ECEN474: (Analog) VLSI Circuit Design
Fall 2011

Lecture 14: Simple OTA

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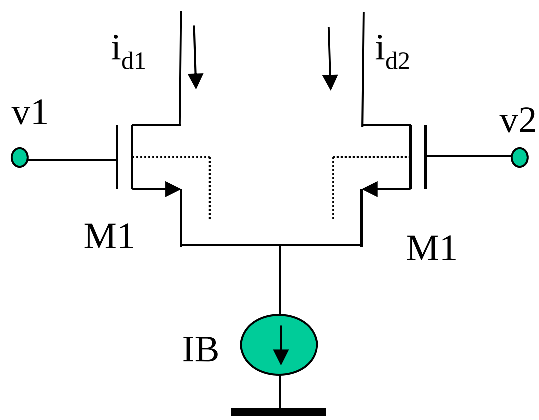
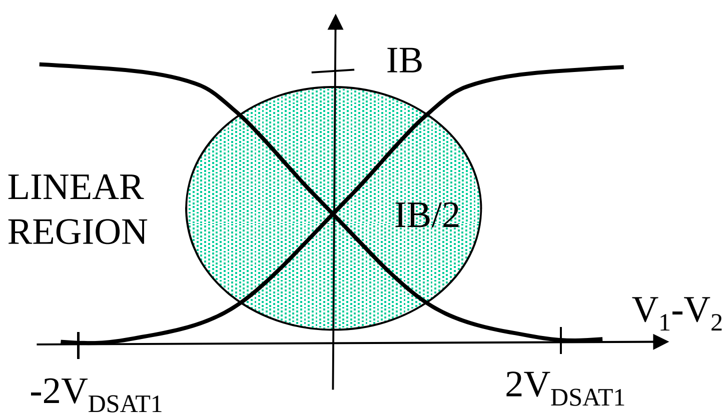
Agenda

- Simple OTA Parameters

Differential Pair





If both transistors are saturated

$$i_{d1} + i_{d2} = IB$$

$$i_{d1} = \frac{\mu_n C_{OX}}{2} \frac{W}{L} \left(V_{gs1} - V_T\right)^2$$

$$i_{d2} = \frac{\mu_n C_{OX}}{2} \frac{W}{L} \left(V_{gs2} - V_T\right)^2$$

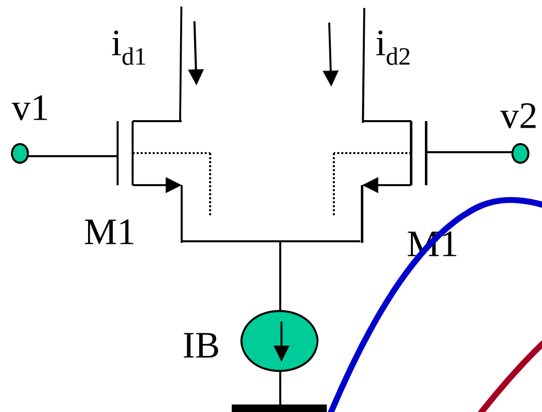
Solving these equations ==>

For Derivation, see Sedra/Smith Ch8

$$i_{d1} = \frac{IB}{2} + \frac{g_{m1}(v_1 - v_2)}{2} \sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}$$

$$i_{d2} = \frac{IB}{2} - \frac{g_{m1}(v_1 - v_2)}{2} \sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}$$

Differential Pair



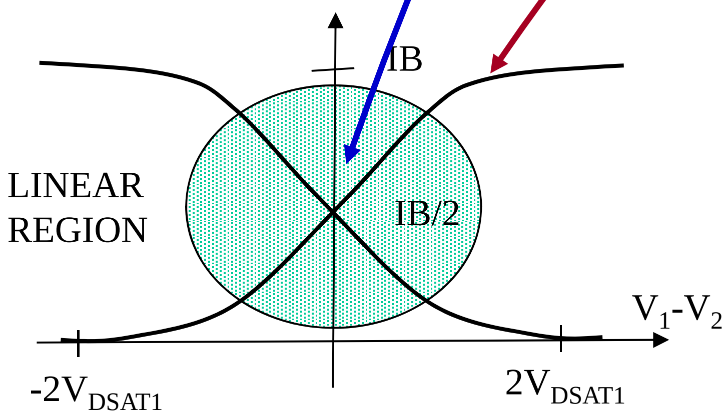
Is a non-linear circuit

$$i_{d1}-i_{d2}=g_{m1}(v_1-v_2)\sqrt{1-\left(\frac{v_1-v_2}{2V_{DSAT1}}\right)^2}$$

