10

Transit System Reliability

10.1 INTRODUCTION

Reliability is an important consideration in the planning, design, and operation of transit systems. The discussion in this chapter is focused on track bound transit systems; the principles can, however, be applied to other types of transit systems as well. The term track bound is used here to describe systems whose vehicles are captive on a common track. This includes steel wheel on steel rail, rubber wheel on concrete guideway, and magnetically levitated vehicles. In the case of a road system, the failure of a vehicle affects the concerned vehicle and some delay may be caused to the other vehicles. The effect on the system is, however, more or less localized since the failed vehicle can be pulled to the side or bypassed by the other vehicles. The bypass capability of the track bound systems, on the other hand is extremely limited. The failure of a single vehicle in such systems could affect or immobilize the upstream vehicles and depending upon network configuration, the degrading or immobilizing effect could spread over the entire or a major part of the system. This serial effect makes the reliability an all the more important consideration in track bound transit systems.

Reliability is important for both the transit operator and the passengers. Lower reliability means increased unscheduled maintenance and decreased equipment availability. If availability is low, more vehicles are needed to meet the passenger demand but even with more vehicles, system performance may not be satisfactory. More vehicles can increase system availability but do not decrease the incidence of system failures. Reliability is important to passengers as it reflects the ability of a transit system to keep operating schedules.

Traditionally, the transit operators have been relying on warranties to assure the procurement of reliable equipment. The warranties are, however, more like maintenance or service contracts and do not necessarily serve as deterrents to system unreliability. Warranty makes the manufacturer pay for repairs during a limited period of time but once the warranty period is over, unreliability becomes the headache of the transit operator. The operators are now realizing that economical reliability can be built into the systems only during the design, development, and manufacturing

stage of the equipment. The emphasis now appears to be shifting towards specifying reliability targets and implementing reliability assurance programs during design and development. Reliability models provide a useful tool for the analysis of design configurations. These models for the components of a transit system and the techniques of combining these models are described in this chapter.

The models are discussed with regard to a specific system configuration in which the track is a single loop (Figure 10.1) with off-line stations. From the view point of reliability modeling, this configuration can be generally regarded equivalent to a single two way route in which the vehicles move on one track in one direction and at the terminal station they are switched on to the other track for the opposite direction. The methodology illustrated by these models can be employed to develop models for more individual applications.

A transit system consists of the following subsystems:

1. Vehicle fleet.
2. Passenger stations.
3. Substations for power supply.
4. Command and control.
5. Guideway.

These subsystems, in turn, may be divided into what is termed components in this chapter. Models for each subsystem are described.

10.2 BASIC THEORY

The models consider the total and partial failure modes of the vehicle, the spare vehicle, and vehicles on maintenance. These models are then combined with the models for the other subsystems. There is a considerable statistical dependence between the various modes of failure, repair, retrieval, and maintenance. The state space approach [9, 10] is, therefore, used in developing these models.

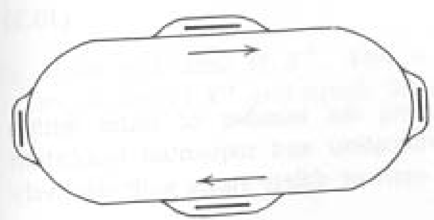


Figure 10.1 A single loop.

10.2.1 State Space Approach

In this approach, the possible states of the system (state space) and the modes of transiting from one state to another are identified. Each mode of interstate transition is assigned a specific value called its interstate transition rate. The state equations can be written using the frequency balancing approach [10]. The frequency of transiting from state i to state j is defined as the expected transition rate from state i to state j and is given by $P_i\lambda_{ij}$. The frequency balancing approach states that the rate of change of the probability of being in state i equals the frequency of transiting into state i from all the remaining states minus the frequency of transiting out of state i, that is,

$$\sum_j P_j(t)\lambda_{ji} - P_i(t)\sum_j \lambda_{ij} = \dot{P}_i(t) \tag{10.1}$$

In the equilibrium condition, $\dot{P}_i(t)=0$, and therefore (10.1) reduces to

$$\sum_j P_j\lambda_{ji} - P_i\sum_j \lambda_{ij} = 0 \tag{10.2}$$

that is, the frequency of encountering state i equals the frequency of encountering the rest of state space from state i. For n states there are n equations and they are linearly dependent; any equation can be obtained from the remaining $(n-1)$ equations. Any $(n-1)$ equations together with the total probability equation,

$$\sum P_i = 1$$

can be solved to obtain the state probabilities. These state probabilities can be used to obtain the reliability measures.

The number of states tend to be large due to the size and complexity of the transit system. The models can be reduced using the concept of equivalent transition rate [10], which under the equilibrium condition is given by

$\lambda^e_{X^-X^+}$ = the equivalent transition rate from subset X^- to subset X^+

$$= \sum_{i\in X^-}\sum_{j\in X^+} \frac{P_i\lambda_{ij}}{\sum_{i\in X^-} P_i} \tag{10.3}$$

The other techniques used for keeping the number of states within manageable limits are state space truncation and sequential truncation [10]. These techniques systematically omit or delete states with relatively low probabilities.

10.2.2 Measures of Reliability

Transit systems, like other commercial or public systems, are designed to meet a certain demand. Therefore, the reliability of transit systems can be viewed in two ways. The system is comprised of hardware, software, and the human interface, although in most of the reliability studies only hardware and computer-based software (if any) are considered. One way of looking at the reliability is in terms of the system deficiencies. The measures relating to this approach are termed system-based reliability measures. It is also desirable to know how these system deficiencies relate to the inability of the system to satisfy the demand and the corresponding measures are called the demand-based reliability measures. Obviously both of these types of measures are interrelated.

System-Based Measures. Reliability indices are usually defined in terms of success or failure. Many complex systems like transit systems or electric power systems have, however, several levels of failure and it is, therefore, appropriate to define the calculated reliability measures in terms of subset X^+, which may contain a specific number of system states. This subset defines an event or a particular mode of degradation of the system. The various modes or levels of system degradation can, therefore, be represented by suitably defining the elements of X^+. As an example X^+ may be used to represent the system states having the number of failed passenger stations greater than a particular number. The following measures defined on X^+ have been used in this chapter.

1. *Probability of X^+.* This can be defined as the limiting value of the time spent in X^+ as a fraction of the total operating time and is given by

$$P_+ = \sum_{i\in X^+} P_i \tag{10.4}$$

2. *Frequency of encountering X^+.* This is the mean number of occurrences of X^+ per unit of the operating time and can be calculated by

$$f_+ = \sum_{j\in X^-} P_j \sum_{i\in X^+} \lambda_{ji} \tag{10.5}$$

3. *Mean cycle time of X^+.* This is the mean time between successive encounters of X^+ and equals the reciprocal of f_+, that is,

$$T_+ = \frac{1}{f_+} \tag{10.6}$$

4. *Mean duration of* X^+. This is the expected time of stay in X^+ in one cycle (one cycle constitutes X^+ and X^-) and is given by

$$d_+ = \frac{P_+}{f_+}$$

$$= P_+ T_+ \qquad (10.7)$$

Demand Based Measures [*13*]. The primary purpose of a transit system is to move passengers between the various points of a network. It is, therefore, important to have a measure of reliability as perceived by the passengers. As an example [7] consider a jeep having an MTBF of approximately 260 hours [5]. If this vehicle were driven on the average for 30 miles/day at an average speed of 15 miles/hour the average interval between two failures would be 130 calendar days, whereas this interval in terms of operating time would be only about 11 days. Coming back to transit systems, consider a system that breaks down on an average of every 15 calendar days with the average down time duration of half an hour. A passenger who travels for only, say, 15 minutes twice a day will not be affected by every system failure. Assuming uniform service level of 16 hours/day operation, the passenger may be affected on the average approximately by every seventh failure and therefore will tend to see the system failing on the average approximately three times a year. The perception of the failure is further affected by several factors such as whether the delay has to be tolerated in a comfortable, airconditioned environment or in a hot stuffy vehicle and the personal temperament of the passenger. Media reports and the stories of system failures told by other passengers add something to the direct exposure to failures. It can be appreciated that it is extremely difficult to measure or predict the passengers perception of the failures. Nevertheless suitable measures related to the impact of system failures on the passengers can be devised.

The ability of a transit system to continue providing transportation services as scheduled or advertised may be termed as the operational reliability. In this definition, it is assumed that schedules are set within normal capabilities of the system. If the transit system cannot provide scheduled service when every subsystem is working normally, it is a problem of planning, scheduling, or operations management. A failure in a subsystem, however, causes a perturbation that may affect the system's ability to provide adequate service. The impact of failures on the schedule-keeping ability of a transit system is the concern of operational reliability. The demand based measures of reliability are, therefore, also the measures of operational reliability. Some measures are described here.

DELAY, THE BASIC MEASURE OF OPERATIONAL RELIABILITY [11]. So long as a passenger can get from one station to another in a comfortable, safe, and timely manner, a failure occurring in a system does not bother him. A minor delay may be hardly noticed, but if the passengers suffer long delays or if the delay is too frequent, the transit system would appear unreliable to the passengers. The failure-induced delay is, therefore, a meaningful measure of operational reliability. The delay may be incurred at the following points.

Departure points. When a passenger arrives at a departure point, the system may be inoperative or operative in a degraded mode. This adds to the waiting time, making the total travel time longer.

Delay during travel. The passengers on board the vehicles may suffer delay due to the breakdown of a subsystem. The delay can be broadly classified into three categories:

1. Minor delay $< x$ units of time.
2. Major delay $> x$ units of time.
3. Entrapment, a major delay requiring passenger evacuation.

There does not appear to be enough relevant data on the passenger intolerance of delays and this appears to be a useful although difficult area for investigation. Despite extensive investigations, some judgment will always be involved in fixing the value of x. Some basic quantitative measures of operational reliability in terms of delay are discussed below [11].

1. $P_D(x_1, x_2)$, that is, the probability that a passenger will encounter a delay, $> x_1$ and $< x_2$, on a trip. The length of the delay is defined by the interval (x_1, x_2). The interval $(0, x)$ means a delay less than x, whereas (x, ∞) means delay longer than x.
2. MTBD, the mean time between two successive delays suffered by a passenger.
3. Expected value of delay.

Using the frequency concept of probability, $P_D(x_1, x_2)$ can also be interpreted as the limiting value of the proportion of the trips having a delay (x_1, x_2) to the total number of trips. Suppose that a person makes a large number of trips of varying lengths. If the trips on which he incurred a delay of say greater than 5 minutes were counted and then divided by the total number of trips, it would approximate $P_D(5, \infty)$. The probability can be further converted into MTBD by knowing the number of trips in a year. It will also be desirable to compute the expected value of delay. Data, however, may not be available to compute these measures. The calculation of these measures could be simplified by relating the delay to the vehicles rather than the passengers. In such a case the ratio of vehicle delayed trips to the total trips would be calculated.

Many measures of operational reliability can be defined [2–4]. These measures should reflect the delay incurred or travel time lost by the passengers or vehicles. The real difficulties lie, however, not in defining measures of operational reliability but in developing suitable analysis techniques and obtaining valid data for calculating these measures.

