ECEN474: (Analog) VLSI Circuit Design
Fall 2011

Lecture 11: Frequency Response (cont.)

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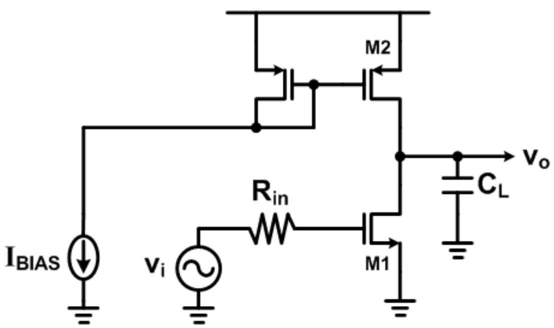
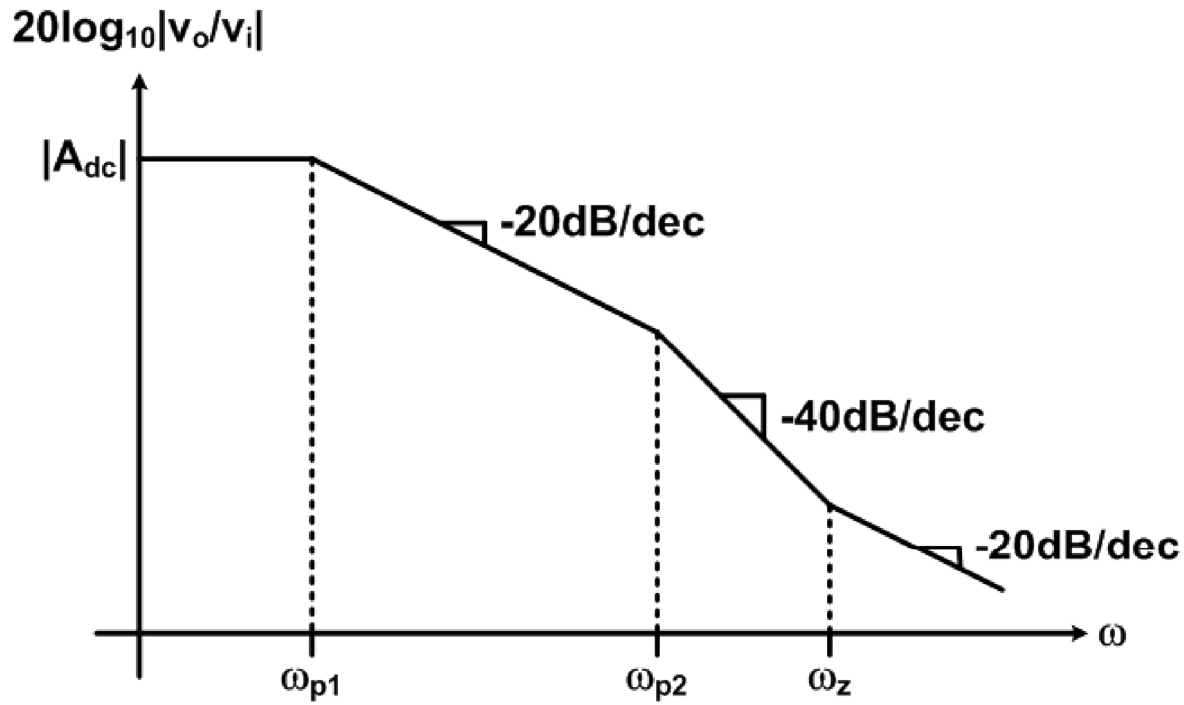
Agenda

- Common-Source Amp Input Impedance
- Common-Drain Amp Frequency Response
- Differential Pairs

Common-Source Amp Frequency Response





Large R_{in} **Small R_{in}**

$$A_{dc} = -g_m r_o \qquad \omega_z = \frac{g_{m1}}{C_{gd1}}$$

Large R_{in}:

$$\omega_{p1} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1} r_o\right)\right]}$$

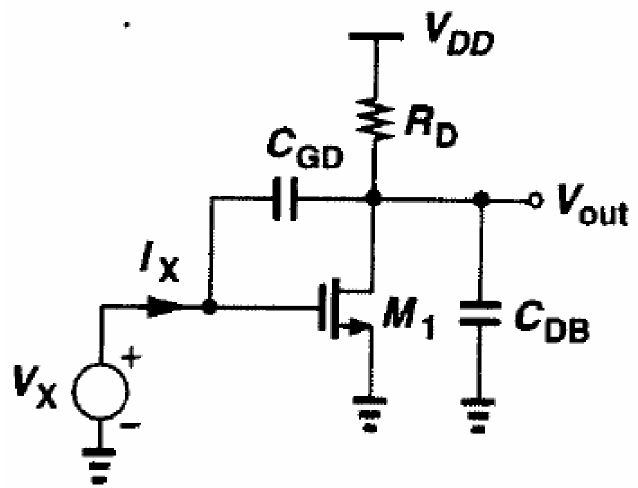
$$\omega_{p2} \cong -\frac{g_{m1} C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o}$$

