# 9

# Power System Reliability

## 9.1 INTRODUCTION

A primary requirement of a modern electric power system is a reasonable ability to satisfy the customer load requirements. In some electric utilities this involves generation, transmission, and distribution facilities. In others, the responsibility may extend over a part of the total facility. A complete power system is, however, composed of generation, transmission, and distribution facilities each one of which contributes its own inherent difficulties to the problem of satisfying customer requirement. A power system should be designed and expansion facilities planned so that it can perform its intended function with a reasonable risk. The risk of power interruption or capacity shortage can be reduced by providing more redundancy in the transmission and distribution networks and enough reserve generating capacity. There has, however, to be a trade off between reliability of power supply and the cost involved. Reliability models provide a means of carrying out this trade off.

There has been considerable growth in the techniques for the quantitative evaluation of the reliability of power systems. Because of the structural similarity of the various power systems, a number of generic reliability techniques have been developed for planning, design, and operation of these systems. This ensemble of concepts, indices, and methods is generally referred to as the power system reliability. Numerous papers [12] and two books [9, 11] have been written on this subject. This chapter gives a compact and unified approach to power system reliability and references to more detailed discussions are provided.

The major areas of a power system are generation, transmission, and distribution. For determining the reliability indices, the entire power system is not considered. Although conceptually possible, the complexity and dimensionality make this task rather impractical at present. It appears, however, that this task will become possible in the future partly by developments in better techniques of modeling and partly due to an increase in both the speed and power of computers. At present, however, the major divisions in power system reliability are the generating capacity reliability, bulk power system reliability, interconnected systems reliability, and the reliability of distributions systems.





## 9.2 GENERATING CAPACITY RELIABILITY

Generating capacity reliability evaluation can be considered in two basic forms, which may be designated static reserve and operating reserve requirements. The static reserve studies are concerned with determining the installed reserve capacity sufficient to provide for unplanned and planned outages of generating units and uncertainties in the forecast load. The operating reserve consists of spinning or quick starting units and is a capacity that must be available to meet load changes and also capable of satisfying the loss of some portion of generating capacity. Whereas the static reserve is of primary concern to the planning engineer, the operating reserve provides assistance in decisions on daily operation of the power system. Ideally both of these areas must be investigated at planning level but once a decision has been reached, the operating reserve becomes an operating problem. This section is concerned with the static reserve area and the operating reserve is described in Section 3.

Generating capacity reliability studies assume the transmission network to be perfectly reliable and capable of transferring the energy from any generation point to the load point. This amounts to the assumption that all the generating units and loads are connected across a single bus. The assessment is basically concerned with the certainty with which the system load can be satisfied by the generation facilities. The three basic steps involved are [17]:

1. A model describing the probabilistic behaviour of capacity outages is developed first. This is referred to as the "generation system model."
2. The probabilistic nature of the daily load curve is incorporated into a "demand model" or "load model."
3. The generation and load models are then merged or convolved to give a "generation reserve model," which depicts the expected occurrence of surplus capacity and capacity deficiencies. Several indices are defined on the generation reserve as measures of generating capacity reliability.

### 9.2.1 Generation System Model

*Model of a Single Unit.* A generating unit, especially a large thermal one, may have several different capacity levels. The consideration of partial or derated capacity states is not a major problem; however, in order to illustrate the basic approach each unit is assumed to exist either in an up (full capacity) or in a down (zero capacity) state. This binary model can be characterized by the following parameters:

$c$ = capacity of the unit in MW
$m$ = mean up time of the unit
$r$ = mean down time of the unit

Using these parameters, the model of the single unit can be expressed as follows:

$$\Pr(\text{capacity out}=0)=\frac{m}{m+r}$$

$$=\frac{\mu}{\lambda+\mu} \tag{9.1}$$

$$\Pr(\text{capacity out}=c)=\frac{r}{m+r}$$

$$=\frac{\lambda}{\lambda+\mu} \tag{9.2}$$

$$\text{Fr}(\text{capacity out}=0)=\text{Fr}(\text{capacity out}=c)$$

$$=\frac{\lambda\mu}{\lambda+\mu} \tag{9.3}$$

where

$\lambda,\mu$ = the reciprocals of $m$ and $r$ and are called the failure and repair rates of the unit

$\Pr(\cdot),\text{Fr}(\cdot)$ = the steady-state probability and frequency of $(\cdot)$, respectively.

Equations 9.1–9.3 are independent of the form of the probability density function of the up and down times. Equation 9.2 can be recognized as the unavailability of the unit and has been traditionally called "forced outage rate" and defined as [13]

$$\text{FOR}=\frac{\text{forced outage hours}}{\text{in-service hours}+\text{forced outage hours}} \tag{9.4}$$

*System Model.* The generation system consists of many units and they are assumed statistically independent. Such a system can have many possible capacity levels and the reliability measures of the following form are required.

1. *Pr*(lost system capacity $\geqslant x$), the steady-state probability of lost capacity equal to or greater than $x$ MW. The state, $\geqslant x$ is commonly called cumulative state as compared with the exact state, that is, equal to $x$ MW.
2. *Fr*(lost system capacity $\geqslant x$), the mean frequency of encountering the cumulative state of $x$ or more MW lost capacity. The reciprocal of this function gives the mean cycle time, that is, the mean time between two successive encounters of this state.



The problem for a small system can be solved by enumerating the possible system states, calculating the probabilities by the multiplication rule, and then determining the subset probabilities by addition. The frequencies can be readily calculated using the frequency balancing approach [19]. This method is practicable, however, only as long as the number of units comprising the system is relatively small. A generation system may have several hundred units and therefore this approach is not feasible. The generation system model is typically developed by the sequential addition of units.

*Algorithm for Unit Addition.* This algorithm [20] is basic to system model building by sequential unit addition. The system is assumed to consist of binary units, that is, it can exist either in an up (full capability) or down (zero capability) state. There is, however, no inherent limitation in developing similar algorithms for multistate units using the same methodology. The assumption of binary units is for keeping the discussion simple. It has been already shown in reference 26 that the probability density functions of up and down times do not effect the steady-state probabilities and mean frequencies when the units are assumed statistically independent. The following notation is used:

$\overline{C}_i$ = $i$th lost capacity state. Capacity states are assumed to be arranged in an increasing order of lost capacity, $\overline{C}_{i+1}>\overline{C}_i$

$P_i, f_i$ = steady-state probability and mean frequency of lost capacity $\geqslant \overline{C}_i$

$c_k$ = capacity of the $k$th unit

$\lambda_k = 1/m_k$

$m_k$ = mean up time of unit $k$

$\mu_k = 1/r_k$

$p_k=\mu_k/(\lambda_k+\mu_k)$,

$\begin{Bmatrix} P_{(i+k)} \\ f_{(i+k)} \end{Bmatrix}$ = steady-state probability and mean frequency of lost capacity $\geqslant(\overline{C}_i+c_k)$

$N_k$ = number of capacity states for the $k$ unit system

$r_k$ = mean down time of unit $k$

a = to "after," that is, after the unit addition or after the unit removal (superscript)

The process of model building is started with a single unit and then each unit is added in turn. The model for a single unit is, of course, quite straightforward. Now assume that a system model exists for $(k-1)$ units and it is required to add the $k$th unit. Since the unit is assumed to exist either in the up state (lost capacity = 0) or in the down state (lost capacity $=c_k$), two groups of system states would be obtained after unit addition, $\{\overline{C}_i+0\}$ and $\{\overline{C}_i+c_k\}$, $i=1,2,\ldots,N$, (see Figure 9.1). The states in the

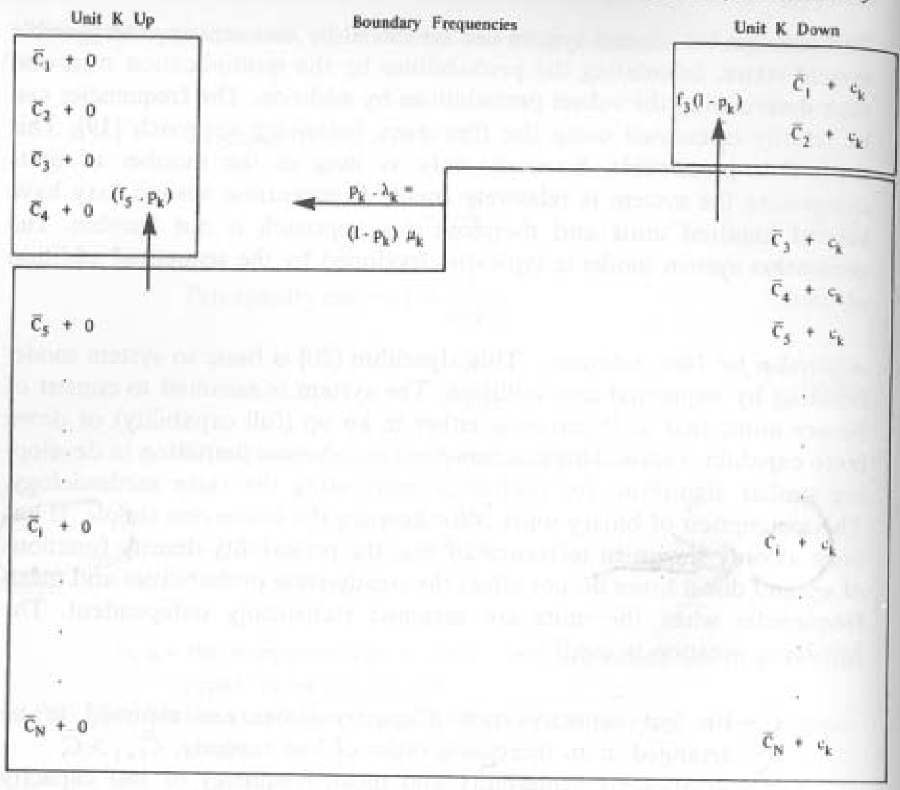


**Figure 9.1** State frequency diagram for unit addition.

former group are termed "existing lost capacity states" and those in the latter "generated lost capacity states." As an example consider $\bar{C}_5$ such that $(\bar{C}_3 + c_k) \geq \bar{C}_5$ and $(\bar{C}_2 + c_k) < \bar{C}_5$. The boundary for this state is shown in Figure 9.1 and the associated frequency is

$$= f_5 p_k + f_3(1-p_k) + (P_3 - P_5) p_k \lambda_k \tag{9.5}$$

In general, modified cumulative probability and frequency of existing lost capacity state $i$ is given by

$$P_i^m = P_i^o p_k + P_j^o (1-p_k) \tag{9.6}$$

and

$$f_i^m = f_i^o \cdot p_k + f_j^o (1-p_k) + (P_j^o - P_i^o) p_k \cdot \lambda_k, \tag{9.7}$$

such that

$$\bar{C}_j \geq (\bar{C}_i - c_k)$$



Superscript $o$ refers to the old values of probabilities and frequencies and superscript $m$ refers to the modified values after unit addition. The probabilities and frequencies of generated lost capacity states are similarly given by

$$P_{(i+k)} = P_i^o (1-p_k) + P_j^o \cdot p_k \tag{9.8}$$

and

$$f_{(i+k)} = f_i^o (1-p_k) + f_j^o p_k + (P_i^o - P_j^o) p_k \lambda_k \tag{9.9}$$

such that

$$\bar{C}_j \geq (\bar{C}_i + c_k)$$

The flow diagram for the computer implementation of this algorithm is provided in reference 20. The procedure described in reference 20 accomplishes calculation and state reordering at the same time and is very fast.

*Algorithm for Unit Removal.* Several times it may be necessary to remove a unit from the system model. For example, during the period of a year different units are on scheduled maintenance and therefore the same system model cannot be used for the whole period. The year can be divided into a number of intervals during which the units on scheduled maintenance stay the same and a single system model can be used. These system models can be derived from the master system model by removing the units on maintenance.

