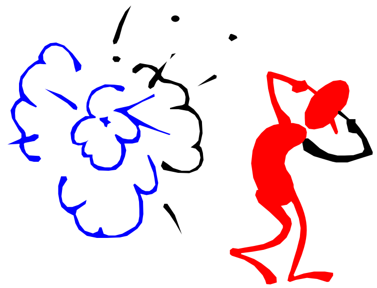

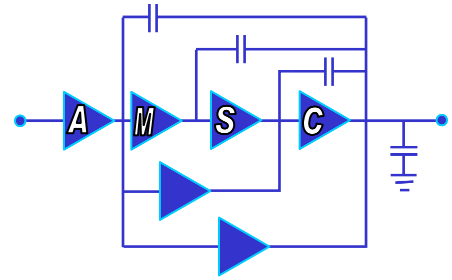

Noise

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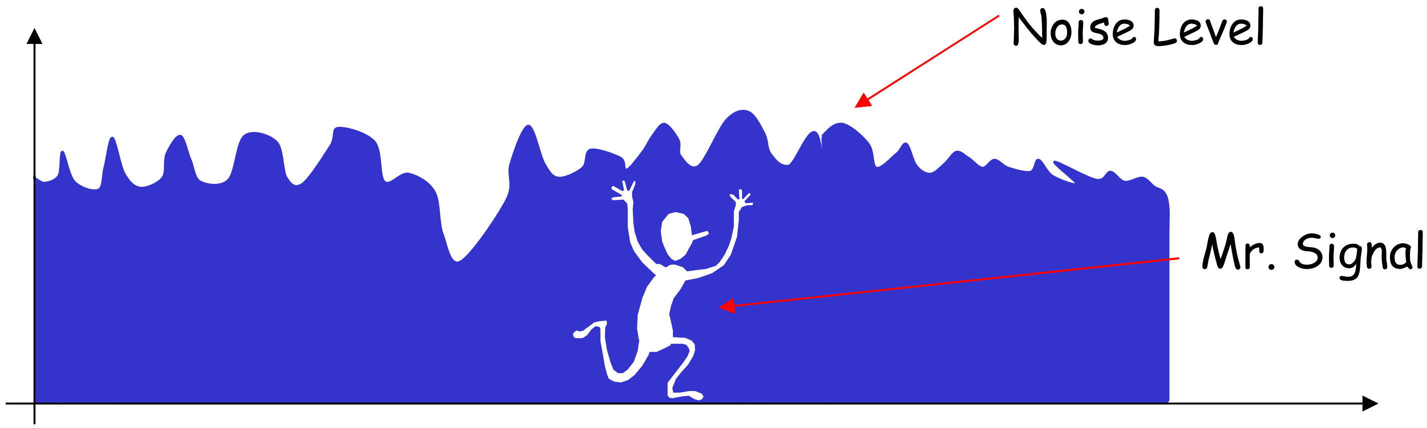
NOISE

- **NOISE** limits the minimum signal level that a circuit can process with acceptable quality.



Can you identify the signal buried in the noise?

How does the minimal signal must be with respect to the noise level?

- Let us consider the street noise, can one predict the (exact) noise at any time?

  No, because it is a random process

- If you decide to blow your car's horn every 5 minutes, then you can say this signal is deterministic.

- So, how can we incorporate noise in circuit design?

  - Observe the noise for a long time

  - Construct a "statistical model"

