Automatic Tuning of Coupled Inductor Filters

R. Balog and P.T. Krein
Grainger Center for Electric Machinery and Electromechanics
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801, USA

Abstract: A coupled inductor filter is presented with interesting notch mode filtering characteristics. Although this configuration has been long established, it has received little attention recently. The new contribution of this paper is to provide an automatic tuning process that can make use of the frequency domain characteristics of the coupled inductor building block. In a sense, this becomes an “active” filter technique that uses passive elements for implementation. The combination offers potential performance improvements in comparison with active filters.

Introduction

Coupled-inductor and other integrated-magnetic techniques have existed for many years [1],[2]. However, the topic is usually incorporated into a discussion of new power converter topologies or approached from a purely magnetic design perspective. In many cases the level of complexity in the treatment of the coupled magnetic device obscures the theory. Unlike most common electrical networks, coupled-inductor techniques involve simultaneous energy transfer through electrical and magnetic pathways. Because of this, it is helpful to approach the coupled inductor from a circuit–based filtering perspective. Whereas previously successful implementations of coupled inductors were often ill-received due to the perceived sensitivity of operation resulting in the tuning the magnetic device to tweak the performance, tuning a filter network is more familiar to most engineers than tuning the coupling in a magnetic device.

The technique described in this paper replaces a smoothing choke with a “smoothing transformer” (a coupled inductor) and a dc blocking capacitor. Together, this is a basic filter building block shown in Fig. 1. Because these components are a linear two-port filter, the building block can be implemented in any dc circuit to reduce the ripple current wherever a choke is currently used. Thus it may be applied to the dc input of a converter, a dc output, or an internal dc link in applications such as motor drives or HVDC transmission.

Fig. 1: Coupled inductor filter block

Dc power distribution systems are one interesting application of coupled inductor filters. It is a well known problem that instabilities can occur on dc power system due to interaction between the dc-dc power converters, passive filter components, and the dc bus itself. These instabilities can occur at high frequencies initiated by the switching action of the converter, or at low frequencies stimulated by “beat frequencies” that occur when two or more converters operate at proximal frequencies. A review of the literature reveals a large body of work establishing stability criterion [3]-[6]. Much of this work has involved design requirements on passive filter networks, or proposals for active filtering schemes. The tunable coupled inductor filter offers the ability to tune the filter network to minimize the propagation of disturbances between the power converter and the distribution bus.

This paper introduces a filter block that uses a coupled inductive network and ripple correlation control to automatically tune the frequency response of the filter. In a sense, the approach provides an “active” filter technique that uses passive elements for implementation. The combination offers potential loss and performance improvements in comparison with active filters.

Coupled inductor filter building block

The technique described in this paper takes advantage of a smoothing choke that is implemented with a coupled inductor. The addition of a dc blocking capacitor, as in Fig. 1, forms a two port linear filter that can be analyzed in the usual way [2]. In a typical application, such as a dc power supply, the port labeled $V_n$ or “noisy port” is connected to the output of a switching circuit and contains a desired dc component with superimposed ac noise. Assuming an infinite and ideal capacitor, the entire ripple component of the input voltage appears across the $L_{ac}$ winding. With perfect coupling and a unity turns ratio, the ripple component will be transferred to the $L_{dc}$ winding. Kirchhoff’s voltage law tells us that only the dc component of the input voltage will be present at $V_q$, the output or “quiet port.”