LOCE OR LOCP. As noted earlier it is not difficult to define more sophisticated measures of operational reliability. The harder part is the data and subsequent synthesis of this data to calculate the measure. Any measure may prove to be satisfactory so long as it reasonably reflects the ability of the system to provide adequate transportation service. One simpler measure may be called "Loss of Capacity Expectation" (LOCE) or "Loss of Capacity Probability" (LOCP) and can be defined as the probability that the system will not have enough capacity to meet the demand adequately. This can also be interpreted as the expected value of time during which the system cannot meet the demand. This will include the periods of degraded operation, for example, not enough vehicles being available or some other subsystem failure causing deficient operation. The LOCE is computed as

$$\text{LOCE} = \sum_i P_i Q_i \tag{10.8}$$

where P_i = probability of the system being in state i
Q_i = probability that the system will not be able to meet demand in state i

The LOCE gives the same weight to all system deficiencies irrespective of the magnitude of the impact and it is therefore likely to be a conservative measure. This approach is identical to the use of LOLE (loss of load expectation) in power generation planning studies [1] by the electric power utilities.

10.3 VEHICLE SYSTEM MODEL

This section first describes the model for a single vehicle and then for the system of vehicles. The vehicle includes the body structure and all onboard equipment carried by the vehicle. A vehicle can have several modes of failure. For the sake of simplicity, however, each component of the vehicle is assumed to have two modes of failure.

Retrieval or Total Failure Mode. With this type of failure, the vehicle is immobilized on the track and it cannot move on its own. External assistance is normally required for clearing it from the track. This kind of failure is severe and causes serious delay to the passengers as not only the affected vehicle is stuck but the upstream vehicles also come to a halt.

Partial Failure Mode. This type of failure causes degradation of vehicle performance but the vehicle is not immobilized and can clear the track on its own. Such failures cause system slow down but do not interrupt the service. When the defective vehicle clears the track by getting into a siding or into the maintenance yard, the normal flow of traffic is resumed.

10.3.1 Vehicle Model

The reliability model for a single track bound transit vehicle is shown in Figure 10.2, where the following notation is used:

$\lambda_{i1}, \lambda_{i2}$ = the retrieval and partial mode failure rates of the ith component, that is,

λ_{i1} = 1/(mean time between retrieval mode failures of the ith component)

λ_{i2} = 1/(mean time between partial mode failures of the ith component)

μ_r = retrieval rate = 1/mean time to retrieve a vehicle

μ_p = partial failure recovery rate = 1/mean time to clear a partially failed vehicle

μ_i = repair rate of the ith component = 1/mean time to repair the ith component

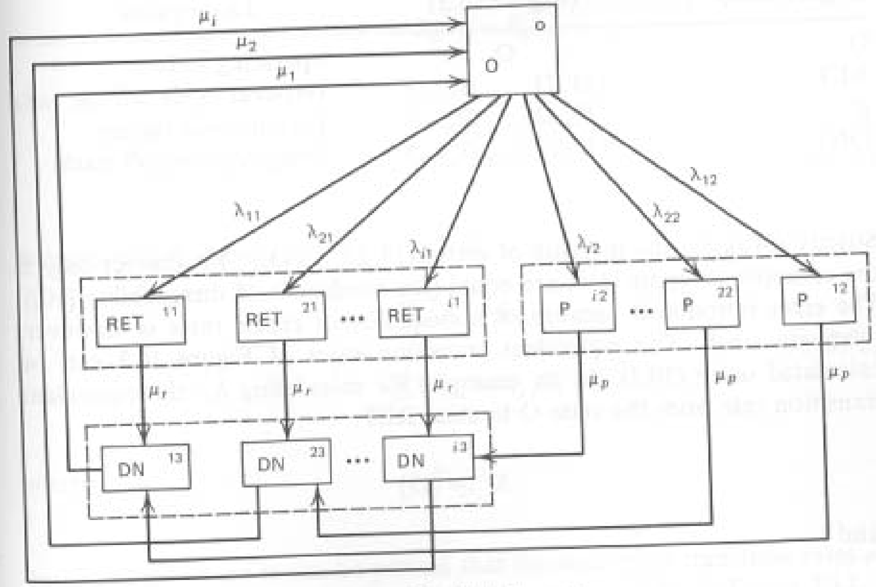


Figure 10.2 Vehicle model.

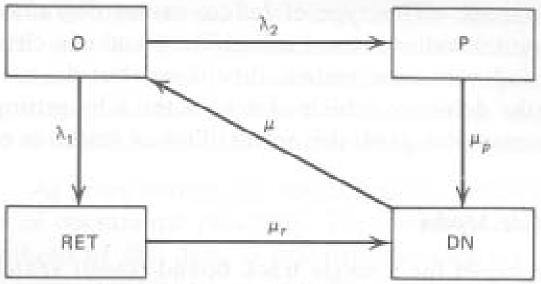


Figure 10.3 Equivalent vehicle model.

The normally operating state of the vehicle is denoted by "O." From this state, the vehicle can fail into a retrieval mode failure (denoted by RET) or a partial mode failure, denoted by P. The vehicle is either retrieved from the track or in the case of partial failure it clears the track on its own power and passes into the down state (denoted DN) in the maintenance area. The vehicle is then repaired back into its normally operating state O. It should be noted that the mean retrieval time is assumed to be the same for failures originating from different components. The same is true for the mean time to clear a partial mode failure.

The vehicle states can be grouped using the concept of equivalent transition rate defined in (10.3). The equivalent model is shown in Figure 10.3 where

Equivalent state (Figure 10.3)	Original states (Figure 10.2)	Description
O	O	operating state
RET	$(11,21,\ldots,i1,\ldots)$	retrieval mode failure
P	$(12,22,\ldots,i2,\ldots)$	partial mode failure
DN	$(13,23,\ldots,i3,\ldots)$	being repaired off track

Strictly speaking, the merging of states $(13,23,\ldots,i3,\ldots)$ is correct only if the component repair rates are equal (see conditions of mergeability [10]). The error introduced because of nonequality of repair rates is, however, relatively small. The equivalent transition rates of Figure 10.3 can be calculated using (10.3). As an example, for calculating λ_1, the equivalent transition rate from the state O to state RET,

$$X^- = (O)$$

and

$$X^+ = (11,21,\ldots,i1,\ldots)$$

and therefore

$$\lambda_1 = \sum_i \frac{P_0 \lambda_{i1}}{P_0} = \sum_i \lambda_{i1} \tag{10.9}$$

Similarly for μ, the equivalent transition rate from the state DN to the state O,

$$X^- = (13,23,\ldots,i3,\ldots)$$

and

$$X^+ = (O)$$

Therefore

$$\mu = \frac{\sum_i P_{i3}\mu_i}{\sum_i P_{i3}} \tag{10.10}$$

Now

$$P_{i3}\cdot\mu_i = P_{i1}\mu_r + P_{i2}\mu_p = P_0\lambda_{i1} + P_0\lambda_{i2}$$

that is,

$$P_{i3} = \frac{(\lambda_{i1}+\lambda_{i2})P_0}{\mu_i} \tag{10.11}$$

Substituting P_{i3} from (10.11) into (10.10)

$$\mu = \frac{\sum_i (\lambda_{i1}+\lambda_{i2})}{\sum_i (\lambda_{i1}+\lambda_{i2})T_i} \tag{10.12}$$

where

$$T_i = \frac{1}{\mu_i}.$$

In a similar fashion it can be proved that the equivalent transition rates μ_r and μ_p in Figure 10.3 have the same values as μ_r and μ_p in Figure 10.2.

10.3.2 Vehicle System Model

The state space model of a single vehicle is described in section 10.3.1 and this section now describes the model for the system of vehicles, that is, all the passenger carrying vehicles in the system. The model is based on the following assumptions:

1. Only the operating vehicles are liable to fail and the vehicles on standby or maintenance are not subject to failure. This assumption results in assigning zero failure rates to the vehicles being maintained or in standby mode. The failure rate of a cold standby can be generally assumed zero. If the vehicles are in a warm standby mode, that is, partially powered, this assumption is valid only so long as the failure rate in warm standby mode is small as compared with that of the operating vehicle.
2. The failure of a single vehicle in the retrieval mode causes the whole system of vehicles to be down. The duration of the down time of a vehicle is considered from the time it comes to a halt to time of resuming normal operation. The down time of all the vehicles, the directly affected vehicle and the vehicles coming to a halt as a result of blocking, is assumed to be the same. This assumption was made because of short headways and a relatively small loop length. In a larger loop and longer headways, all the vehicles may not be equally affected and a correction to models may be needed.
3. A vehicle in a partial failure mode is assumed to be removed from the system as soon as possible after the occurrence of the failure and therefore the probability of another unit failing during this period or the partial mode passing into full mode is assumed zero.
4. So long as there is even one standby, a unit on which maintenance is completed will be interchanged with an operating or standby unit. When, however, no spares are left, the unit passes directly from maintenance into the operating mode, without going on stand-by.

A section of the state transition diagram of the model for the system of vehicles is shown in Figure 10.4, where n, s, and m denote the number of operating, spare, and on-maintenance (preventive) units, respectively. The bigger squares represent the operating states and the small squares denote the corresponding partial and retrieval mode states. The index in the top left corner of the operating state is the state number and the number of failed units is indicated in the lower left corner. The first column of states has m units on maintenance and is called group m in Figure 10.4, the second column is called group $(m-1)$ and has $(m-1)$ units on maintenance and so on. Only two groups are shown in Figure 10.4. In the state

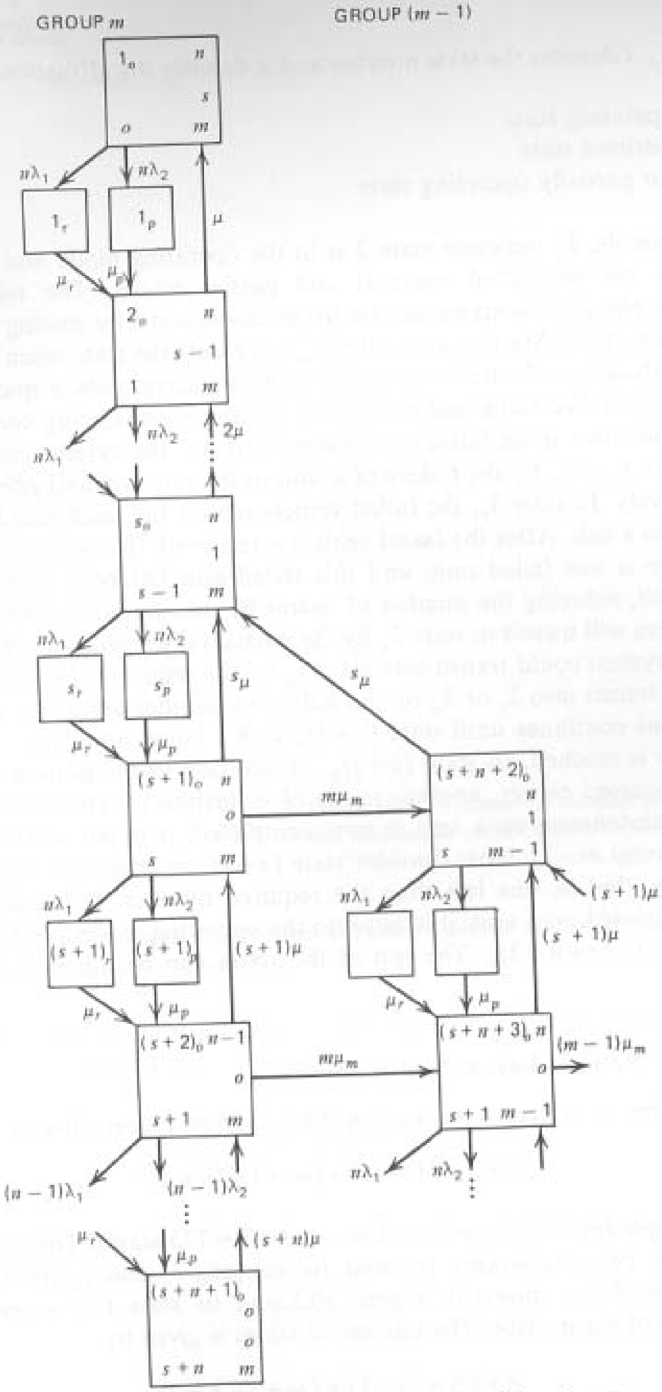


Figure 10.4 A section of vehicle system state transition diagram.

