3

Fundamental Concepts in Reliability Engineering

3.1 INTRODUCTION

This chapter briefly presents the fundamental concepts in reliability engineering such as general reliability function, redundant networks, reliability evaluation techniques, reliability apportionment, and failure mode and effect analysis.

In this chapter, a brief discussion on reliability evaluation techniques such as binomial, Markov processes (state space approach), decomposition, minimal cut set, and network reduction is presented. The delta-star technique is presented in a more detailed form.

3.2 GENERAL RELIABILITY FUNCTION

3.2.1 General Concepts

Suppose n_0 identical components are under test, after time t, $n_f(t)$ fail and $n_s(t)$ survive. The reliability function $R(t)$ is defined by

$$R(t)=\frac{n_s(t)}{n_s(t)+n_f(t)} \tag{3.1}$$

since

$$n_s(t)+n_f(t)=n_0$$

the equation becomes

$$R(t)=\frac{n_s(t)}{n_0} \tag{3.2}$$

since

$$R(t)+F(t)=1$$

$$R(t)=1-F(t) \tag{3.3}$$

where $F(t)$ the failure probability at time t. To obtain failure probability $F(t)$, substitute (3.2) into (3.3), and subtract it from unity, that is,

$$F(t)=1-\frac{n_s(t)}{n_0}$$

since

$$n_s(t)+n_f(t)=n_0$$

$$F(t)=1-\left(\frac{n_0-n_f(t)}{n_0}\right)=\frac{n_f(t)}{n_0} \tag{3.4}$$

By using the above result in relationship (3.3) we get

$$R(t)=1-F(t)=1-\frac{n_f(t)}{n_0} \tag{3.5}$$

the derivative of (3.5) with respect to time t is

$$\frac{dR(t)}{dt}=-\frac{1}{n_0}\frac{dn_f(t)}{dt} \tag{3.6}$$

In the limiting case, as dt approaches zero, the expression (3.6) is the instantaneous failure density function $f(t)$, that is,

$$\frac{1}{n_0}\frac{dn_f(t)}{dt}\to f(t)$$

Therefore, expression (3.6) becomes

$$\frac{dR(t)}{dt}=-f(t) \tag{3.7}$$

By using relationship (3.2) the other form of (3.6) may be written as

$$\frac{dn_f(t)}{dt}=-n_0\frac{dR(t)}{dt}=\frac{dn_s(t)}{dt} \tag{3.8}$$

3.2.2 Instantaneous Failure or Hazard Rate

If we divide both sides of (3.8) by $n_s(t)$, we get

$$\frac{1}{n_s(t)}\frac{dn_f(t)}{dt}=\frac{-n_0}{n_s(t)}\frac{dR(t)}{dt} \tag{3.9}$$

This is equal to the hazard rate $\lambda(t)$, that is,

$$\frac{1}{n_s(t)}\frac{dn_f(t)}{dt}=-\frac{n_0}{n_s(t)}\frac{dR(t)}{dt}=\lambda(t) \tag{3.10}$$

By substituting (3.2) and (3.7) into (3.10), we get an expression for the hazard rate (instantaneous rate):

$$\lambda(t)=-\frac{1}{R(t)}\frac{dR(t)}{dt}=\frac{f(t)}{R(t)} \tag{3.11}$$

3.2.3 Reliability Function

Equation 3.11 may be rewritten in the following form:

$$\frac{-dR(t)}{R(t)}=\lambda(t)\,dt \tag{3.12}$$

By integrating both sides of (3.12) over the time range 0 to t, we get

$$\int_0^t \lambda(t)\,dt=-\int_1^{R(t)}\frac{1}{R(t)}\,dR(t)$$

For the known initial condition that at $t=0$, $R(t)=1$ the above integral expression becomes

$$\ln R(t)=-\int_0^t \lambda(t)\,dt \tag{3.13}$$

The following general reliability function is obtained from (3.13):

$$R(t)=e^{-\int_0^t \lambda(t)\,dt} \tag{3.14}$$

Where $\lambda(t)$ is the time-dependent failure rate or instantaneous failure rate. It is also called the hazard rate. The above expression is a general reliability function. In other words, it can be used to obtain a component reliability for any known failure time distribution.

3.3 BATHTUB HAZARD RATE CURVE

This hazard rate curve, shown in Figure 3.1, is regarded as a typical hazard rate curve, especially when representing the failure behavior of electronic components. Mechanical components may or may not follow this type of failure pattern.

As shown in Figure 3.1 the decreasing hazard rate is sometimes called the "burn-in period." There are also several other names for this period such as debugging period, infant mortality period, break-in period. Occurrence of failures during this period is normally attributed to design or manufacturing defects.

The constant part of this bathtub hazard rate is called the "useful period," which begins just after the infant mortality period and ends just before the "wear-out period."

The wear-out period begins when an equipment or component has aged or bypassed its useful operating life. Consequently, the number of failures during this time begin to increase. Failures that occur during the useful life are called "random failures" because they occur randomly or in another word unpredictably.

The hazard rate shown in Figure 3.1 can be represented by the following function [15]:

$$\lambda(t)=k\lambda ct^{c-1}+(1-k)bt^{b-1}\beta e^{\beta t^{b}} \tag{3.15}$$

for $b, c, \beta, \lambda > 0 \quad 0 \leq k \leq 1 \quad t \geq 0$ and $c=0.5 \quad b=1$

where b, c = shape parameters
β, λ = scale parameters
t = time

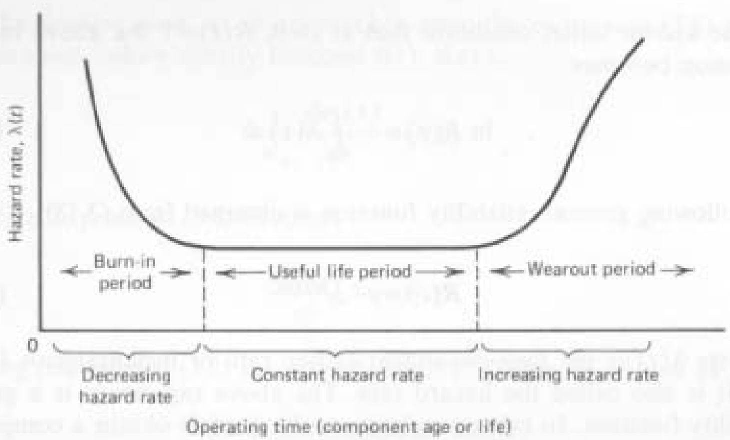

Figure 3.1 Bathtub curve.

