ELEN 607 Analog and Mixed-Signal Center, TAMU Edgar Sánchez-Sinencio





# **Fundamental OTA Linearization Techniques**

The input-output characteristics of an OTA can be expressed as a non-linear polynomial

$$i_{\rm D} = I_D + a_1 v_{in} + a_2 v_{in}^2 + \dots + a_n v_{in}^n$$

For linear applications, the ideal input-output relation for an OTA is:

$$i_{\rm D} = I_D + i_d = I_D + a_1 v_{in}$$

In actual implementations is not possible to obtain the above expression. Thus, we can only aim to reduce as much as possible the coefficients of the higher-order terms, that is  $a_2, a_3, \dots, a_n$ 

In what follows, we will discuss different linearization techniques to minimize these higher order terms.For instance differential outputs can ideally cancel all even coefficients. To be able to make a weakly linear approximation, we can express the polynomial as a Taylor Series, as shown below:

$$i_{d} = i_{d}(v_{in})$$

$$i_{d} = I_{D} + \frac{\partial i_{d}}{\partial v_{in}} \Big|_{Q} v_{in} + \frac{1}{2} \frac{\partial^{2} i_{d}}{\partial v_{in}^{2}} \Big|_{Q} v_{in}^{2} + \frac{1}{6} \frac{\partial^{3} i_{d}}{\partial v_{in}^{3}} \Big|_{Q} v_{in}^{3} + \bullet \bullet$$

for  $\cdot \mathbf{v}_{in} = V \sin(\omega t)$ 

$$i_{d} \cong I_{D} + \left. \frac{1}{4} \frac{\partial^{2} i_{d}}{\partial v_{in}^{2}} \right|_{Q} V^{2} + \left. \frac{\partial i_{d}}{\partial v_{in}} \right|_{Q} + \left. \frac{\partial^{3} i_{d}}{\partial v_{in}^{3}} \right|_{Q} \frac{V^{2}}{8} + \cdots V \sin(\omega t) +$$

$$+ \frac{1}{4} \frac{\partial^2 \mathbf{i}_d}{\partial \mathbf{v}_{\text{in}}^2} + \cdots + \frac{1}{Q} \frac{\partial^2 \mathbf{i}_d}{\partial \mathbf{v}_{\text{in}}^3} + \cdots + \frac{1}{24} \frac{\partial^3 \mathbf{i}_d}{\partial \mathbf{v}_{\text{in}}^3} + \cdots + \frac{1}{Q} \frac{\partial^2 \mathbf{i$$

## **Nonlinear Metrics**



# **ROUGH CLASSIFICATION OF TRANSCONDUCTANCE LINEARIZATION SCHEMES**

- 1. By attenuating the input signal.
- 2. By using negative feedback.
- 3. By strategically adding several non-linear OTAs.
- 4. By connecting a nonlinear function and its inverse non-linear function.

## **OTA Linearization Techniques using attenuation**



$$i_o = a_1(kV) + a_2(kV)^2 + a_3(kV)^3 + \cdots$$



Active attenuation based on a selfcascode transistor

$$k = 1 - 1 / \sqrt{1 + W_1 L_2 / W_2 L_1}$$
 for  $V_{T_1} = V_{T_2}$ 

### **Nonlinearity Reduction**

1. Input Attenuation

**Potential Implementations :** 

- Floating Gate





k can be frequency dependent



- For discrete components, a voltage divider is possible :



### 2. Linearization by using negative feedback.



Example: A source degeneration differential pair

$$\beta = R_{s} \qquad \text{N(Vin)} = \text{gm}$$
$$G_{m} = \frac{g_{m}}{1 + g_{m}R_{s}} = \frac{g_{m}}{1 + N}$$

#### DEGENERATED DIFFERENTIAL PAIRS

 $\Box$  CONCEPT



**>** BASIC PRINCIPLE:

Linearity is determined by the resistor

$$g_m = \frac{I_{01} - I_{02}}{V_d} = \frac{g_m I}{1 + g_{m2} R} \approx \frac{1}{R}$$

> CONDITIONS:

M1, M2 are saturated M1, M2are perfectly matched  $g_{m1}R >> 1$ 

 $(g_{ml}, \text{ intrinsic transconductance of M1, M2})$ 

 $V_{GS}$  OF M1, M2 can be regarded constant

➤ GLOBAL CHARACTERISTICS:

High linearity at the cost of large area occupation.