It can be misleading to think of the smoothing transformer’s windings in terms of a primary and secondary. Rather, it is convenient to denote the windings as “dc” and “ac” indicating their purpose in the circuit: the dc winding carries the heavy direct current (similar to a smoothing choke), while the ac winding carries only a small ac ripple current. Accordingly, the dc winding size is selected appropriately to carry the large dc current while the ac inductor may be wound using a smaller gauge.
The transfer function for the coupled inductor in Fig. 1 is:

\[
\frac{V_s}{V_o} = \frac{1 + s^2 C \cdot L_{ac} \left( 1 - k \cdot \frac{L_{ac}}{L_{dc}} \right)}{1 + s^2 C \cdot L_{dc}} \quad (1)
\]

The structure of the transfer function is similar to that of the second order low pass filter with the exception of a frequency dependent term \( s^2 C L_{ac} \) that appears in both the numerator and denominator. A nulling parameter \( k_{null} \) can be defined such that the numerator simplifies to unity. The value is:

\[
k_{null} = \frac{L_{ac}}{L_{dc}} \quad (2)
\]

When the magnetic coupling coefficient \( k \) matches \( k_{null} \), the coupled inductor filter degenerates into a second order low pass filter. The frequency response is determined by \( L_{ac} \) and \( C \). When the coupling coefficient \( k \) is not the same as \( k_{null} \), the location of the transfer function zero depends on the relationship of the actual coupling coefficient \( k \) to the calculated \( k_{null} \) as illustrated in Fig. 2. For \( k < k_{null} \) the filter has a characteristic notch or transmission zero.

The frequency of the notch can be found by considering the numerator of (1). \( L_{ac} \) and \( C \) form a series resonant circuit, reducing the impedance of that branch to zero for the frequency given by:

\[
\omega_{notch} = \frac{1}{\sqrt{C L_{ac} \left( 1 - k \cdot \frac{L_{ac}}{L_{dc}} \right)}} \quad (3)
\]

The notch mode offers the interesting possibility of significant attenuation at the notch frequency.

**Tuned Notch Mode**

Consider a conventional buck converter with an output smoothing inductor and capacitor. Adding a second winding with a series capacitor to the original magnetic core realizes a coupled inductor filter block. Fig. 3 and Fig. 4 compare the output ripple voltage of the second order low pass filter to that produced by the coupled inductor filter. In Fig. 4 the value of the capacitor in series with the \( L_{ac} \) winding is selected to create a notch at the switching frequency of the converter. The result is significant attenuation of the fundamental switching harmonic. With the energy spectral density shifted into higher frequencies, further filtering can be achieved with minimal effort.

It has been found both analytically and experimentally that the notch mode coupled inductor filter minimizes the output rms ripple when tuned so that switching occurs at the notch frequency.

**Tuning Techniques**

Sensitivity to variations in the values of the passive circuit elements causes the exact frequency of the notch to vary according to (3). Further, the high \( Q \) of the notch means that even modest component tolerances can cause the frequency of the notch to deviate from the nominal design and reduce or eliminate the desired filtering effect. This implies a need for a tuning operation to achieve the maximum performance from the coupled inductor filter.

Two methods are available for aligning the notch frequency to the converter switching frequency: tuning the converter switching frequency to the filter notch and tuning the filter notch to the converter frequency. Conventionally either tuning technique is often performed as an “end-of-line” step in the manufacturing process. However, this cannot compensate for drift due to aging or thermal cycling. The addition of automatic control creates an auto-tuning passive filter block that can track changes in system operation.
Ripple Correlation Control

A promising candidate for auto-tuning is the ripple correlation control method [7]. Given a function with a unique minimum (or maximum), such as that in Fig. 5, the function $J(x)$ can be minimized by applying the correlation control law:

$$u(t) = -k \int_0^t \frac{dJ(x(\tau))}{d\tau} \cdot \frac{dx}{d\tau} d\tau$$

(5)

Here $dx/dt$ is a measured ripple signal on the variable $x$. The integral can be rewritten in terms of a chain rule, $dJ/dt = (dP/dx)(dx/dt)$, which, in the presence of ripple, shows that (5) will drive input $u(t)$ to minimize function $J(x)$. In previous work [8], the function $J(x)$ has been presented as total power to be maximum or system losses to be minimized while $x(t)$ was a voltage or current ripple signal. In the present context, $J(x)$ represents some measure of filter performance such as output voltage ripple, while $x(t)$ is the associated time varying ripple signal.

