ECEN 607 (ESS)

HOW TO APPROXIMATE AN EXPRESSION FOR 3dB BANDWIDTH OF A MULTIPLE POLE SYSTEM?

Let us consider an all pole system

$$A(s) = \frac{A_o}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})(1 + s/\omega_{p3}) \cdots (1 + s/\omega_{pn})}$$

$$|A(j\omega)| = \frac{A_o}{\left\{\left(1 + (\omega/\omega_{p1})^2\right)\left(1 + (\omega/\omega_{p2})^2\right)\left(1 + (\omega/\omega_{p3})^2\right)\cdots\right\}^{1/2}}$$

The 3dB cutoff frequency ω_{3dB} is defined at magnitude value $A_o/\sqrt{2}$, therefore

$$2 = \left(1 + (\omega_{3dB}/\omega_{p1})^2\right)\left(1 + (\omega_{3dB}/\omega_{p2})^2\right)\left(1 + (\omega_{3dB}/\omega_{p3})^2\right)\cdots$$

Since $\omega_{3dB} < \omega_{p1}, \omega_{p2} \cdots, \omega_{pn}$ one can approximate

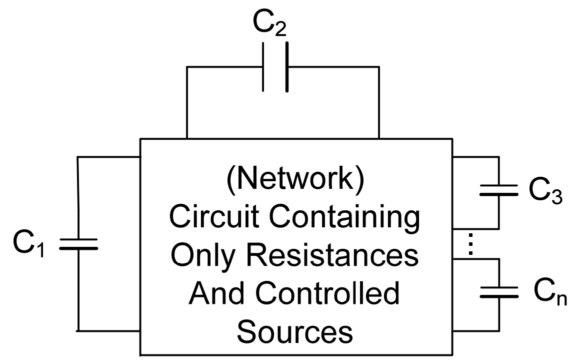
$$2 \cong 1 + \omega_{3dB}^2 \left(\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \frac{1}{\omega_{p3}^2} + \cdots \right)$$

Hence

$$\omega_{3dB}^2 = \frac{1}{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \frac{1}{\omega_{p3}^2} + \cdots} \qquad \omega_{3dB}^2 \cong \frac{1}{\sum_{i=1}^{n} \frac{1}{\omega_{pi}^2}}$$

Another approach to obtain the ω_{3dB} is by means of "The Time - Constant Method".

Consider the network consisting of capacitors, resistors and depending sources



The transfer function of the network above can be expressed as

$$H(s) = \frac{N(s)}{1 + a_1 s + a_2 s^2 + \cdots\cdots}$$

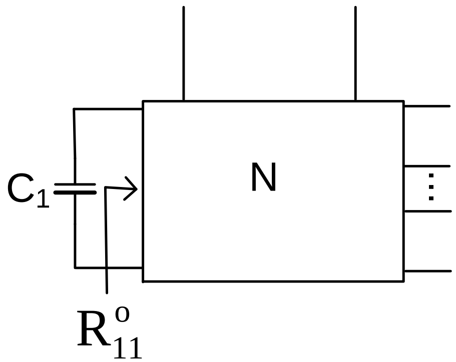
It can be proved that a_1 is the sum of the time constants and a_2 is the sum of the product of time constants.

Lets consider the case of three capacitors,
thus

$$a_1 = R_{11}^{o} C_1 + R_{22}^{o} CL_2 + R_{33}^{o} C_3$$

where $R_{11}^{o}, R_{22}^{o}, R_{33}^{o}$ are the zero frequency resistances seen by C_1, C_2 and C_3 respectively.

