ECEN474: (Analog) VLSI Circuit Design
Fall 2011

Lecture 38: Distortion

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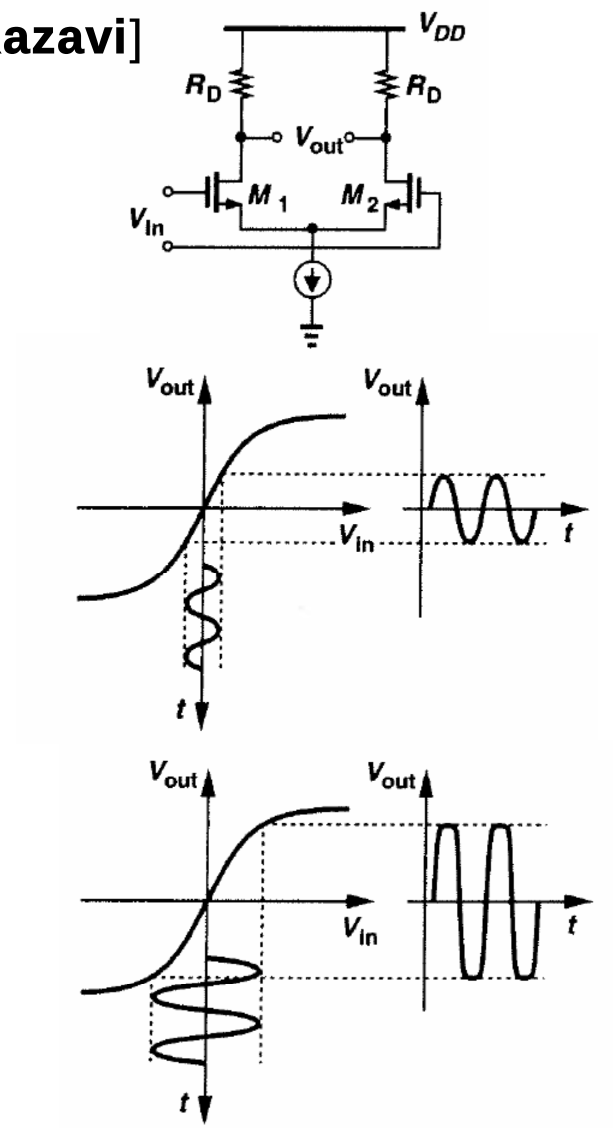
Agenda

- Circuit Nonlinearity
- Harmonic Distortion
- Examples
  - Common-Source Amplifier
  - Differential Pair
- Linearization Techniques
- Course Evaluations

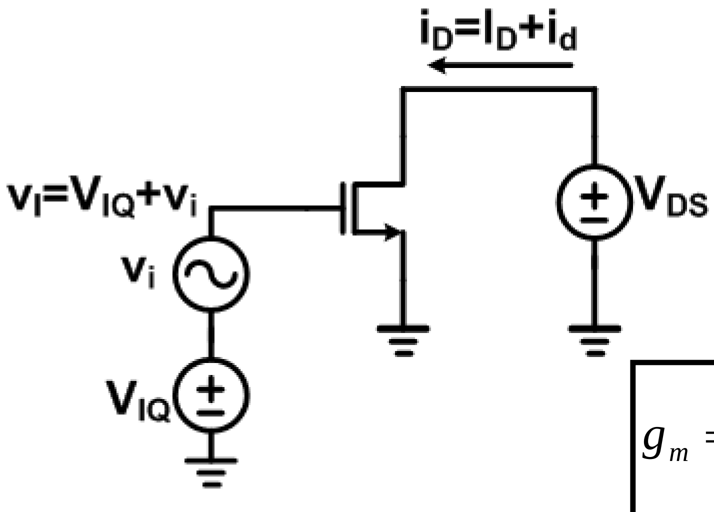
Circuit Nonlinearity

- Circuits generally exhibit nonlinear behavior if the input signal is large enough
  - Linearized small-signal models only model “small-signal” behavior about a given operating point
- This lecture covers low-frequency “memory-less” distortion
  - High-frequency distortion is much more complex
- It also assumes a constant region of operation (saturation)
  - Signal is large enough for distortion, but not large enough to enter triode