3.4 MEAN-TIME-TO-FAILURE (MTTF)

The expected value $E(t)$, in our case MTTF, of a probability density function of the continuous random variable time t is given by

$$E(t)=\text{MTTF}=\int_0^{\infty} tf(t)\,dt \tag{3.16}$$

where $f(t)$ is the failure density function.

Example 1. Suppose, a component failure time follows the exponential failure law. It follows that the component has constant failure rate, λ (i.e., useful life period of the bathtub curve). Find the reliability function and the mean-time-to-failure expressions.

From the known information,

$$f(t)=\lambda e^{-\lambda t} \tag{3.17}$$

and

$$\lambda(t)=\lambda \tag{3.18}$$

To obtain the reliability function substitute (3.18) into (3.19):

$$R(t)=e^{-\int_0^t \lambda\, dt}=e^{-\lambda t} \tag{3.19}$$

In the case of MTTF substitute (3.17) into (3.16) to get

$$\text{MTTF}=\int_0^{\infty} t\lambda e^{-\lambda t} \tag{3.20}$$

The following is obtained by integrating the above expression by parts:

$$\text{MTTF}=\left[-te^{-\lambda t}\right]_0^{\infty}-\left[-\frac{e^{-\lambda t}}{\lambda}\right]_0^{\infty}$$

$$\therefore \text{MTTF}=\frac{1}{\lambda} \tag{3.21}$$

The above expression represents the situation when a component's failure times are exponentially distributed. MTTF is a reciprocal of the constant hazard rate, λ, as given by (3.21).

3.5 RELIABILITY NETWORKS

This section describes the five typical reliability configurations.

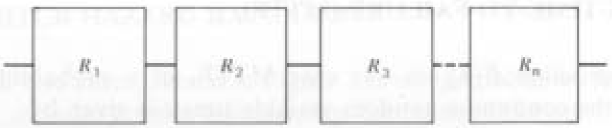

Figure 3.2 A series system block diagram.

3.5.1 Series Structure

This arrangement represents a system whose subsystems or components form a series network. If anyone of the subsystem or component fails, the series system experiences an overall system failure. A typical series system configuration is shown in Figure 3.2.

If the series system component failures are statistically independent, then the reliability R_s of a series system with nonidentical components is given by

$$R_s = \prod_{i=1}^{n} R_i \tag{3.22}$$

where n is the number of components or subsystems and R_i is the reliability of ith component or subsystem.

If the failure times of components are exponentially distributed (i.e., if components have constant failure rates), then the ith component reliability may be obtained from (3.19), that is,

$$R_i(t) = e^{-\lambda_i t} \tag{3.23}$$

By substituting (3.23) into (3.22),

$$R_s(t) = e^{-\sum_{i=1}^{n} \lambda_i t} \tag{3.24}$$

MTTF is given by

$$\text{MTTF} = \int_0^{\infty} e^{-\sum_{i=1}^{n} \lambda_i t}\, dt$$

$$= \frac{1}{\sum_{i=1}^{n} \lambda_i} \tag{3.25}$$

The above expression shows that a series system (MTTF) is the reciprocal of sum of the series network component failure rates.

Example 2. Two nonidentical pumps are required to run a system at a full load. Assume, pump I and II have constant failure rates $\lambda_1 = 0.0001$ failure/hour and $\lambda_2 = 0.0002$ failure/hour, respectively. Calculate this series system mean time to failure and reliability for a 100 hour mission; assume that both the pumps start operating at $t = 0$.

The following series system reliability R_s for a 100 hour mission is computed by using (3.24):

$$R_s(t) = e^{-(\lambda_1 + \lambda_2)t}$$

$$R_s(100) = e^{-(0.0001 + 0.0002)(100)} = 0.97045$$

By utilizing (3.25) we get

$$\text{MTTF} = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{0.0001 + 0.0002} = 3{,}333.3 \text{ hours}$$

3.5.2 Parallel Configuration

This configuration is shown in Figure 3.3. This system will fail if and only if all the units in the system malfunction. The model is based on the assumption that all the system units are active and load sharing. In addition it is assumed that the component failures are statistically independent. A parallel structure reliability R_p with nonidentical units or component reliability is given by

$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{3.26}$$

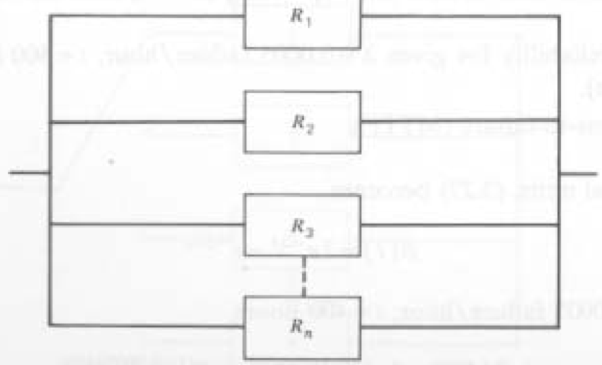

Figure 3.3 A parallel network block diagram.

where n is the number of units. R_i is the reliability of ith component or subsystem.