Small R_{in}:

$$\omega_{p1} = -\frac{1}{r_o\left(C_{gd1} + C_o\right)}$$

$$\omega_{p2} \cong -\frac{1}{R_{in}\left(C_{gs1} + C_{gd1}\right)}$$

Common-Source Amp Input Impedance



[Razavi]

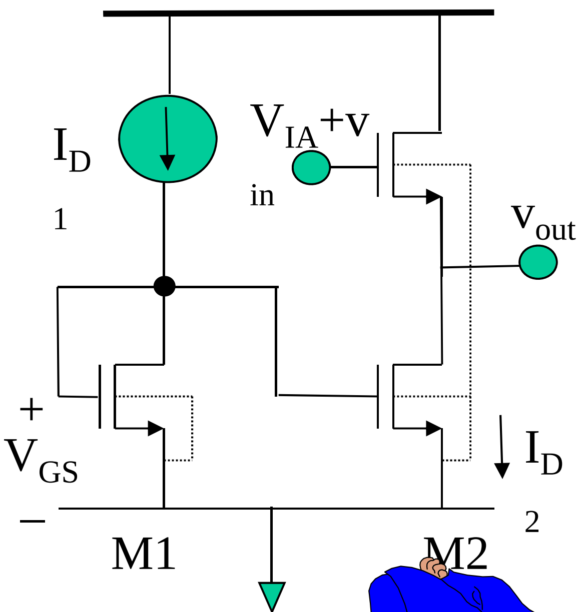
Neglecting Output Cap: $$Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

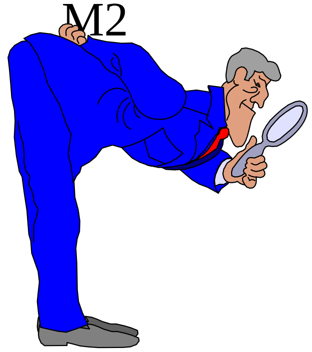
Input impedance is purely capacitive (C_{gs} + Miller C_{gd})

Considering Output Cap: $$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB} s)}$$

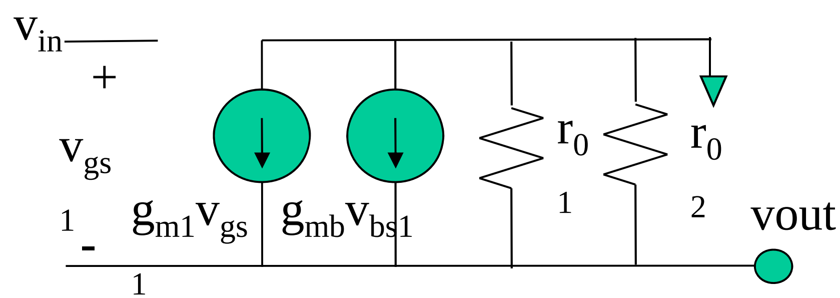
Low frequency is capacitive, but then impedance experiences a zero followed by a second pole

Small signal analysis: Common-drain (source follower) amplifier

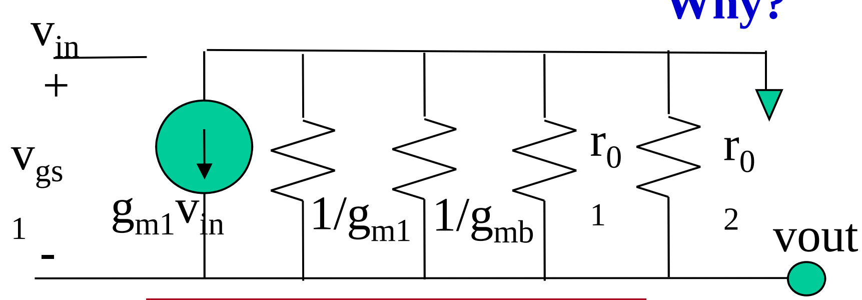




Small signal equivalent circuit



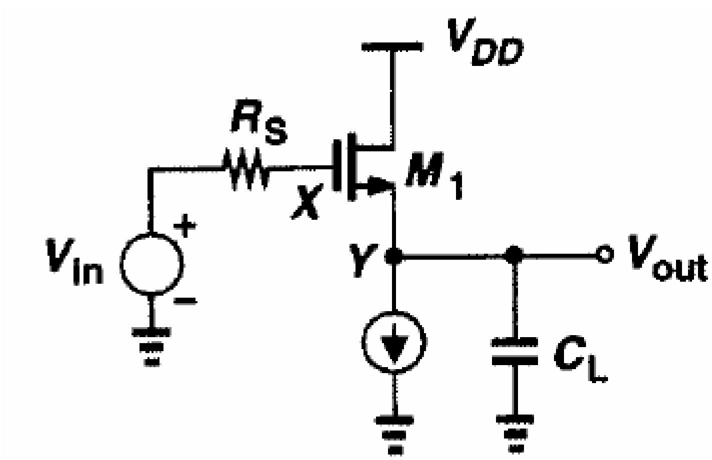
How this is done? Why?