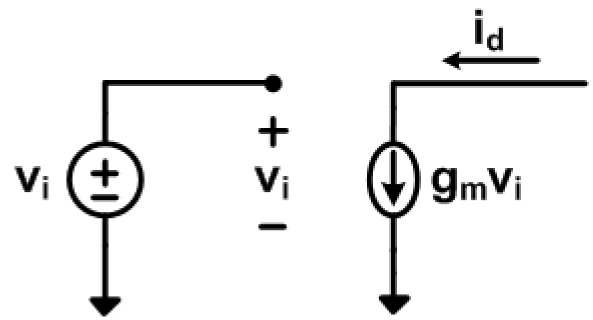
[Razavi]

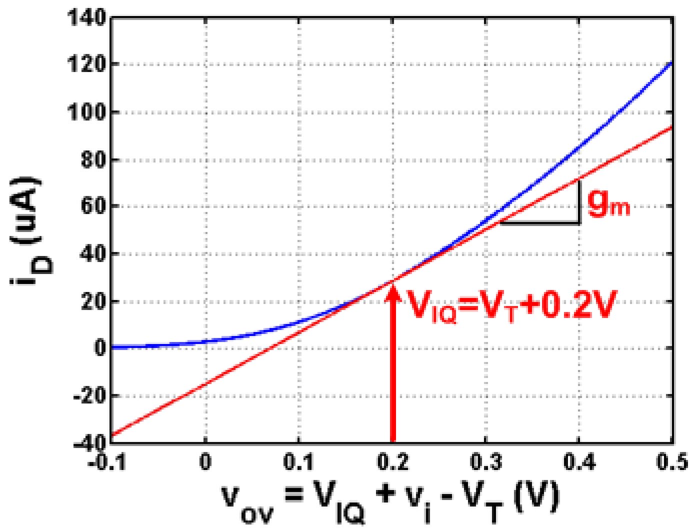


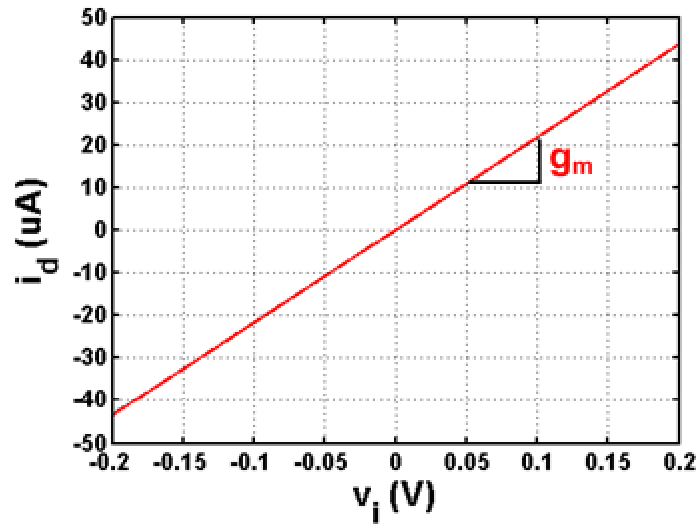
Small-Signal AC Model



$$g_m = \left.\frac{\partial I_D}{\partial V_I}\right|_{V_I=V_{IQ}} = f'(V_{IQ})$$







Taylor Series Model

We can model the nonlinear expression with a Taylor Series at V_{IQ}

$$f(v_I) = f(V_{IQ}) + \frac{f'(V_{IQ})}{1!}(v_I - V_{IQ}) + \frac{f''(V_{IQ})}{2!}(v_I - V_{IQ})^2 + \frac{f'''(V_{IQ})}{3!}(v_I - V_{IQ})^3 + \ldots$$

The small-signal behavior about V_{IQ} **can be modeled with**

$$v_i = v_I - V_{IQ} \quad \textbf{and} \quad i_d = i_D - I_D = f(v_I) - f(V_{IQ})$$

We can express i_d **as**

$$i_d = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

where

$$a_m = \frac{f^{(m)}(V_{IQ})}{m!}$$

$$a_1 = g_m \quad a_2 = \frac{1}{2} g_m' \quad a_3 = \frac{1}{6} g_m''$$

- For hand analysis, it is often sufficient to truncate the Taylor Series after the 3rd to 5th term