Non-linear term

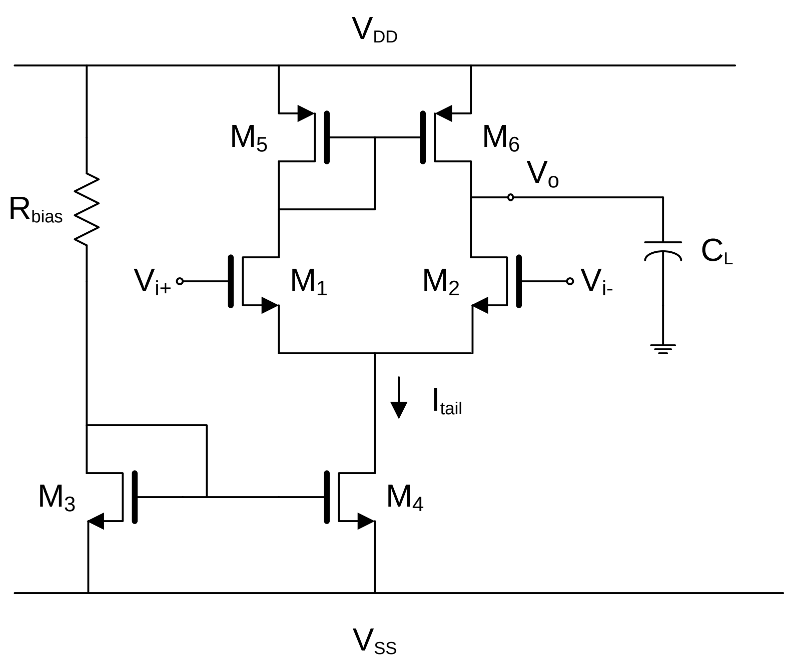
For $v_d=v_1-v_2<V_{DSAT1}$ ==>

$$i_{d1}-i_{d2}=\sqrt{\mu_n C_{OX}\frac{W}{L}I_B}\,(v_1-v_2)$$



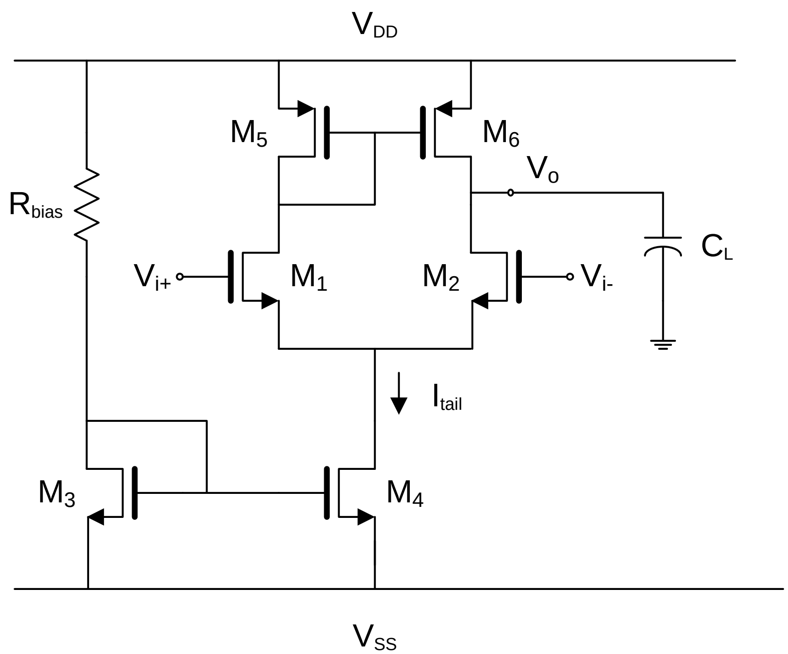
Note:
Linear range increases for large V_{DSAT1}
V_{GS} is also increased (limited by VSS)

Operational Transconductance Amplifier



- Important Parameters
  - Differential Gain
  - Gain-Bandwidth Product
  - Common-Mode Input Range
  - Common-Mode Gain
  - Common-Mode Rejection Ratio (CMRR)
  - Power-Supply Rejection Ratio (PSRR)
  - Slew Rate

OTA Differential Gain



Let $V_{i+} = \dfrac{v_{id}}{2}$ **and** $V_{i-} = -\dfrac{v_{id}}{2}$

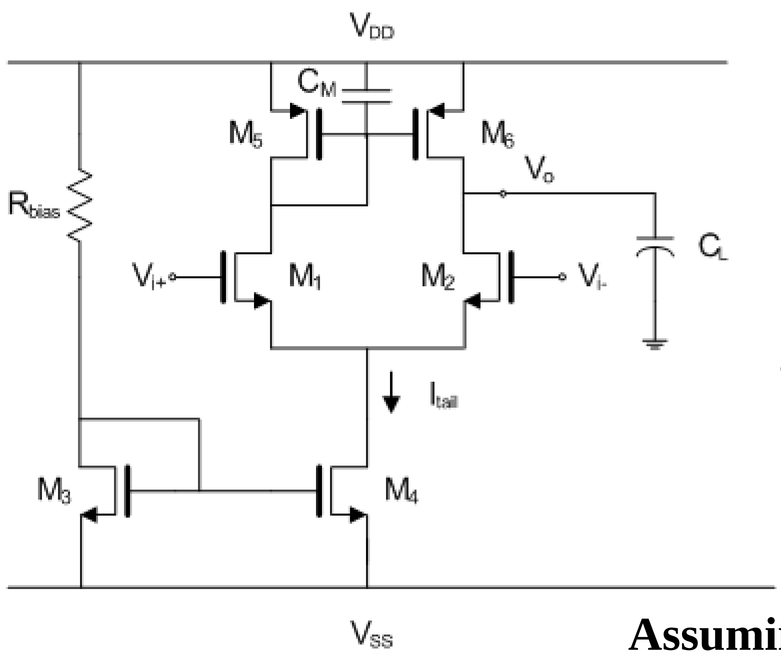
$$v_o = -g_{m2} r_{out}\left(-\frac{v_{id}}{2}\right) - \frac{g_{m1}}{g_{m5}}\left(-g_{m5} r_{out}\right)\left(\frac{v_{id}}{2}\right)$$