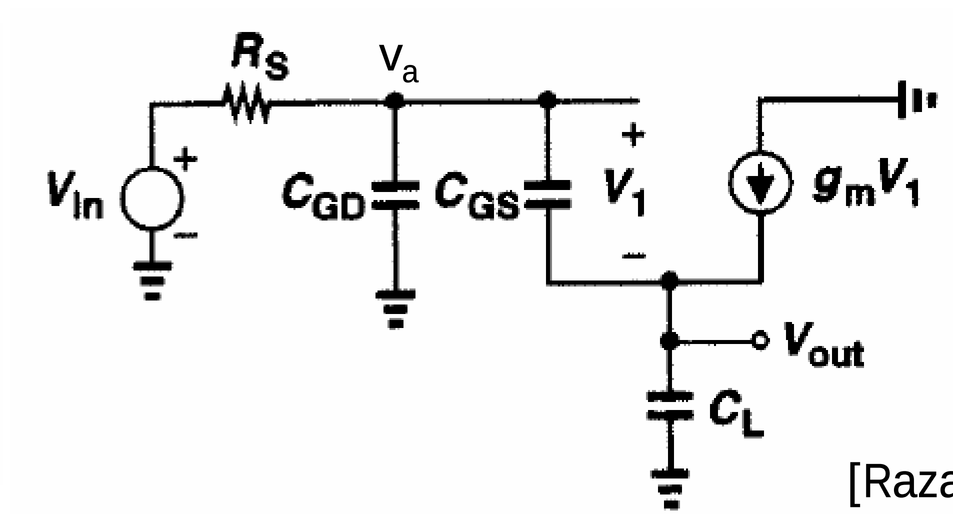


$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{mb} + g_{01} + g_{02}}$$

Common-Drain Amplifier: High Frequency Response

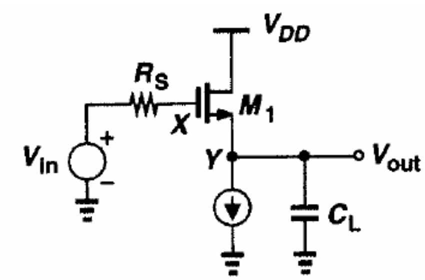
- Simplifying the schematic a bit for SSA
  - Ideal current source load and neglecting transistor r_o and g_{mb} (i.e. $\lambda=\gamma=0$)
  - Will result in an optimistic DC gain estimate

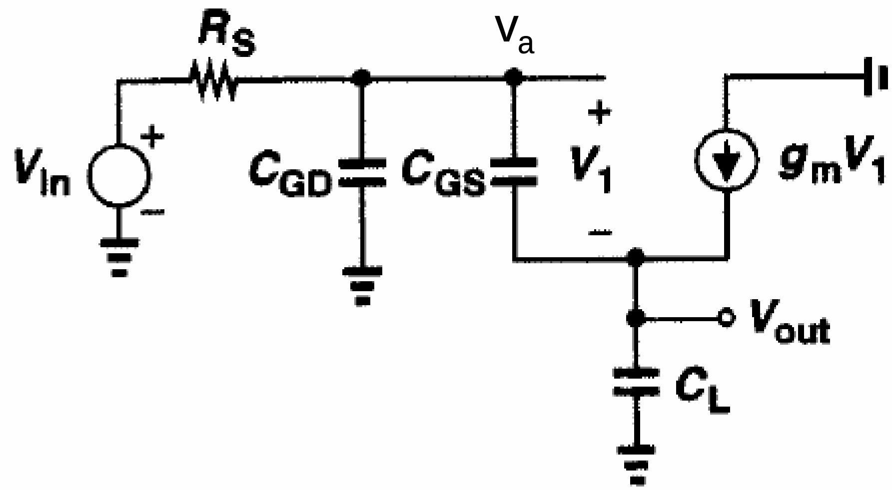




[Razavi]

Common-Drain Amplifier: High Frequency Response





[Razavi]

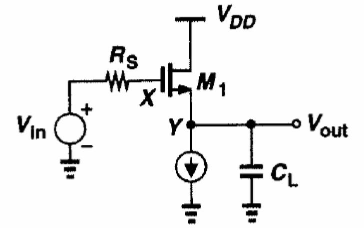
$$\textbf{KCL @ Node } \mathbf{v_a}: (v_a - v_i)G_S + v_a s C_{gd} + (v_a - v_o) s C_{gs} = 0$$

$$\textbf{KCL @ Node } \mathbf{v_o}: (v_o - v_a) s C_{gs} - g_m (v_a - v_o) + v_o s C_L = 0$$

After some algebra:

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Common-Drain Amplifier: High Frequency Response