An alternative to using ripple as the cost function can be found by considering the phase information associated with the notch. The phase response of the coupled inductor filter in the vicinity of the notch is undergoing a sharp transition as shown in Fig. 6. Depending on the $Q$ of the notch it may be difficult to ascertain the exact phase at the notch frequency but it is clear that it is rapidly passing through $-90$ degrees in the vicinity of the notch. Ripple correlation control can be chosen to drive the phase to $-90$ degrees and thus lock in the frequency of the notch. Using this phase relationship rather than a ripple amplitude-based method results in a robust control since the phase transition always occurs regardless of the quality factor of the filter.

Consider a new cost function that is minimized when the phase of the output voltage is $-90$ degrees with respect to the input filter voltage. This condition ensures that the system is operating at or in the neighborhood of the notch frequency.

$$u(t) = -k \int_0^t \text{corr}[a(\tau) \cdot b(\tau)] d\tau$$

(6)

Phase correlation can be performed by a simple mixing operation. Since the voltage on the noisy port of the coupled inductor filter is a switching function, the ripple component of the filter output can be written in terms of the Fourier series of the switching function multiplied by the

$$\sum_{n=1}^\infty a_n \cos(n\omega t + \phi_n)$$

(7)

where $h(n\omega)$ is a complex number representing the filter magnitude and phase and $\theta_n$ is the phase of the $n^{th}$ harmonic. The expression is simplified by associating the magnitude and phase terms:

$$\sum_{n=1}^\infty \cos(n\omega t + \phi_n)$$

(8)

where $\phi_n$ is the composite phase of the filter response and the $n^{th}$ Fourier term.

The output voltage mixed with a local oscillator synchronized to the switching function generates a sum and a difference component:

$$LO \otimes \bar{v}_{out}(t) = \sum_{n=1}^\infty a_n \cos(n\omega t + \phi_n)$$

(9)

The infinite series includes harmonic components at all multiples of the fundamental. For $n=1$, the product of the local oscillator and the fundamental switching component yields a dc term and a double frequency term. A low pass filter rejects the double frequency term while the difference term passes through to produce a dc voltage. The filter also removes all the other frequency components resulting from the mixing of the harmonic switching frequencies with the local oscillator. The low-pass filter is straightforward with the roll-off frequency chosen well below the fundamental. Consider only the sign of the arguments. Then the amplitude term can be ignored, resulting in a dc voltage representation of the correlation of the phase of the ripple voltage fundamental frequency with the phase of the local oscillator:

$$\text{phase correlation} = \cos(\phi_n - \phi_{LO})$$

(10)

The phase correlation is a maximum value when the phase of the local oscillator and the ripple voltage are in phase. In the case of the coupled inductor filter, at the notch the ripple voltage and local oscillator are in quadrature and results in a dc correlation voltage of zero.
The analysis so far has assumed that the local oscillator is spectrally pure, that is, it contains only one frequency. This assumption requires the separate generation of a high quality, low distortion, local oscillator synchronized to the switching function. To reduce the complexity of implementation, it is desirable to use the switching function itself as the local oscillator. Assuming the switching function contains only harmonic frequency components, (9) is re-written with the local oscillator expressed as a Fourier series of the switching function indexed by m:

\[ LO \otimes \tilde{v}_{\text{cor}}(t) = \sum_{n=1}^{\infty} b_n \cos(n \alpha + \phi_n) \sum_{m=1}^{\infty} a_m \cos(n \alpha + \phi_m) \]

\[ = \sum_{n=1}^{\infty} a_n b_n \left[ \cos(n \alpha + \phi_n - m \alpha - \phi_m) + \cos(n \alpha + \phi_n + m \alpha + \phi_m) \right] \]