- The average power noise is predictable

- Most noise sources in circuits exhibit a constant average power

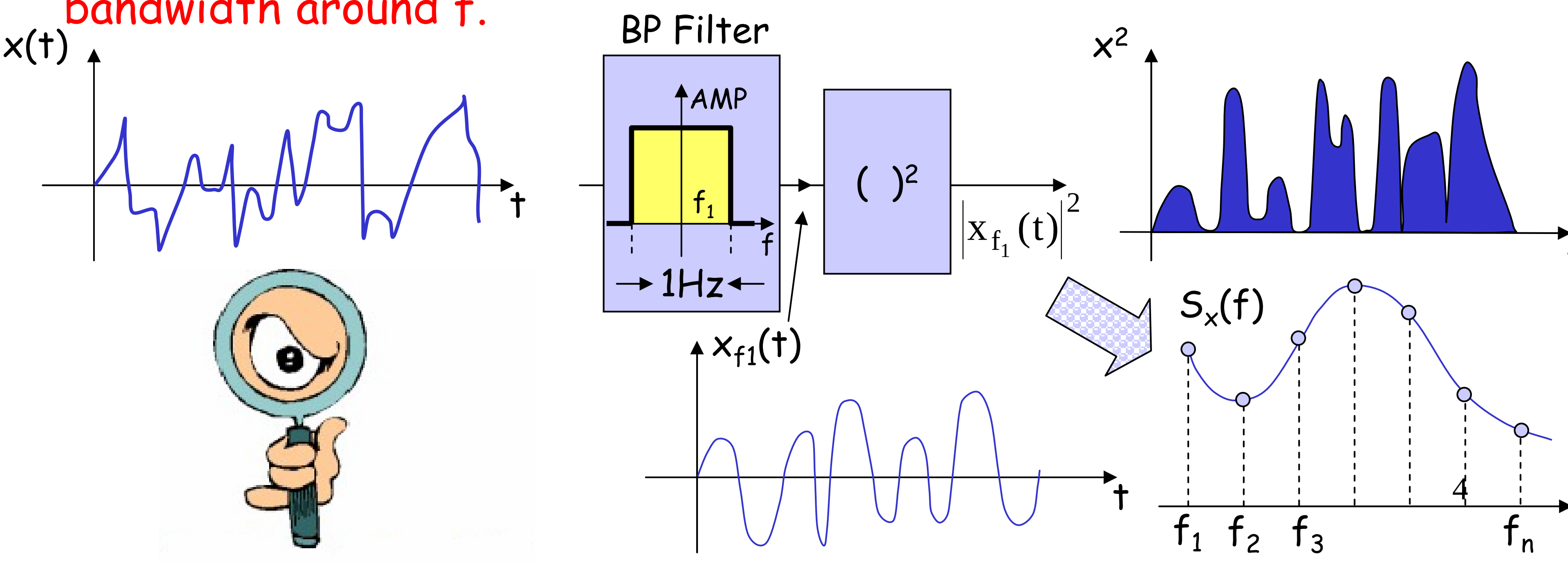
- Average power delivered by a periodic voltage v(t), of period T, to a load resistance R_L is given by

$$P_{av} = \frac{1}{T}\int_{-T/2}^{T/2} \frac{v^2(t)}{R_L} dt = \frac{1}{T}\int_{-T/2}^{T/2} v(t)i(t)dt$$

- For non-periodic signals, T becomes a large quantity

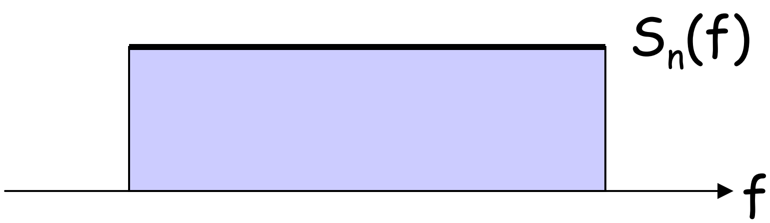
- How much power a signal carries at each frequency is defined by the “power spectral density” (PSD) ($S_x(f)$)

- $S_x(f)$ is defined as the average power carried by x(t) in a one-hertz bandwidth around f.



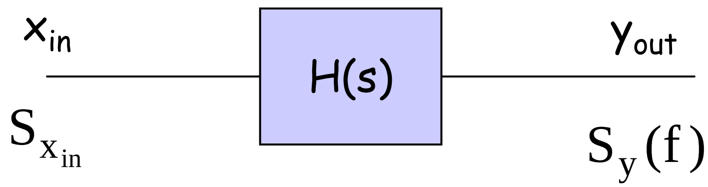
- $S_x(f)$ is expressed in V^2/Hz rather than VI/Hz = w/Hz, in fact $\{S_x(f)\}^{1/2} = v/Hz^{-1/2}$ is often used. Say a filter at 1MHz is equal to 1.414 $nV/Hz^{-1/2}$, means an average power in a 1-Hz bandwidth at 1MHz is equal to $(1.414\times10^{-9})^2V^2=2\times10^{-18}V^2$