index i_x, i denotes the state number and x denotes the affiliation

o = operating state
r = retrieval state
p = for partially operating state

For example, 2_o indicates state 2 is in the operating mode and 2_r and 2_p indicate the associated retrieval and partial modes. The relationships between the various states can be better understood by tracing through a few of the states. Starting at the top, 1_o represents the state when every unit is as it should be. In this state there are n operating units, s spare units to replace the failed units and preventive maintenance is being carried on m units and there is no failed unit. From state 1_o, the system could transit into state 1_r or 1_p by the failure of a unit in the retrieval and partial mode, respectively. In state 1_r, the failed vehicle sits on the track and brings the system to a halt. After the failed vehicle is removed, the system enters state 2_o, there is one failed unit, and this failed unit has been replaced by a spare unit, reducing the number of spares by one. Similarly from state 1_p, the system will transit to state 2_o by the partial failure recovery. From state 2_o, the system could transit into state 1_o by the repair of the failed unit or it could transit into 2_r or 2_p by the failure of another unit. This pattern of transitions continues until state $(s+1)_o$ with s failed units and o units on stand-by is reached. In state $(s+1)_o$, in addition to the pattern of transitions discussed earlier, another mode of transition is introduced, that is, when maintenance on a unit is now completed, it is put in the standby mode (group $m-1$). Now consider state $(s+2)_o$ in which $(n-1)$ units are operating, that is, one less than the required number. If maintenance is now completed on a unit, it is put into the operating mode and the system transits into $(s+n+3)_o$. The rest of the states can be traced in a similar manner.

10.3.3 Reduced Vehicle System Model

The number of system states can be derived from Figure 10.4 as,

$$NVS=3(s-1)+(m+1)(3n+4) \tag{10.13}$$

For example for $n=50$, $s=2$, and $m=4$, $NVS=773$ states. The number of states can be considerably reduced by merging i_r and i_p with i_o. The reduced model is shown in Figure 10.5 and its state i is equivalent to (i_o, i_r, i_p) of Figure 10.4. The number of states is given by

$$RNVS=(s-1)+(m+1)(n+2) \tag{10.14}$$

Now for $n=50$, $s=2$, and $m=4$, $RNVS=261$ as compared with $NVS=773$. For m units on maintenance, there are $(m+1)$ groups of states of Figure

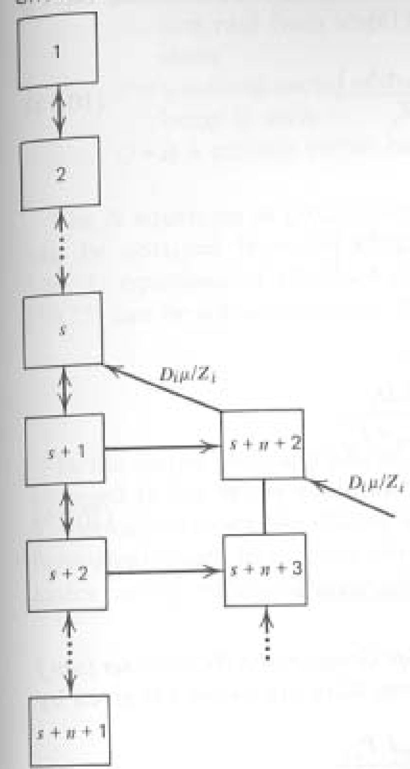


Figure 10.5 A section of reduced state transition diagram of vehicle system.

10.5. The equivalent transition rates between various states of the Figure 10.5 can be calculated using (10.3).

Equivalent Transition Rates within a Particular Group. The transition rate from state i to state $i+1$ is

$$\lambda_{i(i+1)}=\frac{P_{ip}\cdot\mu_p+P_{ir}\cdot\mu_r}{P_{io}+P_{ip}+P_{ir}} \tag{10.15}$$

Now

$$P_{ir}=\frac{O_i\lambda_1}{\mu_r} \tag{10.16}$$

and

$$P_{ip}=\frac{O_i\lambda_2}{\mu_p} \tag{10.17}$$

where O_i is the number of operating units in state i. Substituting for P_{ir} and P_{ip} from (10.16) and (10.17) into (10.15),

$$\lambda_{i(i+1)} = \frac{O_i(\lambda_1+\lambda_2)}{Z_i} \qquad (10.18)$$

where

$$Z_i = 1 + O_i\left(\frac{\lambda_1}{\mu_r} + \frac{\lambda_2}{\mu_p}\right)$$

Similarly

$$\lambda_{i(i-1)} = \frac{P_{io}\mu D_i}{P_{io}+P_{ir}+P_{ip}}$$

$$= \frac{\mu D_i}{Z_i} \qquad (10.19)$$

where D_i is the number of failed units in the state i.

Equivalent Transition Rates from a Particular Group to the Next Group (say j to $j+1$). The equivalent transition rate from state i to $i+n+1$ is given by

$$\mu_{i(i+n+1)} = \frac{m_i\mu_m I_i P_{io}}{P_{io}+P_{ir}+P_{ip}}$$

$$= \frac{m_i\mu_m I_i}{Z_i} \qquad (10.20)$$

where m_i = number of units on maintenance in state i
$\mu_m = 1/T_m$
T_m = mean maintenance time
$I_i = 0$ for number of spares in state $i > 0$
$= 1$ otherwise

The Equivalent Transition Rates From a Particular Group to the Previous Group. Shown in Figure 10.5.

10.3.4 Solution for State Probabilities and Reliability Measures

State Equations. The steady-state equations for the equivalent model (Figure 10.5) can be written using (10.2). In the matrix notation this can be written in the form,

$$AP = O \qquad (10.21)$$

where A = an $N\times N$ matrix such that its ijth term represents the transition rate from state j to state i, N being the total number of states
P = a column vector whose ith term is P_i, that is, the probability of being in state i
O = is a column vector having all elements as zero

The N equations of (10.21) are linearly dependent, that is, any equation can be obtained from the remaining $(N-1)$ equations. Therefore any $(N-1)$ equations of (10.21) together with the total probability equation (10.22) can be solved to obtain P

$$\sum_i P_i = 1.0 \qquad (10.22)$$

In the matrix form any row of A and the corresponding element in O are changed to 1.0 before solution. The linear equations can be solved using numerical methods like Gauss elimination. Once the probabilities of the equivalent states have been determined, the probabilities of the original states can be calculated using the following equations:

$$P_{io} = \frac{P_i}{Z_i} \qquad (10.23)$$

$$P_{ir} = \frac{O_i\lambda_1 P_{io}}{\mu_r} \qquad (10.24)$$

and

$$P_{ip} = \frac{O_i\lambda_2 P_{io}}{\mu_p} \qquad (10.25)$$

Vehicle Exposure Factor. The vehicle system model described in Section 10.3.2 is good for the on-line stations. When, however, the stations are off-line, the retrieval mode failure of a vehicle within the station limits does not immobilize the rest of the vehicles since they can bypass on the express lane. The retrieval state i_r can, therefore, be decomposed into i_{r1} and i_{r2}, representing failure within the station limit and on-line, respectively. The probabilities for these states can be computed as follows:

$$P_{i_{r1}} = P_{ir}\cdot E \qquad (10.26)$$

and

$$P_{i_{r2}} = P_{ir}\cdot(1-E) \qquad (10.27)$$

where E is the exposure factor, defined as the limiting ratio of the unit failures within the station limits to the total failures and is assumed approximately equal to the ratio of the length of the guideway within station limits to the total length of the guideway.

The vehicle failures within the station limits will, however, shut down the station lane and are, therefore, considered a part of the station lane failures. The equivalent failure rate component to be added to the station lane failure rate is calculated by,

$$\lambda_{VS} = \frac{\sum_i P_{io} O_i \lambda_1 E}{\sum_i P_{io} \cdot N_{ST}} \tag{10.28}$$

where N_{ST} is the number of passenger stations.

Vehicle System Reliability Measures. Once the state probabilities have been calculated, the event (subset of states) probabilities, frequencies, and other measures can be calculated using (10.4)–(10.7). There are many ways of defining the events and two of them are described below.

EXACT MEASURES. These measures calculate the probabilities and frequencies of encountering states in which the operating vehicles are equal to a particular number and can be designated as $P(N_o = n)$ and $f(N_o = n)$ where N_o denotes the operating vehicles. The probability of $N_o = n$ can be simply calculated by adding the probabilities of all states having n operating vehicles. The frequency and other measures can be calculated using (10.5)–(10.7).

CUMULATIVE STATE MEASURES. Another way to represent these measures is to calculate the probabilities and frequencies of encountering states in which there are fewer than a particular number of operating vehicles. The equations for these measures are given below.

$$P(N_o < n) = \sum_{i:\, O_i < n} (P_{io} + P_{ir1} + P_{ip}) + \sum_i P_{ir2} \tag{10.29}$$

and

$$f(N_o < n) = P_{ko} \cdot O_k \lambda_1 E + P_{kp} \mu_p + \sum_i P_{io} O_i \lambda_1 (1 - E) \tag{10.30}$$

where state k_o is such that for $(k_o + 1)$, $N_o = n$.

10.3.5 Vehicle System Example

The models described in this chapter have been implemented in a computer program [12]. Starting with the component data, the program gener-

ates the transition rate matrices for the subsystem and system models and then solves these matrix equations to provide with system-based reliability measures.