$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

- From this simplified transfer function:

$$A_{dc} = \frac{g_m}{g_m} = 1 \quad \textbf{(Optimistic)}$$

$$\textbf{Exact} \quad A_{dc} = \frac{g_m}{g_m + g_o + g_{mb}}$$

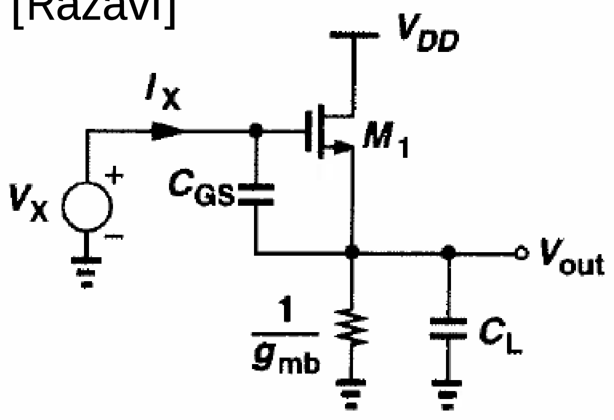
$$\omega_z = -\frac{g_m}{C_{gs}}$$

2 poles, If we assume that they are spaced far apart :

$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \dfrac{C_L + C_{GS}}{g_m}}$$

Common-Drain Amp Input Impedance

[Razavi]



$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

Low Frequency: $Z_{in} \approx \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$

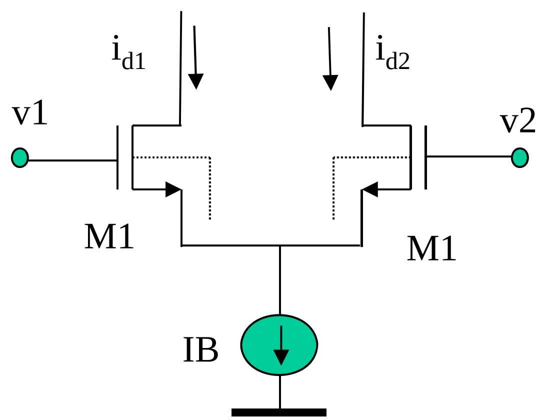
Equivalent to a series capacitve term $C_{gs}\left(\frac{g_{mb}}{g_m + g_{mb}}\right)$ **and resistive term** $\frac{1}{g_{mb}}$

High Frequency: $Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS}C_L s^2}$

Series combination of C_{gs} **and** C_L **and a negative resistance term** $\left(-\frac{g_m}{C_{gs}C_L\omega^2}\right)$

The negative resistance term can be utilized in oscillator design

Differential Pair: Linear range is limited to 2 V_{DSAT}

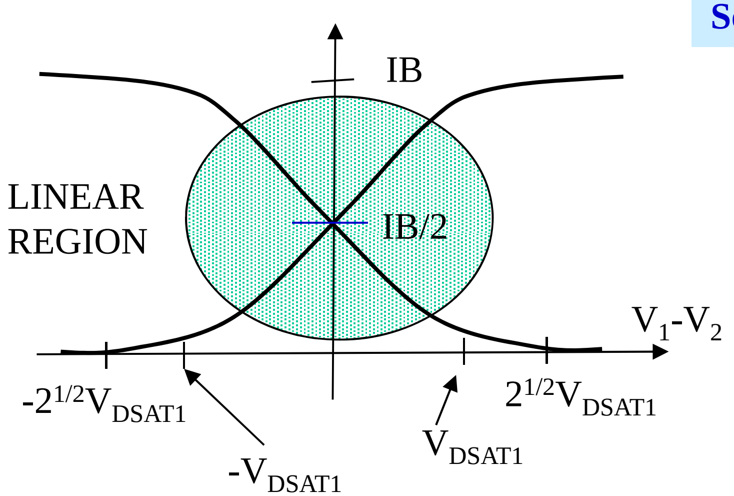


If both transistors are saturated and IB is ideal ($r_{IB}=\infty$)

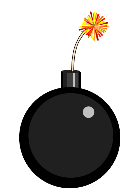
$$i_{d1} + i_{d2} = IB$$

$$i_{d1} = \frac{\mu_n C_{OX}}{2}\frac{W}{L}\left(V_{gs1} - V_T\right)^2$$

$$i_{d2} = \frac{\mu_n C_{OX}}{2}\frac{W}{L}\left(V_{gs2} - V_T\right)^2$$



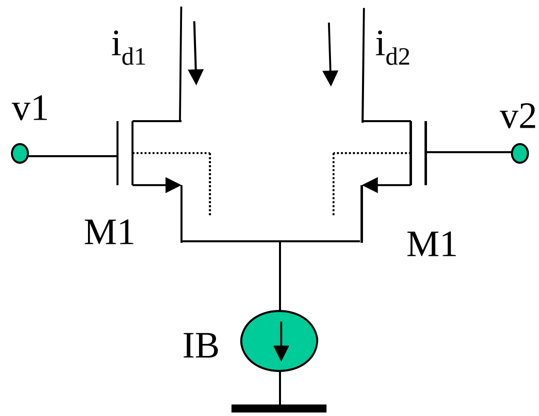
Solving these equations => Valid for $|v_1 - v_2| < 2^{1/2} V_{DSAT1}$

