure probability

$$Q_o=1-(1-q_o)^2 \tag{A.8}$$

and similarly for short-mode failure probability

$$Q_s=q_s^2 \tag{A.9}$$

At $n=3$. According to Table A2, open mode failure terms

$$3Pq_o^2+3P^2q_o+6Pq_oq_s+3q_o^2q_s+3q_s^2q_o+q_o^3 \tag{A.10}$$

By substituting for P in (A.10), open-mode failure probability

$$Q_o=1-(1-q_o)^3 \tag{A.11}$$

and in the same way for short-mode failure

$$Q_s=q_s^3 \tag{A.12}$$

In the case of identical components, according to (A.8) and (A.11), the general form of open mode system failure

$$Q_o=1-(1-q_o)^n \tag{A.13}$$

Similarly, for nonidentical components

$$Q_o=1-\prod_{i=1}^{n}(1-q_{oi}) \tag{A.14}$$

In the case of short mode system failure for identical components

$$Q_s = q_s^n \tag{A.15}$$

and for the nonidentical elements case

$$Q_s = \prod_{i=1}^{n} q_{si} \tag{A.16}$$

A.2 PARALLEL STRUCTURE

A.2.1 Reliability Expression

Let the number of parallel elements $m=2$

$$\therefore (P+q_o+q_s)^2 = 1$$

Thus

$$P^2+2Pq_o+2Pq_s+q_o^2+2q_oq_s+q_s^2=1$$

The reliability terms with inspection From Table A1

$$PP+q_oP+Pq_o=p^2+2Pq_o \tag{A.17}$$

Since $P=1-q_s-q_o$, therefore

$$R=(1-q_o)^2-q_s^2 \tag{A.18}$$

At $m=3$

$$\therefore (P+q_o+q_s)^3=1$$

Thus

$$P^3+3P^2q_o+3P^2q_s+3Pq_o^2+3Pq_s^2+6Pq_oq_s+q_o^3+3q_o^2q_s+3q_oq_s^2+q_s^3=1$$

By inspecting Table A2, system reliability

$$=P^3+3q_o^2P+3P^2q_o \tag{A.19}$$

Replace P with (A.7); therefore

$$R=(1-q_s)^3-q_o^3 \tag{A.20}$$

With the aid of (A.18) and (A.20), the general system reliability formula for identical elements connected in the parallel configuration becomes as follows:

$$R=(1-q_s)^m-q_o^m \tag{A.21}$$

The above equations for nonidentical elements may be rewritten as

$$R=\prod_{i=1}^{m}(1-q_{si})-\prod_{i=1}^{m}q_{oi} \tag{A.22}$$

A.2.2 Failure Probabilities

For $m=2$. Collected short-mode failure terms from Table A1

$$=q_sP+q_sq_o+q_sq_s+q_oq_s+Pq_s=q_s^2+2Pq_s+2q_oq_s$$

Thus:

$$\therefore Q_s=1-(1-q_s)^2 \tag{A.23}$$

Similarly, for open-mode failure

$$Q_o=q_o^2 \tag{A.24}$$

At $m=3$. Short-mode system failure terms from Table A2

$$=q_s^3+3Pq_s^2+6Pq_oq_s+3q_sq_o^2+2q_oq_o^2+3q_sq_o^2+3q_s^2q_o$$

Thus

$$Q_s=1-(1-q_s)^3 \tag{A.25}$$

Similarly, in the case of open mode failure

$$Q_o=q_o^3 \tag{A.26}$$

As seen from (A.23) and (A.25), the short-mode failure equation for identical elements can be generalized as follows:

$$Q_s=1-(1-q_s)^m \tag{A.27}$$

Similarly, for the nonidentical elements case

$$Q_s=1-\prod_{i=1}^{m}(1-q_{si}) \tag{A.28}$$

The corresponding generalized open mode system failure probability

$$Q_o = q_o^m \qquad \text{identical components} \tag{A.29}$$

and

$$Q_o = \prod_{i=1}^{m} q_{oi} \qquad \text{nonidentical components} \tag{A.30}$$