If the component failure rates are constant, then by substituting (3.19) into (3.26),

$$R_p(t) = 1 - \prod_{i=1}^{n} (1 - e^{-\lambda_i t}) \tag{3.27}$$

MTTF is obtained by integrating (3.27) over the interval [0, ∞],

$$\text{MTTF} = \int_0^{\infty} R_p(t)\,dt = \int_0^{\infty} \left\{ 1 - \prod_{i=1}^{n} (1 - e^{-\lambda_i t}) \right\} dt$$

$$= \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \cdots + \frac{1}{\lambda_n} \right) - \left(\frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_1 + \lambda_3} + \cdots \right)$$

$$+ \left(\frac{1}{\lambda_1 + \lambda_2 + \lambda_3} + \frac{1}{\lambda_1 + \lambda_2 + \lambda_4} + \cdots \right)$$

$$+ (-1)^{n+1} \frac{1}{\sum_{i=1}^{n} \lambda_i} \tag{3.28}$$

For identical components, the above equation reduces to

$$\text{MTTF} = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i} \tag{3.29}$$

Example 3. Suppose two identical motors are operating in a redundant configuration. If either of the motors fails, the remaining motor can still operate at the full system load. Assume both motors are identical and their failure rates are constant. In addition, motor failures are statistically independent. If both motors start operating at $t=0$, find the following:

1. System reliability for given $\lambda = 0.0005$ failure/hour, $t = 400$ hours (mission time).
2. Mean-time-to-failure (MTTF).

For identical units, (3.27) becomes

$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

since $\lambda = 0.0005$ failure/hour, $t = 400$ hours

$$\therefore R(400) = 2e^{-(0.0005)(400)} - e^{-(2 \times 0.0005)(400)}$$

$$= 0.9671$$

MTTF is obtained from (3.29)

$$\text{MTTF} = \frac{1}{\lambda}\left(1 + \frac{1}{2}\right) = \frac{3}{2}\frac{1}{\lambda}$$

$$= \frac{1.5}{0.0005} = 3{,}000 \text{ hours}$$

3.5.3 Standby Redundancy

← computers on board?

This type of redundancy represents a situation with one operating and n units as standbys. The standby redundancy arrangement is shown in Figure 3.4. Unlike a parallel network where all units in the configuration are active, the standby units are not active.

The system reliability of the $(n+1)$ unit, in which one unit is operating and n units on the standby mission until the operating unit fails, is given by

$$R_{st}(t) = \sum_{i=0}^{n} \frac{(\lambda t)^i e^{-\lambda t}}{i!} \tag{3.30}$$

The above equation is true if the following are true:

1. The switching arrangement is perfect.
2. The units are identical.
3. The units failure rates are constant.
4. The standby units are as good as new.
5. The unit failures are statistically independent.

In the case of $(n+1)$, nonidentical units whose failure time density functions are different, the standby redundant system failure density is

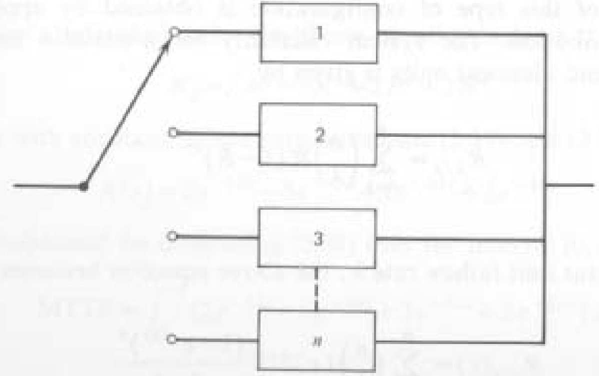

Figure 3.4 A standby redundancy model.

given by

$$f_{st}(t)=\int_{y_n=0}^{t}\int_{y_{n-1}=0}^{y_n}\cdots\int_{y_1=0}^{y_2} f_1(y_1)f_2(y_2-y_1)\cdots f_{n+1}(t-y_n)\, dy_1\, dy_2\cdots dy_n \qquad (3.31)$$

Thus, the system reliability can be obtained by integrating $f_{st}(t)$ over the interval $[t,\infty]$ as follows:

$$R_{st}(t)=\int_t^{\infty} f_{st}(t)\,dt \qquad (3.32)$$

Example 4. Assume that a system contains two identical units, that is, one operating and the other on the standby mission. Furthermore, the units failure rates are constant. In addition assume that the standby unit is as good as new at the beginning of its mission. Evaluate system reliability for a 100-hour mission for given unit failure rate, $\lambda=0.001$ failure/hour.

Equation (3.30) gives us

$$R_{st}(t)=(1+\lambda t)e^{-\lambda t} \qquad (3.33)$$

For known $t=100$ hours, $\lambda=0.001$ failure/hour, the system reliability is

$$=(1+0.1)e^{-0.1}=0.9953$$

3.5.4 k-out of-n Configuration

This is another form of redundancy. It is used where a specified number of units must be good for the system success. The series and parallel configuration in the preceding sections are special cases of this configuration, that is, $k=n$ and $k=1$, respectively.

Reliability of this type of configuration is obtained by applying the binomial distribution. The system reliability for k-out-of-n number of independent and identical units is given by:

$$R_{k/n}=\sum_{i=k}^{n}\binom{n}{i}R^i(1-R)^{n-i} \qquad (3.34)$$

For the constant unit failure rate λ, the above equation becomes

$$R_{k/n}(t)=\sum_{i=k}^{n}\binom{n}{i}(e^{-\lambda t})^i\frac{(1-e^{-\lambda t})^n}{(1-e^{-\lambda t})^i} \qquad (3.35)$$

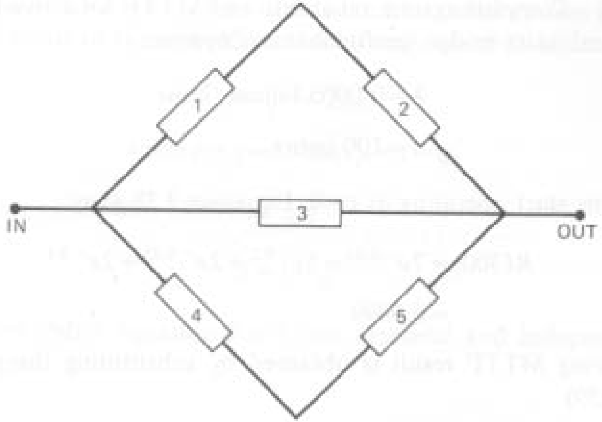

Figure 3.5 A five dissimilar units bridge network.