The vehicle system data for this example is printed out in Table 10.1A. The system consists of 14 vehicles out of which 2 are on maintenance and two are kept as spares. The assumed failure rate and other data are also listed in Table 10.1A. The probabilities of being in various states (see Figure 10.4) are printed in Table 10.1B. The state description on the right-hand side pertains to *io*, that is, the operating state. Column P contains probabilities of the operating states *io* and the associated columns PR, PS and PP give the probabilities of *ir*2, *ir*1, and *ip*. It can be seen that the most significant probability values are for states (10, 2, 2), (10, 1, 2) and the operating states of (10, 1, 1) and (10, 1, 0). The state probabilities are grouped as a function of the exact number of vehicles in Table 10.1C. The second column, "PROB OF OPTG," gives the probability of being in the operating state with number of vehicles indicated in the first column. The third column "FREQ OF OPTG" can be interpreted in either of the two ways: (a) the number of times the system transits out of the state in a day, or (b) the number of times per day the state is entered by the system. The fourth and fifth columns indicate the probabilities and frequencies of encountering the partial operating states. The probability and frequency of the system being in the retrieval state are given at the bottom. The reliability measures arranged in the cumulative form in Table 10.1D, where the reliability measures for $N_o \leq 9$ and downward all are approximately

Table 10.1A *Vehicle system data*

MEAN TIME TO VEHICLE FAILURE(PERMANENT) =	5000.0000	HOURS
MEAN TIME TO VEHICLE FAILURE(PARTIAL MODE) =	500.0000	HOURS
MEAN TIME TO VEHICLE RETRIEVAL =	0.5000	HOURS
MEAN TIME TO CLEAR PARTIAL MODE VEHICLE =	0.2500	HOURS
MEAN TIME TO VEHICLE MAINTENANCE =	3.0000	HOURS
MEAN TIME TO VEHICLE REPAIR =	2.5000	HOURS
NUMBER OF OPERATING VEHICLES =	10	
NUMBER OF SPARE VEHICLES =	2	
NUMBER OF VEHICLES ON MAINTENANCE =	2	
VEHICLE EXPOSURE FACTOR =	0.3000	

Table 10.1B *Vehicle system state probabilities*

THE PROBABILITIES OF BEING IN VARIOUS STATES / STATE DESCRIPTION

STATE NO	PR	PS	P	PP	O	S	M
1	0.658554E-03	0.282237E-03	0.940792D 00	0.470396E-02	10	2	2
2	0.357769E-04	0.153330E-04	0.511099D-01	0.255549E-03	10	1	2
3	0.533779E-06	0.228762E-06	0.762541D-03	0.381271E-05	10	0	2
4	0.564417E-08	0.241893E-08	0.895901D-05	0.403156E-07	9	0	2
5	0.437401E-10	0.187458E-10	0.781074D-07	0.312429E-09	8	0	2
6	0.252282E-12	0.108121E-12	0.514861D-09	0.180201E-11	7	0	2
7	0.108501E-14	0.465003E-15	0.258335D-11	0.775005E-14	6	0	2
8	0.344093E-17	0.147468E-17	0.983122D-14	0.245780E-16	5	0	2
9	0.782828E-20	0.335498E-20	0.279582D-16	0.559163E-19	4	0	2
10	0.121067E-22	0.518858E-23	0.576509D-19	0.864764E-22	3	0	2
11	0.114135E-25	0.489148E-26	0.815247D-22	0.815247E-25	2	0	2
12	0.495558E-29	0.212382E-29	0.707940D-25	0.353970E-28	1	0	2
13	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	0	0	2
14	0.265234E-06	0.113672E-06	0.378906D-03	0.189453E-05	10	1	1
15	0.152215E-06	0.652349E-07	0.217450D-03	0.108725E-05	10	0	1
16	0.351460E-08	0.150626E-08	0.557873D-05	0.251043E-07	9	0	1
17	0.390439E-10	0.167331E-10	0.697212D-07	0.278885E-09	8	0	1
18	0.281618E-12	0.120694E-12	0.574731D-09	0.201156E-11	7	0	1
19	0.141748E-14	0.607490E-15	0.337495D-11	0.101248E-13	6	0	1
20	0.506249E-17	0.216964E-17	0.144642D-13	0.361606E-16	5	0	1
21	0.126534E-19	0.542289E-20	0.451908D-16	0.903815E-19	4	0	1
22	0.211348E-22	0.905779E-23	0.100642D-18	0.150963E-21	3	0	1
23	0.212540E-25	0.910887E-26	0.151815D-21	0.151814E-24	2	0	1
24	0.975243E-29	0.417961E-29	0.139321D-24	0.696602E-28	1	0	1
25	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	0	0	1
26	0.525947E-06	0.225406E-06	0.751354D-03	0.375677E-05	10	1	0
27	0.629002E-08	0.269572E-08	0.898575D-05	0.449287E-07	10	0	0
28	0.327709E-09	0.140447E-09	0.520174D-06	0.234078E-08	9	0	0
29	0.787093E-11	0.337325E-11	0.140552D-07	0.562209E-10	8	0	0
30	0.770138E-13	0.330059E-13	0.157171D-09	0.550099E-12	7	0	0
31	0.465970E-15	0.199702E-15	0.110945D-11	0.332836E-14	6	0	0
32	0.189401E-17	0.811721E-18	0.541147D-14	0.135287E-16	5	0	0
33	0.522363E-20	0.223870E-20	0.186558D-16	0.373117E-19	4	0	0
34	0.943872E-23	0.404516E-23	0.449463D-19	0.674194E-22	3	0	0
35	0.996333E-26	0.427000E-26	0.711667D-22	0.711666E-25	2	0	0
36	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	1	0	0
37	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	0	0	0

Table 10.1C *Exact state probabilities and frequencies of the vehicle system*

THE EXACT STATE PROBS AND FREQS

# OF VEH	PROB OF OPTG	FREQ OF OPTG	PROB PARTIAL MODE	FREQ OF PARTIAL MODE
10	0.994319E 00	0.510543E 00	0.497009E-02	0.477130E 00
9	0.153546E-04	0.692341E-03	0.677606E-07	0.650502E-05
8	0.165949E-06	0.903139E-05	0.647535E-09	0.621633E-07
7	0.128561E-08	0.816761E-07	0.436367E-11	0.418912E-09
6	0.732957E-11	0.531376E-09	0.212033E-13	0.203551E-11
5	0.309791E-13	0.251992E-11	0.742673E-16	0.712966E-14
4	0.962607E-16	0.867087E-14	0.183609E-18	0.176265E-16
3	0.214256E-18	0.211488E-16	0.304859E-21	0.292665E-19
2	0.322797E-21	0.346075E-19	0.304506E-24	0.292325E-22
1	0.228385E-24	0.257839E-22	0.105057E-27	0.100855E-25
0	0.630343E-29	0.000000E 00	0.000000E 00	0.000000E 00

RET STATE PROBABILITY= 0.695705E-03

RET STATE FREQUENCY(PER DAY)= 0.333995E-01

Table 10.1D *Cumulative state probabilities and frequencies of the vehicle system*

OPERATING VEHICLES EQUAL TO OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION DAYS
10	0.100000E 01	0.000000E 00	0.000000E 00	0.000000E 00
9	0.711412E-03	0.338880E-01	0.295089E 02	0.209930E-01
8	0.695990E-03	0.334062E-01	0.299345E 02	0.208341E-01
7	0.695824E-03	0.333996E-01	0.299405E 02	0.208333E-01
6	0.695823E-03	0.333995E-01	0.299405E 02	0.208333E-01
5	0.695823E-03	0.333995E-01	0.299405E 02	0.208333E-01

Table 10.2A *Vehicle system data*

```
MEAN TIME TO VEHICLE FAILURE(PERMANENT) =     5000.0000    HOURS

MEAN TIME TO VEHICLE FAILURE(PARTIAL MODE) =     500.0000    HOURS

MEAN TIME TO VEHICLE RETRIEVAL =     0.5000    HOURS

MEAN TIME TO CLEAR PARTIAL MODE VEHICLE =     0.2500    HOURS

MEAN TIME TO VEHICLE MAINTENANCE =     3.0000    HOURS

MEAN TIME TO VEHICLE REPAIR =     2.5000    HOURS



NUMBER OF OPERATING VEHICLES = 10

NUMBER OF SPARE VEHICLES =  0

NUMBER OF VEHICLES ON MAINTENANCE =  0

VEHICLE EXPOSURE FACTOR = 0.2000
```

equal. This is because of the presence of 2 spares and 2 units on maintenance which also can behave as spares, the probabilities of states with operating units less than 10 are relatively low and therefore are dominated by the retrieval state probability.

The results for another example which is the same as the previous one, except that there are no units spare or on maintenance, are given in Tables 10.2A–D. In Table 10.2D, the reliability measures tend to be almost equal from $N_o < 7$ downward as compared with $N_o < 9$ in Table 10.1D. The probability and frequency of encountering states with a specified number of failures, depends upon the failure rate of units, number of spares, and number of units on maintenance. These relationships will become clearer in the section on sensitivity studies.

10.4 SYSTEM MODEL FOR TRAINS

When the trains are regarded strictly as single units for operation, spares, and maintenance, the vehicle system model described in Section 10.3 can be used. A model, however, is also possible based on a more flexible policy for train formation. The general procedure is the same as for the vehicle system:

1. Generate the possibility space by enumerating and describing the possible system states.
2. Develop the transition rate matrix.
3. Solve for probabilities and calculate reliability measures.

Table 10.2B *Vehicle system state probabilities*

THE PROBABILITIES OF BEING IN VARIOUS STATES

STATE NO	PR	PS	P	PP	STATE DESCRIPTION D	S	M
1	0.658709E-03	0.282304E-03	0.941013D 00	0.470506E-02	10	0	0
2	0.326061E-04	0.139740E-04	0.517557D-01	0.232901E-03	9	0	0
3	0.717334E-06	0.307429E-06	0.128095D-02	0.512381E-05	8	0	0
4	0.920579E-08	0.394534E-08	0.187873D-04	0.657556E-07	7	0	0
5	0.759478E-10	0.325490E-10	0.180828D-06	0.542484E-09	6	0	0
6	0.417713E-12	0.179020E-12	0.119346D-08	0.298366E-11	5	0	0
7	0.153161E-14	0.656405E-15	0.547004D-11	0.109401E-13	4	0	0
8	0.361005E-17	0.154716E-17	0.171907D-13	0.257861E-16	3	0	0
9	0.486145E-20	0.208348E-20	0.347247D-16	0.347246E-19	2	0	0
10	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	1	0	0
11	0.000000E 00	0.000000E 00	0.000000D 00	0.000000E 00	0	0	0

Table 10.2C *Exact state probabilities and frequencies of the vehicle system*

THE EXACT STATE PROBS AND FREQS

# OF VEH	PROB OF OPTG	FREQ OF OPTG	PROB PARTIAL MODE	FREQ OF PARTIAL MODE
10	0.941013E 00	0.496855E 00	0.470506E-02	0.451686E 00
9	0.520380E-01	0.521449E 00	0.232901E-03	0.223585E-01
8	0.129493E-02	0.251354E-01	0.512381E-05	0.491886E-03
7	0.190947E-04	0.548019E-03	0.657556E-07	0.631254E-05
6	0.184773E-06	0.700109E-05	0.542484E-09	0.520785E-07
5	0.122601E-08	0.576014E-07	0.298366E-11	0.286431E-09
4	0.564906E-11	0.316230E-09	0.109401E-13	0.105025E-11
3	0.178471E-13	0.115794E-11	0.257861E-16	0.247546E-14
2	0.362718E-16	0.267052E-14	0.347246E-19	0.333357E-17
1	0.208348E-20	0.000000E 00	0.000000E 00	0.000000E 00
0	0.000000E 00	0.000000E 00	0.000000E 00	0.000000E 00

RET STATE PROBABILITY= 0.691772E-03

RET STATE FREQUENCY(PER DAY)= 0.332180E-01

Table 10.2D *Cumulative state probabilities and frequencies of the vehicle system*

OPERATING VEHICLES EQUAL TO OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION DAYS
10	0.100000E 01	0.000000E 00	0.000000E 00	0.000000E 00
9	0.542823E-01	0.496855E 00	0.201266E 01	0.109252E 00
8	0.201144E-02	0.562123E-01	0.177897E 02	0.357829E-01
7	0.711388E-03	0.337242E-01	0.296523E 02	0.210943E-01
6	0.692228E-03	0.332245E-01	0.300983E 02	0.208349E-01
5	0.692042E-03	0.332180E-01	0.301041E 02	0.208333E-01
4	0.692041E-03	0.332180E-01	0.301042E 02	0.208333E-01
3	0.692041E-03	0.332180E-01	0.301042E 02	0.208333E-01

The state space is generated by a subroutine in the computer program [12] according to the following rules for the train formation.