Harmonic Distortion Analysis

Applying a sinusoidal input signal to the Taylor Series model

$$v_i = V_m \cos(\omega t)$$

$$i_d = a_1 V_m \cos(\omega t) + a_2 [V_m \cos(\omega t)]^2 + a_3 [V_m \cos(\omega t)]^3 + \ldots$$

Recall $\cos^2(x) = \frac{1}{2}[\cos(2x) + 1]$ **and** $\cos^3(x) = \frac{1}{4}[\cos(3x) + 3\cos(x)]$

$$i_d = \left[\frac{1}{2} a_2 V_m^2\right] \quad \textbf{DC Shift}$$

$$+ \left[a_1 V_m + \frac{3}{4} a_3 V_m^3\right] \cos(\omega t) \quad \textbf{Fundamental Component}$$

$$+ \left[\frac{1}{2} a_2 V_m^2\right] \cos(2\omega t) + \left[\frac{1}{4} a_3 V_m^3\right] \cos(3\omega t) + \ldots.. \quad \textbf{Harmonic Components}$$

- The quadratic term a_2 generates a 2^{nd} harmonic tone and a DC shift
- The cubic term a_3 generates a third harmonic term and modifies the fundamental tone amplitude with gain compression ($a_3<0$) or gain expansion ($a_3>0$)

Harmonic Distortion Metrics

$$HD_2 = \frac{\textbf{second harmonic distortion signal amplitude}}{\textbf{fundamental signal amplitude}}$$

$$HD_3 = \frac{\textbf{third harmonic distortion signal amplitude}}{\textbf{fundamental signal amplitude}}$$

Including only terms up to third order :

$$HD_2 \cong \left| \frac{\frac{1}{2} a_2 V_m^2}{a_1 V_m + \frac{3}{4} a_3 V_m^3} \right| \cong \frac{1}{2} \left| \frac{a_2}{a_1} \right| V_m$$

$$HD_3 \cong \left| \frac{\frac{1}{4} a_3 V_m^3}{a_1 V_m + \frac{3}{4} a_3 V_m^3} \right| \cong \frac{1}{4} \left| \frac{a_3}{a_1} \right| V_m^2$$

- Generally the harmonic distortion is quantified in dB, taking $20\log_{10}$ of the above ratios

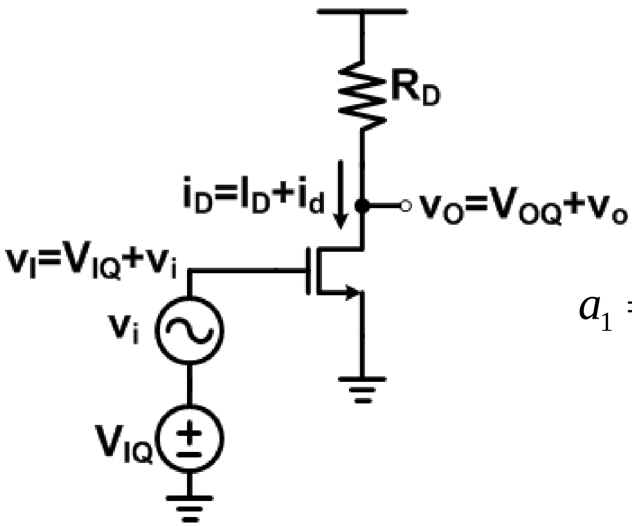
Total Harmonic Distortion

$$THD = \frac{\textbf{total power of distortion signals}}{\textbf{fundamental signal power}}$$

$$= HD_2^2 + HD_3^2 + HD_4^2 + \ldots$$

- Often dominated by the HD_2 and/or HD_3 term
- Typical requirements
  - High-quality audio (CD) 0.01% (-80dB)
  - Video 0.1% (-60dB)

Common-Source Amplifier Distortion



How large can v_i be to satisy a given THD spec?

$$i_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left(V_{IQ} + v_i - V_T\right)^2$$

$$a_1 = \left.\frac{\partial i_D}{\partial V_I}\right|_{V_I = V_{IQ}} = \mu C_{ox}\frac{W}{L}\left(V_{IQ} - V_T\right) = \mu C_{ox}\frac{W}{L}V_{ov} = \frac{2I_{DQ}}{V_{ov}} = g_m$$

$$a_2 = \left.\frac{1}{2}\frac{\partial^2 i_D}{\partial^2 V_I}\right|_{V_I = V_{IQ}} = \frac{1}{2}\mu C_{ox}\frac{W}{L} = \frac{I_{DQ}}{V_{ov}^2}$$