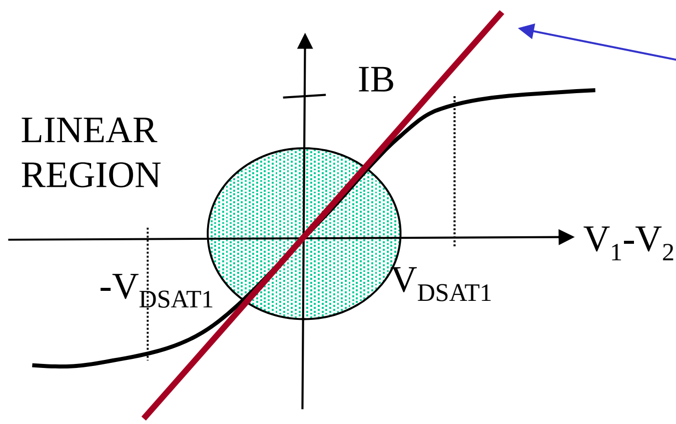


$$i_{d1} = \frac{IB}{2} + \frac{g_{m1}\left(v_1 - v_2\right)}{2}\sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}$$

$$i_{d2} = \frac{IB}{2} - \frac{g_{m1}\left(v_1 - v_2\right)}{2}\sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}$$

Differential Pair





The diff pair is a nonlinear circuit

$$i_{d1} - i_{d2} = g_{m1}(v_1 - v_2)\sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}$$

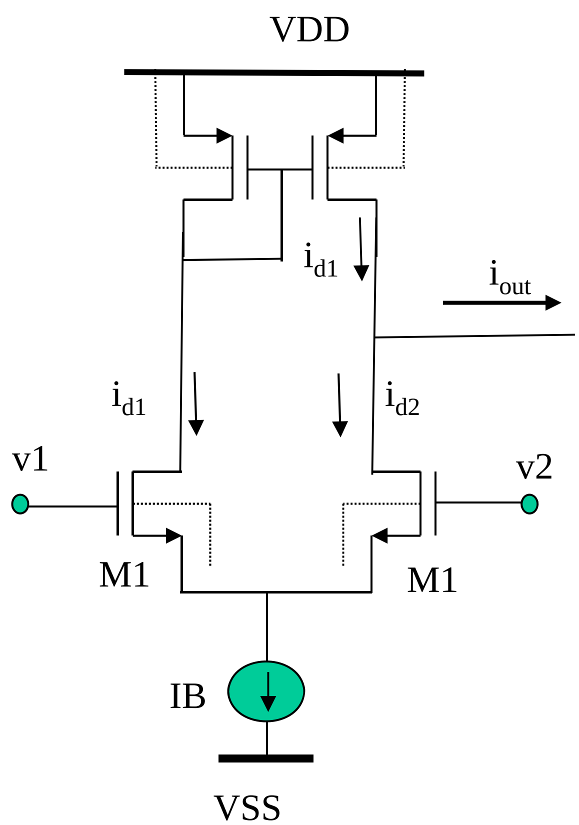
Non-linear term

If $v_d = v_1 - v_2 < V_{DSAT1}$ =>

$$i_{d1} - i_{d2} = \sqrt{\mu_n C_{OX} \frac{W}{L} I_B}\,(v_1 - v_2)$$

Note:
Linear range increases for large V_{DSAT1}
V_{GS} is also increased (limited by VSS)

Basic Operational Transconductance Amplifier



DESIGN CONSIDERATIONS:

- **$V_d = v_1 - v_2 < V_{DSAT}$**
- $V1,2 - VSS > V_{GS1} + V_{DSATB}$

For small signals, ignoring the capacitors:

$$i_{out} = \sqrt{\mu_n C_{OX} \frac{W}{L} I_B}\left(v_1 - v_2\right)$$

or

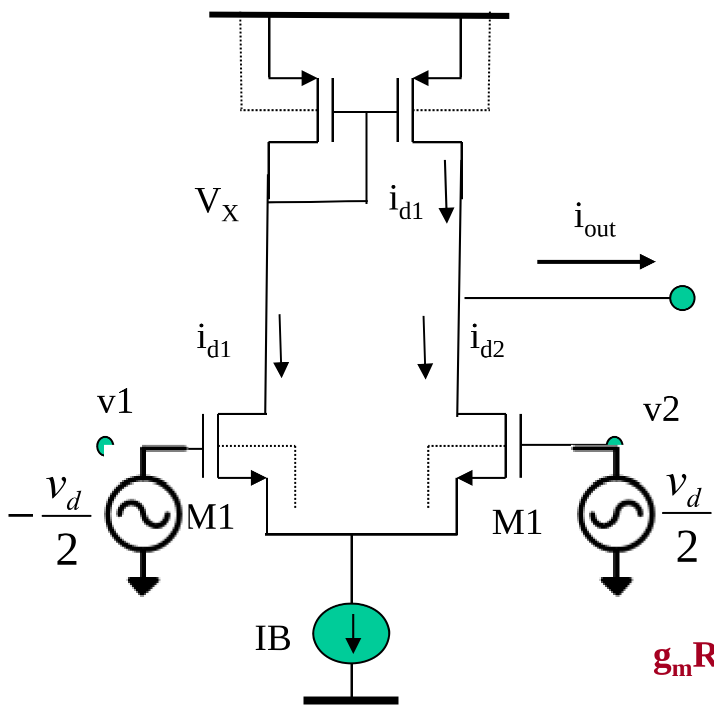
$$i_{out} = g_m\left(v_1 - v_2\right)$$ Sensitive to differential signals

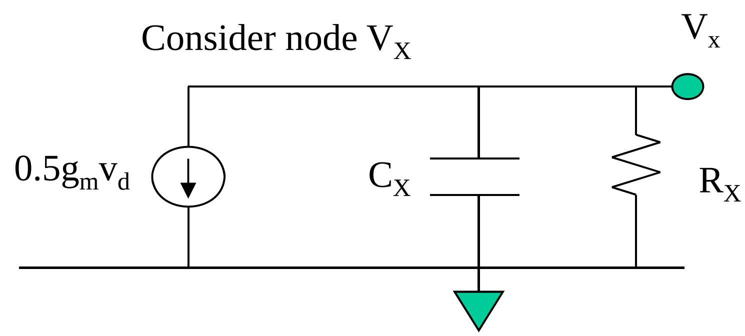
$$v_{out} = g_m r_{out}\left(v_1 - v_2\right)$$

$$r_{out} = r_{o1} II r_{op}$$

For an ideal current source and ignoring the effects of gmb, and transistor mismatches, then $i_{out}=0$ for $v_1=v_2$ ==> rejection to common-mode (noise) signals present at the input!

Frequency Response

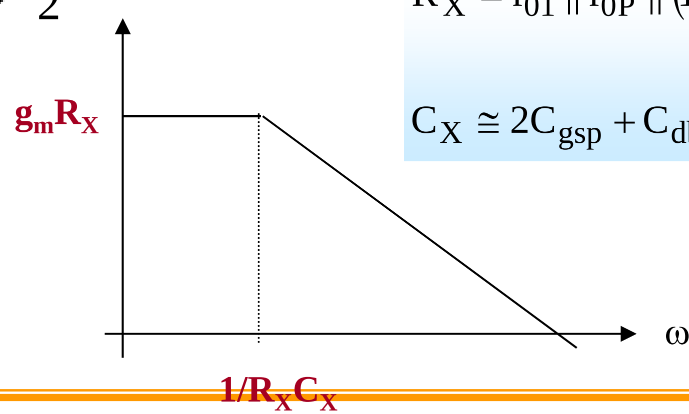




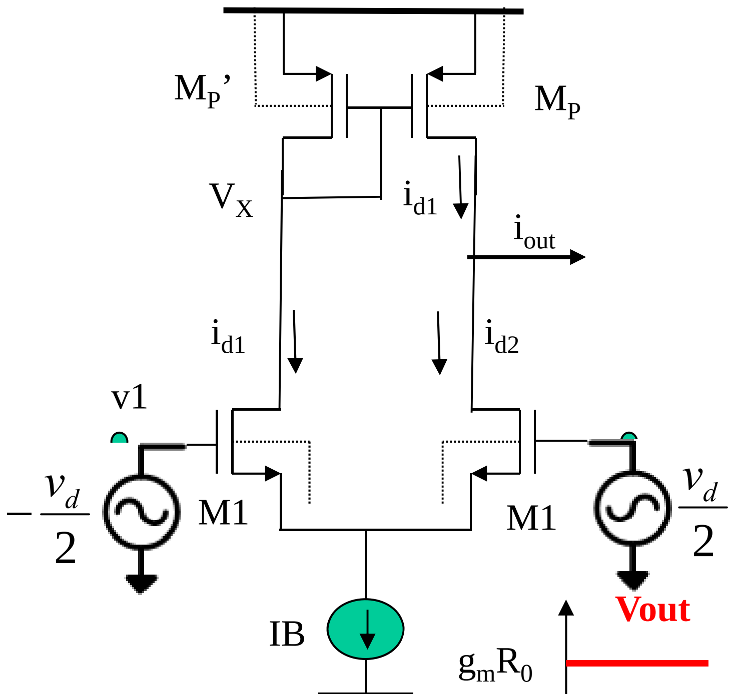
$$V_X = -\frac{0.5 g_m R_X}{1 + S R_X C_X} V_d$$