For example to compute R^o_{11} the circuit shown is used



$$C_2 = C_3 = 0$$

Thus if we obtain R^o_{22} and R^o_{33} in a similar way, one can express a_1 as

$$a_1 = \sum_{i=1}^{n} R^o_{ii} C_i = \sum_{i=1}^{n} \tau_i$$

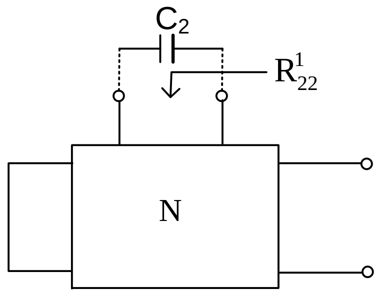
a_1 can be seen as the sum of open - circuit time constants. Thus an approximation of ω_{3dB} can be as

$$\omega_{3dB} \cong \frac{1}{a_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\omega_{p_i}}}$$

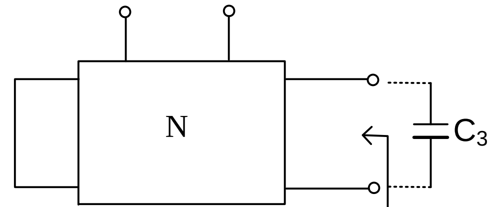
For the sake of completeness let us discuss how to compute a_2.

$$a_2 = R^o_{11} C_1 R^1_{22} C_2 + R^o_{11} C_1 R^1_{33} C_3 + R^o_{22} C_2 R^2_{33} C_3$$

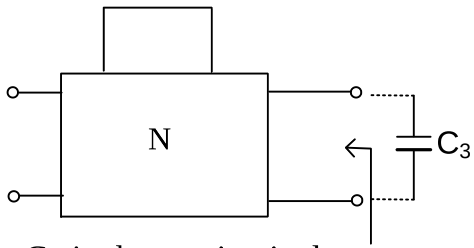
Where R^j_{ii} is the zero - frequency resistance seen by C_i when C_j is short - circuited.



C_1 is short- circuited
C_3 is open circuited



C_1 is short-circuited
C_2 is open circuited



C_2 is short - circuited
C_1 is open circuited

Notation Remarks

R^j_{ii} , ii Indicates the terminals at which the resistance is computed

j denotes the capacitance that is shorted.

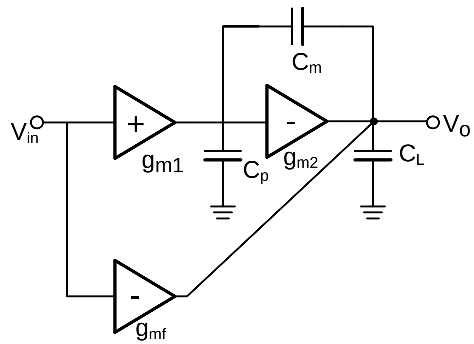
The general form of each term in a_2 is

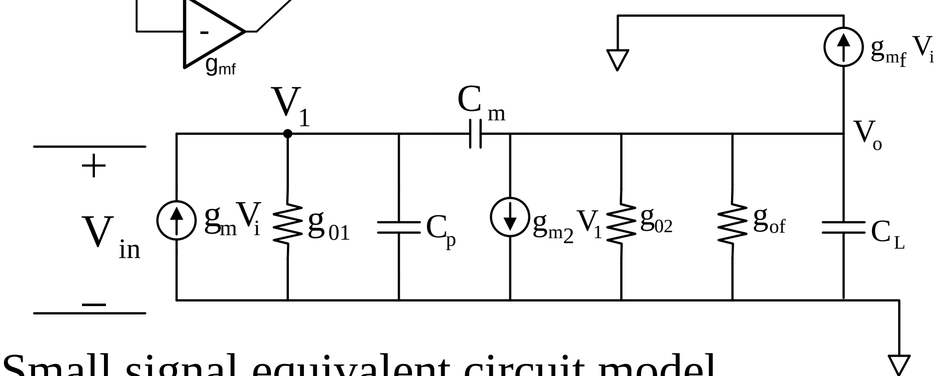
$$R^o_{ii} C_i R^i_{jj} C_j = R^o_{jj} C_j R^j_{ii} C_i$$

For instance if $R^o_{22} C_2 R^2_{33} C_3$ is replaced by $R^o_{33} C_3 R^3_{22} C_2$ the value of a_2 is not modified.

Ref. J. Millman and A. Grabel "Microelectronics", 2nd Edition, New York, 1987.