By design $g_{m1} = g_{m2}$ **and** $g_{m5} = g_{m6}$

$$v_o = g_{m1} r_{out} v_{id}$$

$$A_{DM} = \frac{v_o}{v_{id}} = g_{m1} r_{out} = \frac{g_{m1}}{g_{o6} + g_{o2}}$$

OTA Gain-Bandwidth Product (GBW)



$$A_{DM} = \frac{g_{m1}}{g_{o6} + g_{o2}}$$

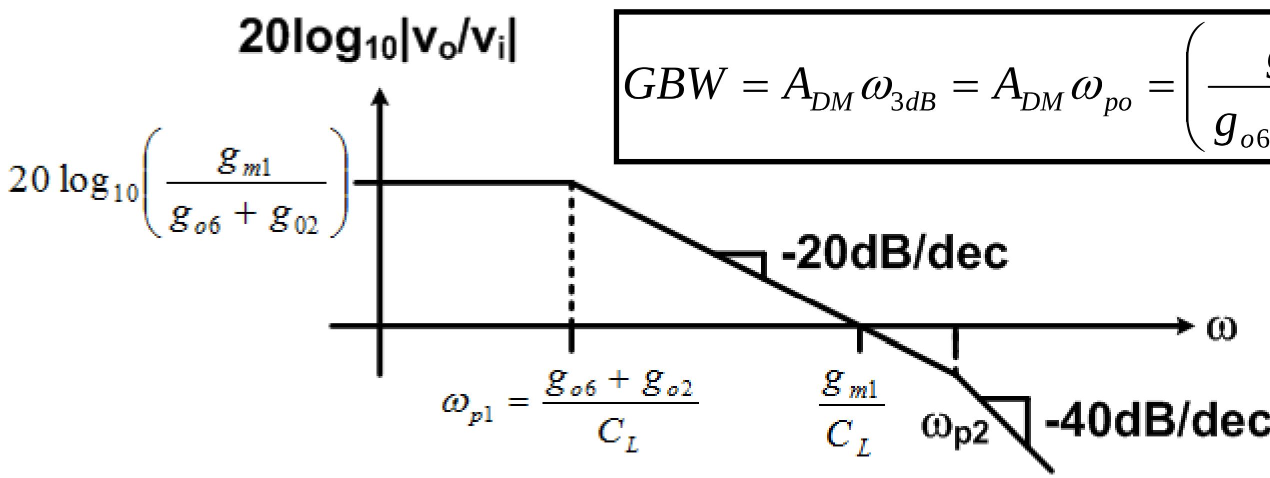
The circuit will have 2 poles

ω_{po} **at the output node and** ω_{pm} **at the "mirror" node**

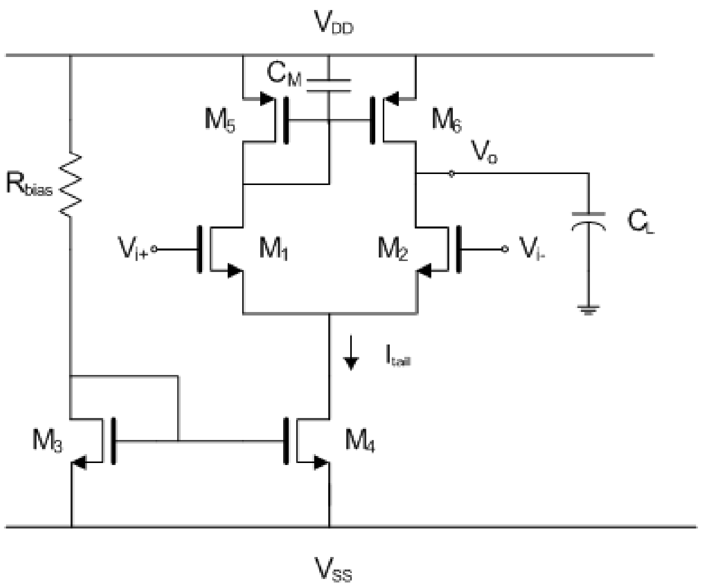
$$\omega_{po} \approx \frac{g_{o6} + g_{o2}}{C_L}, \quad \omega_{pm} \approx \frac{g_{m5}}{C_M}$$

Assuming the poles are widely spaced and ω_{po} **dominates**

$$GBW = A_{DM}\,\omega_{3dB} = A_{DM}\,\omega_{po} = \left(\frac{g_{m1}}{g_{o6} + g_{o2}}\right)\left(\frac{g_{o6} + g_{o2}}{C_L}\right) = \frac{g_{m1}}{C_L}$$