Unit removal is the reverse of the process of unit addition described by (9.6)–(9.9). To reconstruct the system model prior to the addition of unit $k$, (9.8) and (9.9) can be modified. Since $\bar{C}_j$ is equal to or just greater than $\bar{C}_i + c_k$,

$$P^o_{(i+k)} = P_j^o \tag{9.10}$$

and

$$f^o_{(i+k)} = f_j^o \tag{9.11}$$

Substituting (9.10) and (9.11) into (9.8) and (9.9) and replacing subscripts $(i+k)$ and $i$ by $i$ and $j$, respectively,

$$f_i^m = f_i^o \cdot p_k + f_j^o (1-p_k) + (P_j^o - P_i^o) p_k \lambda_k \tag{9.12}$$

and

$$P_i^m = P_i^o p_k + P_j^o \cdot (1-p_k) \tag{9.13}$$

such that

$$\overline{C}_j \geqslant (\overline{C}_i - c_k)$$

Equations 9.12 and 9.13 are the same as (9.6) and (9.7). Probabilities and frequencies of the old system model are therefore,

$$P_i^o = \frac{[P_i^m - P_j^o(1-p_k)]}{p_k} \tag{9.14}$$

and

$$f_i^o = \frac{[f_i^m - f_j^o(1-p_k) + (P_i^o - P_j^o)p_k\lambda_k]}{p_k} \tag{9.15}$$

such that

$$\overline{C}_j \geqslant \overline{C}_i - c_k$$

The algorithm is started with $P_1^o = 1$ and $f_1^o = 0$. After each unit removal, the system model may contain sets of states with different lost capacity values but the same probabilities and frequencies. In these sets, all the states except the last one should be deleted to obtain the exact system model prior to the addition of unit $k$. The flow diagram for the computer implementation of this algorithm is given in reference 20.

### 9.2.2 Load and Generation Reserve

*Load Model for Loss of Load Expectation.* One of the indices used in static reserve studies is loss of load probability (LOLP) or loss of load expectation (LOLE). The load model generally employed for LOLE calculation is of the shape shown in Figure 9.2. This cumulative load curve indicates the time for which the load is more than a specified level in MW. This curve is either from hourly load durations or more generally from the daily peaks. In the latter case, it indicates the number of days on which the peak exceeded a specified value. The LOLE is an expected value and is given by [6].

$$\text{LOLE} = \sum p_i t_i \tag{9.16}$$

where $p_i$ = probability of a capacity outage equal to $c_i$

$t_i$ = number of time units, in the study period, that a capacity outage of $c_i$ would result in a loss of load



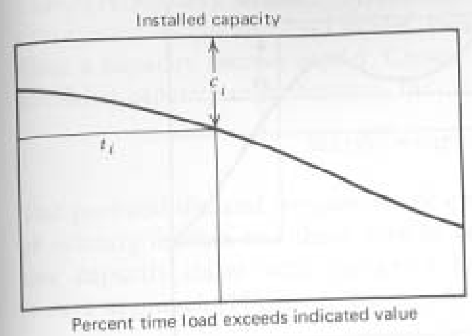


**Figure 9.2** Cumulative load duration curve.

The values of $p_i$ can be obtained from the generation system model discussed in Section 9.2.1 by the following relationship,

$$p_i = P_i - P_{i+1}$$

The LOLE is expressed in days/year, that is, the expected time in days that the load would not be met by the capacity in a period of 1 year. The magnitude of load loss is not considered. The reciprocal of LOLE in years/day is often used as a reliability measure; however, it does tend to obscure the fact that LOLE is a simple expectation.

*Load Model for Frequency and Duration.* The load model for loss of load probability method does not adequately reflect the shape of the daily load variation curve. Reference 16 suggested a load model introducing an exposure factor to indicate that the peak load does not persist for the entire day. The mean duration at a particular load level, usually about 80% of the daily peak, is assumed to be exposure factor. This amounts to approximating the daily load variation curve as shown in Figure 9.3, and taking the mean of $e_i$ as the exposure factor. The load model, then is assumed to consist of a random sequence of $N$ load states, each of which is followed by a low load state (see Figure 9.4). The state transition diagram for this load model is shown in Figure 9.5 and some parameters are given below.

| | |
|---|---|
| Description of load levels, MW | $L_i, i=1,2,\ldots,N$ <br> $L_1 > L_2 > \cdots > L_n$ |
| Number of occurrences of $L_i$ | $n_i$ |
| Interval length | $D = \sum_{i=1}^{N} n_i$ |
| Exposure factor, that is, the mean duration of $L_i$ | $e < 1$ days |

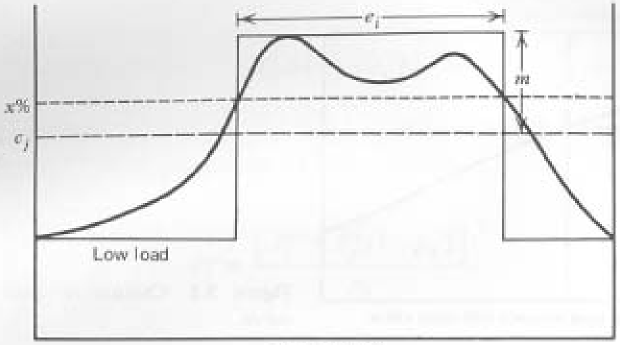


**Figure 9.3** The daily load variation curve.

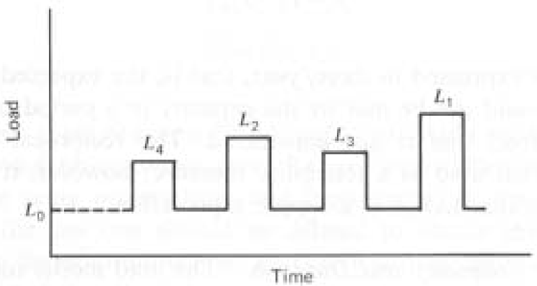


**Figure 9.4** The basic load model.

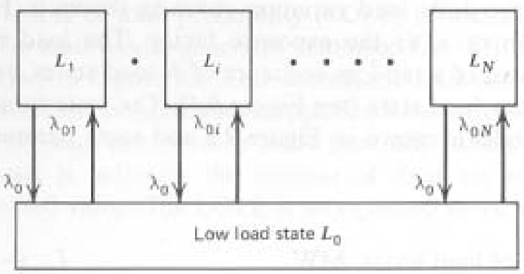


**Figure 9.5** State transition diagram of the basic load model.





CAPACITY RESERVE MODEL. Assuming stochastic independence of generation system model and load model, they may be combined or convolved to form a capacity reserve model. Capacity reserve or margin is an excess of available capacity over demand, that is

$$\text{margin} = \text{capacity} - \text{load}$$

The probabilities and frequencies of cumulative margins (margin $\leqslant M$) are of primary interest and these may be computed by combining the cumulative capacity states with the exact load states. Using the margin state matrix approach [18], it can be proved that

$$A_M = \sum_{i=1}^{N+1} P_v A_i \tag{9.17}$$

where $A_M$ = steady-state probability of margin $\leqslant M$

$$A_i = n_i e / D \tag{9.18}$$

$P_v$ = probability of capacity $\leqslant C_v$ such that $C_v \leqslant (L_i + M)$

$N+1 \equiv$ the low load state, $L_0$

The margin $\leqslant M$ could be encountered either by the change in the system capacity or the change in system load. The contributions to $f_M$, the frequency of encountering a margin $\leqslant M$, by these two respective modes of transitions are termed the generation system transitions and the load model transitions [18]

$$f_M = f_M^g + f_M^l \tag{9.19}$$

where

$$f_M^g = \sum_{i=1}^{N+1} f_v A_i \tag{9.20}$$

and

$$f_M^l = \sum_{i=1}^{N} \frac{n_i}{D} (P_v - P_{v0}) \tag{9.21}$$

It should be noted that in (9.17)–(9.21), $v$ is a function of load level $L_i$ selected and is so determined that $C_v \leqslant (L_i + M)$. The probability $P_{v0}$ is that of capacity level corresponding to low load level such that $C_{v0} \leqslant (L_0 + M)$.

*The CVEF and MLEF Load Models.* The single exposure factor load model discussed previously assumes a random sequence of daily peaks.

This is generally not true and there is likely to be a strong sequential correlation between the daily peaks. The choice of the exposure factor is arbitrary and its nature is questionable. For example if the capacity level $c_j$ exists for the day shown in Figure 9.3, then according to this load model a negative margin of $m$ will exist for a duration $e_i$. This obviously is not an accurate representation.

Load models for accurate representation of the system load were developed in reference 18 and described in the related publications [7, 8]. In these models, the expected daily load variation curve is approximated by the mean durations at various load levels determined as a percentage of the daily peak. The expected daily curve may or may not be symmetrical. It is shown in references 7, 8, and 18 that the asymmetry does not effect the steady state probability and frequency of encountering a margin. The MLEF (Multilevel exposure factor) representation amounts to approximating the daily load variation curve as shown in Figure 9.6. For a bimodal curve this means transferring segments like $d$ to $d'$. This does not alter the steady-state availability of the failure state, but the frequency is slightly decreased and the mean duration slightly increased. For example, for a given capacity level $c_j$, the load exceeds the available capacity twice in a day with durations $e_i$ and $d$, respectively. Using this approximation, the load exceeds $c_j$ once with duration $(e_i+d)$. Such an approximation may even give more realistic indices.

Whereas the MLEF representation assumes discrete variations in the exposure factor, the CVEF (continuously varying exposure factor) model assumes the exposure factor as a continuously varying function of the percentage of daily peak. The continuous approximation does not involve any additional difficulty and relatively little extra computational effort is required.

CAPACITY RESERVE MODEL. Four types of load models are described in references 7, 8, and 18 depending upon the manner in which low load is taken into account and the sequential interdependence of the daily peaks.

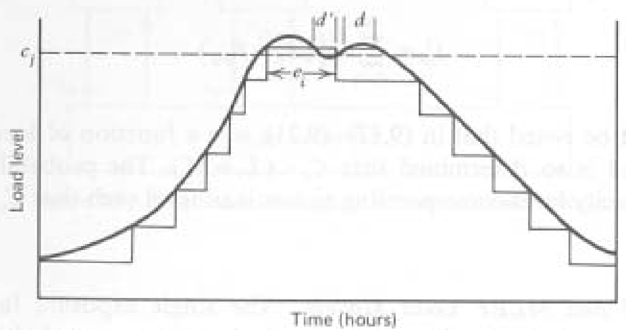


**Figure 9.6** The discrete approximation of a continuously varying daily load curve.



The four possible combinations are

1. Low load assumed same on all days and the sequence of daily peaks assumed random.
2. Same as (1) above except that there is sequential interdependence between daily loads.
3. Low load different on different days and the sequence of daily loads random.
4. Same as (3) except that there is sequential interdependence between daily peaks.