Low transconductance (less suited for high frequency applications)

#### DEGENERATED DIFFERENTIAL PAIRS-IMPLEMENTATIONS

Single Most in Ohmic Region As a Resistor (TSIV86)



• Tuning via  $V_C$ •  $V_C >> \longrightarrow$ MR in ohmic region •  $\left(\frac{W}{L}\right)_{1,2} >> \rightarrow$  $V_{GE1,2} >> \sim coust$ 

≻Two Most in Ohmic Region as a Resistor (KRUM88)

• Tuning via I<sub>O</sub>



### Transconductance linearization schemes via source degeneration.





Active Source Degeneration topologies; (a) and (b) transistors biased on triode region and (c) with saturated transistors.

#### Table 1. Main characteristics for differential pair based OTAs. $N(=G_mR)$ is the source-degeneration factor.

Parameter	Differential Pair	Source Degeneraton
Small-signal transconductance	$G_m = \sqrt{2\mu_n C_{OX}} \sqrt{\frac{W_n}{L_n}} \sqrt{I_B}$	$G_{m,sd} = \frac{G_m}{1+N}$
Third Harmonic Distortion (HD3)	$HD3 = \frac{1}{32} \left( \frac{v_{id}}{V_{DSAT}} \right)^2$	$\left(\frac{1}{1+N}\right)^2 HD3$
Input referred thermal noise density	$\frac{16}{3} \frac{kT}{G_m} \left( 1 + \frac{g_{mp}}{G_m} \right)$	$\frac{16}{3} \frac{kT}{G_{m,sd}} \left( 1 + \frac{g_{mp} + \left(\frac{N}{1+N}\right)^2 g_{mn}}{G_{m,sd}} \right)$
Dynamic Range	$DR = \sqrt{\frac{(HD3)(C_L)}{NF}} 10^{11} V_{DSAT}$	$\sqrt{\frac{NF}{NF_{sd}}(1+N)DR}$
Current consumption*	21B	$2(1+N)I_B$
Transistor dimensions*		$(1+N)\frac{W}{L}$

\* For comparison, same transconductance and same  $V_{DSAT}$  are fixed for both structures. NF=  $NF_{sd}$  for N=0

Reference/Figure	Transconductance	Properties
Fig. (a)	$\frac{g_{m1}}{1 + \frac{\beta_1}{4\beta_3}}$	Low sensitive to common-mode input signals. The linear range is limited to V <sub>in</sub> <v<sub>DSAT, and THD=-50 dB. M1=M2, M3=M4</v<sub>
Fig. (b)	$\frac{g_{m1}}{1+g_{m1}R}$ $R = 1/\mu_o C_{ox} (V_{gs} - V_T)$	Highly sensitive to common-mode input signals. For better linearity large $V_{GS3}$ voltages are required. Large tuning range if $V_G$ is used.
Fig. (c)	$\frac{g_{m1}}{1+g_{m1}/g_{m3}}$ M1=M2	Low sensitive to common-mode input signals. Limited linearity improvement, HD3 reduces by -12 dB. More silicon area is required.



$$i = i_n + i_p$$

$$i = \frac{g_{mn}v}{N_n + 1} \sqrt{1 - \left(\frac{v}{2(N_n + 1)V_{DSsatn}}\right)^2} + \frac{g_{mp}v}{N_p + 1} \sqrt{1 - \left(\frac{v}{2(N_p + 1)V_{DSsatp}}\right)^2}$$

$$G_m = \frac{g_{mn}}{N_n + 1} + \frac{g_{mp}}{N_p + 1}$$

 $R_{DS} = \frac{1}{\beta (V_{GS} - V_T)} \qquad \qquad N_n = g_{mn} R_n$  $N_p = g_{mp} R_p$ 





 $f_z = \frac{G_m}{2\pi C_{\rm GDt}}$ 

#### Ref.-Antonio López/Silva-Martinez

**Current Divider Concept to tune Transconductance Amplifier** 



Symmetric Current Divider for Differential Structures



Ref.- Z.Y. Chang, D. Haspeslagh, J. Boxho and D. Macq, "A Highly Linear CMOS Gm-C Bandpass Filter for Video Applications," IEEE CICC, 1996.