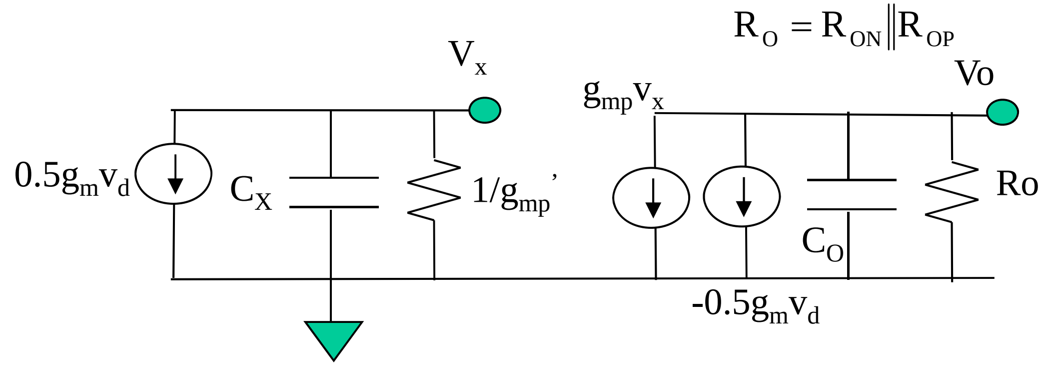
$$R_X = r_{01} \,\|\, r_{0P} \,\|\, (1/g_{mp})$$

$$C_X \cong 2C_{gsp} + C_{dbp} + (1 + \text{miller factor}) C_{dgp} + C_{dbn}$$

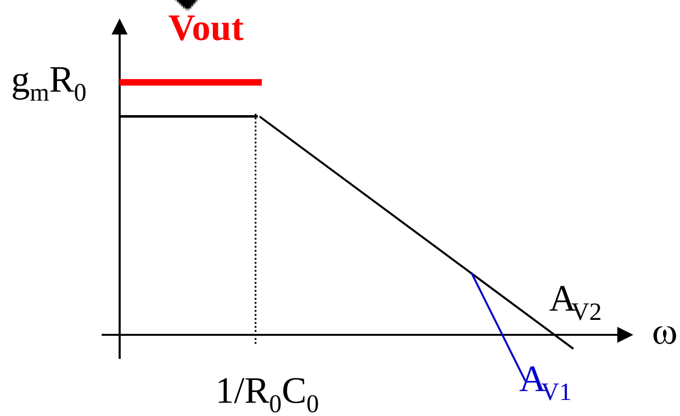


Low Frequency Response





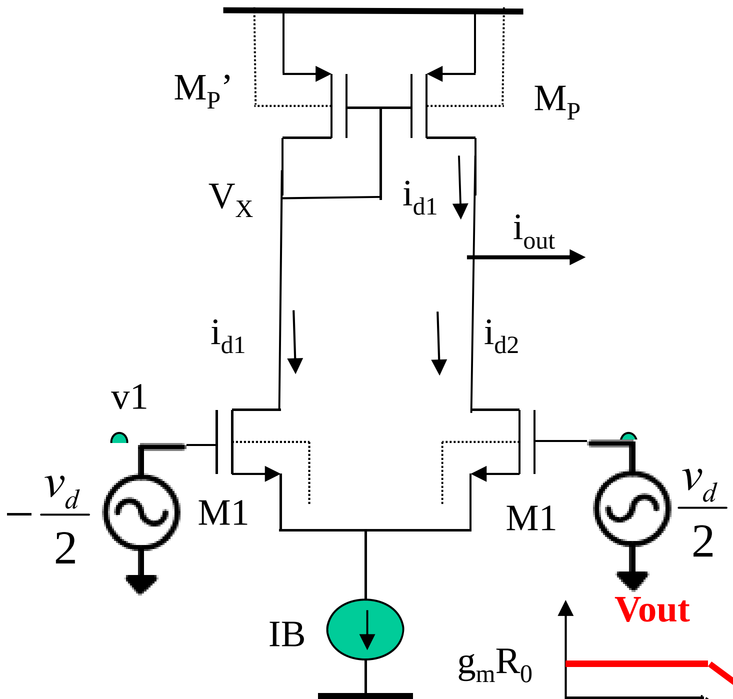
$$v_o = -g_m\left(\frac{v_d}{2}\right)R_o - g_m\left(-\frac{v_d}{2}\right)R_x\left(-g_{mp}R_o\right)$$