Both MLEF and CVEF representations are possible for the four combinations described above. Expression for combination one, using CVEF representation is derived in this section. For other cases, the reader is referred to references 7, 8, and 18. The derivations for the availability and frequency of margin $\leqslant M$ can be understood with reference to the Figure 9.7, which shows exposure factor as a continuously varying function of the daily peak. For a given capacity level $C_v$, the corresponding load level $L$ for a margin $\leqslant M$ can be determined using the relationship

$$C_v - L_v \leqslant M$$

that is,

$$L_v \geqslant C_v - M$$

The expected duration of margin $\leqslant M$ for the $i$th load cycle can be easily shown [18],

$$\sum_{x=0}^{n} P_{v+x}(d_{v+x} - d_{v+x-1}) \tag{9.22}$$

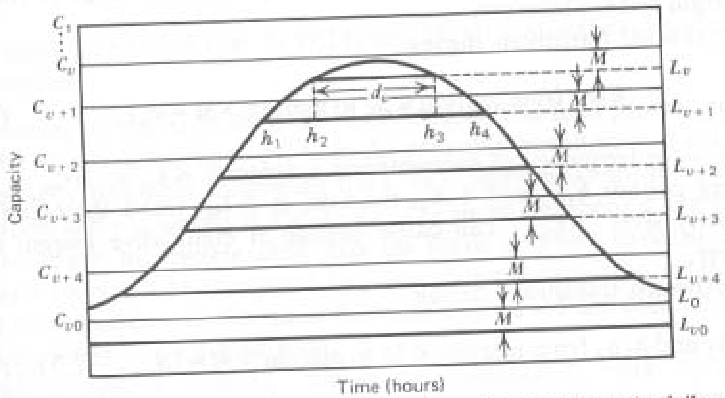


**Figure 9.7** Exposure factor as a continuously varying function of the daily peak.

where $d_{v+x}$ = the mean duration for which the load $\geqslant L_{v+x}$ such that $L_{v+x} = C_{v+x} - M$

$P_{v+x}$ = the probability of capacity $\leqslant C_{v+x}$

In (9.22), $v$ is such that $C_v - M$ makes the first intercept with the daily load cycle and $n$ is such that $C_{v+n} - M$ is at or just below the low load level. Assuming $n_i$ identical peak loads $L_i$, the expected duration of margin $\leqslant M$ in the period of $D$ days

$$= \sum_{i=1}^{N} n_i \sum_{x=0}^{n} P_{v+x}(d_{v+x} - d_{v+x-1})$$

$$= D \cdot A_M \tag{9.23}$$

where $N$ = total number of peak loads, that is, $\sum_{i=1}^{N} n_i = D$

$A_M$ = probability of margin $\leqslant M$

From (9.23)

$$A_M = \frac{1}{D} \sum_{i=1}^{N} n_i \sum_{x=0}^{n} P_{v+x}(d_{v+x} - d_{v+x-1}) \tag{9.24}$$

FREQUENCY OF MARGIN $\leqslant M$. The contribution to the frequency by the generation system transitions can be determined by finding the expected transitions out of the margin states $\leqslant M$. In the period $h_2 h_3$ (Figure 9.7) the load is $\geqslant L_v$ and therefore for the capacity level $C_v$, the margin $\leqslant M$. The generation system may transit, during this period, to a higher capacity making the margin $> M$ or to a lower capacity without change of cumulative margin state. Therefore,

The expected transitions during

$$h_2 h_3 \text{ from margin} \leqslant M \text{ to margin} > M = d_v f_v \tag{9.25}$$

where $f_v$ = the frequency of encountering a capacity $\leqslant C_v$ or $> C_v$.

During periods $h_1 h_2$ and $h_3 h_4$, the load is $\geqslant L_{v+1}$ and the transitions to capacity level $> C_{v+1}$ can cause change of cumulative margin state. Therefore,

The expected transitions during

$$h_1 h_2 \text{ and } h_3 h_4 \text{ from margin} \leqslant M \text{ to margin} > M = (d_{v+1} - d_v) f_{v+1} \tag{9.26}$$



Generalizing from (9.25) and (9.26), the expected transitions due to all the capacity levels, in the $i$th load cycle are

$$\sum_{x=0}^{n} f_{v+x}(d_{v+x} - d_{v+x-1}) \tag{9.27}$$

The expected generation system transitions out of margin $\leqslant M$ during $D$ days are

$$\sum_{i=1}^{N} n_i \sum_{x=0}^{n} f_{v+x}(d_{v+x} - d_{v+x-1}) = f_M^g D \tag{9.28}$$

From (9.28)

$$f_M^g = \frac{1}{D} \sum_{i=1}^{N} n_i \sum_{x=0}^{n} f_{v+x}(d_{v+x} - d_{v+x-1}) \tag{9.29}$$

The load exceeds a given capacity level once a day. Therefore,

$$f_M^l = \sum_{i=1}^{N} \frac{n_i}{D}(P_v - P_{v0})$$

$$= \frac{1}{D} \sum_{i=1}^{N} n_i P_v - P_{v0} \tag{9.30}$$

From (9.29) and (9.30)

$$f_M = f_M^g + f_M^l$$

$$= \frac{1}{D} \sum_{i=1}^{N} n_i \sum_{x=0}^{n} f_{v+x}(d_{v+x} - d_{v+x-1}) + \frac{1}{D} \sum_{i=1}^{N} n_i P_v - P_{v0} \tag{9.31}$$

It should be noted that in (9.24) and (9.29) $v$ is different for different load cycles.

EXTENSION TO THE LOSS OF ENERGY CONCEPT. The load models proposed can be readily extended to evaluate the probable curtailment of energy due to capacity shortages. Assuming the CVEF representation, the daily load variation curve is shown in Figure 9.8. The mean durations $d_j$ are stored in the computer in discrete steps and the mean duration between any two discrete steps can be computed by linear interpolation. The magnitude of load corresponding to the mean duration $d_j$ is called the subload level $L_{ij}$ where $i$ refers to peak load level.

For a capacity level $C_v$, the energy curtailed is equal to the area bounded by the expected load variation curve and the mean duration $d_v$

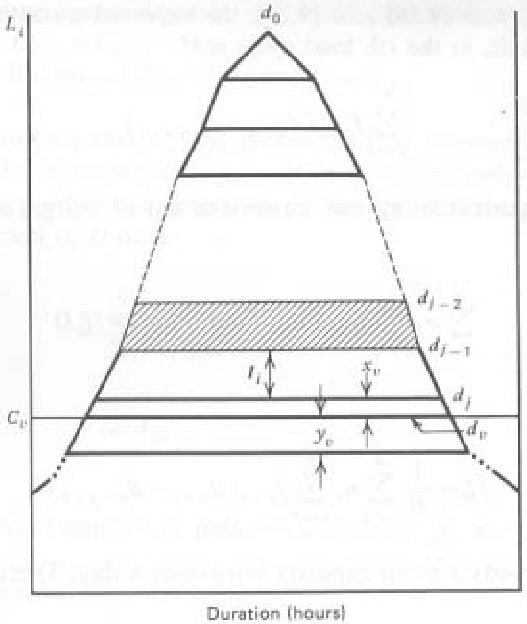


**Figure 9.8** Expected daily load variation curve for the $i$th peak.

for which the load exceeds $C_v$. The area of the shaded portion is

$$=\frac{1}{2}(d_{j-1}+d_{j-2})I_i$$

where $I_i$ = the interval in MW between the successive subload levels, for the $i$th peak load
$= L_i \cdot p/100$

where $L_i$ = the $i$th peak load, MW
$p$ = the interval between successive subload levels as a percentage of the daily peak

The energy curtailed

$$=\frac{1}{2}(d_j+d_v)x_v+\frac{1}{2}(d_j+d_{j-1})I_i+\cdots+\frac{1}{2}(d_0+d_i)I_i$$

$$=\frac{1}{2}\left[(d_v+d_j)(I_i-y_v)+(d_j+d_{j-1})I_i+\cdots+(d_0+d_1)I_i\right]$$

$$=\frac{1}{2}(d_v\cdot x_v-d_j\cdot y_v)+I_i\cdot E_j$$



where

$$E_j=\sum_{k=0}^{J} d_k$$

= the cumulative total of the subload mean durations up to $j$th sublevel such that $L_{ij} \geqslant C_v$

where $L_{ij}$ = the value of the $j$th sublevel of the $i$th peak, MW

The expected curtailment of energy, given $C_v$, $L_i$

$$=P_v'\cdot\left[\frac{1}{2}(d_v\cdot x_v-d_j\cdot y_v)+I_i\cdot E_j\right]$$

The total expected curtailment of energy in a period of $D$ days

$$EN=\sum_{i=1}^{N} n_i\sum_{v=1}^{R} P_v'\left[\frac{1}{2}(d_v\cdot x_v-d_j\cdot y_v)+I_i\cdot E_j\right] \quad (9.32)$$

where $P_v'$ = the exact state availability of capacity level $C_v$
$n_i$ = number of occurrences of peak $L_i$, $1=1,2\ldots N$

such that

$$D=\sum_{i=1}^{N} n_i$$

and $L_{ij} \geqslant C_v$.

The expression (9.32) is suitable for digital computation and its execution is very fast. The value of $EN$ given by this expression may be multiplied by a suitable cost factor in \$/MW-Hr to get the expected loss in dollars. To act as an index of reliability $EN$ must be normalized by dividing it by the total energy required by the system. This quantity is given by the expression

$$ENP=\sum_{i=1}^{N} n_i\left[I_i\left(\frac{1}{2}d_n+E_{n-1}\right)+24(L_i-n\cdot I_i)\right] \quad (9.33)$$

where $n$ = the sublevel just at the low load. This quantity, which is rather small, may be subtracted from unity to get what is conventionally called the Energy Index of Reliability. Thus

$$EIR=1.0-EN/ENP \quad (9.34)$$

SEQUENTIAL CORRELATION OF DAILY PEAKS. A number of models have been proposed and analyzed in references 7 and 18. The essential difference between the various load models is the assumptions regarding low load and the sequential correlation of daily peaks. The analysis leads to an interesting conclusion that, if the low load can be considered of constant magnitude, the numerical values of the probability and frequency of margin states are the same whether the sequence of peak is assumed random or correlated. For most systems, the probability of having capacity as low as the low load period is very small and therefore the error in the numerical indices of reliability because of the assumption of constant low load is insignificant. The model discussed here is thus adequate in most situations of interest. If, however, low load cannot be assumed constant, the numerical indices are effected and appropriate load models [7, 18] can be employed.

## 9.3 ASSESSMENT OF OPERATING RESERVE

The operating reserve evaluation is concerned with the ability of the generation system to meet the load within the next few hours. If a generating unit fails, additional capacity can be brought in after a time equal to the start up time of the reserve units. This time known as lead time, delay time or start up time is different for different types of units. It is of the order of a few minutes for hydraulic, gas turbines; for the thermal units on cold standby it may be 4–24 hours. One way of reducing this time is to keep the boilers banked; these units are called hot reserve units. The reserve connected to bus ready to take load is called spinning reserve. This spinning reserve together with the rapid start and hot reserve units is called operating reserve. The basic problem is to decide how much reserve to have so that the load can be satisfied with reasonable level of risk. Three methods have been proposed for the assessment of the operating reserve and an excellent review of these is provided in reference 15. These methods are briefly discussed in this section.

### 9.3.1 *Basic PJM Method*

This method was first described in 1963 by a group associated with the Pennsylvania–New Jersey–Maryland interconnection [2]. The index computed is the probability of having insufficient capacity in operation at a future time equal to the time needed to bring in additional generating capacity. It is assumed that there is enough installed capacity and it is just a matter of time before the additional capacity can be brought in to share the load. The present state of the system is assumed known and the start up time of all the stand-by units is considered the same. The procedure for computation is basically similar to the static reserve evaluation. The essential difference is the time.



*Generation System Model.* The concepts will be illustrated using a two state unit. There is, however, no additional difficulty in including units with derated states. The probability of a two-state unit being down at time $T$, given that it is operating at $t=0$ is [23]

$$p_d(T)=\frac{\lambda}{\lambda+\mu}\left[1-e^{-(\lambda+\mu)T}\right] \tag{9.35}$$

where $\lambda$, $\mu$ are the failure and repair rates of the unit.

If $(\lambda+\mu)T\ll 1$, then (9.35) can be approximated as

$$p_d(T)\simeq\frac{\lambda}{\lambda+\mu}\cdot(\lambda+\mu)T$$

$$=\lambda T \tag{9.36}$$

The expression (9.36) can also be obtained by assuming that there is no repair in $(0, T)$ and that $T$ is small. If $T$ is the start up time of additional capacity, then $\lambda T$ is the probability of loosing capacity and not being able to replace it and is called ORR, that is, the outage replacement rate. After computing the ORRs for the units scheduled at time $t=0$, the probability of having various levels of capacity outage at $T$ can be calculated either by the product rule or more generally by the algorithm described for the static reserve evaluation.

*Load Model and Risk Calculation.* The load model for the operating reserve evaluation is the forecast load at $T$. The risk, that is, the probability of having insufficient capacity at $T$, is

$$R=\sum_i Pr(\text{load at } T=L_i)\, Pr(\text{capacity at } T<L_i) \tag{9.37}$$

where $L_i$ is the value of forecast load. The continuous distribution of forecast values is approximated by a discrete distribution. If no uncertainty in forecast load is assumed, then there is only one value of $L_i$ and

$$R=Pr(\text{capacity at } T<\text{load at } T) \tag{9.38}$$

The computed $R$ is then compared with a maximum tolerable risk $R_{ref}$ to decide whether the system is adequately secure. If the computed risk is more or less than the reference value, suitable action can then be taken. At present the selection of $R_{ref}$ is not simple and is generally based on judgment and past experience.