1. When a vehicle in the train fails, the train is removed from service and a spare train, if available, is put into operation as a replacement. If, however, a complete spare train is not available, the defective vehicle is replaced by a good vehicle and the train is put back into operation. If no good vehicle is available, the defective vehicle is removed and the train put back into operation. There is no additional difficulty in modeling with married pairs as they can be treated as single units.
2. Train-consists of different lengths are allowed.
3. In case of no available spares and when all trains are not full length, a vehicle on which maintenance or repair is completed is attached to the train having the least number of vehicles.

In the rules outlined above, the attempt is to keep the maximum vehicle system capacity in the operating condition. Rules 1–3 represent only one policy and models can be similarly developed for other policies. The results of the solution of the train model are grouped in terms of vehicle system capacity, that is, the output format is the same as for the vehicle system model.

10.5 PASSENGER STATION MODEL

The stations are assumed off-line and basically of two types:

1. Stations having one station lane and an express lane for through traffic. These are termed type *A* stations.
2. Stations having two station lanes and an expressway lane, called type *B* stations.

10.5.1 Model for a Single Station

The state transition diagrams for the type *A* and type *B* stations are shown in Figures 10.6 and 10.7 respectively, using the following notation:

U = normal station operation, that is, both the station lane and the express lane are working
D = station lane down
L = failure of the station lane as well as the express lane, complete failure of type *A* station.
$\bar{L}$ = station lane working but express lane down

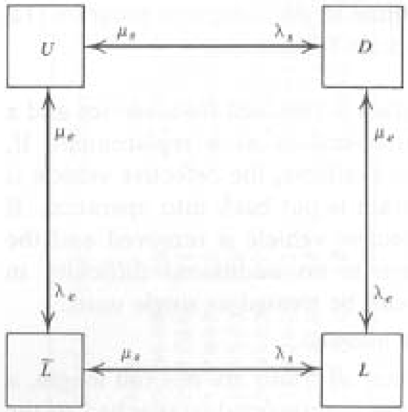


Figure 10.6 State transition diagram of type *A* station.

For the type *B* station:

$2D$ = both the station lanes down
$\bar{L}_1$ = one station lane working and the express lane down
$\bar{L}_2$ = both station lanes working with express lane down
λ_s, μ_s = failure and repair rates of the station lane including the contribution λ_{VS} of the vehicle system failure
λ_e, μ_e = express lane failure and repair rates

The impact of the various station states on the system can be tabulated in Tables 10.3 and 10.4.

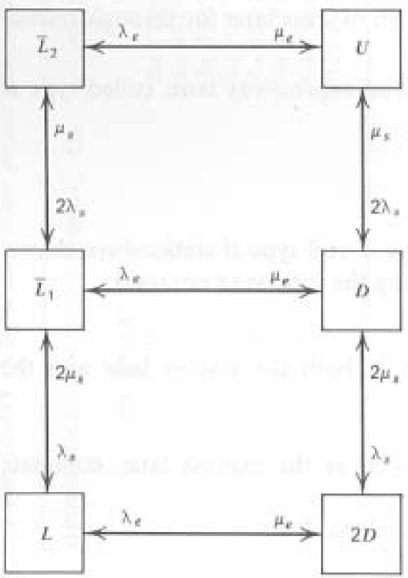


Figure 10.7 State transition diagram of type *B* station.

Table 10.3 *Type A Station*

Station State	Impact on System	Impact on Station
U	Traffic can pass	Passengers can embark/disembark
D	Traffic can pass	Passengers cannot embark/disembark
L	Traffic cannot pass	Passengers cannot embark/disembark
$\bar{L}$	Traffic can pass	Passengers can embark/disembark

10.5.2 *Model for System of Stations*

A transit system may have $N0$ number of type *A* stations and $N1$ number of type *B* stations. The model for the system can be built by combining the models for the individual stations. It is assumed that when at a station both the station lanes and expressway are down and the traffic cannot pass through, no further failure of stations takes place.

The model for the system of stations is built by sequential addition [10]. This is achieved by adding one station at a time, solving for state probabilities, deleting low probability states, and adding the next station. This procedure helps to keep the system states within manageable limits. The process of sequential addition is illustrated in Table 10.5 for three type *A* stations with the following hypothetical data:

mean up time of the station lane = 800 hours
mean down time of the station lane = 2 hours
mean up time of the express lane = 1000 hours
mean down time of the express lane = 2 hours

Table 10.4 *Type B Station*

Station State	Impact on System	Impact on Station
U	Traffic can pass	Passengers can embark/disembark
D	Traffic can pass	Passengers can embark/disembark
$2D$	Traffic can pass	Passengers cannot embark/disembark
L	Traffic cannot pass	Passengers cannot embark/disembark
$\bar{L}_1$	Traffic can pass	Passengers can embark/disembark
$\bar{L}_2$	Traffic can pass	Passengers can embark/disembark

Table 10.5 *Sequential Model Building*

System State	Identical states	No. of stations in state U	D	L	L	Probability
(a) Model of a single station						
1		1	0	0	0	
2		0	1	0	0	
3		0	0	1	0	
4		0	0	0	1	
(b) Addition of a station						
1(1,1)		2	0	0	0	
2(2,1)		1	1	0	0	
3(3,1)		1	0	1	0	
4(4,1)		1	0	0	1	
5(1,2)		1	1	0	0	
6(2,2)		0	2	0	0	
7(3,2)		0	1	1	0	
8(4,2)		0	1	0	1	
9(1,3)		1	0	1	0	
10(2,3)		0	1	1	0	
11(4,3)		0	0	1	1	
12(1,4)		1	0	0	1	
13(2,4)		0	1	0	1	
14(3,4)		0	0	1	1	
15(4,4)		0	0	0	2	
(c) Merging of identical states						
1	1	2	0	0	0	0.991051
2	2,5	1	1	0	0	0.495525×10^{-2}
3	3,9	1	0	1	0	0.992536×10^{-5}
4	4,12	1	0	0	1	0.396420×10^{-2}
5	6	0	2	0	0	0.618994×10^{-5}
6	7,10	0	1	1	0	0.165058×10^{-7}
7	8,13	0	1	0	1	0.990308×10^{-5}
8	11,14	0	0	1	1	0.132036×10^{-7}
9	15	0	0	0	2	0.396090×10^{-5}
(d) Truncation of states with probabilities less than 10^{-5}						
1		2	0	0	0	0.991051
2		1	1	0	0	0.495525×10^{-2}
3		1	0	1	0	0.992536×10^{-5}
4		1	0	0	1	0.396420×10^{-2}
5		0	2	0	0	0.618994×10^{-5}
6		0	1	0	1	0.990308×10^{-5}
7		0	0	0	2	0.396090×10^{-5}
(e) Addition of the third station						
1(1,1)		3	0	0	0	
2(2,1)		2	1	0	0	
3(3,1)		2	0	1	0	
4(4,1)		2	0	0	1	

TABLE 10.5 *(Continued)*

System State	Identical states	No. of stations in state U	D	L	$\bar{L}$	Probability
5(5,1)		1	2	0	0	
6(6,1)		1	1	0	1	
7(7,1)		1	0	0	2	
8(1,2)		2	1	0	0	
9(2,2)		1	2	0	0	
10(3,2)		1	1	1	0	
11(4,2)		1	1	0	1	
12(5,2)		0	3	0	0	
13(6,2)		0	2	0	1	
14(7,2)		0	1	0	2	
15(1,3)		2	0	1	0	
16(2,3)		1	1	1	0	
17(4,3)		1	0	1	1	(3,3) not possible
18(5,3)		0	2	1	0	
19(6,3)		0	1	1	1	
20(7,3)		0	0	1	2	
21(1,4)		2	0	0	1	
22(2,4)		1	1	0	1	
23(3,4)		1	0	1	1	
24(4,4)		1	0	0	2	
25(5,4)		0	2	0	1	
26(6,4)		0	1	0	2	
27(7,4)		0	0	0	3	
(f) Merging of identical states						
1	1	3	0	0	0	0.986606
2	2,8	2	1	0	0	0.739954×10^{-2}
3	3,15	2	0	1	0	0.148435×10^{-4}
4	4,21	2	0	0	1	0.591964×10^{-2}
5	5,9	1	2	0	0	0.184865×10^{-4}
6	6,11,22	1	1	0	1	0.295760×10^{-4}
7	7,24	1	0	0	2	0.118294×10^{-4}
8	10,16	1	1	1	0	0.493508×10^{-7}
9	12	0	3	0	0	0.153901×10^{-7}
10	13,25	0	2	0	1	0.369310×10^{-7}
11	14,26	0	1	0	2	0.295407×10^{-7}
12	17,23	1	0	1	1	0.394773×10^{-7}
13	18	0	2	1	0	0.461670×10^{-10}
14	19	0	1	1	1	0.738569×10^{-10}
15	20	0	0	1	2	0.295387×10^{-10}
16	27	0	0	0	3	0.787643×10^{-8}

Table 10.6 *Model of Three Stations Without Truncation*

System State	State No. as in Table 10.1	No. of stations in U	D	L	L	Probability
1	1	3	0	0	0	0.986606
2	2	2	1	0	0	0.739954×10^{-2}
3	3	2	0	1	0	0.148435×10^{-4}
4	4	2	0	0	1	0.591964×10^{-2}
5	5	1	2	0	0	0.184865×10^{-4}
6	8	1	1	1	0	0.493508×10^{-7}
7	6	1	1	0	1	0.295760×10^{-4}
8	12	1	0	1	1	0.394773×10^{-7}
9	7	1	0	0	2	0.118294×10^{-4}
10	9	0	3	0	0	0.153901×10^{-7}
11	13	0	2	1	0	0.461824×10^{-10}
12	10	0	2	0	1	0.369310×10^{-7}
13	14	0	1	1	1	0.738538×10^{-10}
14	11	0	1	0	2	0.295407×10^{-7}
15	Deleted	0	1	2	0	0.123121×10^{-12}
16	Deleted	0	0	2	1	0.984465×10^{-13}
17	15	0	0	1	2	0.295264×10^{-10}
18	16	0	0	0	3	0.787643×10^{-8}

The model for a single station is represented in Table 10.5*a*. The addition of one more station is shown in Table 10.5*b*. For each state of the station being added, there is a set of system states of Table 10.5*a* except the state (3,3). The state (3,3) representing two stations completely down (state L) is assumed not possible since exposure to failure is assumed zero as soon as one station is completely down. The system states in Table 10.5*b* are numbered in the serial order and the numbers in parenthesis indicate the combination, the first number denoting the system state before addition and the second indicating the state of the station being added. The identical states can now be grouped together as shown in Table 10.5*c*. The states with probabilities less than 10^{-5} (an arbitrary reference) are deleted and the remaining states are shown in Table 10.5*d*, where the state numbers are serial and have no relationship to state numbers in Table 10.5*c*. Tables 10.5*e* and 10.5*f* show the addition to the third station. If a fourth station is now to be added, then the states with probabilities less than 10^{-5} can again be deleted and the procedure repeated. The exact results, without any truncation, are shown in Table 10.6 and are almost identical to Table 10.5*f*. In general, the effect on the results depends on the reference probability value employed for truncation.