Example 5. Assume that in (3.35) $k=2$, $n=3$, and $\lambda=0.0001$ failure/hour. Therefore the system reliability for a 200-hour mission is

$$R_{2/3}(200)=3e^{-(2\times 0.001)200}-2e^{-(3\times 0.001)200}$$
$$=0.9133$$

3.5.5 Bridge Configuration

This network is shown in Figure 3.5. The critical element of the configuration is labeled as "3." For nonidentical and independent units, the five units bridge network reliability equation from reference 27 is

$$R_b=2R_1R_2R_3R_4R_5-R_2R_3R_4R_5-R_1R_3R_4R_5-R_1R_2R_4R_5$$
$$-R_1R_2R_3R_5-R_1R_2R_3R_4+R_1R_3R_5+R_2R_3R_4+R_1R_4+R_2R_5 \qquad (3.36)$$

In this case of identical units, the above equation reduces to

$$R_b=2R^5-5R^4+2R^3+2R^2 \qquad (3.37)$$

For units with constant failure rate, substitute (3.19) into (3.37), that is,

$$R(t)=2e^{-5\lambda t}-5e^{-4\lambda t}+2e^{-3\lambda t}+2e^{-2\lambda t} \qquad (3.38)$$

MTTF is obtained by integrating (3.38) over the interval $[0,\infty]$, that is,

$$\mathrm{MTTF}=\int_0^{\infty}(2e^{-5\lambda t}-5e^{-4\lambda t}+2e^{-3\lambda t}+2e^{-2\lambda t})\,dt$$
$$=\frac{49}{60}\frac{1}{\lambda} \qquad (3.39)$$

Example 6. Compute system reliability and MTTF for a five independent and identical units bridge configuration. Suppose

$$\lambda = 0.0005 \text{ failure/hour}$$

$$t = 100 \text{ hours}$$

and all units start operating at $t=0$. Equation 3.38 gives

$$R(300) = 2e^{-0.25} - 5e^{-0.2} + 2e^{-0.15} + 2e^{-0.1}$$

$$= 0.9999$$

The following MTTF result is obtained by substituting the given failure data in (3.39)

$$\text{MTTF} = \frac{49}{60 \times 0.0005} = 1633.4 \text{ hours}$$

3.6 RELIABILITY EVALUATION TECHNIQUES

This section briefly describes reliability evaluation techniques Readers with no indepth knowledge of these techniques should consult references 36 and 37.

3.6.1 *Binomial Theorem to Evaluate Network Reliability*

This is one of the simplest methods to evaluate system reliability. However, it is only useful for evaluating reliability of simple systems of series and parallel form. For complex systems it is quite a trying task.

The following is always true for the binomial expression in reliability engineering:

$$(p+q)^n = 1 \tag{3.40}$$

where P = the component probability of success
q = the component probability of failure
n = the number of identical components

Example 7. When two identical components form a series or parallel configuration we obtain the resulting equation from (3.40):

$$(p+q)^2 = p^2 + 2pq + q^2 \tag{3.41}$$

Here $p^2 \equiv$ probability of both components operating
$2pq \equiv$ probability of one component failed and one working
$q^2 \equiv$ probability of both components failed

Therefore, the reliability equation of a two unit parallel system is given by the first two terms of the right-hand side of expression (3.41):

$$R = p^2 + 2pq \tag{3.42}$$

$$\text{since } p+q=1, \text{ then } p = 1-q \tag{3.43}$$

By substituting (3.43) into (3.42) we get

$$R = 1 - q^2 \tag{3.44}$$

This is the reliability equation for a two identical and independent unit parallel system.

3.6.2 *State Space Approach (Markov Processes)*

The state space approach is a very general approach and can generally handle more cases than any other method. It can be used when the components are independent as well as for systems involving dependent failure and repair modes. There is no conceptual difficulty in incorporating multistate components and modeling common cause failures.

The method proceeds by the enumeration of system states. The state probabilities are then calculated and the steady-state reliability measures can be calculated using the frequency balancing approach [39]. The pertinent relationships are given below:

1. Unavailability or the probability of failure is given by

$$P_f = \sum_{i \in F} p_i \tag{3.45}$$

where p_i = probability of being in state i
F = subset of failure states

2. Frequency of failure [39] of encountering subset F is given by

$$f_f = \sum_{i \in S-F} p_i \sum_{j \in F} \lambda_{ij} \tag{3.46}$$

where S = system state space
λ_{ij} = transition rate from state i to state j

3. Mean duration of failure state is [39]

$$d_f = \frac{P_f}{f_f} \tag{3.47}$$

When the components are independent, the state probabilities can be obtained using the multiplication rule. When, however, dependent failure or repair modes are involved, the state probabilities have to be obtained by solving a set of linear algebraic equations. Probably the only serious problem, particularly when constant transition rates are assumed, with this approach is that for large systems, it could become unmanageable. In many situations, the problem can be handled using a computer-generated state transition matrix and reducing the size of the state space by truncation, sequential truncation, and using the concept of state merging. These techniques are discussed in reference 39. Examples of the application of these techniques for large systems can be found in reference 40.