In the case of \( n=m=1 \) the result is identical to (9). Similarly, for \( n \neq m \) there is no dc frequency term. However, for harmonic of the switching frequency where \( n=m \) there is now a dc component. The result is the introduction of additional phase error terms not present with the single frequency local oscillator. Collectively this can be referred to as the discriminator error:

\[ \text{phase correlation} = \cos(\phi_n - \phi_m) + \sum_{n=2}^{\infty} \cos(\phi_n - \phi_m) \]

\[ = \cos(\phi_n - \phi_m) + \left( \text{discriminator error} \right) \]

The discriminator error will cause the correlation process to convergence to a frequency where the filter phase is something other than –90 degrees. Depending on the magnitude of the discrimination error term and the Q of the filter this frequency may be far from the actual notch and reduce the performance of the filter. The designer must weight the trade-off between allowable convergence error and the ease of implementation of not needing a separate LO.

It has been shown [7] that the correlation control law in (5) converges to the minimum of \( J(x) \) if the function is unimodal, i.e., has a unique global minimum. In general, the phase of a filter network crosses –90 degrees at more than one frequency, as shown in Fig. 6, and therefore \( J(x) \) can have more than one minimum. In order to guarantee that the correlation control converges to the desired frequency, the problem must be constrained such that the frequency \( f \in F = [F_{\text{min}}, F_{\text{max}}] \) where \( F_{\text{min}} \) and \( F_{\text{max}} \) are determined by the tolerance of the filter components and the coupling coefficient \( k \). As Fig. 6 suggests, this is not a severe limitation. Without special design effort, a tuning range on the order of 10:1 can be achieved.

**Application and results**

Consider again the buck converter with the coupled inductor output filter with the frequency response shown in Fig. 6. The nominal system frequency was selected to be 50kHz to correspond with the notch frequency. Ripple correlation control was used to tune the switching frequency to the notch frequency by using the phase correlation between the switching function and the output ripple voltage as shown in Fig. 7.

**Fig. 7: Auto-tuning of switching frequency on a buck converter**

**Fig. 8: Block diagram of correlation control**

The block diagram for the ripple correlation control is shown in Fig. 8 and consists of three sections: the differentiator that operates on the ripple signal, the phase discriminator that produces a voltage proportional to the difference in phases between the time derivative of the ripple signal and a local oscillator, and the correlation integrator that generates the control signal \( u(t) \).

The gradient information of the cost function is required to determine the direction to drive the system change. When applied to the phase correlation process, the time derivative of the ripple signal is taken. The ideal differentiator circuit has a magnitude response with a positive slope and unity gain at 1 rad/sec and, if implemented by an inverting op-amp, a constant –90 degree phase shift. This operation emphasizes higher frequencies which results in extreme susceptibility to noise. In addition, the –90 degree phase shift must now be compensated for in the phase discriminator. Further, the correlation process is only concerned with the relative phase at the switching and notch frequencies and in fact the presence of energy at lower frequencies can propagate through the system and result in oscillations in \( u(t) \).

A more robust design for the differentiator stage is to limit the bandwidth of the circuit. A two pole differentiator can be designed to have positive gain from 1 kHz to about 1.2 MHz. The phase contribution of each poles results in a net phase shift through the differentiator of –180 degrees at frequencies in the neighborhood of the notch. An additional advantage to bandwidth limiting the differentiator is that by carefully choosing the second pole frequency, the switching function itself can be used instead of a dedicated local oscillator.

The phase error from the discriminator in the prototype circuit was observed to be negative for \( f_{\text{switch}} \) greater than \( f_{\text{notch}} \). From the Bode plot for the coupled inductor filter, \( \phi_n - \phi_{LO} \) is greater than –90 degrees for \( f_{\text{switch}} \) greater than \( f_{\text{notch}} \). The differentiator circuit contributes an additional 180 degrees of phase shift in the neighborhood of 50kHz so
the phase discriminator expression can be re-written as:

\[
\text{phase error} = a_o \cos(-180 + \phi_o - \phi_{lo}) / |\omega| \text{MHz}
\]

(13)

The ideal integrator has a positive gain for extremely low frequencies, unity gain at 1 rad/sec, and a constant -90 degree phase shift. At this point the phase shift is irrelevant and the output of the integral control ideally has only low frequency components. The location of the system zero will determine how quickly the integral controller responds to a disturbance and the gain term will determine the rate of convergence to the equilibrium point.