OTA Common-Mode Input Range



- Common-mode input range set by transistor saturation conditions
  - Low-end set by tail current source saturation

$$V_{icm} \geq V_{SS} + V_{DSAT4} + V_{GS1} = V_{SS} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_4}} + \sqrt{\frac{I_{tail}}{\mu_n C_{ox} \frac{W}{L}_1}} + V_{Tn1}$$

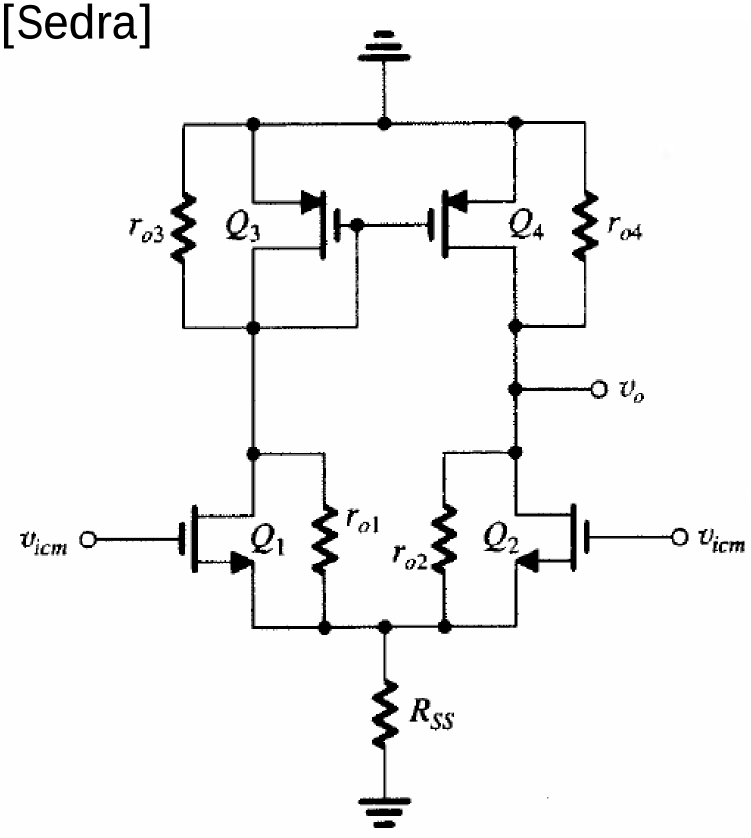
- High-end set by differential pair saturation

$$V_{icm} \leq V_{ocm} + V_{T1} = V_{DD} - V_{GS5} + V_{T1} = V_{DD} - \left(\sqrt{\frac{I_{tail}}{\mu_p C_{ox} \frac{W}{L}_5}} + \left| V_{Tp5} \right| \right) + V_{Tn1}$$

$$\boxed{V_{SS} + \sqrt{\frac{2I_{tail}}{\mu_n C_{ox} \frac{W}{L}_4}} + \sqrt{\frac{I_{tail}}{\mu_n C_{ox} \frac{W}{L}_1}} + V_{Tn1} \leq V_{icm} \leq V_{DD} - \left(\sqrt{\frac{I_{tail}}{\mu_p C_{ox} \frac{W}{L}_5}} + \left| V_{Tp5} \right| \right) + V_{Tn1}}$$

OTA Common-Mode Gain

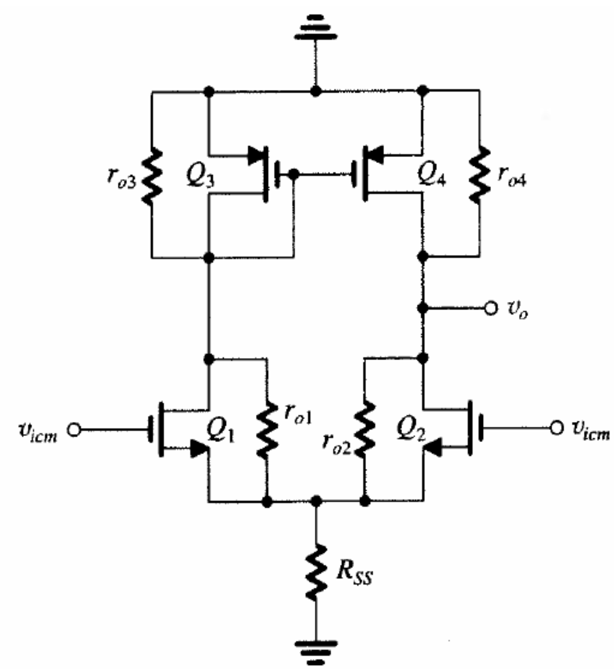
[Sedra]