### 9.3.2 *Modified PJM Method*

The basic PJM method can be modified to incorporate the effect of rapid start and hot reserve units [10]. The models for the rapid start and hot

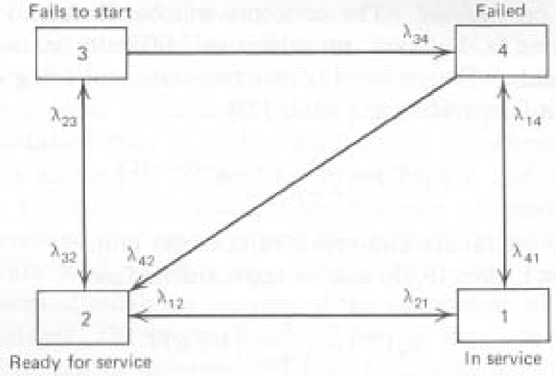


**Figure 9.9** A 4-state model for rapid start units.

reserve units are shown in Figures 9.9 and 9.10. The state transition diagrams are self-explanatory and the transition rates are given by

$$\lambda_{ij} = \frac{n_{ij}}{T_i} \tag{9.39}$$

where $\lambda_{ij}$ = transition rate from state $i$ to state $j$
$n_{ij}$ = number of transitions from state $i$ to state $j$ during the time $T_i$ spent in state $i$

Assuming the times in the various states to be exponentially distributed, the state differential equations can be written as

$$\dot{P}_i(t) = \sum_{j \neq i} P_j(t)\lambda_{ji} - P_i(t) \sum_{j \neq i} \lambda_{ij} \tag{9.40}$$

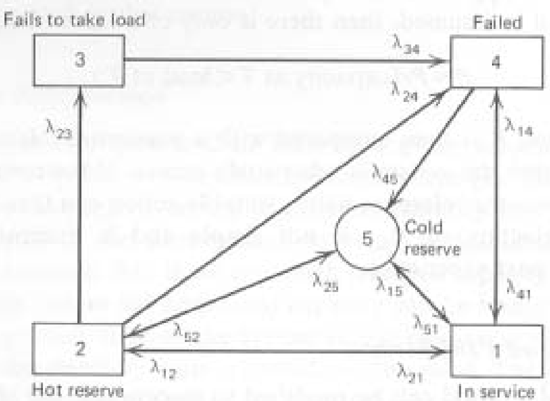


**Figure 9.10** A 5-state model for hot reserve units.



The set of differential equations (9.40) can be solved using the initial conditions to calculate the probabilities of being in various states. Denoting the start up times of rapid start, hot reserve, and cold reserve units by $t_r$, $t_h$, and $t_c$ respectively, the modified PJM method proceeds in the following steps:

1. Using the scheduled units, that is, the units on line at $t=0$, the probability of having insufficient generation at $t_r$ is computed using the basic PJM approach, that is,

   $R(t_r)$ = risk during the interval $(0, t_r)$

   = probability of having insufficient generation at $t_r$

2. During $(0, t_r)$ the rapid start units are assumed to be in ready for service with probability of unity. Using the differential equations (9.40) and initial condition, $P_2(t_r^+) = 1 \cdot 0$, the probabilities of finding the rapid start units in states 3 and 4 at time $t_h$ are determined. Denoting these probabilities by $P_{23}$ and $P_{24}$, the probability of having the rapid start unit failed at $t_h$ is $(P_{23}+P_{24})$. These probabilities of the rapid start units are combined with the probabilities of scheduled units at $t_h$ to develop the generation model at $t_h$. The generation model at $t_h$ consists of the units in operation at $t=0$ and the rapid start units that are assumed to become available at $t=t_r^+$. This generation model is combined with the load at $t_h$ to find the probability

   $R(t_h)$ = probability of having insufficient generation at $t_h$

   Also $R(t_r^+)$ is computed by considering all the units in operation at $t=0$ and using their probabilities along with the probabilities of the rapid start units at $t=t_r^+$, which is, in fact, zero time for these units.

3. At time $t_h^+$, the hot reserve units are assumed to become available. The probability of insufficient capacity at $t_h^+$ is calculated by modifying the generation model at $t_h$ by combining with the probabilities of hot reserve units at $t_h^+$ which is, in effect, zero time for these units since in the interval $(0, t_h)$, the hot reserve units are assumed in the hot reserve state with a probability of unity. The probabilities of hot reserve units at $t_c$ are computed with $P_2(t_h^+) = 1.0$ and using differential equations or matrix multiplication technique [23]. The generation system at $t_c$, therefore, consists of (a) units in operation at $t=0$, having operated for $t_c$, (b) rapid start units having operated for $(t_c - t_r)$ and (c) the hot reserve units having operated for $(t_c - t_h)$. This generation model is then combined with the load at $t_c$ to find the probability of insufficient capacity at $t_c$, that is, $R(t_c)$. The risk at $t_c$ according to reference 10 is

given by

$$R = R(t_r) + [R(t_h) - R(t_r^+)] + [R(t_c) - R(t_h^+)] \tag{9.41}$$

The risk given by (9.41) is used as an index in the modified PJM method. It appears, however, difficult to assign a physical significance to this index. Also it appears that reference 10 does not incorporate the models for rapid start and hot reserve units in a realistic manner. Denoting $P_{ij}(t)$ as the probability of being in state $j$ starting in state $i$, reference 10 appears to compute the probability of a rapid start unit as

$$P_d(t) = P_{23}(t) + P_{24}(t) \tag{9.42}$$

and

$$P_u(t) = 1 - P_d(t) \tag{9.43}$$

where $P_u(t)$, $P_d(t)$ are the probabilities of the rapid start units being up and down, respectively, at time $t$. The initial condition assumed is $P_2(0) = 1.0$, which does not reflect the fact that at $t=0$, the unit was commanded to start. Reference 1 suggests an improvement in the procedure for incorporating the effect of rapid start and hot reserve units. When a rapid start unit, for example is commanded to start, it either starts (state 1) or fails to do so (state 3). Denoting the probability of starting or failing to start by $s$ and $f$, respectively,

$$s = \frac{\lambda_{21}}{\lambda_{21} + \lambda_{23}} \tag{9.44}$$

$$f = \frac{\lambda_{23}}{\lambda_{21} + \lambda_{23}} \tag{9.45}$$

The probabilities of various states are now computed with the probabilities of starting in states 2 and 3 as $s$ and $f$, respectively. After computing $P_{ij}(t)$, the probability of the unit being down and up given the period of need are computed as

$$P_{\text{out}} = \frac{P_{23}(t) + P_{24}(t)}{P_N} \tag{9.46}$$

and

$$P_{\text{in}} = \frac{P_{21}(t)}{P_N} \tag{9.47}$$

where

$$P_N = P_{21}(t) + P_{23}(t) + P_{24}(t)$$



The probabilities calculated by (9.46) and (9.47) are then used in the risk computation instead of $P_d(t)$ and $P_u(t)$ given by (9.42) and (9.43).

### 9.3.3 Security Function Method

The security function method was proposed in reference 14 and later modified and expanded in several publications [15]. Basically this method calculates the probability of system trouble as a function of time. The time span of computation is the lead time required for the modification of the system operating configuration to achieve improved system security. The form of security function suggested in reference 14 is

$$S(t) = \sum_i P_i(t) W_i(t) \tag{9.48}$$

where $P_i(t)$ = probability of the system being in state $i$ at time $t$
$W_i(t)$ = probability that the system configuration of state $i$ results in system trouble

Equation 9.48 in its general form can be applied to the entire set of components comprising a bulk power system. When applied to the operating reserve problem $S(t)$ indicates the probability of insufficient capacity at time $t$ into future. The function $S(t)$ is examined for a time period equal to the lead time, that is, the time to start and synchronize additional capacity. If the security function is exceeding a predefined reference value, then a decision to start additional capacity can be taken. Likewise if the system appears too secure, appropriate generating capacity may be taken out for economic operation. This method treats the standby generators in a rational manner and in conformity with the normal operating practices. In the modified PJM method, the standby generators are started only when a scheduled unit fails. The amount of standby generators are shut down when the scheduled unit has been repaired. In contrast to the modified PJM method, there is no difficulty in interpreting the security function $S(t)$. It should be noted that when only the generating system is considered and when $t$ is the shortest start up time for stand-by generators then the risk obtained by the security function method is the same as the basic PJM method.

### 9.3.4 Frequency and Fractional Duration Method

The frequency and duration method for short-term reliability calculation originated in reference 18 and is also described in reference 22. The previously described methods calculate the pointwise probability of capacity deficiency. Although the security function method calculates $S(t)$ over the entire period, the total interval is not considered at a time. The frequency and duration method in addition to the pointwise probability of generation deficiency, also calculates two additional interval related indices, interval frequency, and fractional duration.

*Basic Concepts.* The entire sample space $X$ can be partitioned into disjoint subsets $X^+$ and $X^-$. Whenever the system enters any state contained in $X^+$, this subset of states is said to have been encountered. The following indices can now be defined.

TIME SPECIFIC PROBABILITY OF $X^+$. This is the probability of the system being in any state contained in $X^+$ at time $t$,

$$P_+(t) = \sum_{i\in X^+} P_i(t) \tag{9.49}$$

where $P_i(t)$ is the probability of being in state $i$ at time $t$. When $X^+$ is constituted by states indicating system trouble, (9.49) becomes identical with (9.48).

FRACTIONAL DURATION. The fractional duration of $X^+$ in the interval $(t_1, t_2)$ is defined as the expected proportion of $(t_1, t_2)$ spent in $X^+$. Denoting fractional duration by $D_+(t_1, t_2)$,

$$D_+(t_1,t_2) = \frac{1}{t_2-t_1}\sum_{i\in X^+}\int_{t_1}^{t_2} P_i(t)\,dt$$

$$= \frac{\int_{t_1}^{t_2} P_+(t)}{t_2-t_1} \tag{9.50}$$

INTERVAL FREQUENCY. The interval frequency $F_+(t_1, t_2)$ is defined as the expected number of encounters of $X^+$ in $(t_1, t_2)$.

$$F_+(t_1,t_2) = \sum_{i\in X^-}\int_{t_1}^{t_2} P_i(t)\sum_{j\in X^+}\lambda_{ij}\,dt \tag{9.51}$$

where $\lambda_{ij}$ is the constant transition rate from state $i$ to state $j$.

*Application to Operating Reserve.* The relationships (9.49)–(9.51) are general and can be applied to the entire system or parts thereof. The application of these concepts to operating reserve evaluation involves the following two basic steps.

GENERATION SYSTEM MODEL. The generation system model depicts the time-specific probability, the fractional duration and the interval frequency as functions of the cumulative capacity outages, that is, capacity outages equal to or greater than specific values. The numerical techniques for developing generation system model are described in references 18 and 22.



GENERATION RESERVE MODEL. The load is assumed to be forecast with probability one and to stay constant over the hourly intervals. If, however, the load is forecast with a certain probability distribution, there is no additional difficulty in incorporating this and also if a closer representation is required, the intervals over which the load is assumed constant can be made as small as desired. Since the load is assumed to exist at a certain number of discrete levels and as the capacity states are also discrete, the operating or generation reserve which is capacity minus the load would also exist in discrete levels. This can be illustrated by assuming the load for four hours as

| Hour | 0–1 | 1–2 | 2–3 | 3–4 |
|---|---|---|---|---|
| Load | 20 | 40 | 50 | 60 |

The hours will now be indicated by interval numbers, for example, 0–1 will be denoted by interval #1. This forecast load is combined with the generation model, the resulting generation reserve will be as shown in Table 9.1. The boundary of any cumulative margin, that is, a margin equal to or less than a specified value can now be drawn. The boundary of capacity deficient states, for example, is shown in Table 9.1. If the interval frequency of encountering capacity deficiency is to be determined, then $F_+(0,4)$ will be determined where $X^+$ will contain all the states below the thick line.