- Another spectrum is the "White Spectrum" or white noise



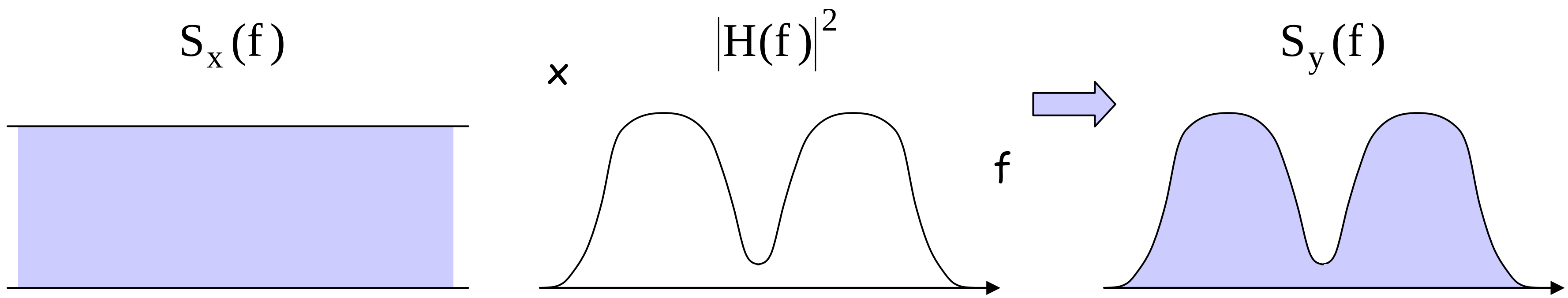
In practice is band limited.

- How do you determine the output spectrum of a linear, time-invariant system with transfer function, H(s)?



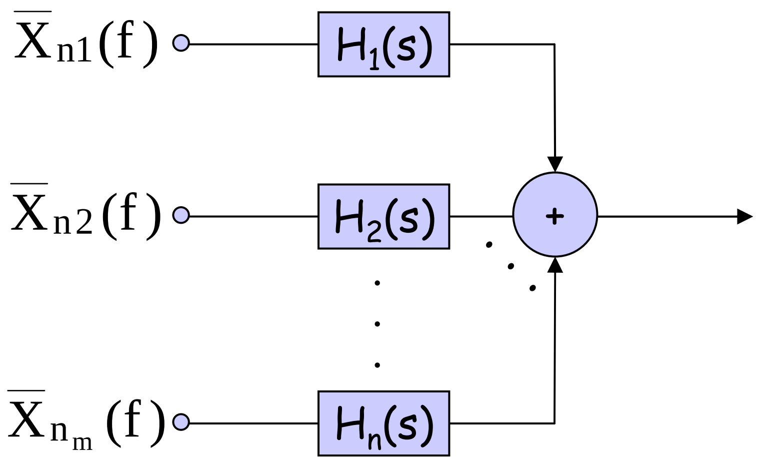
$$S_y(f) = S_x(f) \times |H(f)|^2$$

- How is the shape of the output spectrum?



- A practical example is the telephone bandwidth, where BW of $S_{xin}(f)$ is between 0-20KHz, BW of H(f) is 4KHz and consequently BW of $S_{yout}(f)$ is 4 KHz.

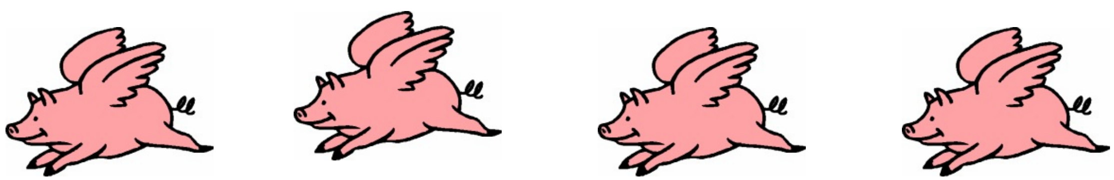
If there are more than two noise sources, how do you compute their total effects?



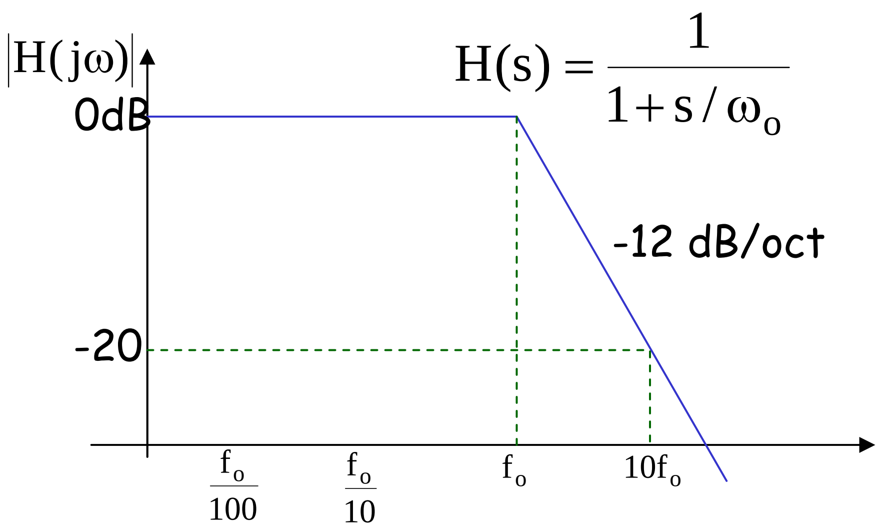
$$S_y(f) = \sum \left|H_i(j2\pi f)\right|^2 \overline{X}_{ni}^2(f)$$