3.6.3 Network Reduction Technique

It is a simple and useful procedure for systems consisting of series and parallel subsystems. Configurations such as bridge networks can be analyzed using delta-star conversions [12, 13, 41]. Some approximation, however, is involved in the use of these techniques [41]. The technique consists of sequentially reducing the parallel and series configurations to equivalent units until the whole network reduces to a single unit. The bridge configurations can be converted to series and parallel equivalents by using delta-star conversions or decomposition approach.

The primary advantage of this method is that it is easy to understand and apply; however, generally it is not suitable for considering degraded failure modes of components and systems. The independence of components has to be generally assumed.

3.6.4 Decomposition Method

This method decomposes a complex system into simpler subsystems by the application of conditional probability and conditional frequency theorems [37]. The reliability measures of simpler subsystems are calculated and then combined to obtain the results for the system. This method can be used to simplify both the state space, as well as the network approach. Examples of application in both of these areas can be found in reference 37. The success of the method depends upon the choice of the key component, that is, the component used for decomposing the network. If this component is not judiciously chosen, the final results will be the same, but the computations could be far more tedious. For a relatively complex network, the choice of proper key components to decompose the system into series parallel configurations can be a trying task.

3.6.5 Minimal Cut Set Method

A general approach to the solution of reliability block diagrams is based on minimal cut sets or minimal tie sets. This approach is very suitable for computer application. The minimal cut sets of a reliability block diagram

can be identified using special algorithms. Once the minimal cut sets have been enumerated, the reliability measures can be calculated using the following relationships:

$$P_f = P(\bar{C}_1) + \cdots + P(\bar{C}_m) - \left[P(\bar{C}_1 \cap \bar{C}_2) + \cdots + P\Big(\underset{i \neq j}{\bar{C}_i \cap \bar{C}_j}\Big) \right]$$
$$\cdots (-1)^{m-1}\left[P(\bar{C}_1 \cap \cdots \cap \bar{C}_m) \right] \tag{3.48}$$

and

$$f_f = P(\bar{C}_1)\bar{\mu}_1 + \cdots + P(\bar{C}_m)\bar{\mu}_m - P(\bar{C}_1 \cap \bar{C}_2)\bar{\mu}_{1+2}$$
$$+ \cdots + P\Big(\underset{i \neq j}{\bar{C}_i \cap \bar{C}_j}\Big)\bar{\mu}_{1+j}\Big)$$
$$\cdots (-1)^{m-1}\left[P(\bar{C}_1 \cap \cdots \cap \bar{C}_m)\bar{\mu}_{1+\cdots+m} \right) \tag{3.49}$$

where $C_i, \bar{C}_i$ = the minimal cut set i and failure of components in C_i, respectively

μ_i = the repair rate of component i

$\bar{\mu}_{i+j+k}$ = the sum of μ_j over all $j \in (C_i \cup C_j \cup C_k)$, that is the sum of repair rates of the components which belong to any or all of C_i, C_j, C_k

P_f = probability of failure

f_f = frequency of failure

As in the case of all network methods, this technique is not suitable for incorporating degraded modes of operation. For m minimal cut sets, the number of terms to be evaluated is 2^m. This could create computational problems, which could be partly alleviated using the concept of probability and frequency bounds [42]. Similarly, one may obtain tie sets for a complex system (described in detail in reference 37).

3.6.6 Delta-Star Technique

To analyze a complex structure such as bridge, the delta-star transformation [12, 13] easily transforms the configuration into series and parallel combinations. We derive the delta-star equivalent formulas by obtaining the equivalent legs of the block diagrams of Figure 3.6.

Consider, for example, three components of a system with reliabilities R_{AC}, R_{AB}, and R_{CB} connected to form the delta configuration shown in

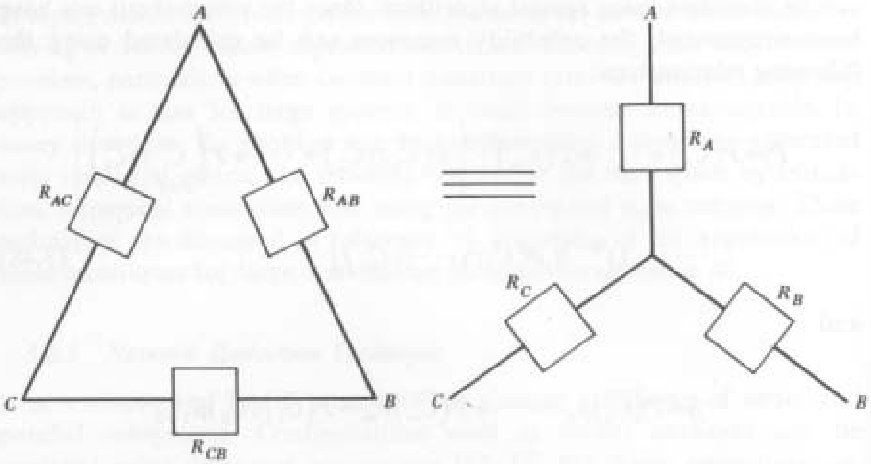

Figure 3.6 Delta-star reliability equivalent.

Figure 3.6. This configuration yields the star equivalent with reliabilities R_A, R_B and R_C.

Now consider the transformation steps indicated by Figure 3.7 to derive the delta to star equivalent. The application of independent event probability laws to components connected in series and parallel combinations as shown in Figure 3.7(*a–c*) will yield (3.55), (3.56), and (3.57), respectively:

For a simple independent series, the total system reliability is given by

$$R_T = R_A R_B \cdots R_N \tag{3.50}$$

where $R_A, R_B \cdots R_N$ are the reliabilities of the N components.

The simple independent parallel case yields the total system failure probability as

$$F_T = F_A F_B \cdots F_N = (1-R_A)(1-R_B)\cdots(1-R_N) \tag{3.51}$$

where $F_A, F_B \cdots F_N$ are the unreliabilities of the N components.