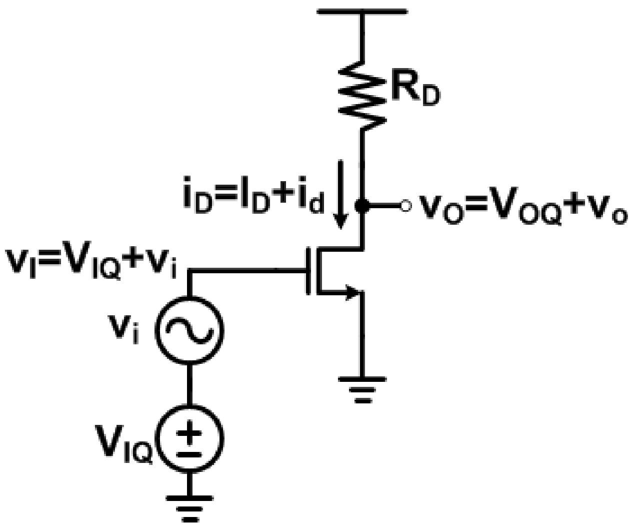
$$a_3 = \left.\frac{1}{6}\frac{\partial^3 i_D}{\partial^3 V_I}\right|_{V_I = V_{IQ}} = 0$$ **higher order a_m terms will also be zero**

$$HD_2 \cong \frac{1}{2}\left|\frac{a_2}{a_1}\right|V_m = \frac{1}{2}\left(\frac{I_{DQ}}{V_{ov}^2}\right)\left(\frac{V_{ov}}{2I_{DQ}}\right)V_m = \frac{1}{4}\frac{V_m}{V_{ov}} \qquad HD_3 \cong \frac{1}{6}\left|\frac{a_3}{a_1}\right|V_m^2 = 0$$

For 1% (-40dB) THD :

$$\frac{1}{4}\frac{V_m}{V_{ov}} \le 0.01 \Rightarrow V_m \le 0.04V_{ov}$$

Common-Source Amplifier Distortion



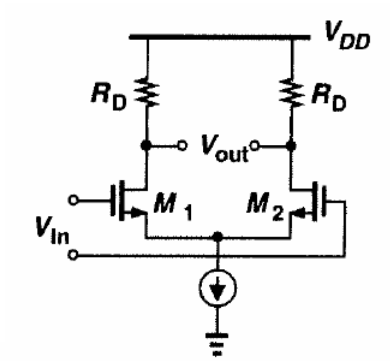
For 1% (-40dB) THD :

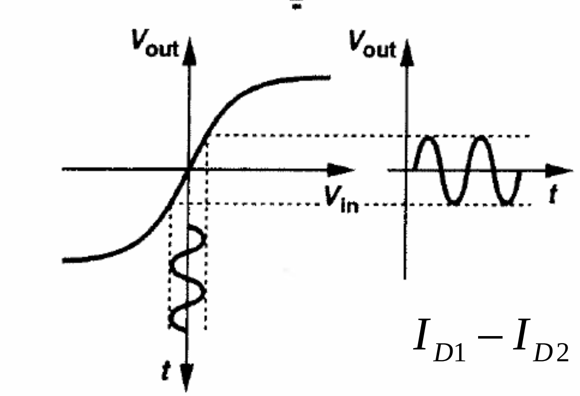
$$\frac{1}{4}\frac{V_m}{V_{ov}} \le 0.01 \Rightarrow V_m \le 0.04 V_{ov}$$

If $V_{ov} = 1\text{V}$**, then** $V_m \le 40\text{mV}$

- For an ideal square-law MOSFET, a_3 and higher terms are zero
- However, real MOSFETs deviate from this ideal behavior and will display higher order harmonics