OPEN – CIRCUIT TIME CONSTANT TECHNIQUE: A NESTED Gm-C EXAMPLE

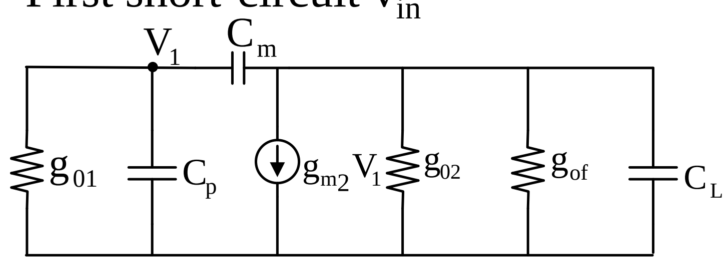




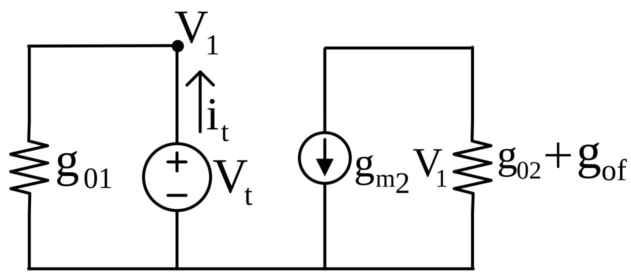
Small signal equivalent circuit model

There are three capacitors, therefore 3 times constants.

First short-circuit v_{in}



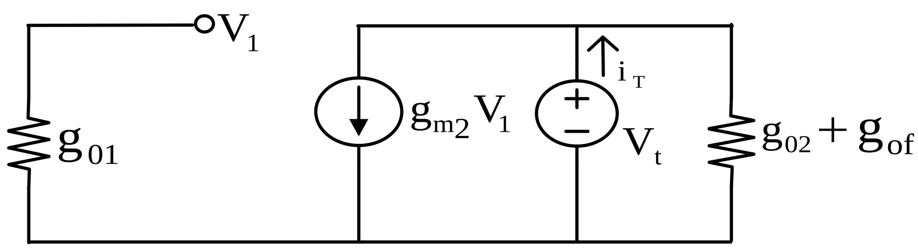
Time Constant Associated With C_p



Therefore

$$R_t = R_{thevenin} = 1/g_{01} = R_p^o \text{ , } \tau_p = C_p/g_{01}$$

Time Constant Associated with C_L

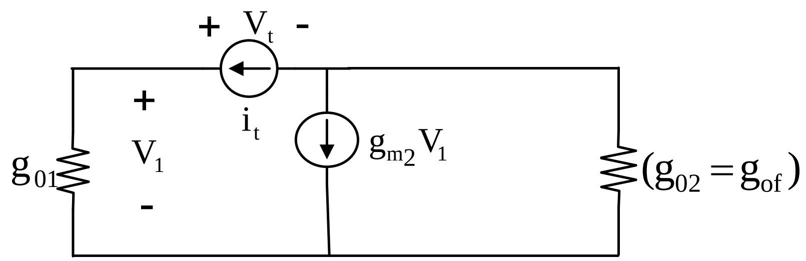


$V_1 = 0$

Thus $$R_T = \frac{1}{g_{02} + g_{of}}$$

$$\tau_L = \frac{C_L}{g_{02} + g_{of}}$$

Time Constant Associated with C_m



$$V_1 = i_t / g_{o1}$$

KVL

$$V_t = V_1 + \left(g_{m2} V_1 + i_t\right) / \left(g_{o2} + g_{of}\right)$$

Thus

$$V_t = i_t \left\{ \left[g_{o1}^{-1} + \left(g_{o2} + g_{of}\right)^{-1} \right] + \frac{g_{m2}}{g_{o1}} \frac{1}{g_{o2} + g_{of}} \right\}$$

$$R_T = \frac{V_t}{i_t} = \frac{1}{g_{o1}} + \frac{1}{g_{o2} + g_{of}} \left(1 + \frac{g_{m2}}{g_{o1}} \right) = R_m^o$$

$$\tau_m = C_m \left[\frac{1}{g_{o1}} + \frac{1}{g_{o2} + g_{of}} \left(1 + \frac{g_{m2}}{g_{o1}} \right) \right]$$