Applying (3.50) and (3.51) to the legs presented in Figure 3.7 their corresponding relationships are

$$R_A R_B = 1-(1-R_{AB})(1-R_{AC}R_{CB}) \tag{3.52}$$

$$R_B R_C = 1-(1-R_{CB})(1-R_{AC}R_{AB}) \tag{3.53}$$

$$R_A R_C = 1-(1-R_{AC})(1-R_{CB}R_{AB}) \tag{3.54}$$

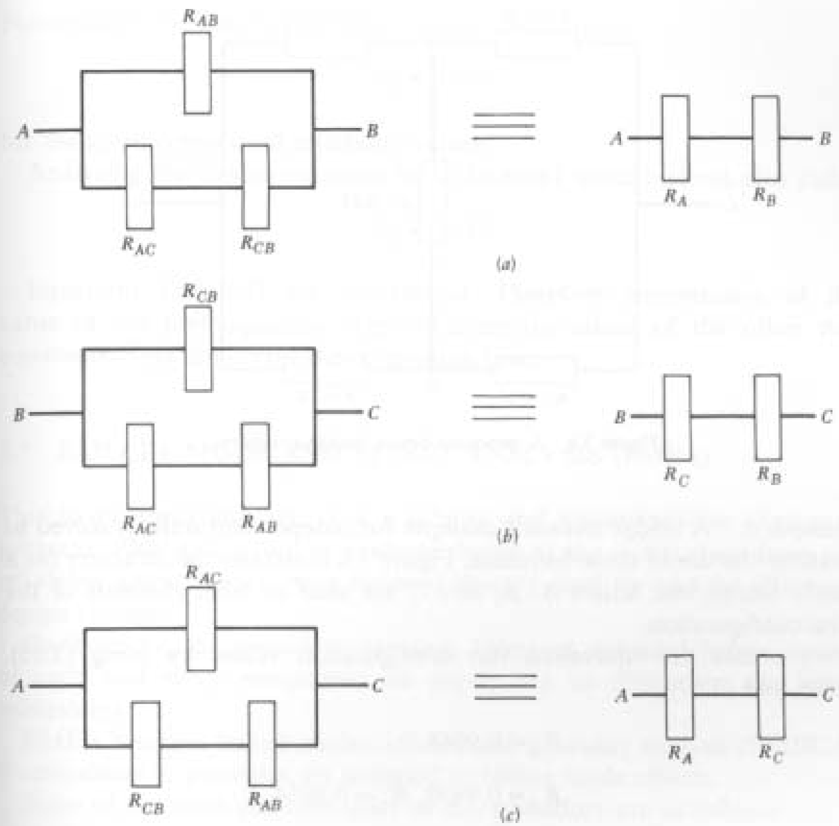

Figure 3.7 Delta-star equivalent legs.

From these three simultaneous equations the following delta-to-star relationships result:

$$R_A = \sqrt{\frac{[1-(1-R_{AC})(1-R_{CB}R_{AB})][1-(1-R_{CB})(1-R_{AC}R_{AB})]}{[1-(1-R_{AB})(1-R_{AC}R_{CB})]}} \tag{3.55}$$

$$R_B = \sqrt{\frac{[1-(1-R_{AB})(1-R_{AC}R_{CB})][1-(1-R_{CB})(1-R_{AC}R_{AB})]}{[1-(1-R_{AC})(1-R_{CB}R_{AB})]}} \tag{3.56}$$

$$R_C = \sqrt{\frac{[1-(1-R_{AC})(1-R_{CB}R_{AB})][1-(1-R_{AB})(1-R_{AC}R_{CB})]}{[1-(1-R_{CB})(1-R_{AC}R_{AB})]}} \tag{3.57}$$

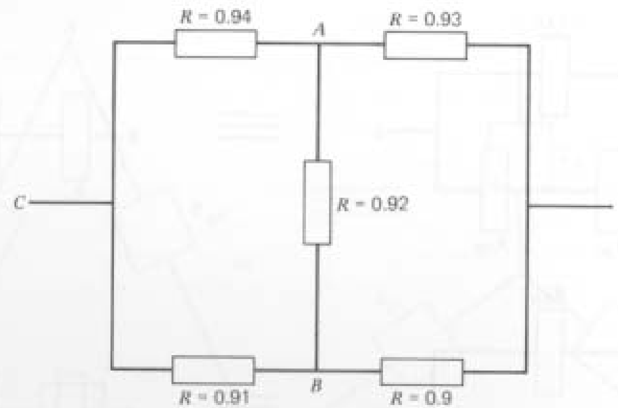

Figure 3.8 A two-state device bridge structure.

Example 8. A bridge network example for independent units is solved to illustrate the use of these formulas. Figure 3.8 illustrates the structure for a simple bridge; the letters A, B, and C are used to label elements of the delta configuration.

We obtain the equivalent star configuration values by using (3.55), (3.56), and (3.57).

$$\therefore R_A = 0.9948$$

$$R_B = 0.9930, \; R_C = 0.9954$$

The network shown in Figure 3.8 may be expressed as its equivalent as shown in Figure 3.9. The reliability equation for this structure is

$$R_T = [1-(1-R_1 R_A)(1-R_2 R_B)]R_C$$

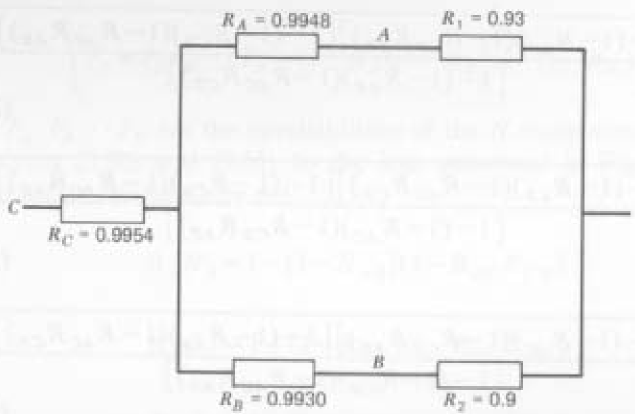

Figure 3.9 A transformed two-state device structure.