In general, denoting the capacity associated with the $i$th cumulative capacity outage state by $C_i$, the boundary for cumulative reserve margin $M$, that is, a margin equal to or less than $M$ MW, corresponding to the load during the $j$th interval $L_j$ is fixed by the relationship

$$C_i - L_j \leqslant M$$

That is,

$$C_i \leqslant L_j + M \tag{9.52}$$

**Table 9.1** *The Generation Reserve Model of the Example*

| | Interval # | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Load | 20 | 40 | 50 | 60 |
| Cumulative Capacity Outage | | | | |
| 0 | 55 | 35 | 25 | 15 |
| 25 | 30 | 10 | 0 | −10 |
| 50 | 5 | −15 | −25 | −35 |
| 75 | −20 | −40 | −50 | −60 |

The expressions for the different indices can be written using the following notation:

$T$ = the lead time
$P_i(t)$ = the probability of the $i$th cumulative capacity outage state at time $t$
$D_i(t_1, t_2)$,
$F_i(t_1, t_2)$ = the fractional duration and the interval frequency of encountering the $i$th cumulative capacity outage state in the interval $(t_1, t_2)$
$P_M(T)$ = the probability of the cumulative margin $M$ at the end of lead time
$D_M(0, T)$,
$F_M(0, T)$ = the fractional duration and the interval frequency of encountering the cumulative margin $M$ in the interval $(0, T)$, that is, during the lead time.

*Probability.* The expression for time specific probability of the cumulative margin $M$ is straightforward:

$$P_M(T) = P_i(T) \tag{9.53}$$

such that

$$C_i \leqslant L(T) + M$$

where $L(T)$ = the load at time $T$.

*The Fractional Duration.* Denoting the time at the end of $j$th interval by $t_j$,

$$D_M(0, T) = \sum_{j=1}^{m} D_i(t_{j-1}, t_j) \tag{9.54}$$

where $m$ is the total number of intervals in the lead time $T$, and the cumulative capacity outage state $i$ during the interval $j$ is determined using (9.52).

*The Interval Frequency.* The state of margin equal to or less than $M$ may be encountered either due to a decrease in capacity or increase in load. There are, therefore, two components of interval frequency, the generation system transition $F_M^g(0, T)$ and the load transitions, $F_M^l(0, T)$ such that

$$F_M(0, T) = F_M^g(0, T) + F_M^l(0, T)$$



Now

$$F_M^g(0, T) = \sum_{j=1}^{m} F_i(t_{j-1}, t_j)$$

and

$$F_M^l(0, T) = \sum_{j=1}^{m} \left[ P_k(t_j) - P_i(t_j) \right]$$

such that $C_i \leqslant M + L_j$ at the end of the $j$th interval and

$C_k \leqslant M + L_j'$ at the beginning of the $(j+1)$th interval.

Finally,

$$F_M(0, T) = \sum_{j=1}^{m} F_i(t_{j-1}, t_j) + \sum_{j=1}^{m} \beta\left[ P_k(t_j) - P_i(t_j) \right] \tag{9.55}$$

where $\beta = 1$ if $[P_k(t_j) - P_i(t_j)]$ is positive
$= 0$ otherwise

Expressions 9.53–9.55 can be used to determine the three indices for cumulative margin $M$. If $M$ is such that it defines the capacity deficiency states the three indices are the three risk indices.

## 9.4 INTERCONNECTED SYSTEMS

The interconnection of a power system to one or more power systems generally improves the generating capacity reliability. When the power system suffers a loss of load, assistance is generally available from the systems to which it is interconnected. The benefits of interconnection result from the diversity in the occurrence of the peak loads and the outages of capacity in different systems.

In evaluating the reliability of interconnected systems, the load and generation in each system are assumed to be connected to a common bus and the tie lines are assumed to connect these buses together. This is shown schematically in Figure 9.11. This means that within each system, the transmission system is assumed capable of transferring the available generation to the points of demand. Also when needed generation is made available to a system from a neighboring system, it is assumed that the intratransmission system can then properly distribute this capacity.

The type of agreement with the assisting systems effects the evaluation of the reliability of an interconnected system. This discussion assumes a

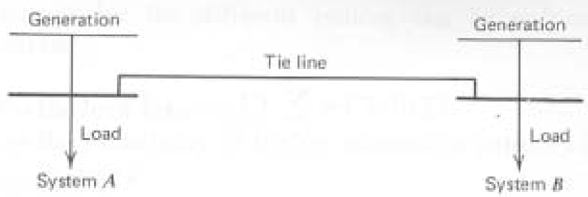


**Figure 9.11** Schematic diagram of system $A$ connected to system $B$.

basic agreement that one system helps the other as much as it can without curtailing its own load. The concept may, however, be easily extended to cover other agreements. The discussion in this section is limited to a system $A$ connected to system $B$, which is generally called a two area problem. For a detailed discussion of this problem and that on multi-area problems, the reader is referred to references 17, 18, and 3–5. The methods of reliability analysis discussed in this section were developed in references 17 and 18 and later described in references 3–5.

### 9.4.1 *Independent Load Models*

The relationships for the probability and frequency of negative margins (load loss) in system $A$ connected to system $B$ are developed assuming the generation and load models in the two systems to be stochastically independent. Assuming the capacity and load in each system to exist at discrete levels, the margin state, which is capacity available less the load on the system, would also exist at discrete levels in each system. In Figure 9.12, $M_a$, $M_b$ contain the margin states in systems $A$ and $B$, respectively,

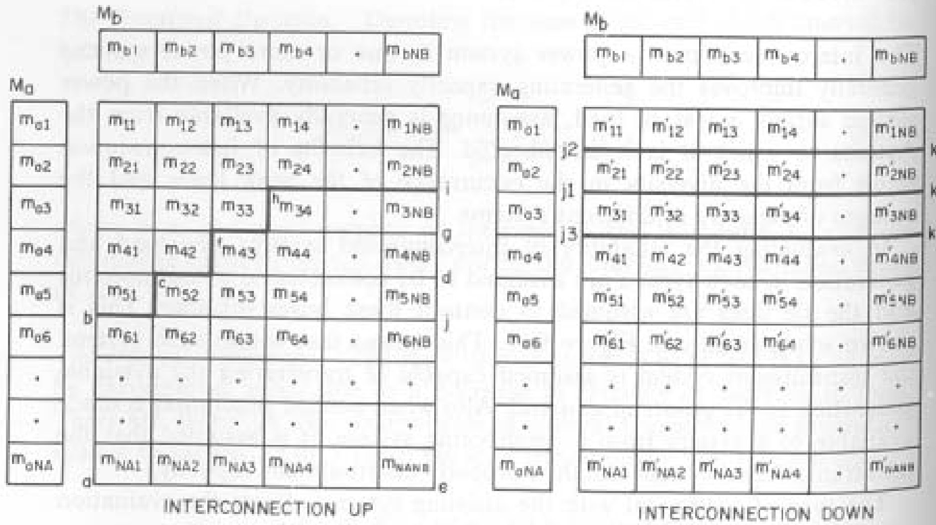


**Figure 9.12** Effective margin states matrices for system $A$ connected to system $B$.



without including the effect of interconnection assistance. These states are arranged in the order of decreasing reserve, that is,

$$m_{a1} > m_{a2} > \cdots > m_{ai} > \cdots > m_{aNa}$$

and

$$m_{b1} > m_{b2} > \cdots > m_{bj} > \cdots > m_{bNB}$$

The effective margin states in system $A$, that is, when the tie line is in operation, are given by the elements of matrix $M$,

$$m_{ij} = m_{ai} + h_{ij} \tag{9.56}$$

where $h_{ij}$ is either the help available to system $A$ from system $B$ or it is the help required by system $B$ from system $A$. In the latter case $h_{ij}$ has a negative sign. If no assistance is possible from one system to the other, $h_{ij} = 0$. The maximum of $h_{ij}$ is limited by the tie line capability. The effective margin states in system $A$ while the tie line is out are given by the elements of $M'$,

$$m'_{ij} = m_{ai} \tag{9.57}$$

Equations 9.56 and 9.57 define the boundaries in $M$ and $M'$, respectively, of any effective cumulative margin. In this discussion, $m$ with proper subscript represents an exact margin and $M$ with the same subscript denotes the corresponding cumulative margin, for example, $M_{ij}$ means a margin equal to or less than $m_{ij}$. The probability and frequency equations for the negative cumulative margin are derived and for the equations for any margin, positive or negative, the reader is referred to reference 3.

*Probability of Margin Equal or Less Than* N. The probability of a margin equal to or less than $N$ is simply the sum of the probabilities of the margins states comprising this cumulative state. The equation for this probability is easily seen to be

$$P_N = \sum_{l,k} \left[ P_{a(l)} - P_{a(l+1)} \right] \left( A_{ab} P_{b(k)} + \bar{A}_{ab} \right) \tag{9.58}$$

where $P_{a(l)}, P_{b(k)}$ = the probabilities of cumulative margins $M_{a(l)}$ and $M_{b(k)}$, respectively

$A_{ab}$ = the availability of the tie line between $A$ and $B$

$\bar{A}_{ab} = 1 - A_{ab}$

$l, k$ = the indices defining the boundary of the margins less than or equal to $N$. The index $l$ indicates the margin in array $M_a$ and $k$ indicates the corresponding margin in array $M_a$ to give an effective margin $\leq N$. For example in $M_b$ to give an effective margin $\leq N$. For example the boundary in Figure 9.12 is identified by $(NA, 1)$, $(NA-1, 1), \ldots, (6,1), (5,2), (4,3)$, and $(3,4)$.

Equation (9.58) can be easily seen to be an application of conditional probability theorem [23].

*Frequency*. Define

$f_{a(l)}, f_{b(k)}$ = the frequencies of encountering the cumulative margin states $M_{a(l)}$ and $M_{b(k)}$, respectively

$\lambda_{ab}, \mu_{ab}$ = the mean failure and repair rates of the tie line

and

$f_N$ = the frequency of encountering an effective margin in system $A$, equal or less than $N$

System $A$ can transit from one effective margin state to another in any of the following ways.

1. *Capacity or load transitions in System A*. System $A$ will shift vertically in $M$ when the interconnection is in operation and in $M'$ when the interconnection is on forced outage.
2. *Capacity or load transitions in System B*. Due to transitions in system $B$, system $A$ will transfer horizontally from one effective state to another in the matrix $M$, when the interconnection is up. With the interconnection in the down state, the system $A$ will transfer horizontally in the matrix $M'$. These latter transitions do not ultimately reflect into the effective operation of system $A$.
3. *Failure or repair of the interconnection*. When the interconnection fails or is repaired, system $A$ will transit from a state in $M'$ to the corresponding state in $M$ and vice versa.

The frequency of encountering a cumulative margin in system $A$ equals the expected transitions per unit time across the boundary defining that cumulative margin plus the transitions per unit time associated with the boundary states. The boundary states are a subset of the cumulative margin when the interconnection is down but leave this set as a result of repair of the interconnection.

Summing the frequency due to the three modes of transition gives the following relationship.

$$f_N = \sum_{l,k} \{ [f_{a(l)} - f_{a(l+1)}][A_{ab}P_{b(k)} + \bar{A}_{ab}] + [P_{a(l)} - P_{a(l+1)}](f_{b(k)} + [1 - P_{b(k)}]\lambda_{ab})A_{ab} \} \tag{9.59}$$



Equation 9.59 can be easily derived by the application of the conditional frequency formula [23],

$$f_N = Fr(N/TL\ \text{Up})A_{ab} + Fr(N/TL\ \text{Down})A_{ab} + [P(N/TL\ \text{Down}) - P(N/TL\ \text{Up})]A_{ab}\lambda_{ab} \tag{9.60}$$

where

$Fr(\cdot), P(\cdot)$ are the frequency and probability of $(\cdot)$.