#### **Balanced Nonlinearity Cancellation (no even power terms)**



- CMR is poor
- The technique requires perfect matching of components and voltages.

# 3. Subtraction of Polynomials. This is Another Balanced Nonlinearity Cancellation.



Implementations:



Single Device.  $V_A$  is a levelshiftingvoltagekeeping thetransistor in saturation



**Balanced Configuration** 

3. General transconductance linearization by non-linear terms sum cancellation techniques



(a) using single multipliers by a constant ( $V_A$ ), (b) using single-quadrant devices.  $V_1$ = -  $V_2$ .



Transconductance (A) based on Fig (a); (B) based on Fig. (b). Note that in (A) the input signal might need a DC bias. Analog and Mixed Signal Center, TAMU

LINEAR OTA USING FEEDBACK CONTROL ON COMMON SOURCE NODE



Ref.- J. Sevenhans and M. Van Paemel, "Novel CMOS Linear OTA Using Feedback Control on Common Source Node, "Elect. Letters, pp. 1873-1874, 26<sup>th</sup> Sept. 1996.

#### **Nonlinearity Cancellation Using Function Compensation**

CM

 $I_{SS} = I_B + kV^2$ 



$$I_{SS} = I_B + F_1(V)$$
  
Where  $F_1(V)$  is chosen such that,  
$$I_0 = F(V, I_B + F_1(V)) = VF_2(I_B) = G_m V$$

Example 1. Linearization via bias current modulation of a differential pair operating in strong inversion.

$$I_{out} = K \Delta V \sqrt{2I_{SS} / K - \Delta V^2} \quad ; K = \mu_o C_{ox} \frac{W}{2L} \tag{1}$$

Goal is to add  $I_M = K \Delta V^2 / 2$  to  $I_{SS}$ , thus

$$I_m = K \Delta V^2 / 2$$

$$\frac{2(I_{SS} + I_M)}{K} = \frac{2I_{SS}}{K} + \frac{K \Delta V^2}{K} - \Delta V^2 = \frac{2I_{SS}}{K}$$
(2)

(2) into (1)

$$I_{out} = K \Delta V \sqrt{2I_{SS}/K} = \sqrt{2KI_{SS}} \ \Delta V = G_m \Delta V \tag{3}$$

Where  $G_m$  is the linearized transconductance, and  $\Delta V$  is the input differential voltage.

How to generate  $I_M = K \Delta V^2 / 2$ ?



Complete  $\Delta V^2$  circuit structure

E2



Single-Ended OTA with the Tail Current and  $K\Delta^2$  Circuit Generator

Ref. – T. Inoue, F. Ueno, Y. Aramaki, O. Matsumoto and M. Suefuji, "A Design of CMOS OTA's Using Simple Linearizing Techniques and Their Application to High-Frequency Continuous-Time Filters," 2000 IEEE.

#### Nedungadi Transconductor

[A. Nedungadi and T. R. Viswanathan, "Design of Linear CMOS Transconductor Elements," *IEEE Transactions on Circuits and Systems*, vol. CAS-31, No. 10, pp. 891-894, October 1984]