Numerically the value of the total bridge reliability is

$$R_T = 0.987$$

for the given component reliability values.

Analyzing this bridge structure with the event space method also yields

$$R_T = 0.987$$

Equations 3.55–3.57 are interrelated. Therefore computation of the value of the first equation helps to compute values of the other two equations. This minimizes the computing time.

3.7 FAILURE MODE AND EFFECT ANALYSIS (FMEA)

This is an important step in a reliability and maintainability assurance program. FMEA is a tool to evaluate design at the initial stage from the reliability aspect. This criteria helps to identify need for and the effects of design change.

Furthermore, the procedure demands listing of potential failure modes of each and every component on paper and its effects on the listed subsystems.

FMEA becomes failure modes, effects, and criticality analysis (FMECA) if criticalities or priorities are assigned to failure mode effects.

Some of the main characteristics of this procedure are as follows:

1. This is a routine upward procedure that begins from the detailed level.
2. By evaluating failure effects of each component, the whole system is screened completely.
3. It improves communication among design interface personnel.
4. It identifies weak areas in a system design and indicates areas where further or detailed analysis are desirable.

Some of the main steps to perform FMEA are shown in Figure 3.10.

3.8 RELIABILITY APPORTIONMENT

To achieve the required reliability of a complex system, it is a routine procedure to set reliability targets for subsystems. Its main advantage is that once the individual subsystem reliability goal is achieved, then the overall system goal will automatically be fulfilled.

The process to set such reliability goals is known as the reliability apportionment. Normally this is accomplished before the key design or development decisions are made.

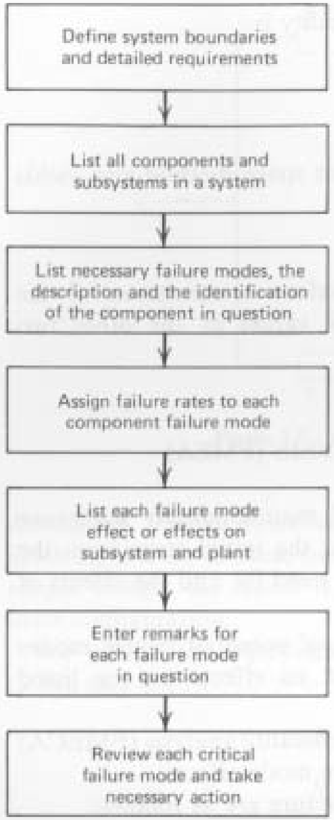

Figure 3.10 FMEA flow chart.

Some of the reliability apportionment techniques are described in the following sections.

3.8.1 Reliability/Cost Models

Before applying this reliability/cost procedure, the relationship between reliability and cost must be known for each subsystem to meet system reliability goal at minimum cost. However the main drawback of this procedure is the lack of availability of cost data, that is, cost at a given level of reliability.

3.8.2 Similar Familiar Systems Reliability Apportionment Approach

This approach is based upon the familiarity of the designer with similar systems or subsystems. Its main weakness stems from the fact that the reliability and life cycle cost of earlier similar designs have to be assumed adequate when designing new systems. By applying this technique the

failure data collected on similar systems from the various sources can be utilized.

3.8.3 Factors of Influence Method

This procedure is purely based upon the following important factors that effect the system in question:

1. *Complexity/Time*. In the case of complexity it relates to the number of subsystem parts, whereas time is related to the relative operational time during the total functional period.
2. *Environmental Factor*. This concerns each subsystem's operating environmental conditions such as temperature, vibration, humidity. In other words it deals with susceptibility or exposure of subsystems to such environmental conditions.
3. *State-of-the-Art*. This factor takes care of advancement in the state-of-the-art for a particular subsystem or component.
4. *Subsystem Failure Criticality*. This factor includes the criticality effect of a subsystem failure on the system. For example, the failure of some auxiliary instruments in an aircraft may not be as critical as the failure of engine.

When applying this factor of influence procedure, each and every subsystem is rated with respect to the influential factors, and one can assign a number between 1 and 10, where 1 is allocated to a subsystem least affected by the factor in question and 10 is allocated to a subsystem most affected by the factors of influence. Thus reliability can be allocated by using the weight of these assigned numbers for all factors.

3.8.4 Combined Familiar Systems and Factor of Influence Method

Both the familiar systems and factors of influence methods have their weakness when they are used individually. However, combining the two methods produces better results because data are used from the similar subsystems as well as when new subsystems are designed under different factors of influence.

REFERENCES

1. Amstadter, B. L., *Reliability Mathematics*, McGraw-Hill, New York, 1971.
2. Von Alven, W. H. (Ed.), *Reliability Engineering*, Prentice-Hall, Englewood Cliffs, NJ, 1964.
3. Barlow, R. E. and F. Proschan, *Mathematical Theory of Reliability*, Wiley, New York, 1965.