$TL$ = Tie line

Now

$$Fr(N/TL\ \text{up}) = \sum_{l,k} \{ [f_{a(l)} - f_{a(l+1)}]P_{b(k)} + [P_{a(l)} - P_{a(l+1)}]f_{b(k)} \} \tag{9.61}$$

$$Fr(N/TL\ \text{down}) = \sum_{l,k} [f_{a(l)} - f_{a(l+1)}] \tag{9.62}$$

and

$$P(N/TL\ \text{Down}) - P(N/TL\ \text{Up}) = \sum_{l,k} (1 - P_{b(k)}) \tag{9.63}$$

It can be easily seen that substitution of (9.61)–(9.63) into (9.60) will yield (9.59).

### 9.4.2 Correlated Load Models

In the discussion in Section 9.4.1., the load models in the two systems are assumed statistically independent. It is, however, more likely that the loads in the two interconnected systems will bear a correlation. This section develops the relationships for the probability and frequency of a cumulative margin, assuming the loads in the two systems to be perfectly correlated.

Let

$(L_{ax}, L_{bx})$ = perfectly correlated load levels in systems $A$ and $B$ where $x = 1, 2, \ldots, n$

$M_a^x, M_b^x$ = the reserve margin state arrays of systems $A$ and $B$ corresponding to the load condition $(L_{ax}, L_{bx})$

The margin state arrays $M_a^x$ and $M_b^x$ can be obtained by subtracting loads $L_{ax}$ and $L_{bx}$ from the capacity states of the systems $A$ and $B$, respectively. The arrays $M_a^x$ and $M_b^x$ are shown in Figure 9.13 where the margin states are arranged in the decreasing order of magnitude.


**Figure 9.13** Effective margin states matrices for system $A$ connected to system $B$.

As before $c$ and $m^x$ with proper subscripts represent exact capacity and conditional exact margin state and $C$ and $M^x$ with the same subscript denote the corresponding cumulative states. For example, $C_{ai}$ means a capacity equal to or less than $c_{ai}$, and similarly $M^x_{ij}$ means a margin equal to or less than $m^x_{ij}$. The effective margin states in system $A$ given the load condition $(L_{ax}, L_{bx})$ and the interconnection in the up state are given by



the elements of matrix $M^x$ of Figure 9.13,

$$m^x_{ij} = m^x_{ai} + h^x_{ij} \tag{9.64}$$

where $h^x_{ij}$ is either the help available to system $A$ from system $B$ or it is the help required by system $B$ from system $A$, given the load condition $(L_{ax}, L_{bx})$.

The effective margin states in system $A$ given the load condition $(L_{ax}, L_{bx})$ and with the interconnection down are given by the elements of matrix $M^{1x}$

$$m^{1x}_{ij} = m^x_{ai} \tag{9.65}$$

Equations 9.64 and 9.65 define the boundaries of effective cumulative margin states in $M^x$ and $M^{1x}$, respectively. The effective margin states in system $A$, given the low load condition $(L_{ao}, L_{bo})$ and the interconnection in the up state are given by the matrix $M^o$ (see Figure 9.13), and the effective margin states for the above condition with the interconnection in the down state are given by $M^{1o}$.

*Probability of Margin Equal or Less Than* N. Using (9.58)

$$P_{(N/x)} = \sum_{lx,\,kx} \left[ P_{a(l)} - P_{a(l+1)} \right]\left[ A_{ab} P_{b(k)} + \bar{A}_{ab} \right] \tag{9.66}$$

where $P_{(N/x)}$ = probability of an effective margin $\leq N$, given the load condition $(L_{ax}, L_{bx})$

$lx, kx$ = the indices defining the boundary of $N$ in $M^x$, for example, if the boundary is "$abcfhie$", the indices are $(NA,1)-(6,1)$, $(5,2)$, $(4,3)$, and $(3,4)$ (see Figure 9.13)

$P_{a(l)}, P_{b(k)}$ = probabilities of $M^x_{al}$ and $M^x_{bk}$, given the load condition $(L_{ax}, L_{bx})$

Since

$$m^x_{al} = c_{al} - L_{ax}$$

and

$$m^x_{bk} = c_{bk} - L_{bx}$$

it can be seen that $P_{a(l)}, P_{b(k)}$ are probabilities of $C_{al}$ and $C_{bk}$, respectively.

Using conditional probability, for $n$ load levels and low load level $(x=0)$,

$$A_N = \sum_{x=0}^{n} A_x P_{N/x} \tag{9.67}$$

where $A_x$ = probability of load condition $(L_{ax}, L_{bx})$.

*Frequency.* The two different modes of state transition in system $A$ for a given load condition $(L_{ax}, L_{bx})$ are

1. The generation system transitions in system $A$ or system $B$.
2. The state transition due to the failure or the repair of the tie line.

The frequency of encountering any cumulative effective margin in system $A$ due to these two modes can be determined using (9.59),

$$f_{(N/x)} = \sum_{lx,\,kx} \left\{ \left(f_{a(l)} - f_{a(l+1)}\right)\left[A_{ab}\cdot P_{b(k)} + \bar{A}_{ab}\right] + \left[P_{a(l)} - P_{a(l+1)}\right]\left[f_{b(k)} + \left[1 - P_{b(k)}\right]\lambda_{ab}\right]A_{ab}\right\} \quad (9.68)$$

where $f_{(N/x)}$ = the frequency of encountering an effective margin $< N$, given the load condition $(L_{ax}, L_{bx})$

$f_{a(l)}, f_{b(k)}$ = the frequencies of encountering $M_{al}^{x}$ and $M_{bk}^{x}$, respectively, given the load condition $(L_{ax}, L_{bx})$
= the frequencies of encountering $C_{al}$ and $C_{bk}$, respectively

The frequency of encountering the effective margin $< N$ with the load condition $(L_{ax}, L_{bx})$ is

$$f_N^x = A_x \cdot f_{(N/x)}$$

The frequency due to the two modes listed previously is given by the summation over all the load conditions, that is,

$$f_{CN} = \sum_{x=1}^{n} f_N^x + f_N^o$$

This $f_{CN}$ represents the contribution to $f_N$ (the frequency of encountering the margin $< N$) due to the generation transitions and also takes into account transitions due to failure or repair of the interconnection. It does not, however, take into account the contribution due to the load transitions.

THE CONTRIBUTION DUE TO THE LOAD TRANSITIONS. The peak loads are assumed to be followed by the low load period. Thus the system can transit from a load condition $(L_{ai}, L_{bi})$ to $(L_{ao}, L_{bo})$ and again to some load condition $(L_{aj}, L_{bj})$. It should be kept in mind that there are no interpeak transitions, that is, the system cannot transit directly from $(L_{ai}, L_{bi})$ to $(L_{aj}, L_{bj})$. The contribution to $f_N$ will thus result from the transition of system $A$ from a given load condition to the low load condition and vice versa.



Let the boundary of $N$ be represented by "$a\,b\,c\,f\,h\,i\,e$" and "$j\,l\,k\,l$" in $M^x$ and $M^{1x}$, respectively. The corresponding boundary in $M^o$ and $M^{1o}$ may be represented by "$q\,r\,s\,t\,u\,v\,w$" and "$p\,l\,q\,l$". The boundary states as a result of the load transitions are those states included in the boundaries "$a\,b\,c\,f\,h\,i\,e$" and "$j\,l\,k\,l$" but not in "$q\,r\,s\,t\,u\,v\,w$" and "$p\,l\,q\,l$". Let these states be represented by a set $S$. The contribution due to the transition from load condition $(L_{ax}, L_{bx})$ to the low load condition $(L_{ao}, L_{bo})$ are given by

$$f_x = \sum_{s\in S} A_{(s/x)}\cdot A_x \cdot \lambda_{xo}$$

where $A_{(s/x)}$ = the probability of a boundary state $s\in S$, given the load condition $(L_{ax}, L_{bx})$

$\lambda_{xo}$ = the transition rate from the load condition $(L_{ax}, L_{bx})$ to $(L_{ao}, L_{bo})$

It can be seen that

$$\sum_{s\in S} A_{(s/x)} = A_{(N/x)} - A_{(N/o)}$$

Therefore,

$$f_x = \left[A_{(N/x)} - A_{(N/o)}\right]\cdot A_x \cdot \lambda_{xo}$$

The total contribution can be found by summation over all the load conditions, and is

$$\sum_{x=1}^{n} f_x$$

Adding this to $f_{CN}$

$$f_N = \sum_{x=1}^{n} \left\{ f_{(N/x)}\cdot A_x + \left[A_{(N/x)} - A_{(N/o)}\right]\cdot A_x \cdot \lambda_{xo}\right\} + f_N^o$$

$$= \sum_{x=1}^{n} A_x \cdot \left\{ f_{(N/x)} + \left[A_{(N/x)} - A_{(N/o)}\right]\lambda_{xo}\right\} + f_N^o \quad (9.69)$$

The step by step procedure for evaluating the reliability indices in system $A$, with correlated load models is outlined as follows.

1. The low load condition $(L_{ao}, L_{bo})$ is selected first and from each capacity state in $C_a$, $L_{ao}$ is subtracted to obtain $M_a^o$. $M_b^o$ is obtained in a similar manner. The boundary of the failure state in $M^o$ and $M^{1o}$ can be fixed using (9.64) and (9.65).

2. $A_{(N/o)}$ and $f_{(N/o)}$ are evaluated using equations (9.67) and (9.68). Here $N$ represents the effective margin $\leqslant$ the first negative margin in $M^o$ or $M^{lo}$. If the low loads in the two systems are assumed zero,

$$A_{(N/o)}=0$$

$$f_{(N/o)}=0$$

3. $A_{(n/x)}$ and $f_{(N/x)}$ for every load condition are evaluated in the same manner as outlined for the low load state.
4. The probability and the frequency of the failure state in system $A$ can be finally found using the (9.67) and (9.69). Assuming the low load level in both the systems to be zero, these equations simplify to

$$A_{-}=\sum_{x=1}^{n} A_x \cdot A_{(N/x)}$$

and

$$f_{-}=\sum_{x=1}^{n} A_x\left[f_{(N/x)}+A_{(N/x)}/e\right]$$

where $e$ = the load exposure factor.

### 9.4.3 *System Studies*

The techniques described in Sections 9.4.1. and 9.4.2. have been implemented in a computer program [21] and several studies based on this program have been reported in references 17, 18, and 3–5. A typical study, the effect of tie line capacity on risk level in system $A$, is reported here.

A system designated $A$ is assumed to be connected to an identical system $B$ by a single tie line. The mean failure and repair rates of the tie line are assumed to be 0.01 and 2.5 per day, respectively. The description of the generation system and load model in each system is provided in Table 9.2.

**Table 9.2** *Generation System*

| No. of Identical Units | Unit Size (MW) | Mean Down Time (Years) | Mean Up Time (Years) |
|---|---|---|---|
| 1 | 250 | 0.06 | 2.94 |
| 3 | 150 | ↓ | ↑ |
| 2 | 100 | | |
| 4 | 75 | | |
| 9 | 50 | | |
| 3 | 25 | | |

Total number of units = 22
Total installed capacity = 1725 MW



*Load System*
Exposure factor = 0.5 day
Period = 20 days

| Load Condition (x) | (MW, MW) | No. of Occurrences |
|---|---|---|
| 1 | (1450, 1450) | 8 |
| 2 | (1255, 1255) | 4 |
| 3 | (1155, 1155) | 4 |
| 4 | (1080, 1080) | 4 |

The low load level in both the systems was assumed to be at zero MW.