Differential Pair Distortion





[**Razavi**]

$$I_{D1}-I_{D2}=\mu C_{ox}\frac{W}{L}V_{OV}v_i\sqrt{1-\frac{v_i^2}{4V_{OV}^2}}=\mu C_{ox}\frac{W}{L}V_{OV}v_i\sqrt{1-x}$$

where $x=\dfrac{v_i^2}{4V_{OV}^2}$

Performing a Taylor Expansion w/ only 1 term

$$\sqrt{1-x}\Rightarrow 1-\frac{x}{2}$$

$$I_{D1}-I_{D2}\cong \mu C_{ox}\frac{W}{L}V_{OV}v_i\left[1-\frac{v_i^2}{8V_{OV}^2}\right]=\mu C_{ox}\frac{W}{L}V_{OV}\left[V_m\cos(\omega t)-\frac{V_m^3\cos^3(\omega t)}{8V_{OV}^2}\right]$$

$$I_{D1}-I_{D2}\cong g_m\left[V_m-\frac{3V_m^2}{32V_{OV}^2}\right]\cos(\omega t)-g_m\frac{V_m^3\cos(3\omega t)}{32V_{OV}^2}$$

$$HD_3=\frac{V_m^2}{32V_{OV}^2}$$

Linearization Techniques

- Source Degeneration

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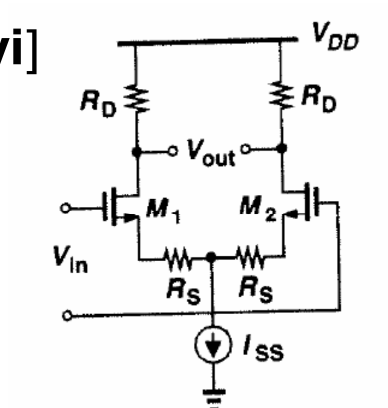
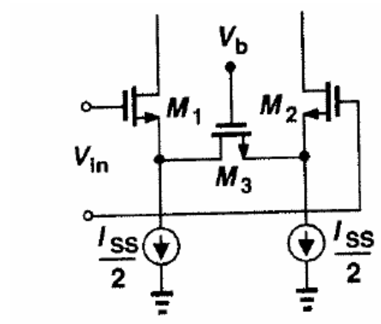
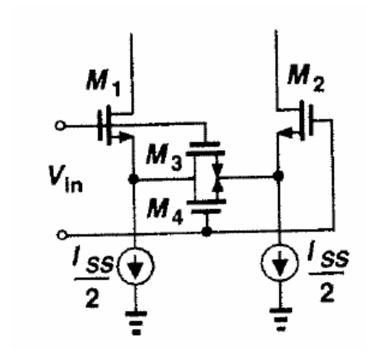
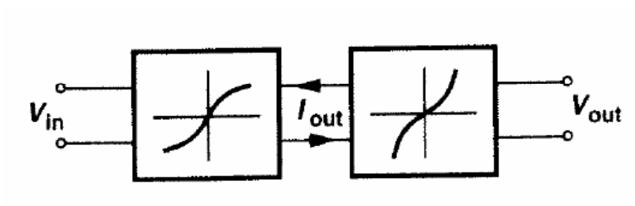
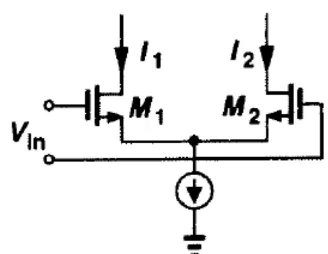


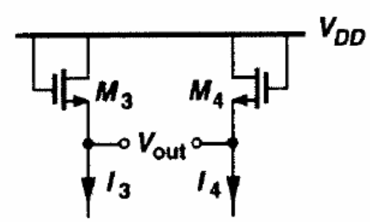


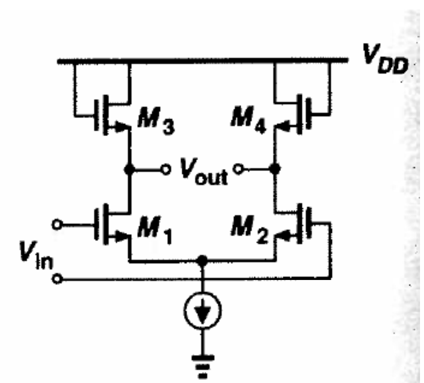


- Cascaded Nonlinear Stages









Next Time

- Exam 3 Review Session