$\overline{x}_{n1}(f)$, $\overline{x}_{n2}(f)$, … are uncorrelated noise sources

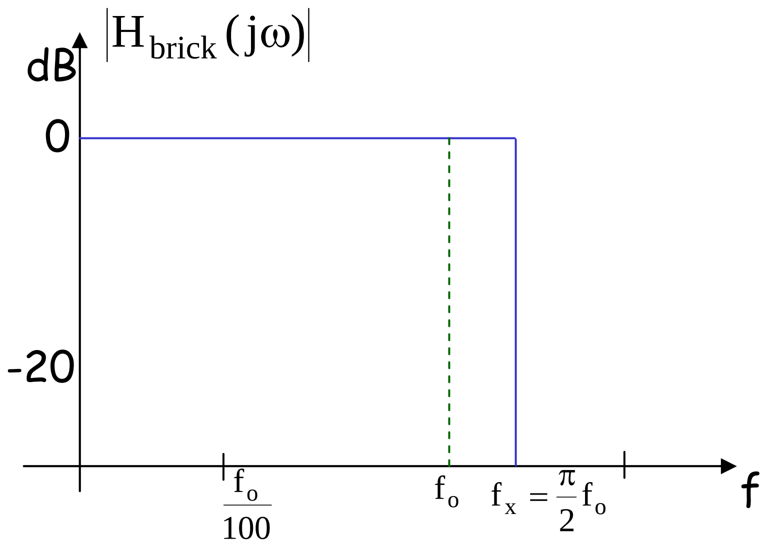
Superposition is applied to obtain the total output spectrum.



Noise Bandwidth



First - Order LP



Brick - Wall Filter

Assume an input signal, $V_{ni}(f)=V_{nw}$ (constant)

$$\overline{V}^2_{n,out(rms)} = \int_o^\infty \overline{V}^2_{ni}(f)\left|H(2\pi fj)\right|^2 df = \int_o^\infty \frac{\overline{V}^2_{nw}}{1+\left(f/f_o\right)^2} df = \overline{V}_{nw} f_o \tan^{-1}\left(\frac{f}{f_o}\right)\Bigg|_o^\infty = \frac{\overline{V}_{nw}\pi f_o}{2}$$

If this same input signal, V_{nw}, is applied to the (brick-wall) filter

$$\overline{V}^2_{brick(rms)} = \int_o^{f_x} \overline{V}^2_{nw} df = \overline{V}^2_{nw} f_x \quad , \quad \text{since} \quad V_{n,out} = V_{brick} \quad , \quad \text{then} \quad f_x = \frac{\pi f_o}{2}$$

Equivalent Noise Bandwidth

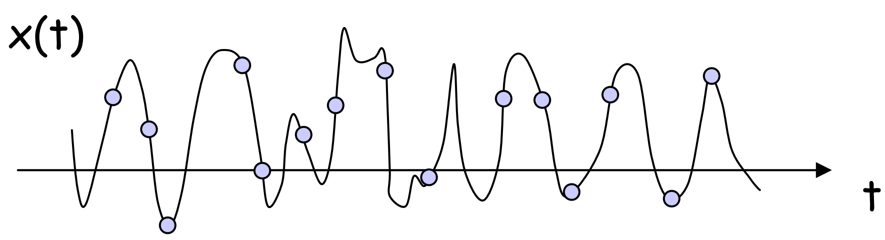
$$\Delta f = \frac{1}{A_{vo}^2} \int_o^{\infty} |A_v(f)|^2 df$$

