Fundamentals on Boost (Step Up) Converters

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Application of Texas Instruments TPS61020

AC24V (un-regulated)

Diode Rectifier → Buck Converter → 3.3V/0.5A

BOOST TPS61020

Control System including Wifi

AA alkaline battery Backup Power

Home Automation – Thermostat

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Boost Converters
Fancy Electric Cars
Boost Converters: Applications

12V or 24V nominal $V_{BATT}$

Boost Converter

1.25A

1.25A

51V (125W)

Reservoir cap

Injector coils

Full custom design resource: PMP8532
Review of Battery Charger Topologies, Charging Power Levels, and Infrastructure for Plug-In Electric and Hybrid Vehicles

Murat Yilmaz, Member, IEEE, and Philip T. Krein, Fellow, IEEE
Boost Converters: Principles

- Output voltage is larger than Vin, but output current is smaller
- Pin is approximately equal to Pout

- If the MOSFET gate driver sticks in the “on” position, then there is a short circuit to ground through the MOSFET
- If the load is disconnected during operation when switch is in “off” position, then L continues delivering current to the right and very quickly charges C and may generate very large output voltage.
Boost Converters: Principles

Switch closed for DT seconds: Charging phase

\[
\frac{di_L}{dt} = \frac{V_{in}}{L}
\]

- If the switch stays closed for very long time, the current continues increasing, only limited by the switch and/or battery!
If the Switch remains opened for \((1 - D)T\) seconds

\[
\frac{di_L}{dt} = \frac{V_{in} - V_{out}}{L}
\]

- We assume continuous conduction; \(i_L(t_0) > 0\).
- In this graph, diode’s voltage drop is ignored; it can be easily incorporated replacing \(V_{out}\) by \(V_{out} + V_D\) when computing the inductance’s current variation.
For $\phi_1$ the switch is on during the time period $t_0 \leq t \leq t_1$

$$i_L = \frac{v_B}{L} (t - t_0) + I_0 \quad ; \quad I_0 = i_L(t_0)$$

$$i_L(t = t_1) = \frac{v_B}{L} (t_1 - t_0) + I_0$$

For $\phi_2$ the switch is off, then

$$i_L(t) \approx I_1 + \frac{1}{L} (v_B - v_0)(t - t_1)$$
In the following derivation, it is assumed that \( \Delta v_o \) is small

\[
i_L(t_2) \approx I_1 + \frac{1}{L}(v_B - v_0)(t_2 - t_1) \quad \text{where} \quad I_1 = i_L(t = t_1)
\]

Since \( i_L(t_0) = i_L(t_2) \), then it follows

\[
I_0 = \frac{v_B}{L} (t_1 - t_0) + I_0 + \frac{v_B - v_0}{L} (t_2 - t_1)
\]

or

\[
\frac{v_B}{L} (t_2 - t_0) = \frac{v_0}{L} (t_2 - t_1)
\]

\[
v_0 = \frac{T_{ck}}{t_2-t_1} v_B \quad \text{then} \quad \frac{v_0}{v_B} = \frac{1}{1-D}
\]
Let us define $\frac{t_1-t_0}{T_{ck}} = D$; (when switch is closed)

$$I_{RMS}^2 = I_{AV}^2 + \frac{I_{pk-pk}^2}{12}$$

$$\lambda_1 = \frac{2I_{pk}}{DT_{ck}}; \lambda_2 = -\frac{2I_{pk}}{(1-D)T_{ck}}$$

$$\lambda_1 = \frac{v_B}{L}; \lambda_2 = \frac{v_{in} - v_{out}}{L}$$
Average Current and Voltage

\[ \lambda_1 = \frac{2I_{pk}}{DT_{ck}} = \frac{v_B}{L} \]

\[ I_{pk} = \frac{Dv_B}{2L}T_{ck} \]

\[ \frac{v_{0,AV}}{v_B} = \frac{1}{1 - D} \]

Diode is forward biased

\[ i_D \]

\[ i_{L(pk)} \]

\[ i_C \]

\[ i_{out} \]

\[ V_{in} \]

\[ V_{out} \]

\[ v_{out, Aver} = \frac{1}{1 - D} \quad v_{in}I_L = v_{out, Aver} \cdot I_{out} \quad or \quad \frac{v_{in}}{v_{out, Aver}} = \frac{I_{out}}{I_L} = 1-D \]

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Boost Converters
**Output Voltage Ripple**

**Charging phase:** $i_C = i_D - i_R$

**Discharging phase:** $i_C = -i_{load}$

$$C \Delta v_0 = \Delta Q = I_{load} D T_{ck}$$

Therefore, the output ripple can be estimated as:

$$\Delta v_0 = \left( \frac{I_{load\_average}}{C f_{ck}} \right) D$$

Diode is reverse biased
During charging phase

\[ v_L = V_{in}, \quad \frac{di_L}{dt} = \frac{V_{in}}{L} \]

During the discharging phase

\[ v_L = V_{in} - V_{out}, \quad \frac{di_L}{dt} = \frac{V_{in} - V_{out}}{L} \]

Current flowing through the inductor:

\[ I_{Lrms}^2 = I_{avg}^2 + \frac{1}{12} I_{pp}^2 = I_{in}^2 + \frac{1}{12} (\Delta I_L^2) \]

Worst case: \( \Delta I_L = 2I_{\text{average}} \)

\[ I_{Lrms}^2 = I_{in}^2 + \frac{1}{12} (2\Delta I_{in})^2 = \frac{4}{3} I_{in}^2 \]

\[ I_{Lrms} = \frac{2}{\sqrt{3}} I_{in} \]
Summary of results

- Voltage Gain
  \[ \frac{v_0}{v_{in}} = \frac{1}{1 - D} \]