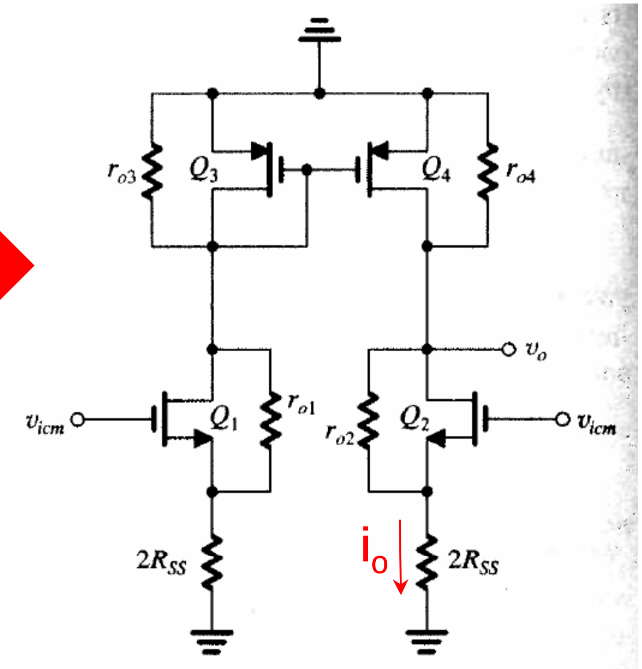


- Ideally, common-mode perturbations are suppressed by the differential amplifier, i.e. $A_{cm} = 0$
- Finite common-mode gain exists due to amplifier asymmetries and finite tail current source impedance
- Note transistor numbers are different from previous slides, as I borrow figures from Sedra/Smith text

OTA Common-Mode Gain

[Sedra]





$$v_s = v_{icm}\frac{(2R_{SS} \| r_{o1})}{(2R_{SS} \| r_{o1}) + (1/g_{m1})}$$

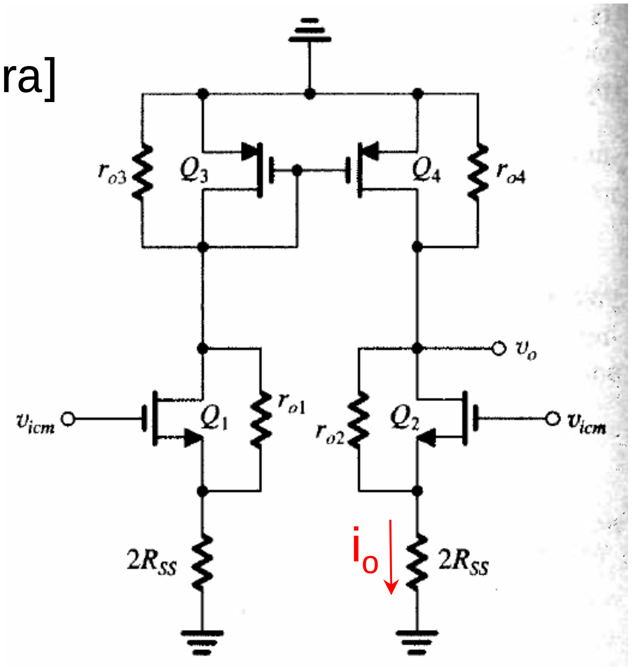
$$\simeq v_{icm}$$

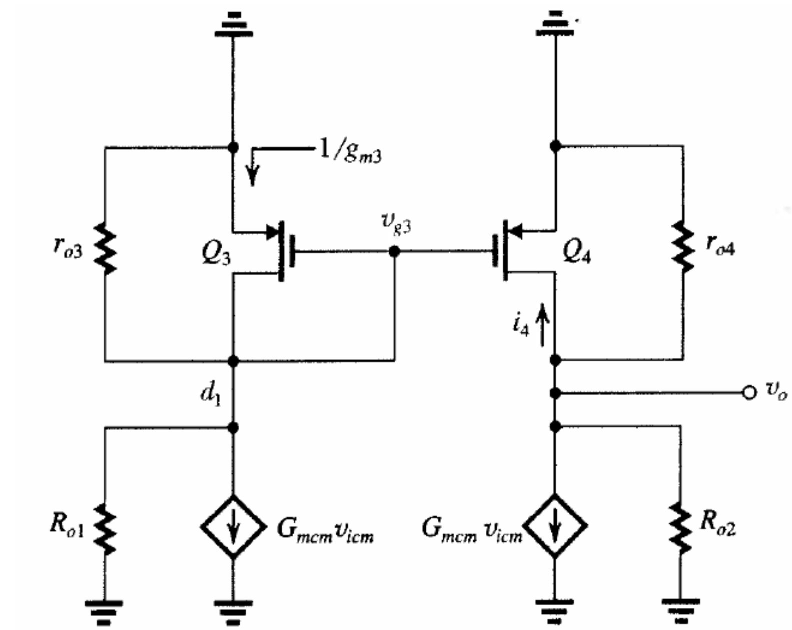
$$i_o = \frac{v_{icm}}{2R_{SS}}$$

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{1}{2R_{SS}}$$

OTA Common-Mode Gain

[Sedra]