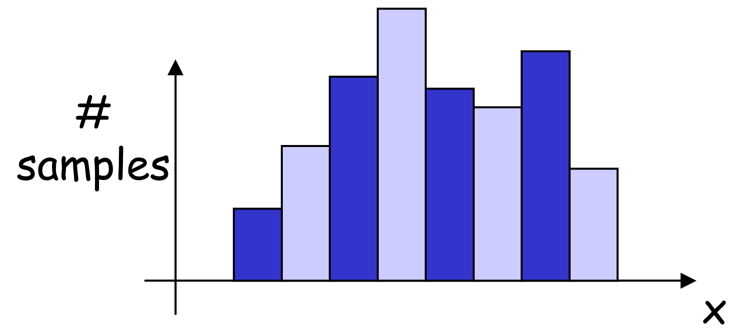
Examples

$$A_v(f) = \frac{1}{1 + j\, f / f_{3dB}} \quad , \quad \Delta f = \int_o^{\infty} \frac{df}{1 + \left(f / f_{3dB} \right)^2} = 1.571\, f_{3dB}$$

$$A_v(f) = \left| \frac{1}{1 + j\, f / f_{3dB}} \right|^2 \quad , \quad \Delta f = \int_o^{\infty} \left| \frac{1}{1 + \left(f / f_{3dB} \right)^2} \right| df = 1.22\ f_a = 1.22 \times 0.6436\ f_{3dB}$$

- Amplitude Distribution of Noise





Probability Density Function (PDF), the distribution of x(t) is

$$p_x(x)dx = \text{probability of } x < X < x + dx$$

- An important example of PDFs is the Gaussian (or normal) Distribution.

$$p_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2}$$

Where σ and m are the standard deviation and mean of the distribution, respectively.

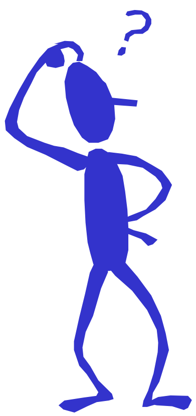
- How do you determine the total average power due to two or more noise sources?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [x_1(t) + x_2(t)]^2 \, dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (x_1^2(t)dt + x_2^2(t)dt + 2x_1(t)x_2(t)dt$$

$$P_{av} = P_{av_1} + P_{av_2} + \underbrace{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 2x_1(t)x_2(t)dt}_{= 0 \,?\quad \text{When?}}$$

If noise sources are uncorrelated the third term is zero.

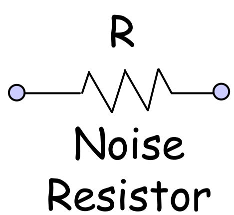
Example of uncorrelated and correlated noises is that of spectators in a sports stadium.