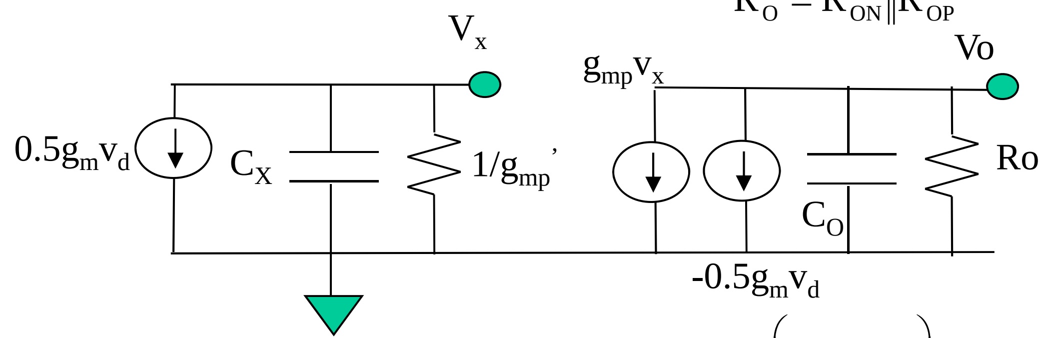
$$R_x \approx \frac{1}{g_{mp}}$$

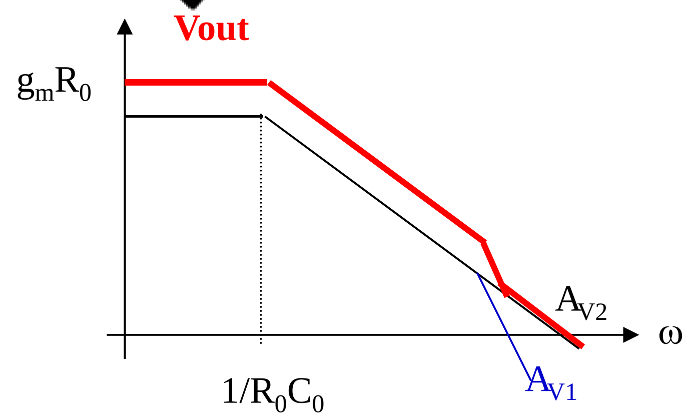
$$v_o = -g_m v_d R_o$$

Frequency Response



$$R_O = R_{ON} \| R_{OP}$$





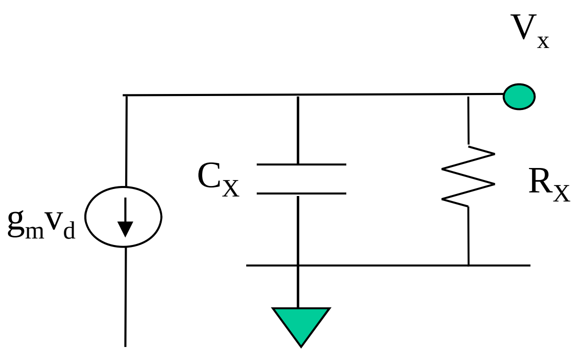
$$v_o = -g_m\left(\frac{v_d}{2}\right)\frac{R_o}{1+sR_oC_o} - g_m\left(\frac{v_d}{2}\right)\left(\frac{1/g_{mp}}{1+s\frac{C_x}{g_{mp}}}\right)\frac{g_{mp}R_o}{1+sR_oC_o}$$

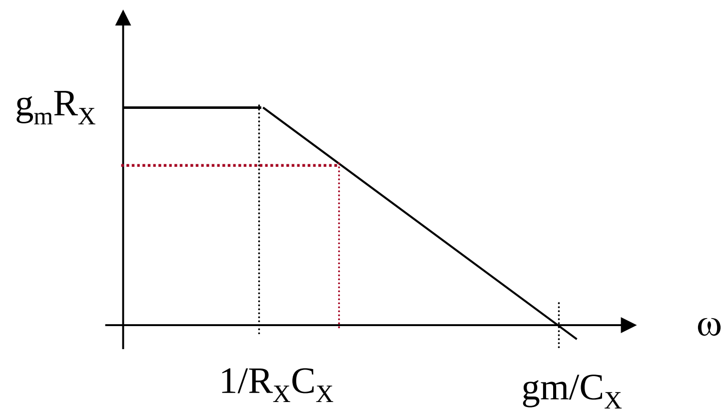
$$v_o = -g_mR_o\left(\frac{v_d}{2}\right)\frac{1}{1+sR_oC_o}\left[1+\frac{1}{1+s\frac{C_x}{g_{mp}}}\right]$$

$$v_o = -g_mR_o\left(\frac{v_d}{2}\right)\frac{1}{1+sR_oC_o}\left[\frac{2+s\frac{C_x}{g_{mp}}}{1+s\frac{C_x}{g_{mp}}}\right]$$

Frequency Response

Poles are associated with both resistors and capacitors





$$V_X = \frac{g_m R_X}{1 + S R_X C_X} V_d$$

IMPORTANT REMARKS !!!!

- DC-GAIN IS PROPORTIONAL TO R_X
- POLE FREQUENCY IS PROPORTIONAL TO $1/C_X R_X$
- GAIN-BANDWIDTH PRODUCT ($=gm/C_X$) IS CONSTANT

TRADEOFF BETWEEN GAIN AND BANDWIDTH

Next Time

- Single-Stage Amplifiers (cont.)
  - Common-Gate
  - Cascode Stage

- Noise