The phase correlation process provides the input for the integral control. For a frequency below the notch the phase is less than -90 degree and results in a negative correlation value. The integral control with negative gain in (6) will respond by driving the control input \( u(t) \) positive and command the voltage controlled oscillator (VCO) to increase the frequency. As the frequency converges to the notch, the phase error converges to zero and the rate of change of the control \( u(t) \) slows. At the notch, \( u(t) \) becomes a constant value. Similarly, for an initial frequency greater than the notch, the phase error is positive. The control law will generate a negative \( u(t) \) and command the VCO to lower the frequency.

Initially the control input \( u(t) \) was disconnected from the VCO and the converter manually tuned to a frequency near the notch. The small positive phase error, consistent with a frequency just below the notch, caused integrator capacitor voltage windup. After the control input was connected to the VCO, the large initial condition on \( u(t) \) forced the converter switching frequency to abruptly change to a maximum value of 75.46 kHz and the phase error to correspondingly jump negative as shown in Fig. 9. The correlation control began to reduce the switching frequency by driving the phase error to zero. For the correlation gain chosen, after approximately six seconds the system had reached an equilibrium frequency of 50.10 kHz. The measured notch frequency from the Bode plot for this filter is 50.159 kHz.

In steady state, the phase error signal had a ripple of about 75mV pk-pk at a frequency much less than the switching frequency, Fig. 10, representing hunting as the system oscillated about the minimum of the cost function. The average value of the phase error signal is positive confirming that the equilibrium frequency of the correlation function is slightly less than the notch due to discriminator error contributed by harmonic frequencies.

Saturable Inductor

The previous method integrates the coupled inductor into the control of the switching converter to tune the frequency of operation. A second method of tuning treats the coupled inductor as a stand-alone automatically tuned filter. In order to tune the notch response of the passive filter it is proposed to add a small saturable inductor in the shunt branch of the filter as in Fig. 11. A separate dc coil on the saturable inductor permits real-time electrical adjustment of \( L_{tune} \) as shown in Fig. 12.

The transfer function of the tunable coupled inductor filter is:

\[
\frac{V_o}{V_n} = \frac{1 + (j\omega)^2 C_o L_o \left(1 - k \frac{L_{dc}}{L_o} + \frac{L_{tune}}{L_o}\right)}{1 + (j\omega)^2 C_o (L_o + L_{tune})}
\]

(4)

The transmission zero of the filter can now be adjusted by control of \( L_{tune} \). Experimental results confirm the (nearly linear) ability to control the notch frequency by controlled saturation of the tuning inductance. Fig. 13 illustrates the

![Fig. 9: Convergence of the correlation control](#)

![Fig. 10: \( u(t) \) hunting about optimal frequency](#)

![Fig. 11: Buck converter with auto-tuning coupled inductor filter](#)
tuning of the notch frequency. In the automatic tuning method, the control function \( u(t) \) adjusts \( L_{\text{tune}} \) via \( V_{\text{control}} \) and therefore the notch frequency.

**Conclusion**

A control strategy has been introduced to implementing an auto-tuning coupled inductor filter block. Correlation control was shown as the means of tuning the notch frequency of the coupled inductor filter. The approach can be used to eliminate a specific frequency, such as switching frequency from the output of a dc-dc converter in a distributed dc power system application. In future work, the auto-tuning coupled inductor filter will be investigated as the input filter for power converter modules in a distributed dc power system.

**Acknowledgement**

This research is supported through the International Telecommunications Energy Conference (INTELEC) Fellowship program.

**References**