Thus the approximated ω_{3dB} becomes

$$\omega_{3dB} = \frac{1}{\tau_p + \tau_L + \tau_m}$$

$$\omega_{3dB} = \frac{1}{\frac{C_m + C_p}{g_{o1}} + C_m\left[\frac{1}{g_{o2} + g_{of}}\left(1 + \frac{g_{m2}}{g_{o1}}\right)\right] + \frac{C_L}{g_{o2} + g_{of}}}$$

Note that $\tau_p << \tau_m$, thus one can approximate

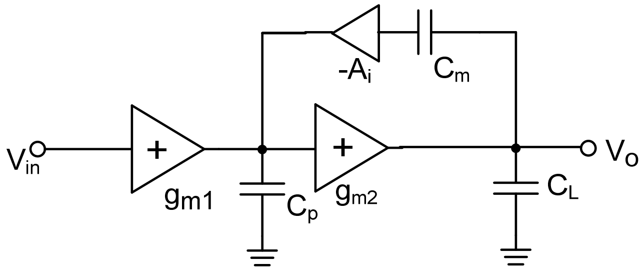
$$\omega_{3dB} \cong \frac{1}{\tau_m + \tau_L}$$

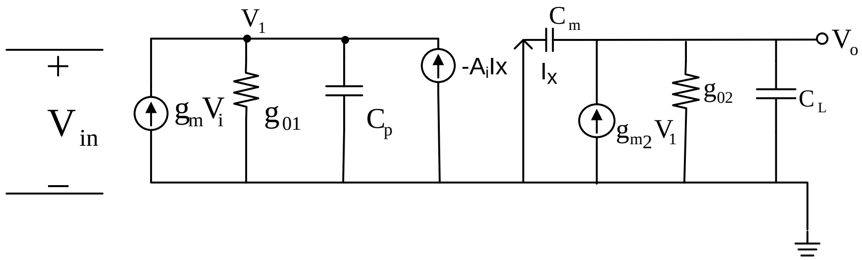
Also

$$\omega_m = \frac{1}{\tau_m} \cong \frac{g_{o2} + g_{of}}{C_m\left(1 + \frac{g_{m2}}{g_{o1}}\right)} \cong \frac{g_{o2} + g_{of}}{C_m \frac{g_{m2}}{g_{o1}}}$$

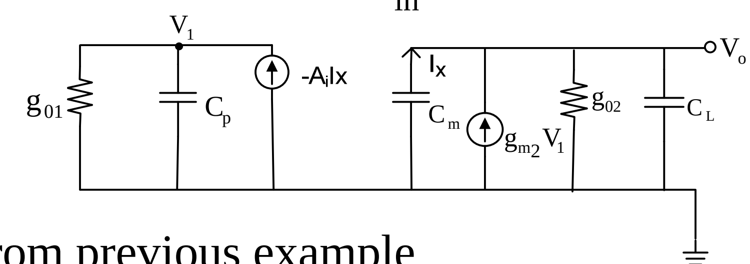
$$\omega_L = \frac{1}{\tau_L} = \frac{g_{o2} + g_{of}}{C_L}$$