10.6 MODELS FOR OTHER SUBSYSTEMS

10.6.1 Power Substations Model

A single power station is assumed to have only two states, up (i.e., the substation is working) and down (i.e., the substation is failed). The system of substations is, however, assumed as an m/n configuration; that is, the system is good if m out of n substations are working. In other words the assumption means that so long as m out of n substations are working, the power supply is adequate to keep the system running. When, however, one more substation fails, the system either completely fails or goes into severe degradation. The state transition diagram for the substations model is shown in Figure 10.8, where λ_{ss} and μ_{ss} are the substation failure and repair rates respectively.

10.6.2 Guideway Model

Guideway consists of the structure, power rails, and any other equipment whose failure would incapacitate the guideway. As an example in magnetic

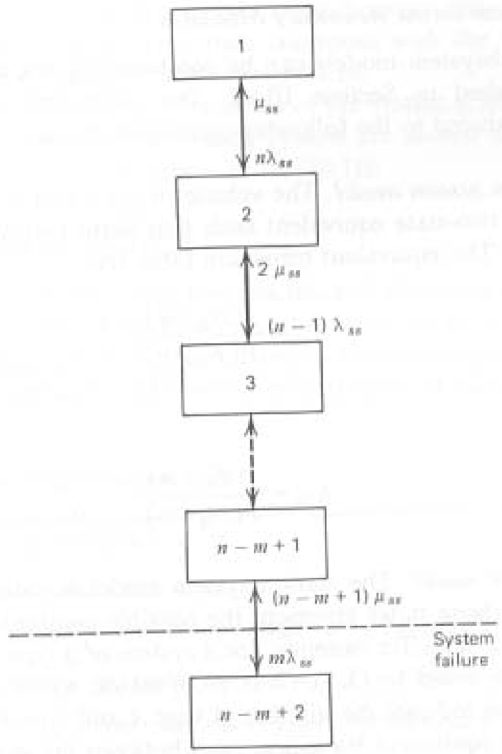


Figure 10.8 State transition diagram for power substations model.

levitation and linear induction propulsion, the guideway would include suspension armature rail and LIM rail. The guideway is assumed to be a two-state system: Up state when the guideway is in satisfactory condition and down state when a failure of guideway interrupts the flow of traffic.

10.6.3 Command and Control System Model

Command and control includes all equipment associated with the control of vehicle movement, except the vehicle-borne equipment, which is regarded as a part of the vehicle. Like the guideway, the command and control is also assumed a two-state system.

10.7 TOTAL SYSTEM MODEL

The models for various subsystems have been described in Sections 10.3–10.6. These models can be combined to give the reliability measures of the entire transit system.

10.7.1 System-Based Reliability Measures

The various subsystem models can be combined by the sequential model building described in Section 10.5.2. The subsystem models, for this purpose, are reduced to the following equivalent forms:

1. *Vehicle/train system model*. The vehicle or train system model is represented by a two-state equivalent such that State $1=(N_0>n)$ and State $2=(N_0\leq n)$. The equivalent transition rates are

$$\lambda_{12}=\frac{f(N_0\leq n)}{P(N_0>n)}$$

and

$$\lambda_{21}=\frac{f(N_0\leq n)}{P(N_0\leq n)}$$

2. *Station system model*. The station system model is reduced to a multistate model whose states represent the possible combination of stations in the working state, for example, for a system of 3 type A and 1 type B stations, there could be (3,1), (2,1), (0,0) states, where the numbers in the parenthesis indicate the number of type A and type B stations in the up state. The equivalent transition rates between the various states can be determined using (10.3).

3. *Substation Model*.
   State $1=(\text{number of up stations}\geq m)$
   State $2=(\text{number of up stations}<m)$
   The equivalent transition rates are

$$\lambda_{12}=P(N_{ss}=m)\cdot m\frac{\lambda_{ss}}{P(N_{ss}\geq m)}$$

$$\lambda_{21}=\frac{P(N_{ss}=m-1)(n-m+1)\mu_{ss}}{P(N_{ss}<m)}$$

where N_{ss} is the number of working substations.

The station system model, the substation system model, the command and control model and the guideway model are combined together and the resulting states are grouped as $(N_A=i, N_B=j)$ where N_A and N_B indicate the number of type A and type B stations up, respectively. The state $N_A=0$, $N_B=0$ includes the condition of having the substation system, guideway or command and control down. Therefore, $(N_A=0, N_B=0)$ does not necessarily mean that all the stations are failed. It really means that the stations are not available for embarcation because the system is not moving. This combined model is then combined with the vehicle system model to give the measure for the entire system.

The printout of the reliability measures of the system is shown in Tables 10.1I–10.1L. The data for the vehicle system are shown in Table 10.1A and for other subsystems in Tables 10.1E–10.1H.

10.7.2 Including Demand [13]

The calculation of probabilities, frequencies, and the mean duration of the various deficient states of the system is illustrated in Section 10.7.1. These system deficiencies can be further related to the delays that they cause to the vehicles or passengers and then the probabilities of these delays can be

Table 10.1E *Passenger stations data*

```
NUMBER OF TYPE A STATIONS =  3
NUMBER OF TYPE B STATIONS =  1
MEAN TIME TO FAILURE OF THE STATION  =    3260.88    HOURS
MEAN TIME TO REPAIR OF THE STATION =        1.97    HOURS
MEAN TIME TO FAILURE OF  THE EXPRESSWAY =466666.80    HOURS
MEAN TIME TO REPAIR OF THE EXPRESSWAY =       10.00    HOURS
```

Table 10.1F *Power substations data and results*

```
NUMBER OF SUB STATIONS = 4

MINIMUM NUMBER OF SUB STATIONS REQUIRED FOR OPERATION = 2

MEAN TIME TO SUB STATION FAILURE = 20000.00   HOURS

MEAN TIME TO SUB STATION REPAIR =      0.50   HOURS



SUB STATION SYSTEM AVAILABILITY = 0.1000000 01

FREQUENCY OF ENCOUNTERING THE DOWN STATE = 0.899910D-11   PER DAY

CYCLE TIME TO ENCOUNTER DOWN STATE = 0.111122D 12   DAYS

MEAN DURATION OF DOWN STATE = 0.692571D-02   DAYS
```

Table 10.1G *Command and control system data and results*

```
MEAN TIME TO COMMAND AND CONTROL FAILURE = 1666.67   HOURS

MEAN TIME TO COMMAND AND CONTROL REPAIR =    2.58   HOURS



COMMAND AND CONTROL AVAILABILITY = 0.998454E 00

FREQUENCY OF ENCOUNTERING DOWN STATE = 0.143777E-01   PER DAY

CYCLE TIME TO ENCOUNTER DOWN STATE = 0.695521E 02   DAYS

MEAN DURATION OF DOWN STATE = 0.107500E 00   DAYS
```

Table 10.1H *Guideway data and results*

```
MEAN TIME TO GUIDE WAY FAILURE = 50000.00   HOURS

MEAN TIME TO GUIDE WAY REPAIR =    10.00   HOURS



GUIDE WAY AVAILABILITY = 0.999800E 00

FREQUENCY OF ENCOUNTERING DOWN STATE = 0.479904E-03   PER DAY

CYCLE TIME TO ENCOUNTER OF DOWN STATE = 0.208375E 04   DAYS

MEAN DURATION OF DOWN STATE = 0.416447E 00   DAYS
```

Table 10.1I *System reliability measures*

# OF TYPE A STATIONS UP	#OF TYPE B STATIONS UP	# OF VEHICLES GREATER THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	9	0.9957400 00	0.705775E-01	0.141688E 02	14.108
2	1	9	0.1803290-02	0.220835E-01	0.452827E 02	0.082
1	1	9	0.1087630-05	0.265535E-04	0.376597E 05	0.041
0	0	9	0.1744130-02	0.149114E-01	0.670628E 02	0.117

# OF TYPE A STATIONS UP	# OF TYPE B STATIONS UP	# OF VEHICLES EQUAL OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	9	0.7088860-03	0.337939E-01	0.295911E 02	0.021
2	1	9	0.1283800-05	0.768316E-04	0.130155E 05	0.017
1	1	9	0.7743020-09	0.557615E-07	0.179335E 08	0.014
0	0	9	0.1241680-05	0.697208E-04	0.143429E 05	0.018

Table 10.1J *System reliability measures*

# OF TYPE A STATIONS UP	#OF TYPE B STATIONS UP	# OF VEHICLES GREATER THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	8	0.9957550 00	0.700980E-01	0.142657E 02	14.205
2	1	8	0.1803320-02	0.220830E-01	0.452838E 02	0.082
1	1	8	0.1087640-05	0.265534E-04	0.376599E 05	0.041
0	0	8	0.1744160-02	0.149108E-01	0.670655E 02	0.117

# OF TYPE A STATIONS UP	# OF TYPE B STATIONS UP	# OF VEHICLES EQUAL OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	8	0.6935190-03	0.333132E-01	0.300181E 02	0.021
2	1	8	0.1255970-05	0.756222E-04	0.132236E 05	0.017
1	1	8	0.7575170-09	0.548279E-07	0.182389E 08	0.014
0	0	8	0.1214760-05	0.686506E-04	0.145665E 05	0.018

Table 10.1K *System reliability measures*

# OF TYPE A STATIONS UP	#OF TYPE B STATIONS UP	# OF VEHICLES GREATER THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	7	0.9957560 00	0.700914E-01	0.142671E 02	14.207
2	1	7	0.1803320-02	0.220830E-01	0.452838E 02	0.082
1	1	7	0.1087640-05	0.265534E-04	0.376599E 05	0.041
0	0	7	0.1744160-02	0.149108E-01	0.670656E 02	0.117

# OF TYPE A STATIONS UP	# OF TYPE B STATIONS UP	# OF VEHICLES EQUAL OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	7	0.6933530-03	0.333066E-01	0.300240E 02	0.021
2	1	7	0.1255670-05	0.756066E-04	0.132264E 05	0.017
1	1	7	0.7573360-09	0.548162E-07	0.182428E 08	0.014
0	0	7	0.1214470-05	0.686366E-04	0.145695E 05	0.018