The study was carried out by varying the tie capability from 25 to 625 MW. The mean failure and repair rates of the tie were maintained at 0.01 and 2.5 per day. The results of this study are shown in Figures 9.14 and 9.15. The curves representing the correlated load models are labeled 1 and those corresponding to the independent load models by 2. With the lower

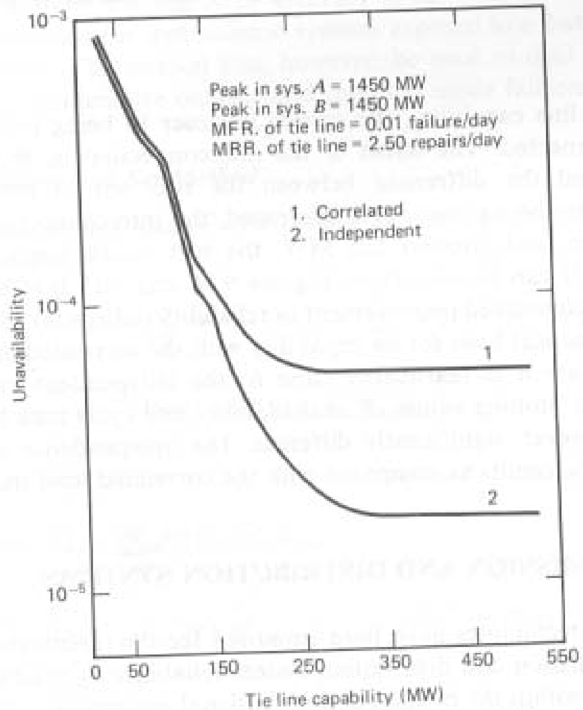


**Figure 9.14** Variation of risk level (unavailability) in system $A$ with the variation of tie line capability.

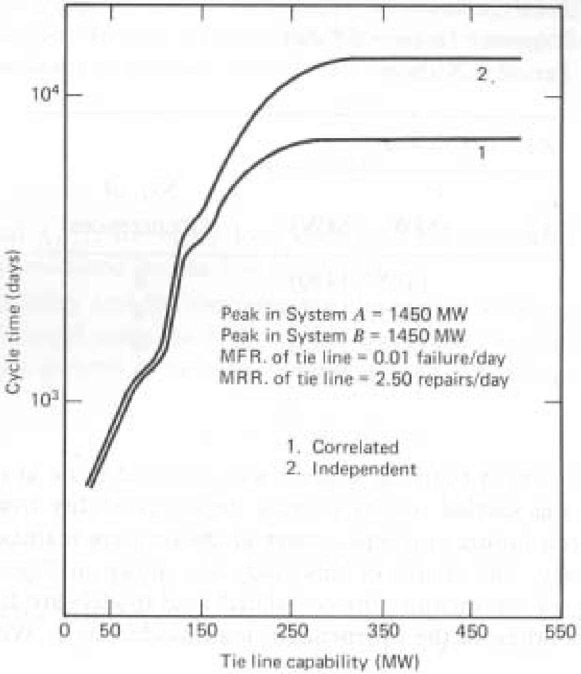


**Figure 9.15** Variation of risk level (cycle time) in system $A$ with the variation of tie line capacity.

values of tie line capability, the system is closer to being isolated rather than interconnected. The effect of the interconnection is, therefore, not significant and the difference between the two sets of results is not discernible. As the tie capacity is increased, the interconnection becomes more effective, and, around 125 MW, the two results begin to deviate significantly. It can be seen from Figures 9.14 and 9.15 that beyond 250 MW there is no marked improvement in reliability indices for curve 1. This is then the practical limit for tie capability with the correlated load models and in this case it is reasonably close to the independent load models condition. The limiting values of unavailability and cycle time for the two cases are, however, significantly different. The independence assumption gives optimistic results as compared with the correlated load models.

## 9.5 TRANSMISSION AND DISTRIBUTION SYSTEMS

A number of techniques have been proposed for the quantitative evaluation of transmission and distribution system reliability. It is now generally accepted that within the bounds of distributional assumptions, the Markov approach is the most accurate. If the fluctuating environment is not



included in the analysis, the transmission system elements can be considered independent and the probabilities and frequencies calculated directly and simply. In the case of independent components, cut set or tie set methods can also be effectively utilized. When, however, the fluctuating environment, stormy and normal weather, is considered, the statistical behavior of the components cannot be regarded independent and the solution of $2^{n+1}$ linear algebraic equations is required, where $n$ is the number of components.

When the number of components is large, the number of linear equations becomes unmanageable. Methods like state merging, state space truncation, and sequential truncation have been proposed for alleviating this problem and are described in reference 19. The most efficient method for dealing with transmission and distribution systems, involving dependent modes like the fluctuating environment is the Markov Cut Set method [27]. This method is a combination of the cut set and Markov methods. This composite approach consists in decomposing the system by cut sets and then using Markov processes and frequency balancing concepts for the calculation of the terms in the cut set expansion. The Markov process of only the cut set members is considered and, therefore, a limited number of equations need to be solved at a time. A very useful feature of this approach is that both time-specific and steady-state probabilities and frequencies of system failure can be calculated. It is also possible to control and measure the degree of accuracy of the results. The use of this method is illustrated for transmission systems exposed to a 2-state fluctuating environment. The method can, however, be used to deal with dependence due to maintenance outages and common mode failures.

### 9.5.1 *Minimal Cut Set Method*

The equations for the steady-state probability and average frequency of system failure are

$$P_f = \sum_i Pr(C_i) - \sum_{i<j} Pr(C_i \cap C_j) + \sum_{i<j<k} Pr(C_i \cap C_j \cap C_k) \cdots (-1)^{m-1} Pr(C_1 \cap C_2 \cap \cdots \cap C_m) \tag{9.70}$$

and

$$f_f = \sum_i Pr(C_i)\bar{\mu}_i - \sum_{i<j} Pr(C_i \cap C_j)\bar{\mu}_{i+j} + \sum_{i<j<k} Pr(C_i \cap C_i \cap C_k)\bar{\mu}_{i+j+k} \cdots (-1)^{m-1} Pr(C_1 \cap C_2 \cap \cdots C_m) \bar{\mu}_{1+2+\cdots+m} \tag{9.71}$$

when $P_f, f_f$ = probability and frequency that the system has failed
$C_i$ = cut set $i$ and also the event: all components of $C_i$ are failed
$m$ = number of cut sets
$Pr(C_i \cap C_j)$ = probability of the components of both $C_i$ and $C_j$ failed
$\mu_j$ = repair rate of component $j$
$\bar{\mu}_{i+k+l} = \sum \mu_j$
$j \in (C_i \cup C_k \cup C_l)$

The min cut sets can be calculated using Failure Mode and Effect Analysis and for some well-defined reliability block diagrams, specific algorithms are also available. The min cut sets are defined as sets of minimum number of components whose outage will result in loss of continuity for the system. Once the min cut sets have been determined the probability and frequency of system failure can be determined using (9.70) and (9.71). The mean duration of failure state can be determined using,

$$d_f = \frac{P_f}{f_f}. \tag{9.72}$$

For practical applications, $\lambda_j/\mu_j \ll 1$ and the upper bounds [24] to probability and frequency of failure give results very close to the exact values. These upper bounds are

$$P_{fu} = \sum_i Pr(C_i) \tag{9.73}$$

and

$$f_{fu} = \sum_i Pr(C_i)\bar{\mu}_i \tag{9.74}$$

where $P_{fu}$, $f_{fu}$ are the first upper bounds to probability and frequency of system failure.

The interval in which $P_f$ and $f_f$ lie can be determined by calculating the lower bounds as well,

$$P_{fl} = \sum_i Pr(C_i) - \sum_{i<j} Pr(C_i \cap C_j) \tag{9.75}$$

and

$$f_{fl} = \sum_i Pr(C_i)\bar{\mu}_i - \sum_{i<j} Pr(C_i \cap C_j)\bar{\mu}_{i+j} \tag{9.76}$$

Increasingly closer upper and lower bounds to $P_f$ and $f_f$ can be obtained by the successive addition of odd and even order terms [24]. It should, however, be reiterated that for almost all the practical systems, the component failure rates are much smaller than the repair rates and therefore (9.73) and (9.74) give results very close to the exact results.



Expression 9.70 for the probability of system failure is true whether or not the components are independent. Equation 9.71 for the frequency of system failure also does not depend on the independence of components so long as there is no restriction on the repair of a failed component [25]. This restrictive assumption can also be eliminated by a more general expression for $f_f$ as shown subsequently. The basic quantities to be calculated in (9.70) and (9.71) are the probabilities of cut sets and their intersections. When the components are assumed independent, these quantities can be calculated by the simple product rule. It is this problem that has restricted the application of cut set methods to the systems where components can be assumed independent. A method is proposed in the next section for calculating these probabilities when the components cannot be assumed independent and this method, therefore, significantly extends the capability of the cut set approach to the transmission and distribution systems.

### 9.5.2 The Markov Cut Set Method

This section describes a method for calculating the probabilities and frequencies of cut sets and their intersections when the components cannot be regarded statistically independent.

*Concept of Equivalent Subset.* The proposed method depends upon the relationship between a minimal cut set and the equivalent subset of the system state space. In a cut set $C_i$, having components $l$ and $m$ as members, if $l$ and $m$ fail, the system will be failed irrespective of the states of the other components of the system. The failure of the members of $C_i$ is equivalent to the system being in subset $S_i$ of the state space $S$, where

$S_i \equiv \{s_j$: in the state $s_j$, the components $l$ and $m$ are failed and the other components exist in either state$\}$

The state $s_i^v$ in which members of $C_i$ are failed and all the other components are good is called the vertex state of subset $S_i$. The system can transit from the vertex state either upward (in the sense of less components in the failed state) by the repair of the members of $C_i$ or it can transit downward (in the sense of more components failed) by the successive failures of the nonmembers of $C_i$. The subset $S_i$ consists of states generated by the downward transitions from $s_i^v$.

It can be seen from the discussion that

$$Pr(C_i) = Pr(S_i) \tag{9.77}$$

where $Pr(S_i)$ = probability of the system being in $S_i$

$$= \sum_{j: s_j \in S_i} Pr(s_j) \qquad (9.78)$$

where $Pr(s_j)$ = probability of being in system state $j$.

The frequency formula, not dependent on component independence, can also now be stated in terms of $S_i$,

$$f_f = \sum_i F(S_i) - \sum_{i<j} F(S_i \cap S_j)$$

$$+ \sum_{i<j<k} F(S_i \cap S_j \cap S_k) - \cdots$$

$$(-1)^{m-1} F(S_1 \cap S_2 \cap \cdots \cap S_m) \qquad (9.79)$$

where $F(S_i)$ = frequency of encountering subset $S_i$.

Equation 9.79 is true whether or not the components are statistically independent.

*The Method.* The problem of calculating the probabilities and frequencies of cut sets and their intersection can be transformed into that of determining these values for the corresponding equivalent subsets using (9.77)–(9.79). It is now proposed that the Markov and frequency balancing approach be used for the calculation of probabilities and frequencies of equivalent subsets.

Consider, for example, subset $S_i$ equivalent of cut set $C_i$. If $Pr(S_i)$ and $F(S_i)$ could be calculated from transition rate matrix of only those components that are members of $C_i$, this would present a big step forward. This is because in a large network, the number of elements in a minimal cut set is generally much smaller than the number of components in the whole network. The number of components to be dealt with at a time can also be kept within reasonable limits by excluding minimal cut sets beyond a specified order. As an example assume that the number of components in a system is 50. The total number of states, if the equations for the entire system are to be solved is $2^{51}$, that is, $225 \times 10^{13}$ when the 50 components are exposed to the same 2-state fluctuating environment. Now if the largest cut set to be considered is of the order 5, then only $2^{5+1} = 64$ states need to be dealt at a time using the purposed method. The solution of 64 linear equations on a modern digital computer is a trivial task.

It can be shown that in a system exposed to a 2-state fluctuating environment $Pr(S_i)$ and $F(S_i)$ can indeed be calculated by developing the transition rate matrix for the elements of $C_i$ only and neglecting other components of the system.