$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{1}{2R_{SS}}$$

$$R_{o1} = 2R_{SS} + r_{o1} + (g_{m1}r_{o1})(2R_{SS})$$

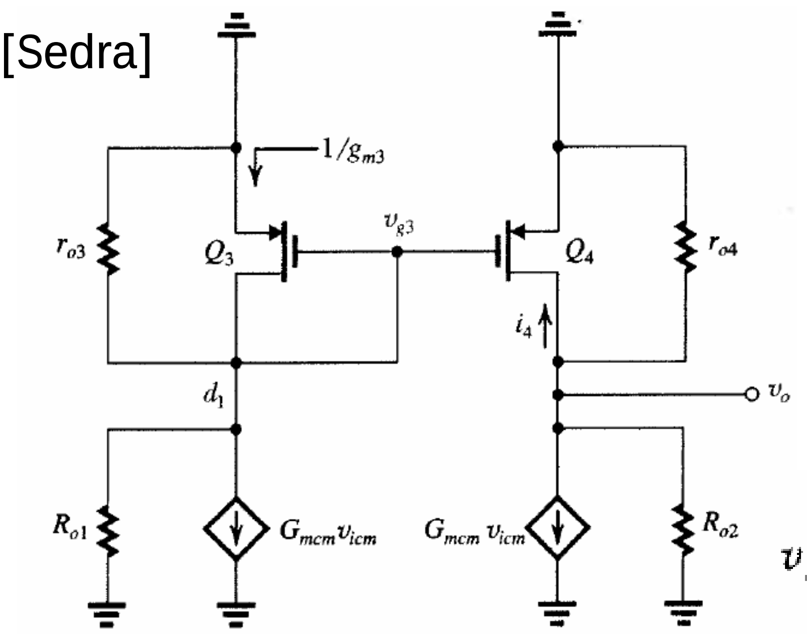
$$R_{o2} = 2R_{SS} + r_{o2} + (g_{m2}r_{o2})(2R_{SS})$$

$$v_{g3} = -G_{mcm}v_{icm}\left(R_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}}\right)$$

$$i_4 = -g_{m4}G_{mcm}v_{icm}\left(R_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}}\right)$$

OTA Common-Mode Gain

[Sedra]



$$i_4 = -g_{m4}G_{mcm}v_{icm}\left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)$$

KCL at v_o

$$G_{mcm}v_{icm} + i_4 + \frac{v_o}{R_{o2}} + \frac{v_o}{r_{o4}} = 0$$

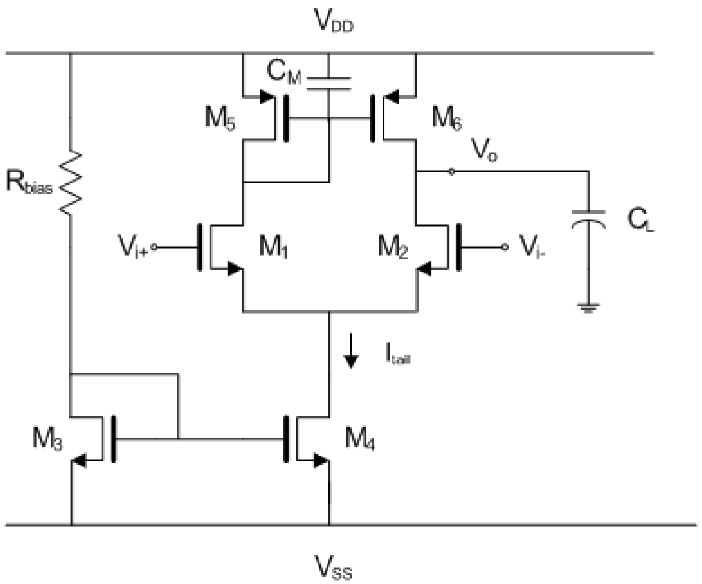
$$v_o = -v_{icm}\frac{r_{o4} \| R_{o2}}{2R_{SS}}\left[1 - g_{m4}\left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)\right]$$

Since $R_{o2} >> r_{o4}$ and $R_{o1} >> r_{o3}$ and $g_{m3} = g_{m4}$

$$A_{cm} \equiv \frac{v_o}{v_{icm}} \simeq -\frac{r_{o4}}{2R_{SS}}\,\frac{1}{1 + g_{m3}r_{o3}}$$

$$A_{cm} \simeq -\frac{1}{2g_{m3}R_{SS}}$$

OTA Common-Mode Gain & Rejection Ration (CMRR)



$$A_{cm} = \frac{v_o}{v_{CM}} \approx -\frac{1}{2g_{m6}r_{o4}}$$

- To improve (lower) common-mode gain, we need a high tail current impedance

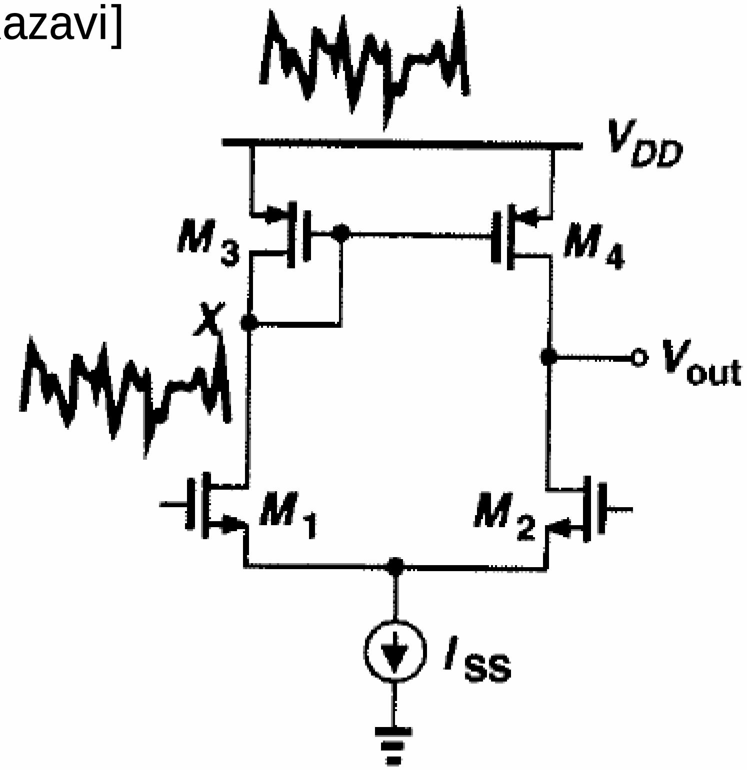
- An amplifier figure-of-merit is the common-mode rejection ratio (CMRR)