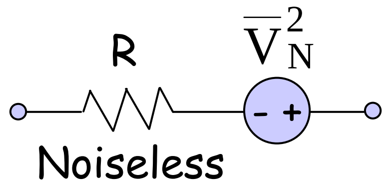


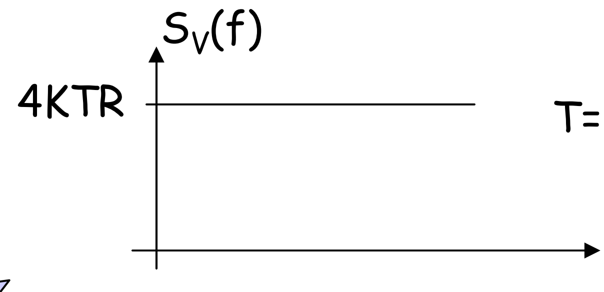
Noise Types

- Device Electronics Noise
- Environmental Noise

- Thermal Noise

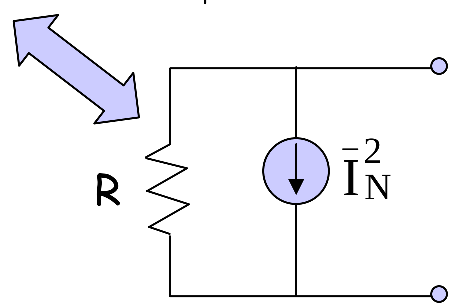






T=Temperature

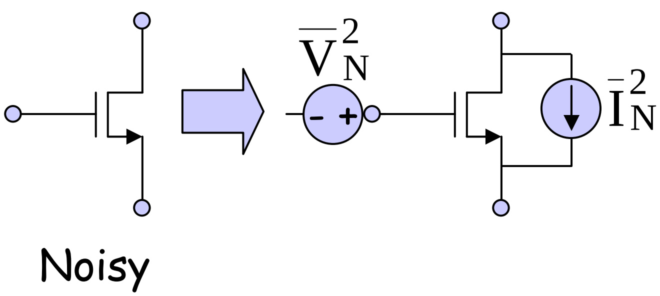
$$\overline{V_N^2} = 4KTR$$



$$\overline{I_N^2} = \frac{4KT}{R}$$

Example

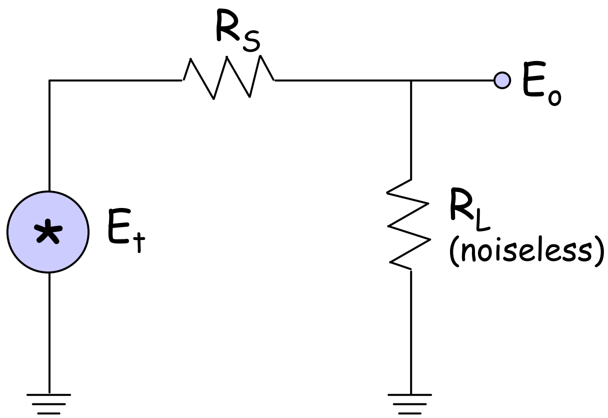
$$R = 50\Omega \quad T = 300^\circ K \quad \Rightarrow \quad \overline{V_N^2} = 8.28\times10^{-19}\, V^2/Hz \quad \Rightarrow \quad (0.91nV/\sqrt{Hz})^2$$



$$\overline{I_N^2} = 4KT\left(\frac{2}{3}\right)g_m \quad \Rightarrow$$ Thermal (White) Noise

$$\overline{V_N^2} = \frac{K_{dev}}{WLC_{ox}f} \quad \Rightarrow$$ Flicker Noise

Noise Voltage



Power Supplied to R_L is

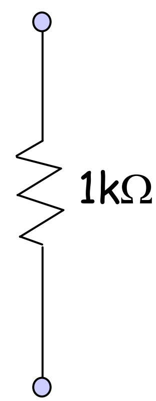
$$P_t = I_o E_o = \frac{E_o^2}{R_L}$$

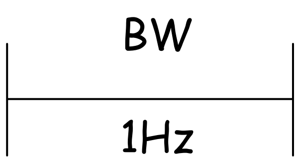
$$P_t \Big|_{R_S = R_L = R} = \frac{E_t^2}{4R} = KT\Delta f \quad , \quad E_o = \frac{E_t}{2R} R = \frac{E_t}{2}$$

Then

$$E_t = \sqrt{4kTR\Delta f} \quad (V_{rms})$$

$$E_t \Big|_{\substack{R = 1K\Omega \\ \Delta f = 1Hz}} = 4n \ V_{rms}$$



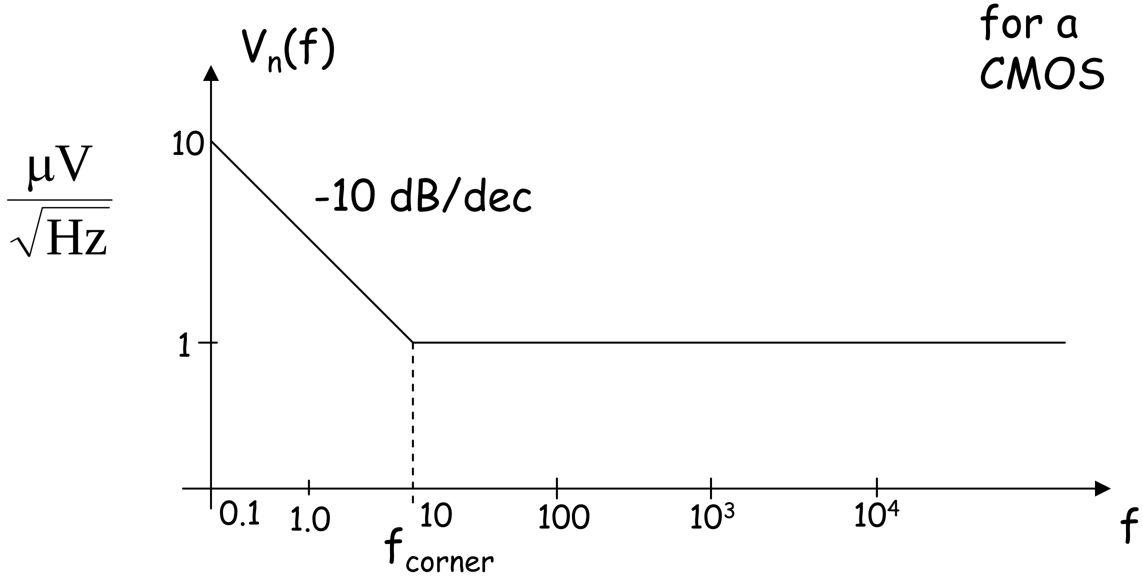


- Flicker or 1/f Noise

PSD is given by

$$\overline{V_n^2}(f) = \frac{K_V^2}{f} = \left(\frac{K_V}{\sqrt{f}}\right)^2 \Bigg| = \frac{K}{C_{ox}WL}\frac{1}{f}$$