- Voltage gain considering power Efficiency
  \[ \frac{v_0}{\eta v_B} = \frac{1}{1 - D} \quad \eta = \text{power efficiency} \]

- Inductance’s current ripple
  \[ \Delta I = 2I_{pk} = \frac{\eta v_B}{L} DT_{ck} \]

- Maximum current @ inductor
  \[ I_{Lmax} = I_{AV} + I_{pk} = I_{AV} + \frac{\eta v_B}{2L} DT_{ck} \]

- Switch current
  \[ I_{SWmax} = I_{Lmax} = I_{Dmax} \]

- Inductance’s current
  \[ I_{Lmax} = \frac{I_0}{1 - D} + \frac{\eta v_B}{2L} DT_{ck} = \frac{v_0}{R_L (1 - D)} + \frac{\eta v_B}{2L} DT_{ck} \]
Design Considerations

The switch must be able to manage the maximum current

\[ I_{SW\text{max}} = \frac{I_{0AV}}{1-D} + \frac{\eta v_B}{2L} DT_{ck} \]

Since \( \frac{v_0}{\eta v_B} = \frac{1}{1-D} \) \( \Rightarrow D = 1 - \frac{\eta v_B}{v_0} \)

**L-current ripple:**

\[ 2I_{pk} = \frac{\eta v_B}{L} DT_{ck} \]

\[ L = \frac{\eta v_B}{2I_{pk}} DT_{ck} = \frac{\eta v_B \left(1 - \frac{\eta v_B}{v_0}\right)}{\Delta I_L f_{ck}} \]

\[ L \approx \frac{v_B \left(1 - \frac{v_B}{v_0}\right)}{\Delta I_L f_{ck}} \]
Current Ripple:
\[ \Delta I \equiv 0.2 - 0.4 \times I_{LAV} \]

LIN: Ratio of Inductance ripple to Average current
\[ \text{LIN} = \frac{\Delta I}{I_{LAV}} = \{0.2 - 0.4\} \]

Load Capacitance
\[ \Delta v_0 = \frac{D I_{LAV} (1 - D) T_{ck}}{C} = \frac{I_{OUT,AV}}{C f_{ck}} \cdot D \]

Computation of load capacitance
\[ C \geq \left( \frac{I_{OUT,AV}}{\Delta v_0 f_{ck}} \right) D \]
AC analysis is tricky!

System operates under large signal conditions and it is very non-linear!

AC is for small signal sinewave waveforms! This system is not even close to that!
Similarly to any AC analysis, we have to assume small signal variations, and define an operating point!

Output of the PWM is duty cycle!

This chart includes resistive losses, otherwise, D=0.7 should result in V-D conversion ratio of approximately 4.

\[
\frac{v_o}{V_i} = \frac{1}{1 - D}
\]

\[
\frac{v_o}{DutyCycle} = \frac{1}{(1 - D)^2}
\]
Small signal model (TI)

Figure 1. Boost Converter Block Diagram

\[ G_{dv} = \frac{\hat{V}_O}{d} = G_{do} \left( 1 + \frac{S}{\omega_{Z1}} \right) \left( 1 - \frac{S}{\omega_{RHP-zero}} \right) \left( 1 + \frac{S}{\omega_0 \cdot Q} + \frac{S^2}{\omega_0^2} \right) \]

Where

\[ G_{do} \approx \frac{V_{IN}}{(1-D)^2} = \frac{V_o^2}{V_{IN}} \]

\[ \omega_{Z1} = \frac{1}{r_C \cdot C} \]

\[ \omega_{RHP-zero} \approx \frac{(1-D)^2 \cdot (R-r_l)}{L} \approx \frac{R}{L} \cdot \left( \frac{V_{IN}}{V_o} \right)^2 \quad \text{or} \quad f_{RHP-zero} \approx \frac{R}{2\pi \cdot L} \left( \frac{V_{IN}}{V_o} \right)^2 \]

\[ \omega_0 \approx \frac{1}{\sqrt{L \cdot C}} \sqrt{\frac{r_l + (1-D)^2 \cdot R}{R}} \approx \frac{1}{\sqrt{L \cdot C}} \cdot \frac{V_{IN}}{V_o} \quad \text{or} \quad f_0 \approx \frac{1}{2\pi \cdot \sqrt{LC}} \cdot \frac{V_{IN}}{V_o} \]

\[ Q \approx \frac{\omega_0}{\frac{r_l}{L} + \frac{1}{C \times (R+r_c)}} \]
Figure 7. Varying error signal generates a pulse-width-modulated switch signal.
Figure 9. Current-mode pulse-width modulation.
Figure 8. The MAX1932 provides an integrated boost circuit with voltage-mode control.
Figure 10. The MAX668 for current-mode-controlled boost circuit.
**Equivalent Impedance**

\[ V_{out} = \frac{V_{in}}{1 - D} \]

\[ I_{out} = (1 - D)I_{in} \]

\[ R_o = \frac{V_{out}}{I_{out}} \]

\[ R_{eq} = \frac{V_{in}}{I_{in}} = \frac{(1 - D)V_{out}}{I_{out}} = \frac{(1 - D)^2 V_{out}}{I_{out}} = (1 - D)^2 R_o \]
Final Advice:
Check websites and look for tutorials of the following companies:

- Texas Instruments
- Maxim
- Linear Technology
- And many others