Range:

$$\begin{split} I_1 &= (n+1) I \\ I_2 &= 0 \end{split} \Rightarrow \Delta V = \sqrt{\frac{I}{k} (n+1)} \\ I_1 &= 0 \\ I_2 &= (n+1) I \end{aligned} \Rightarrow \Delta V = -\sqrt{\frac{I}{k} \left(1 + \frac{1}{n}\right)} \end{split}$$



$$\begin{split} I_1 &= k \left[ \frac{n \Delta V}{n+1} + \sqrt{\frac{I}{k}} \sqrt{1 - \frac{kn \Delta V^2}{(n+1)^2 I}} \right]^2 \\ I_2 &= k \left[ \frac{-n \Delta V}{n+1} + \sqrt{\frac{I}{k}} \sqrt{1 - \frac{kn \Delta V^2}{(n+1)^2 I}} \right]^2 \\ \Rightarrow I_o &= I_1 - I_2 \Rightarrow \begin{cases} I_o &= g_m \Delta V \sqrt{1 - \beta \Delta V^2} \\ g_m &= \frac{4n}{n+1} \sqrt{Ik} \quad , \quad \beta &= \frac{k}{I} \frac{n}{(n+1)^2} \end{cases} \\ \text{Range:} \quad |\Delta V| \leq \sqrt{\frac{I}{k} \left(1 + \frac{1}{n}\right)} \\ \text{For} \quad \Delta V &= \sqrt{\frac{I}{k} \left(1 + \frac{1}{n}\right)} \quad \Rightarrow \quad I_o &= \frac{4n}{n+1} I \end{split}$$





$$I_{1} = k \left[ \frac{n\Delta V}{n+1} + \sqrt{\frac{I}{k}} \sqrt{1 - \frac{kn\Delta V^{2}}{(n+1)^{2}I}} \right]^{2}$$
$$I_{2} = k \left[ \frac{-n\Delta V}{n+1} + \sqrt{\frac{I}{k}} \sqrt{1 - \frac{kn\Delta V^{2}}{(n+1)^{2}I}} \right]^{2} \right\} \Rightarrow$$

$$\Rightarrow \quad I_a = I_1 + I_2 = 2k \frac{n(n-1)}{(n+1)^2} \Delta V^2 + 2I \quad , \quad \Delta V \leq \sqrt{\frac{I}{k} \left(1 + \frac{1}{n}\right)}$$

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Nedungadi Transconductor



Transistors M1 and M2 form a simple differential pair biased by

$$I_{ss} = aI - (aI - I_a) = I_a = 2k \frac{n(n-1)}{(n+1)^2} \Delta V^2 + 2I$$

Consequently,

~

$$\begin{split} I_o &= \Delta V \sqrt{2k' I_{ss} - k'^2 \Delta V^2} = \Delta V \sqrt{k'^2 \Delta V^2 \left(4\gamma \frac{n(n-1)}{(n+1)^2}\right) + 4k' I} \quad , \quad \gamma = \frac{k}{k'} \\ 4\gamma n(n-1) - (n+1)^2 &= 0 \quad \Rightarrow \quad I_o = 2\Delta V \sqrt{k' I} \quad \begin{cases} g_m = 2\sqrt{k' I} \\ |\Delta V| \le \min\left(\sqrt{\frac{I}{k}\left(1 + \frac{1}{n}\right)}, 2\sqrt{\frac{I}{k'}}\right) \end{cases} \end{split}$$

Examples:

If

$$\begin{split} \gamma &= 1 \quad \Rightarrow \quad n = 1 + \frac{2\sqrt{3}}{3} = 2.155 \\ n &= 3 \quad \Rightarrow \quad \gamma = \frac{\left(n+1\right)^2}{4n\left(n-1\right)} = \frac{2}{3} \\ aI - I_a &\ge 0 \quad \Leftrightarrow \quad \begin{cases} \Delta V_{max}^2 = \frac{I}{k} \left(\frac{n+1}{n}\right) \quad \to \quad a \ge \frac{4n}{n+1} \\ \Delta V_{max}^2 = \frac{I_a}{k'} \quad \to \quad a \ge 2 \left[\frac{\left(1+4\gamma\right)n^2 + 2n\left(1-2\gamma\right) + 1}{\left(n+1\right)^2}\right] \end{cases} \end{split}$$

4. By cascading a nonlinear function and its inverse non-linear function



Nonlinear Function Combined with its Inverse Nonlinear Function



If bulk – and lambda effects are neglected, it can be shown that  $HD_2 = 0$ .