for a CMOS



1/f noise dominates

Low Frequency

white noise dominates

High Frequency

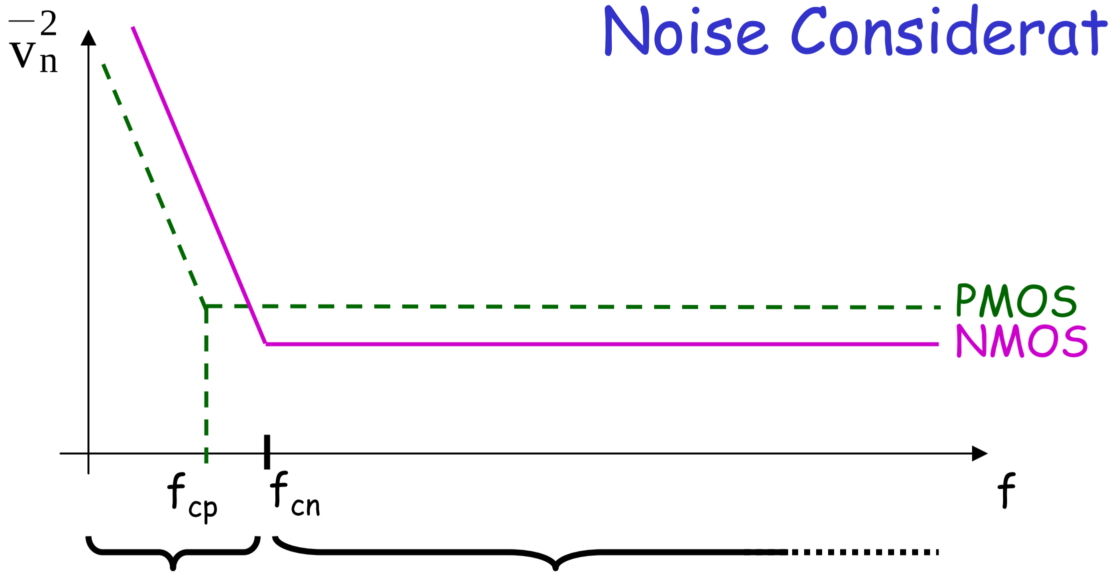
$K \sim 10^{-25} V^2 F$

Equating output currents

$$4KT\left(\frac{2}{3}g_m\right) = \frac{K}{C_{ox}WL}\frac{1}{f_c}g_m^2$$

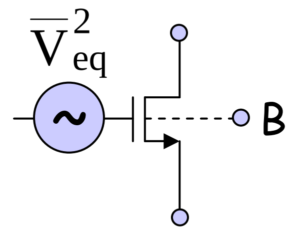
$$f_c = \frac{K}{C_{ox}WL}g_m\frac{3}{8KT}$$

Noise Considerations



Flicker Noise

Thermal or Johnson Noise (White Noise)



$$\eta = \frac{g_{mb}}{g_m}$$

$$\overline{V_N^2} = \overline{V_{eq}^2} = \underbrace{\left\{\frac{KF \cdot \Delta f}{2fC_{ox}K_pWL}\right\}}_{\text{Flicker Noise}} + \underbrace{\frac{8kT(1+\eta)}{3g_m}\Delta f}_{\text{White Noise}}$$

- To reduce Flicker Noise:

  i.e.,

  $$KF_p = 2.7\times10^{-21}\,A$$

  Increase W * L (Area)

  $$KF_n = 4.3\times10^{-21}\,A$$

  To keep same bias point keep same (W/L)

- To reduce White Noise

  Increase g_m. Since $g_m \cong \sqrt{2K_pI_D\left(\frac{W}{L}\right)} = K_p(V_{gs} - V_T)\frac{W}{L}$

  (a) (W/L) ↑ and (I_D) ↑ will increase g_m, power consumption (I_D), and area

  (b) (W/L) ↑ and modify the bias to keep I_D same as before increasing g_m.

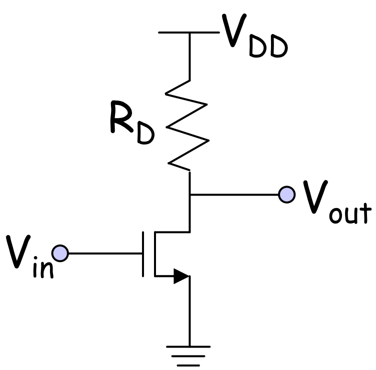
How to compute the total output noise due to individual noise sources?