$$CMRR = 20\log_{10}\left|\frac{A_{dm}}{A_{cm}}\right|$$

$$CMRR = 20\log_{10}\left|\frac{A_{dm}}{A_{cm}}\right| = 20\log_{10}\left|\left(\frac{g_{m1}}{g_{o6}+g_{o2}}\right)(-2g_{m6}r_{o4})\right|$$

OTA Power-Supply Rejection Ration (PSRR)

[Razavi]



$$PSRR^{+} = 20 \cdot \log_{10}\left(\left|\frac{A_{dm}}{v_o / v_{DD}}\right|\right)$$

$$PSRR^{-} = 20 \cdot \log_{10}\left(\left|\frac{A_{dm}}{v_o / v_{SS}}\right|\right)$$

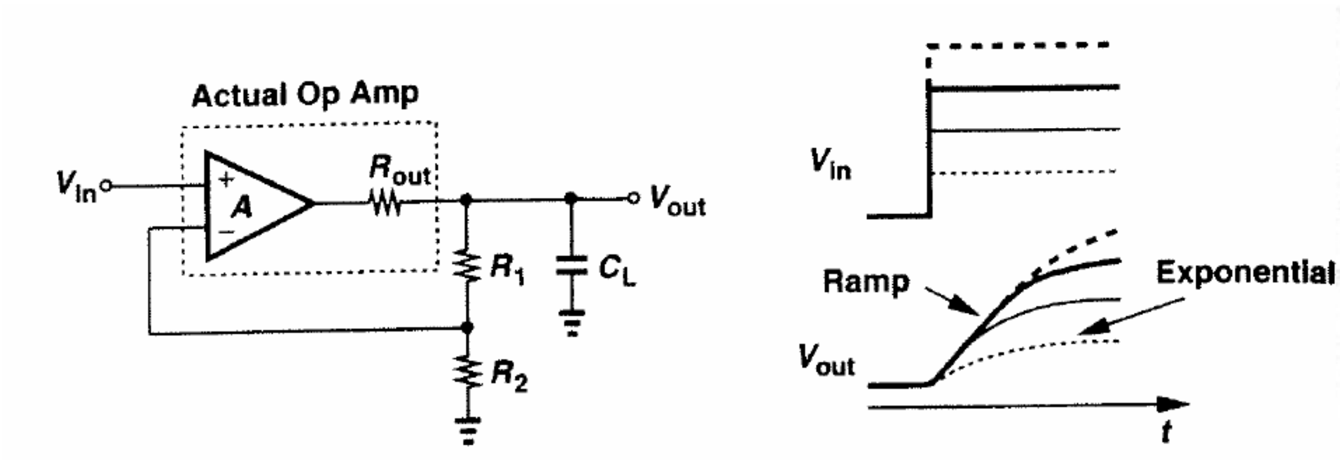
$$A_{dm} = \frac{g_{m1}}{g_{o2} + g_{o4}}$$