4. Barlow, R. E. and F. Proschan, *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, New York, 1975.
5. Bazovsky, I., "Fault Trees, Block Diagrams and Markov Graphs," *Proceedings of the* 1977 *Annual Reliability and Maintainability Symposium, Philadelphia*, IEEE, New York, 1977, pp. 134–141.
6. Bazovsky, I., *Reliability: Theory and Practice*, Prentice-Hall, Englewood Cliffs, NJ, 1961.
7. Billinton, R., *Power-System Reliability Evaluation*, Gordon and Breach, New York, 1970.
8. Billinton, R., R. J. Ringlee, and A. J. Woods, *Power-System Reliability Calculations*, MIT Press, Cambridge, MA, 1973.
9. Blanchard, B. S., Jr. and E. E. Lawery, *Maintainability*, McGraw Hill, New York, 1969.
10. Calabro, S. R., *Reliability Principles and Practice*, McGraw-Hill, New York, 1962.
11. Cox, D. R., *Renewal Theory*, Methuen, London, 1962.
12. Dhillon, B. S., The Analysis of the Reliability of Multi-State Device Networks, Ph.D. Thesis (1975). (Available from National Library of Canada, Ottawa.)
13. Singh (Dhillon), B., and C. L. Proctor, "Reliability Analysis of Multi-State Device Networks." *Proceedings of the* 1976 *Annual Reliability and Maintainability Symposium, Las Vegas*, IEEE, New York, 1976, pp. 31–35.
14. Dhillon, B. S., and C. Singh, "Bibliography of Literature on Fault Trees," *Microelectr. Reliability*, **18**, 501–503 (1978).
15. Dhillon, B. S., "A Hazard Rate Model," *IEEE Trans. Reliability* **R-28**, pp. 150, (1979).
16. Epstein, B., *Mathematical Models for System Reliability*, Student Association Technion, Israel Institute of Technology, Haifa, Israel, 1969.
17. Gnedenko, B. V., Y. K. Belyaev, and A. D. Solovyev, *Mathematical Methods of Reliability Theory*, Academic, New York, 1969.
18. Goldman, A. S., and T. B. Slattery, *Maintainability*, Wiley, New York, 1964.
19. Grouchko, D., (Ed.), *Operations Research and Reliability*, Gordon and Breach, New York, 1969.
20. Haviland, R. T., *Engineering Reliability and Long-Life Design*, Van Nostrand, Princeton, NJ, 1964.
21. Howard, R. A., *Dynamic Probabilistic Systems*, Vols. I and II, Wiley, New York, 1971.
22. Ireson, W. G., (Ed.), *Reliability Handbook*, McGraw-Hill, New York, 1966.
23. Jardine, A. K. S., (Ed.), *Operational Research in Maintenance*, Manchester University Press, Manchester and Barnes and Nobles, New York, 1970.
24. Jorgenson, D. W., J. J. McCall, and R. Radner, *Optimal Replacement Policy*, Rand McNally, Chicago, 1967.
25. Kapur, K. C., and L. R. Lamberson, *Reliability in Engineering Design*, Wiley, New York, 1977.
26. Kozlov, B. A., and I. A. Ushakov, *Reliability Handbook*, Holt, Rinehart and Winston, New York, 1970.
27. Lipp, J. P., "Topology of Switching Elements vs Reliability," *Trans. IRE Reliability Qual. Control*, 7, 21–34 (June 1957).
28. Mann, N. R., R. E. Shafer, and N. D. Singpurwalla, *Methods for Statistical Analysis of Reliability and Life Data*, Wiley, New York, 1974.
29. Morse, P. M., *Queues, Inventories and Maintenance*, Wiley, New York, 1958.
30. Myer, R. H., K. L. Wong, and H. M. Gordy (Ed.), *Reliability Engineering for Electronic Systems*, Wiley, New York, 1964.
31. Pieruschaka, E., *Principles of Reliability*, Prentice-Hall, Englewood Cliffs, NJ, 1963.
32. Polovko, A. M., *Fundamentals of Reliability Theory*, Academic, New York, 1968.

33. *Proceedings of the Annual Reliability and Maintainability Symposia* and its predecessors (available from Annual Reliability and Maintainability Symposium; 6411 Chillum Place NW, Washington, D.C. 20012; or from the IEEE).
34. Roberts, N. H., *Mathematical Methods in Reliability Engineering*, McGraw-Hill, New York, 1965.
35. Sandler, G. H., *System Reliability Engineering*, Englewood Cliffs, NJ, Prentice-Hall, 1963.
36. Shooman, M. L., *Probabilistic Reliability: An Engineering Approach*, McGraw-Hill, New York, 1968.
37. Singh, C., and R. Billinton, *System Reliability Modeling and Evaluation*, Hutchinson, London (1977).
38. Singh, C., *Calculating the Frequency of Boolean Expressing Being 1, IEEE Trans. Reliability*, **R-26**, 354–355 (1977).
39. Singh, C., "Reliability Calculations on Large Systems," *Proceedings of 1975 Annual Reliability and Maintainability Symposium, Washington, D.C.*, IEEE, New York, 1975, pp. 188–193.
40. Singh, C., "Reliability Models for Track Bound Transit Systems," *Proceedings of the 1977 Annual Reliability and Maintainability Symposium, Philadelphia*, IEEE, New York, 1977, pp. 242–247.
41. Singh, C., and M. D. Kankam, "Comments on Closed Form Solutions for Delta-Star and Star-Delta Conversion of Reliability Networks," *IEEE Trans. Reliability*, **R-25**, 336–339 (1976).
42. Singh, C., *On the Behaviour of Failure Frequency Bounds, IEEE Trans. Reliability*, **R-26**, 63–66 (1977).
43. Smith, C. O., *Introduction to Reliability in Design*, McGraw-Hill, New York, (1976).
44. Smith, D. J., *Reliability Engineering*, Pitman, New York, 1973.
45. Smith, D. J., and A. H. Babb, *Maintainability Engineering*, Pitman, New York, 1973.
46. Tsokos, C. P., and I. N. Shimi (Ed.), *The Theory and Applications of Reliability: With Emphasis on Bayesian and Nonparametric Methods*, Academic, New York (1977).
47. Vogt, W. G., and M. H. Mickle (Ed.), *Modeling and Simulation Conference Proceedings*, Instrument Society of America, 1969–1979.
48. Zelen, M., (Ed.), *Statistical Theory of Reliability*, Madison, WI, University of Wisconsin Press, 1963.