Open-Circuit Time Constant for Amplifier with Current Buffer





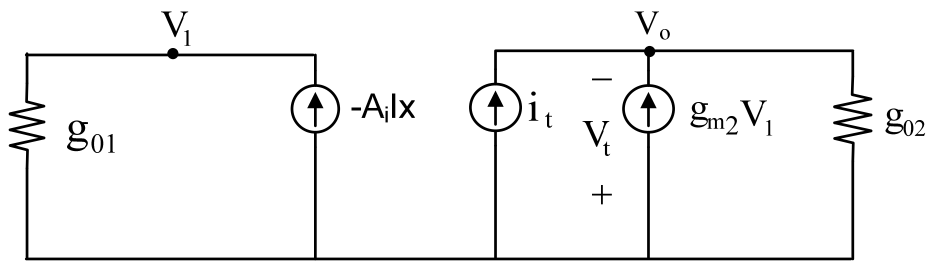
First Short-Circuited V_{in}



From previous example

$$\tau_L = \frac{C_L}{g_{o2}} \text{ and } \tau_p = \frac{C_p}{g_{o1}}$$

Time Constant Associated with C_m is considered next



$$I_x = I_t \quad ; \quad V_o = -V_t$$

$$V_1 = +g_{o1} A_i I_x$$

KCL

$$V_o g_{o2} = g_{m2} V_1 + i_t$$

$$V_t g_{o2} = g_{m2} g_{o1} A_i i_t + i_t$$

$$V_t = \left(g_{m2} g_{o1} A_i + 1\right) \frac{1}{g_{o2}}$$

$$R_T = \frac{1 + g_{m2} g_{o1} A_i}{g_{o2}}$$

$$\tau_m = \frac{1 + g_{m2} g_{o1} A_i}{g_{o2}} C_m$$

$$\omega_m = \frac{g_{o2}}{\left(1 + g_{m2} g_{o1} A_i\right) C_m} \cong \frac{g_{o2}}{g_{m2} g_{o1} A_i C_m}$$

$$\omega_{3dB} \cong \frac{1}{1/\omega_m + 1/\omega_L} = \frac{1}{\tau_m + \tau_L}$$

$$\omega_{3dB} \cong \frac{g_{o2}}{g_{m2} g_{o1} A_i C_m + C_L}$$

Let us illustrate how to compute a_2 using the amplifier shown in Figure of Slide 5. Thus for a two capacitor system*

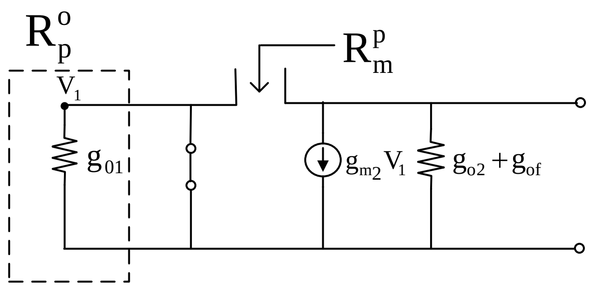
$$a_2 = R_p^o C_p R_m^p C_m = R_m^o C_m R_p^m C_p$$

The open-circuit resistances R_p^o and R_m^o are known since they were derived previously

$$R_m^o = \frac{1}{g_{o1}} + \frac{1}{g_{o2} + g_{of}}\left(1 + \frac{g_{m2}}{g_{o1}}\right)$$

$$R_p^o = \frac{1}{g_{o1}}$$

To illustrate the analysis we will obtain R_m^p and R_p^m using the following circuits

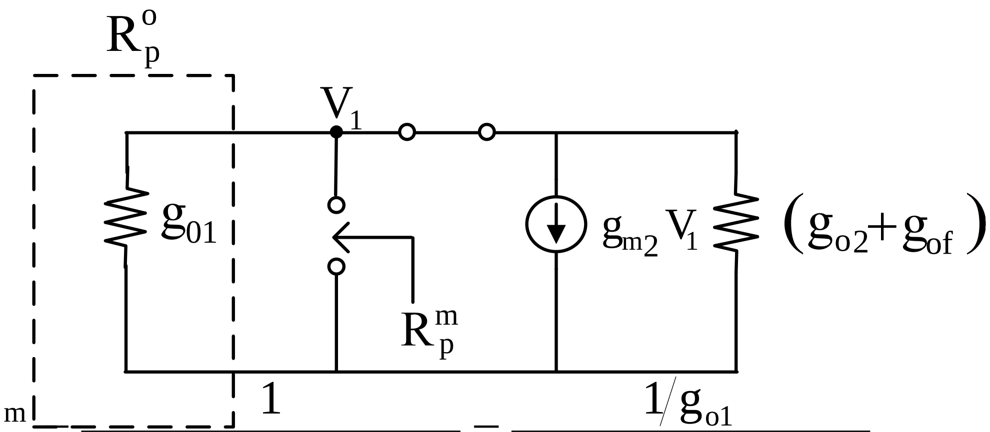


$V_1 = 0$, thus $R_m^p = \dfrac{1}{g_{o2} + g_{of}}$

Therefore

$$a_2 = \frac{1}{g_{o1}} C_p \frac{C_m}{g_{o2} + g_{of}} = \frac{C_p C_m}{g_{o1}(g_{o2} + g_{of})}$$