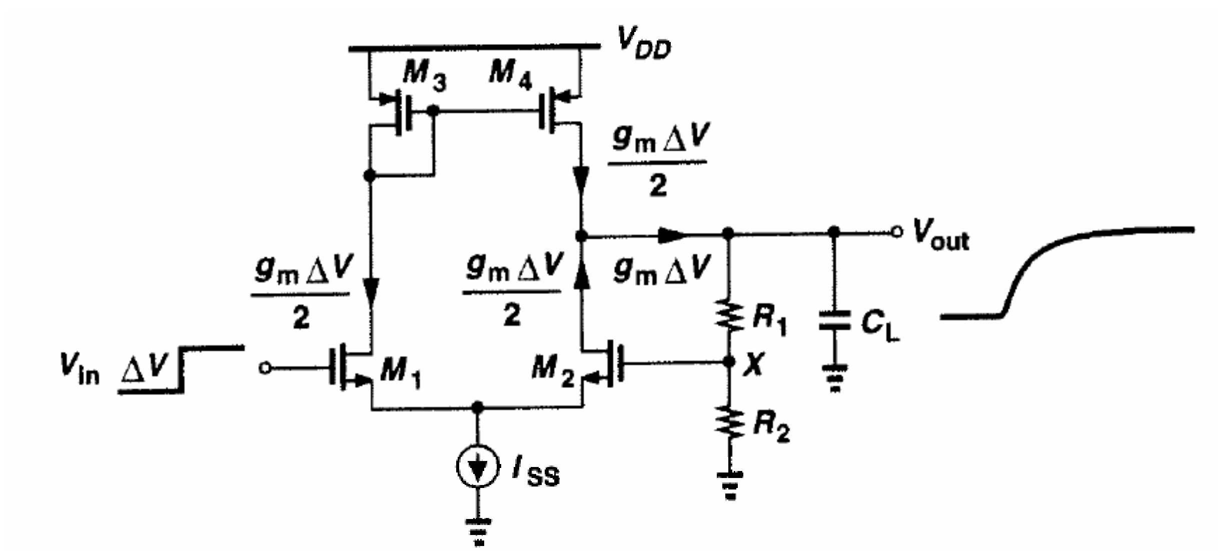
$$A_{vdd} \approx 1$$

$$PSRR^{+} \approx \frac{g_{m1}}{g_{o2} + g_{o4}}$$

OTA Slew Rate

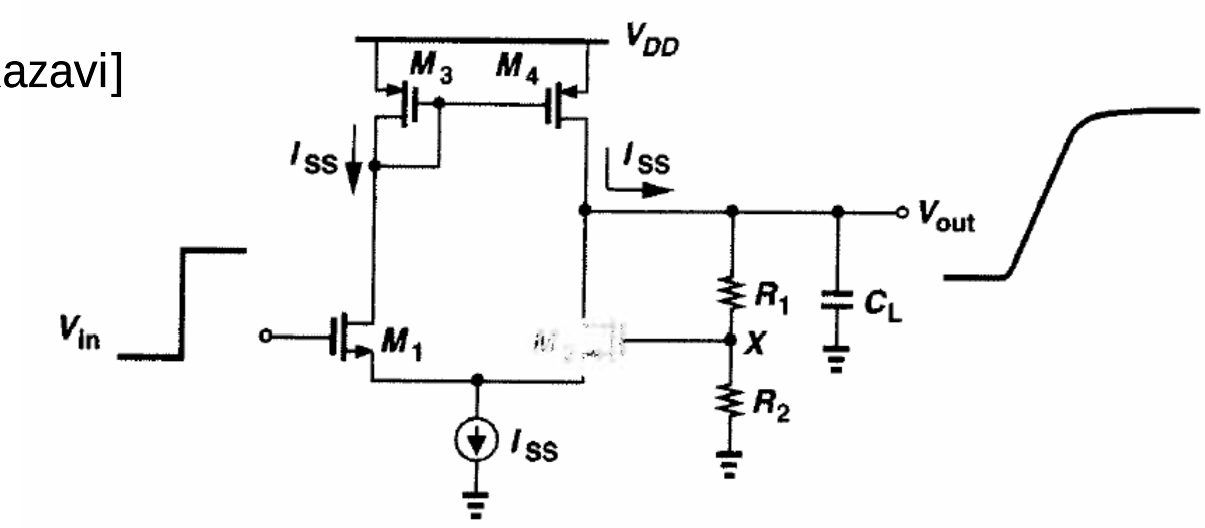
[Razavi]

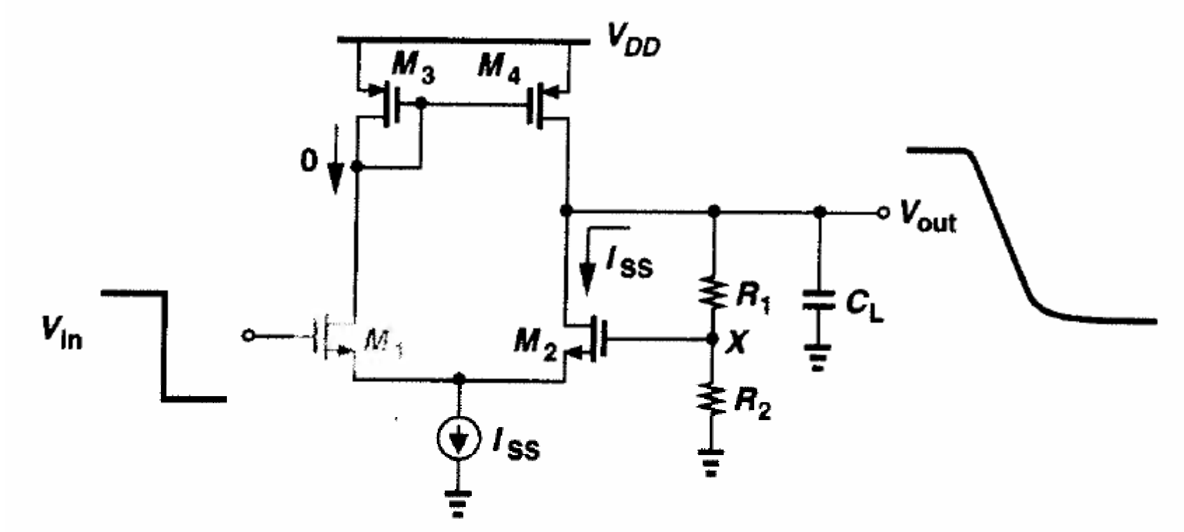




OTA Slew Rate

[Razavi]





$$\textbf{Slew Rate} = \frac{I_{SS}}{C_L}$$

Next Time

- Other OTA Circuits