Table 10.1L *System reliability measures*

# OF TYPE A STATIONS UP	#OF TYPE B STATIONS UP	# OF VEHICLES GREATER THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	6	0.995756D 00	0.700913E-01	0.142671E 02	14.207
2	1	6	0.180332D-02	0.220830E-01	0.452838E 02	0.082
1	1	6	0.108764D-05	0.265534E-04	0.376599E 05	0.041
0	0	6	0.174416D-02	0.149108E-01	0.670656E 02	0.117

# OF TYPE A STATIONS UP	# OF TYPE B STATIONS UP	# OF VEHICLES EQUAL OR LESS THAN	PROBABILITY	FREQUENCY PER DAY	CYCLE TIME DAYS	MEAN DURATION
3	1	6	0.693352D-03	0.333066E-01	0.300241E 02	0.021
2	1	6	0.125566D-05	0.756065E-04	0.132264E 05	0.017
1	1	6	0.757335D-09	0.548162E-07	0.182428E 08	0.014
0	0	6	0.121447D-05	0.686365E-04	0.145695E 05	0.018

calculated. The extent to which such an analysis can be carried out depends upon the information available on the flow of passengers. The calculation of delays can be illustrated by an example. Consider a system such that only vehicle fleet need be considered and the other subsystems can be ignored. The delay could be caused by the following types of system deficiencies,

1. A vehicle could fail in the retrieval mode blocking the flow of vehicles. This condition will last till the failed vehicle has been removed and the system put back into normal operation. Let the probability of being in this state be denoted by P_{RET}.
2. A vehicle could fail in a partial mode, degrading the operation of the system until the vehicle finally clears the system. Let the probability of being in this state be denoted by P_{PR}.
3. The delay could also be caused because the number of vehicles available for service is less than the required number. This condition can result when spare vehicles are not available to replace the failed ones.

These probabilities can be calculated using the models and methods described in this report. These probabilities can be then weighted with the probabilities of delay caused by these conditions. This will yield the probability of delay caused by system deficiencies. The calculation of probabilities of delay by the deficient conditions is not covered in this chapter.

10.8 SYSTEM STUDIES

Some vehicle system sensitivity studies using these models are reported here. The system is assumed to consist of 50 operating vehicles and the relevant data is listed on appropriate figures. The vehicle system state with number of vehicles less than or equal to 49 is considered as the reference state.

10.8.1 Sensitivity of Reliability Indices to the Vehicle MTBF (Retrieval Mode)

The effect of variation in vehicle MTBF on the probability, mean time to encounter and the mean duration of the system state with vehicles ≤ 49 is shown in Figures 10.9–10.11. The reliability indices are plotted for three cases, $(s=2, m=2)$, $(s=0, m=2)$ and $(s=0, m=0)$. The probability of $N_o \leq 49$ is the lowest and the most sensitive in the case of $s=2$, $m=2$. This is because when there are no spare vehicles, the vehicle failure rate in partial mode begins to be effective and since it is 500 hours as compared with 5000 hours for the retrieval mode, the partial failure mode dominates

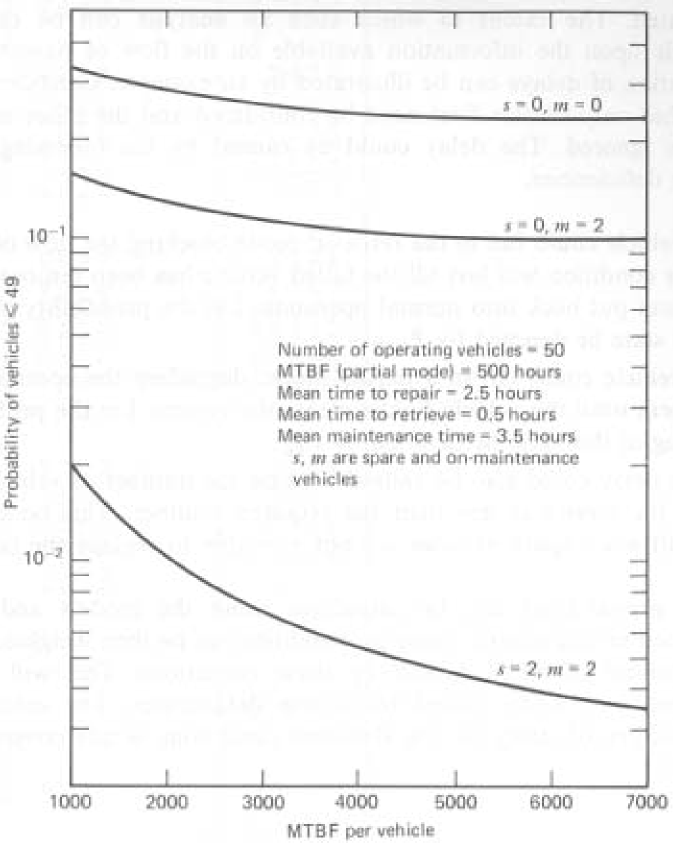


Figure 10.9 System failure probability vs vehicle MTBF.

for $s=0$. The mean time to encounter $N_o \leqslant 49$ is again highest for $s=2$, $m=2$ and shows the most sensitivity to variation in retrieval mode vehicle MTBF. This is because for $s=0$, the partial mode of failure dominates. The mean duration of $N_o \leqslant 49$ is insensitive to the variation in vehicle MTBF since with spare vehicles available to replace the failed ones, this index is more or less determined by the mean time to retrieve.

10.8.2 Sensitivity of Reliability Indices to the Mean Time to Repair a Vehicle

The effect of variation in vehicle MTTR on the reliability indices of the system is shown in Figures 10.12–10.14. The probability and the mean duration of $N_o \leqslant 49$ decrease with the increase in the MTTR and the mean time to encounter $N_o \leqslant 49$ correspondingly increases. For $s=0$, the mean time to encounter $N_o \leqslant 49$ is relatively insensitive to the MTTR. This is because in this case, the system behaves more or less like a series system

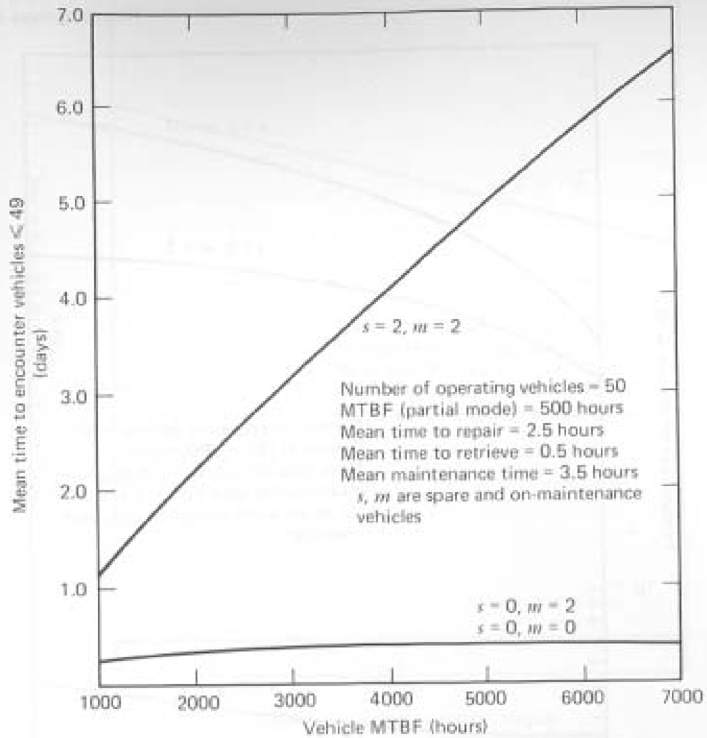
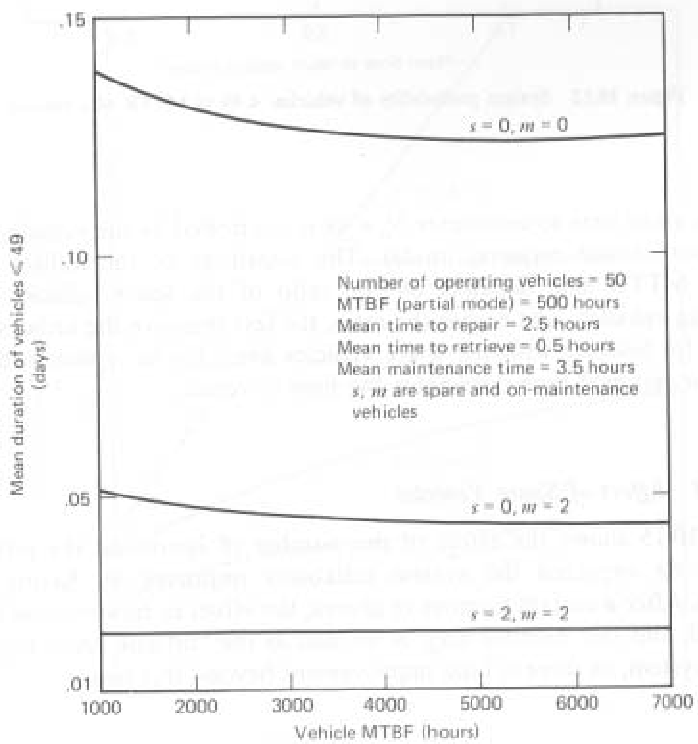


Figure 10.10 System mean time to vehicles < 49 vs vehicle MTBF.



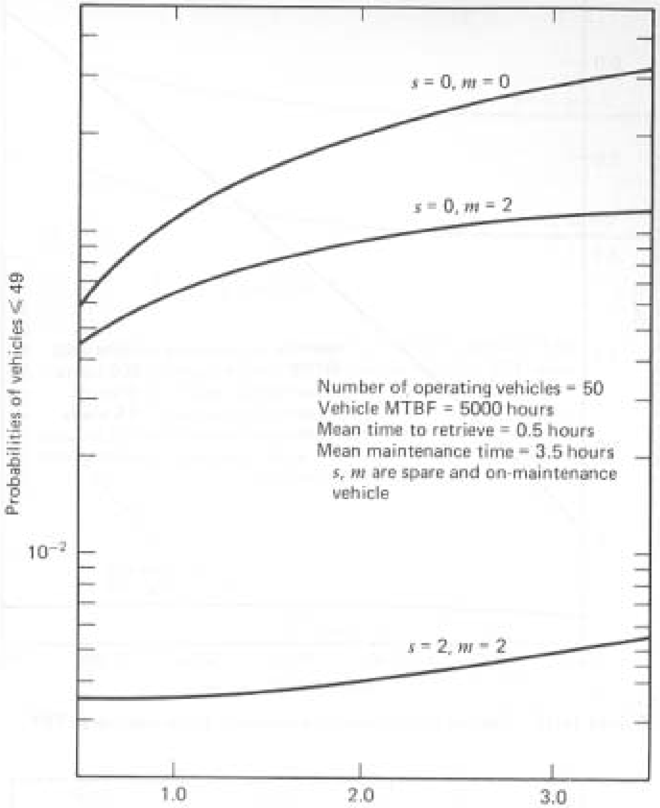


Figure 10.12 System probability of vehicles ≤ 49 vs MTTR of a vehicle.

and the mean time to encounter $N_o \leq 49$ is controlled by the vehicle MTBF (both partial and retrieval mode). The sensitivity of the indices to the vehicle MTTR also depends on the ratio of the spare vehicles to the operating vehicles. The higher the ratio, the less sensitive the indices are to MTTR [6] because with the spare vehicles available to replace the failed ones, the retrieval time dominates the time to repair.