$$H_1 = \frac{V_{o1}}{V_{i1}} \cong -\frac{g_{mA}}{g_{mB}} \qquad \qquad H_2 = \frac{V_{02}}{V_{i2}} = \frac{-g_{m1}g_{m2}}{g_{01}g_{02}}$$

Improved Transconductor via Series-Shunt Feedback (DUPU89)



•  $g_{mI}$  increased by a + A

#### TRIODE TRANSCONDUCTORS

### $\Box$ CONCEPT



► BASIC PRINCIPLE:

Linearity is based on the approximately Linear model of MOSFETs in Ohmic Region

$$I_{oi} = \frac{\beta_i}{Z} \Big[ Z \Big( V_{GS_i} - V_r \Big) V_{DS_i} - V_{DS_i} \Big]; i = 1, Z$$
$$g_m = \beta V_{OS}$$

#### > CONDITIONS:

#### ➤ GLOBAL CHARACTERISTICS:

M1, m2 are perfectly matched

 $V_{DS}\,$  of M1, M2 must be held constant and below  $V_{DS,SAT}$ 

$$V_{DS1} = V_{DS2}$$

Nonlinearity is mainly due to mobility reduction

$$HD_{Z} = 0$$
  $HD_{S} \approx \frac{Q^{2}V_{d}^{2}}{16(1+Q(V_{C}-V_{T}))^{2}}$ 

Well suited for high frequency applications

#### TRIODE TRANSCONDUCTORS - IMPLEMENTATIONS

≻ Cascoding (STEF90)



- M1C M2C in saturation
- $\left(\frac{W}{L}\right)_{ic} \implies \rightarrow$

M1C, M2C act as per-feet source-followers

- Tuning via  ${\rm I}_2$
- M2, M3, M4 constitute 2 feedback loop, which keeps V<sub>og</sub>, constant when V<sub>1</sub> is applied.

Dual

#### ➢ Compensation (NAUT91)



M2 = M3 = M4

•  $V_{GS2}$  is modulated, so that,  $V_{DS1}$  remain constant and equal to  $V_B$ .

> Concept



M1, M2 in strong inversion and saturation

➢ Basic Principle:

Most of the proposed techniques are based on the simple algebraic identity:

$$(a+b)^2 - (a-b)^2 = 4ab$$
  
 $g_m = \beta (V_{GS_1} + V_{GS_2} - ZV_T)$ 

#### ➤ Conditions:

M1, M2 are perfectly matched  $V_{GS1} + V_{GS2}$  must be held constant

 $V_{\text{DS}}$  of M1, M2 above  $V_{\text{DA,SAT}}$ 

➤ Global Characteristics:

Also, nonlinearity is mainly due to mobility reduction

$$HD_Z = \phi \quad HD_3 \approx \frac{\theta V_d^2}{16V_o (Z + \theta V_o)(1 + \theta V_o)^2}$$

Well suited for high frequency applications

Careful layout required

#### Cross-Coupling (KHOR84)



Adaptive Biasing (NEDU84)



• M1 = M2, M3 = M4  
• 
$$\left[\frac{\beta_1}{\beta_3}\right]^{3/2} = \left[\frac{I_{Q1}}{I_{Q2}}\right]^{1/2}$$
  
 $\rightarrow$   
 $g_m = \sqrt{\beta \cdot I_{Q1}} - \sqrt{\beta_3 I_{Q2}}$ 

•  $M1 = M2 = M6 = M\beta_1 7$ M3 = M4 $\beta_3 = n\beta_1$  $\frac{I_{Q1}}{I_{Q2}} = \frac{2}{n+1}$  $2 > \frac{4n}{n+1}$  $g_m = \sqrt{\frac{2\beta_1 I_{Q1}}{2}}$ 

### ➢ Class AB (SEEV87)



$$M1 = M2 = MN$$
$$M3 = M4 = MP$$

•  $V_{Teq} = V_{TN} + V_{TF}$ 

• 
$$\beta_{eq} = \frac{\beta_N \beta_F}{\left(V_{\beta_N}^- + V_{\beta_F}^-\right)^2}$$

 $g_m = \beta_{eq} V_B$