* We will ignore initially C_L



$$R_p^m = \frac{1}{g_{m2}+g_{o2}+g_{of}+g_p^o} = \frac{1/g_{o1}}{\frac{g_{m2}}{g_{o1}}+1+\frac{g_{o2}+g_{of}}{g_{o1}}}$$

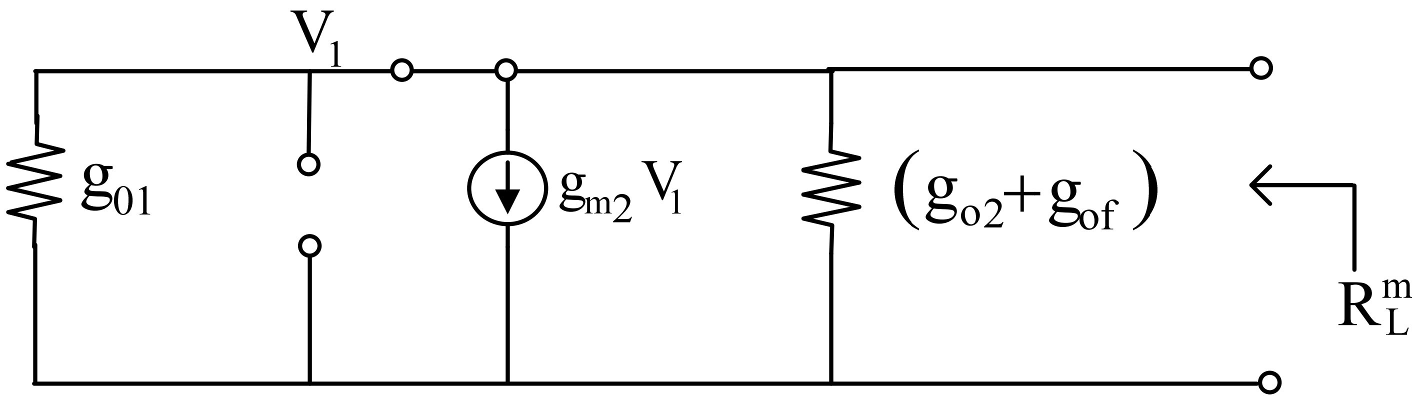
Thus

$$a_2 = R_m^o C_m R_p^m C_p$$

$$a_2 = \left[\frac{1}{g_{o1}}+\frac{1}{g_{o2}+g_{of}}\left(1+\frac{g_{m2}}{g_{o1}}\right)\right] C_m \frac{1/g_{o1}}{1+\frac{g_{m2}}{g_{o1}}+\frac{g_{o2}+g_{of}}{g_{o1}}} C_p$$

$$a_2 \cong \frac{C_p C_m}{g_{o1}(g_{o2}+g_{of})}$$

Let us consider C_L



$$\text{Note } R_L^m = R_p^m = \frac{1}{g_{m2} + g_{o2} + g_{of} + g_{o1}}$$

Thus

$$a_2 = R_m^o C_m R_p^m C_p + R_m^o C_m R_L^m C_L + R_m + R_p^o C_p R_L^p C_L$$

The dominant term of a_2 when $C_p << C_L$, C_m becomes

$$a_2 \cong R_m^o R_L^m C_m C_L$$

Thus

$$a_2 = \left\{ \frac{1}{g_{o1}} + \frac{1}{g_{o2} + g_{of}} \left(1 + \frac{g_{m2}}{g_{o1}} \right) \right\} \frac{\frac{1}{g_{o1}}}{1 + \frac{g_{m2}}{g_{o1}} + \frac{g_{o2} + g_{of}}{g_{o1}}} C_m \, C_L$$

$$a_2 \cong \frac{C_m C_L}{g_{o1}\left(g_{of} + g_{o2}\right)}$$

Ref. 2:
R.T. Howe and C.G. Sodini, “Microelectronics An Integrated Approach”, Prentice Hall, Upper Saddle 1997