10.8.3 Effect of Spare Vehicles

Figure 10.15 shows the effect of the number of spares on the reliability indices. As expected the system reliability improves by having spare vehicles. After a certain number of spares, the effect is, however, incremental small, and this number may be termed as the "infinite spare capacity" for the system, as there is little improvement beyond this point.

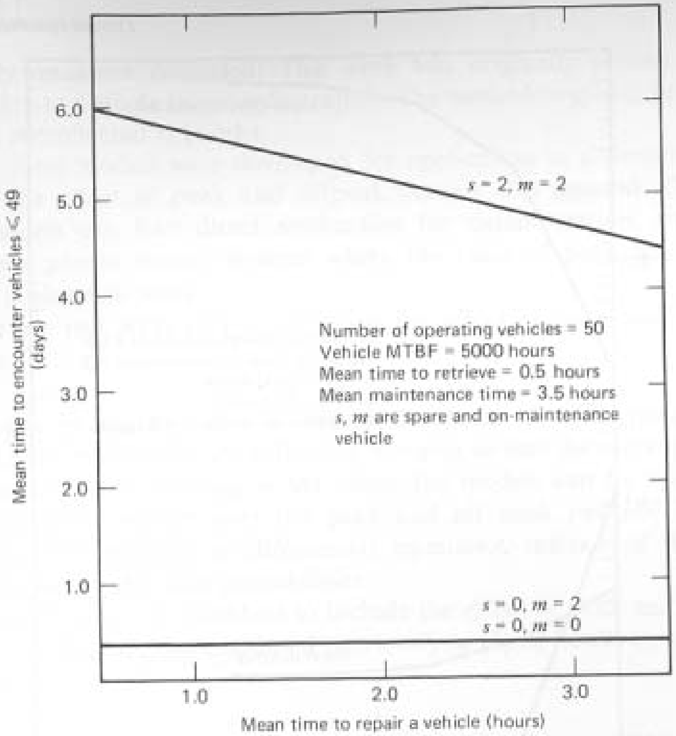


Figure 10.13 System mean time to vehicles ≤ 49 vs vehicle MTTR.

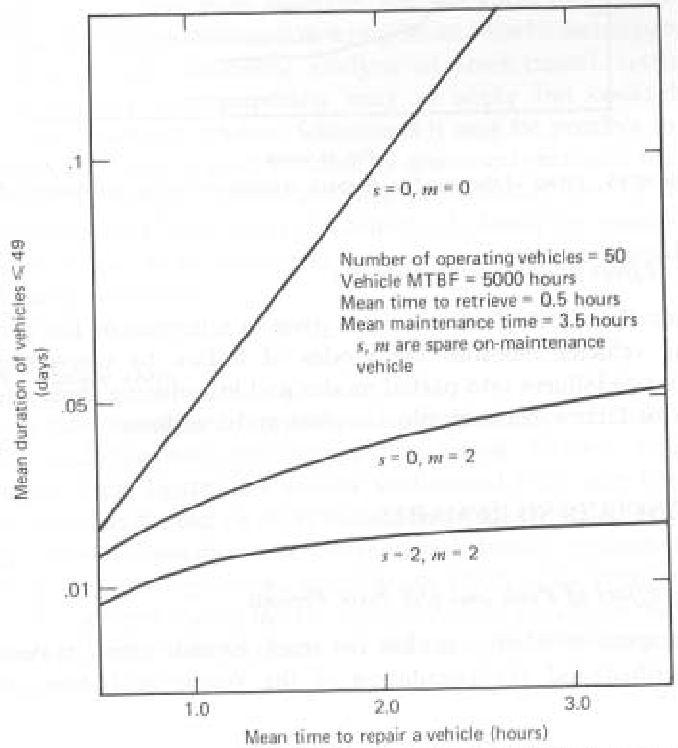


Figure 10.14 System mean duration of vehicles ≤ 49 vs vehicle MTTR.

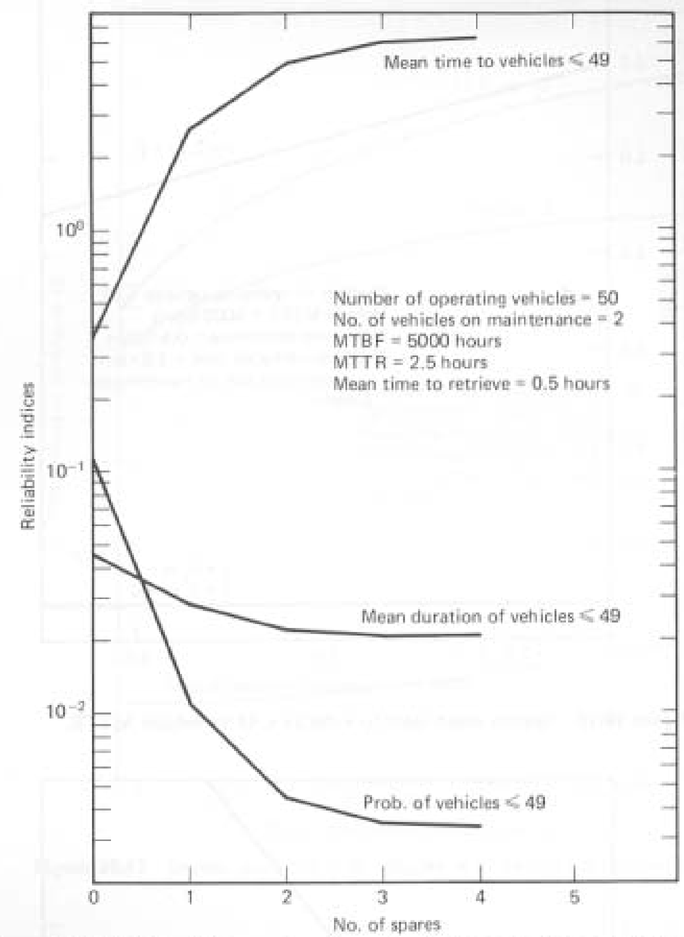


Figure 10.15 Effect of the number of spare vehicles on system reliability indices.

10.8.4 Effect of Entraining Vehicles

A study on the effect of entraining is given in reference 6. The process of entraining vehicles modifies the modes of failure by converting some retrieval mode failures into partial modes and introducing some additional elements for failure, for example, couplers and trainlines.

10.9 CONCLUDING REMARKS

10.9.1 Effect of Peak and Off Peak Periods

The state space reliability models for track bound transit systems have been described and the calculation of the system and demand based

reliability measures discussed. This work was originally carried out for application to a single loop configuration. The methodology can be applied to more complicated networks.

Since these models were developed for application to a demonstration project, the effect of peak and offpeak periods was ignored. Therefore these models can find direct application for demonstration, airport or downtown people mover systems where the ratio of peak to off peak periods is closer to unity.

There are two ways [13] of including the effect of peak and off peak periods:

1. If it can be assumed that all the failed vehicles over the previous day have been repaired by the following morning so that the initial probability vector every morning is the same, the models can be solved in a time-specific manner over the peak and off peak periods. This will involve the solution of differential equations, instead of the linear equations for the state probabilities.
2. The models can be modified to include the effect of peak and off peak periods. The approach outlined previously in (1) is, however, simpler to implement.

10.9.2 Simulation Versus Analytical Method

As the systems become more complex, the analytical techniques become more difficult to apply. Simulation using Monte-Carlo techniques can be used to perform the reliability analysis of track-bound systems. The simulation method is conceptually easy to apply but could be quite expensive for sensitivity studies. Sometimes it may be possible to apply a hybrid approach, that is, part solution by analytical methods and part by simulation. For example, the system base probabilities may be calculated by analytical models and the probabilities of delays by specific system deficiencies calculated by simulation and the two results combined to yield demand based measures.

10.9.3 Failure Data

Subsystem failure rate is an important input parameter for transit system reliability modeling and calculation. For transit systems using newer technologies, these figures are usually synthesized from part failure rates available from handbooks or other data collection and exchange programs. Although such information on conventional transit systems could be available from field experience, no collective effort at national or international level has been visible in this regard. A data collection and analysis activity to fulfill this need has been carried out and the results are reported in references 8 and 14.

10.9.4 Further Work

The simulation techniques are conceptually simpler for calculating the reliability measures but consume considerable computer time, especially when performing sensitivity studies. The analytical methods become quite complicated when applied to complex system configurations but are very suitable for sensitivity analysis. There is a need for further development of the analytical and simulation methods for application to more complex network configurations, taking into account the peak and off peak periods, and including demand in deriving suitable measures of reliability.

REFERENCES

1. Billinton, R. and C. Singh, "Frequency and Duration Method of Generating Capacity Reliability Evaluation," Transactions of the Engineering Institute of Canada, vol. 15, March 1972, pp. I–V.
2. Heiman, D., "Availability—Concepts and Definitions," *Proceedings 1976 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1976.
3. Welker, E. L. and H. H. Buchanan, "Safety—Availability Study Methods Applied to BART," *Proceedings 1975 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1975.
4. Welker, E. L., "Reliability and Availability Assessment Criteria, Data Inputs and Analysis Methods for Mass Transit Systems," *Proceedings 1976 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1976.
5. Westmoreland, M. E., "Reliability Improvement of Land Vehicles," *Proceedings 1973 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1973.
6. Singh, C., R. Billinton, J. H. Parker, and R. Puccini, "Transit System Reliability," presented at 2nd Intersociety Transportation Conference, Denver, 1973.
7. Singh, C., "Some Reliability Data on Conventional Transit Vehicles," internal report Transit Systems R & D Branch, Ministry of Transportation and Communications, Ontario, Canada, Nov. 1974.
8. Singh, C. and M. D. Kankam, "Reliability Data and Analysis for Transit Vehicles," Research Report No. RR217, Transit Systems R & D Branch, Ministry of Transportation and Communications, Ontario, Canada, Jan. 1977.
9. Singh, C., "Reliability Models for Track Bound Transit Systems," *Proceedings 1977 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1977.
10. Singh, C. and R. Billinton, *System Reliability Modelling and Evaluation*, Hutchinson Educational, London, 1977.
11. Singh, C., "Operational Reliability Analysis," internal report Transit Systems R & D Branch, Ministry of Transportation and Communications, Ontario, Canada, Feb. 1975.
12. Singh, C., "Computer Program for Reliability Analysis of Track Bound Transit Systems," information available from the author.
13. Singh, C., "Reliability Analysis of Track Bound Transit Systems," Research Report, Research and Development Division, Ministry of Transportation and Communications, Ontario, Canada, May 1977.
14. Singh, C. and Kankam, M. D., "Failure Data Analysis for Transit Vehicles," *Proceedings of the 1979 Annual Reliability and Maintainability Symposium*, IEEE, New York, 1979.