Let $n_a$ be the number of components comprising $C_i$ and $n_b = n - n_a$ be the remaining components, that is, components that are not members of $C_i$. Here $n$ is the total number of components. The sets of components $n_a$ and $n_b$ will be denoted by $X_a$ and $X_b$, respectively. Now $X_a$ can exist in $2^{n_a}$ number of states in each of the environmental states. The component configuration of each state in either of the two environments is the same but the interstate transition rates in the two weather states are different. For the state space corresponding to $X_a$ the transition rate from configuration $i$ to $j$ will be denoted by $\alpha_{ij}$ and $\beta_{ij}$ in the normal and adverse weather, respectively. The transition rate from a state in the normal weather condition to the corresponding state in the adverse weather condition is assumed as $w_n$ and in the reverse direction as $w_m$. The states associated with $X_a$ are indicated by $a_1, a_2, \ldots, a_p$ in the normal weather and $a_1^s, a_2^s, \ldots, a_p^s$ in the adverse weather condition $p$ being equal to $2^{n_a}$. Similarly the states generated by $X_b$, that is, components not member of $C_i$, are indicated by $b_1, b_2, \ldots, b_q$ in the normal weather condition and $b_1^s, b_2^s, \ldots, b_q^s$ in the adverse weather condition, $q$ being equal to $2^{n_b}$.

The state space of the entire system can now be generated from the state space of $X_a$ and $X_b$. When $X_a$ and $X_b$ are combined, there will be $p \times q$ states in each weather condition. This combination of state $s$ for the normal and adverse weather is shown below.

*Normal Weather State Space*

$$\begin{matrix} b_1a_1 & b_1a_2 & \cdots & b_1a_p \\ b_2a_1 & b_2a_2 & \cdots & b_2a_p \\ \vdots & \vdots & & \vdots \\ b_qa_1 & b_qa_2 & \cdots & b_qa_p \end{matrix}$$

*Adverse Weather State Space*

$$\begin{matrix} b_1^sa_1^s, & b_1^sa_2^s, \ldots, & b_1^sa_p^s \\ b_2^sa_1^s, & b_2^sa_2^s, \ldots, & b_2^sa_p^s \\ \vdots & \vdots & \vdots \\ b_q^sa_1^s, & b_q^sa_2^s, \ldots, & b_q^sa_p^s \end{matrix}$$

The transition rate from $b_ia_j$ to $b_ia_k$ is $\alpha_{jk}$, that is, the transition rate from $a_j$ to $a_k$. The transition rate from $b_ia_j$ to $b_i^sa_j^s$ is $w_n = 1/T_n$ where $T_n$ is the mean duration of the normal weather. The transition rate from $b_i^sa_j^s$ to $b_ia_j$ is likewise $w_m = 1/T_s$ where $T_s$ is the mean duration of the adverse weather.

Let the states be grouped into subsets $D_i$ and $D_i^s$ in the normal and adverse weather conditions, $i = 1, 2, \ldots, p$. These subsets are such that

$$D_i = \{b_1a_i, b_2a_i, \ldots, b_qa_i\}$$

and

$$D_i^s = \{b_1^s a_i^s, b_2^s a_i^s, \ldots, b_q^s a_i^s\}$$

The necessary and sufficient condition [23] for the states to be mergeable into subsets is that for any two subsets $D_i$, $D_j$, the transition rate from each state in subset $D_i$ to each of the states in $D_j$ when summed over all states in $D_j$ is the same for each state in $D_i$ and this equivalent transition rate from $D_i$ to $D_j$ is given by

$$\sum_{j \in D_j} \lambda_{ij}$$

When this condition of mergeability is satisfied between all the subsets taken in pairs, the Markov process of the system is said to be mergeable into these disjoint subsets. Let us apply this condition of mergeability to $D_i$ and $D_i^s$, $i = 1, \ldots, p$.

1. For any two $D_i$, $D_j$, $\Sigma_{j \in D_j} \lambda_{ij}$ is the same for each $i \in D_i$ and is equal to $\alpha_{ij}$ which is the normal weather transition rate from $a_i$ to $a_j$. Therefore the condition of mergeability is satisfied for any $D_i$, $D_j$ and the transition rate from subset $D_i$ to $D_j$ is $\alpha_{ij}$, that is, the transition rate from $a_i$ to $a_j$.
2. In a similar manner the condition of mergeability is satisfied between any two $D_i^s$ and $D_j^s$ and the transition rate from $D_i^s$ to $D_j^s$ is $\beta_{ij}$, that is, the adverse weather transition rate from $a_i^s$ to $a_j^s$.
3. For any pair $D_i$ and $D_i^s$, the condition of mergeability is also satisfied and the transition rate from $D_i$ to $D_i^s$ is $w_n$ and from $D_i^s$ to $D_i$ is $w_m$.

From the preceding discussions, it can be concluded that the Markov process for the system is mergeable into subsets $D_i$, $D_i^s$. It can also be recognized that the merged Markov process is identical to the Markov process for components of $X_a$, that is, components member of cut set $C_i$. Now if the states $a_p$ and $a_p^s$ represent the failure of all the elements of $C_i$ in the normal and adverse weather respectively, then

$D_p$ = {subset of states in the normal weather having members of $C_i$ failed and other components of the system in either failed or good state}

$D_p^s$ = {subset of states in the adverse weather having members of $C_i$ failed and other components of the system in either of states}

That is,

$$S_i = D_p \cup D_p^s = D_p + D_p^s$$

$$D_p \equiv a_p, \; D_p^s \equiv a_p^s$$



The merged process is identical to the process corresponding to the members of cut set $C_i$. Therefore

$$\begin{aligned} Pr(C_i) &= Pr(S_i) \\ &= Pr(D_p) + Pr(D_p^s) \\ &= Pr(a_p) + Pr(a_p^s) \end{aligned} \tag{9.80}$$

$$\begin{aligned} F(S_i) &= F(D_p \cup D_p^s) \\ &= F(a_p \cup a_p^s) \end{aligned} \tag{9.81}$$

Here $F(a_p \cup a_p^s)$ is the frequency of encountering the state where all the elements of $C_i$ are failed and can be readily calculated using frequency balancing concept. For steady-state

$$F(a_p \cup a_p^s) = \sum_{i \neq p} \left[ Pr(a_p)\alpha_{pi} + Pr(a_p^s)\beta_{pi} \right] \tag{9.82}$$

From (9.80) and (9.81) it can be seen that the probability and frequency of a cut set $C_i$, for the system exposed to fluctuating environment, can be calculated by considering the Markov process associated with the members of cut set $C_i$ only and that the transition rate matrix of the entire system need not be generated. Therefore, the terms in (9.70) and (9.71) can be computed by generating the transition rate matrix of the elements of each cut set or intersection at a time and as noted previously these matrices are much smaller in size than the matrix for the entire system. It is to be noted that since the necessary and sufficient condition of mergeability is satisfied, (9.80) and (9.81) can be used for both time specific and steady state.

It becomes, however, obvious that the higher the order of intersections considered, the less advantageous the procedure becomes since the number of components to be considered at a time increases. Therefore, for the successful implementation, the following procedure is suggested.

PROCEDURE

1. Identify the cut sets to be considered. The cut sets having more than $x$ components may be ignored. It will be reasonable to have $x = 5$, since the probability of more than 5 overlapping outages can safely be regarded negligible.
2. Since in all practical systems the component failure rate is much smaller than the repair rate, the upper bound will give an almost exact result. Therefore $P_f$ and $f_f$ can be approximated as

$$P_f \simeq P_{fu} = \sum_i Pr(C_i) \tag{9.83}$$

and

$$f_f \simeq f_{fu} = \sum_i F(S_i) \tag{9.84}$$

3. The terms of (9.83) and (9.84) can be calculated by generating the transition rate matrix of the components of each cut set at a time and computing $Pr(C_i)$ and $F(S_i)$ using (9.80) and (9.81).

It can be seen that if the above procedure is followed only $2^{x+1}$ equations need be solved at a time, $x$ being the number of elements in cut set. If $x=5$, it means 64 equations, which is a trivial task when digital computers are employed. The calculation of the first lower bound will not involve much additional difficulty and can provide insight on the margin of error.

It should be carefully noted that (9.80) and (9.81) for the calculation of the terms of (9.70) and (9.71) or (9.83) and (9.84) are exact. The approximation involved is either in the ignoring of higher order cut sets or using upper bound approximations by (9.83) and (9.84) instead of complete (9.70) and (9.71).

*Comparison with State Space Truncation.* If the cut sets of say order 6 or higher were to be ignored, one might ask, "How is the Markov cut set approach superior to state space truncation when contingencies of order higher than 5 are ignored?" Consider a system of say $n$ components exposed to normal and adverse weather. If contingencies of the order higher than $x$ are ignored, the number of linear equations involved is,

$$=2\sum_{i=0}^{x}\binom{n}{i}$$

If, for example, $n=50$ and $x=5$, the total number of states or corresponding equations is 2369936, which is beyond the capability of the today computers. On the other hand using Markov cut set approach only 64 equations need be considered simultaneously, which for state space truncation, corresponds to considering only single-order contingencies.

APPROXIMATIONS AND EXTENSIONS. The Markov cut set method has been shown to deal in an exact manner with the form of dependency induced by the fluctuating environment. Even though this method may not exactly apply to all forms of dependency, it could provide good approximations for certain limited forms of component dependence due to maintenance outages and common-mode failures.

*Example.* The Markov Cut-set method is illustrated by application to a complex configuration shown in Figure 9.16. The cut sets of this system are identified in Table 9.3. The relevant data is shown in Table 9.4, which



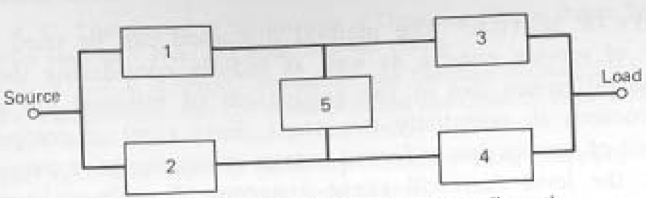


**Figure 9.16** An example of a complex configuration.

also gives the comparison of the results by exact Markov and Markov cut set methods. The results obtained by the Markov cut set method are only negligibly higher than the Markov method. The mean duration of the failure state can be obtained by the (9.72). The error is somewhat higher for higher percentage of failures during adverse weather. This is to be expected since this results in effectively increasing the ratio of failure rate to repair rate. It should be reiterated here that any error introduced by the Markov cut set method is because of the use of upper bound approximation and not in the calculation of terms of the equations for the probability and frequency of failure.

**Table 9.3** *Minimal Cut Sets of Figure 9.16*

| Cut Set | Component Members |
|---|---|
| $C_1$ | 1,2 |
| $C_2$ | 3,4 |
| $C_3$ | 1,4,5 |
| $C_4$ | 2,3,5 |

**Table 9.4** *Comparison of Markov and Markov Cut Set Methods.* Components are assumed identical. Average failure rate = 0.5 failure/year. Normal weather mean duration = 200 hours. Adverse weather mean duration = 1.5 hours.

| Mean Down Time of Each Component (hours) | Failures During Adverse Weather, % | Failure Probability | | Failure Frequency (per year) | |
|---|---|---|---|---|---|
| | | Markov | Markov Cut Set | Markov | Markov Cut Set |
| 5 | 20 | $3.4996\times10^{-7}$ | $3.4996\times10^{-7}$ | $1.2275\times10^{-3}$ | $1.2275\times10^{-3}$ |
| 5 | 80 | $3.3144\times10^{-6}$ | $3.3157\times10^{-6}$ | $1.1674\times10^{-2}$ | $1.1684\times10^{-2}$ |
| 10 | 20 | $1.0736\times10^{-6}$ | $1.0736\times10^{-6}$ | $1.8830\times10^{-3}$ | $1.8831\times10^{-3}$ |
| 10 | 80 | $7.7573\times10^{-6}$ | $7.7633\times10^{-6}$ | $1.3678\times10^{-2}$ | $1.3695\times10^{-2}$ |

ON QUALITY OF SERVICE. The method discussed can be used for both continuity of service criteria as well as indices considering the voltage levels. The difference lies in the calculation of minimal cut sets. The process proceeds in essentially two steps. First a set of components is assumed out of service due to forced outage or maintenance outage. Given this event, the level that will cause unacceptable voltage level is then determined using load flow. Repetition of this procedure for different sets of component outages, then identifies the minimal cut sets in terms of component outages and load levels. Once the minimal cut sets have been identified, the Markov cut set method can be used.

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