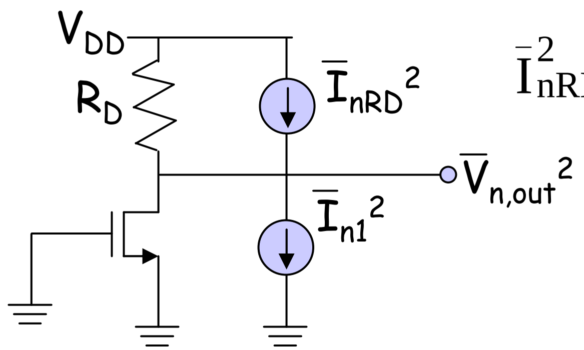
How to compute the input-referred noise?

Example.___

$$\overline{I^2_{n1}} = 4KT\left(2/3\right)g_m + Kg_m^2/(C_{ox}WLf)$$

$$\overline{I^2_{nRD}} = 4KT/R_D$$





$$\overline{V^2_{n,out}} = \left(4KT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\frac{1}{f}g_m^2 + \frac{4KT}{R_D}\right)R_D^2$$

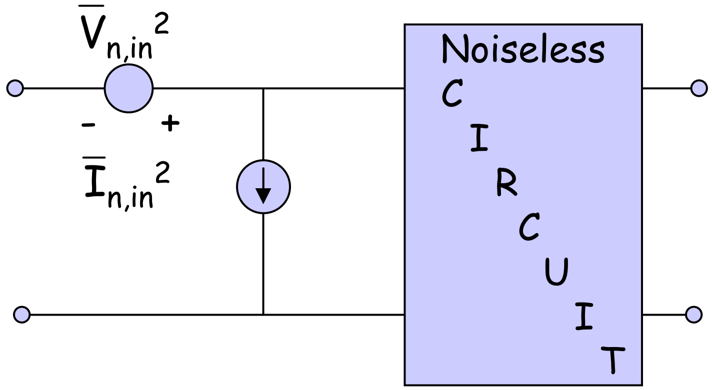
in 1Hz at frequency f

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_V^2} \cong \frac{1}{g_m^2 R_D}\left(\overline{V^2_{n,out}}\right)$$

$$\overline{V^2_{n,in}} = 4KT\frac{2}{3g_m} + \frac{K}{C_{ox}WL}\frac{1}{f} + \frac{4KT}{g_m^2 R_D}$$

→ Good performance parameter

- Can we always use the input-referred noise by a single voltage source in series with the input?



A necessary and sufficient representation

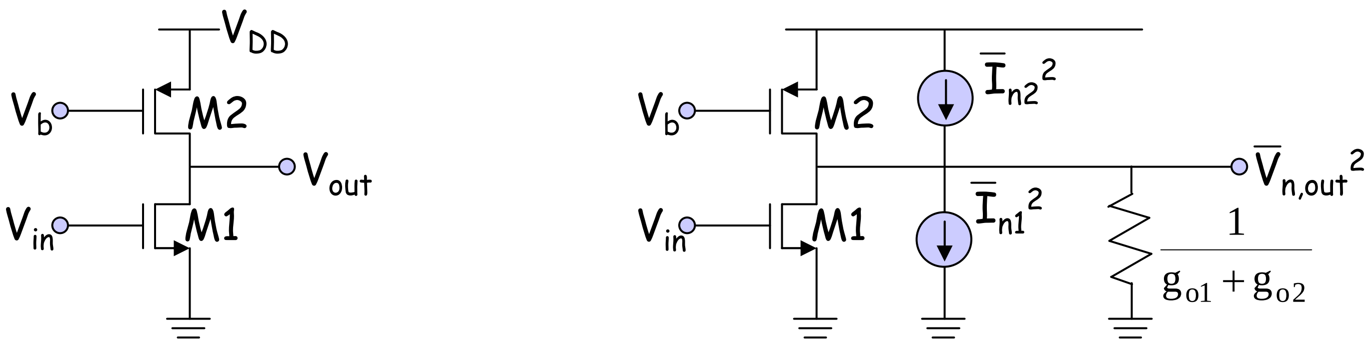
- Simplifications

  - For zero source impedance, $\overline{I^2_{n,in}}$ not affecting the output

  - For infinite source impedance, then $\overline{V^2_{n,\,in}}$ has no effect

- Note that both sources will not count the noise twice.

For the following amplifier and from the point of view, of minimum noise, how the values of g_{m1} and g_{m2} should be set?



Considering the thermal noise:

$$\overline{V_{n,out}^2} = 4KT\left(\frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)\frac{1}{(g_{o1}+g_{o2})^2}$$

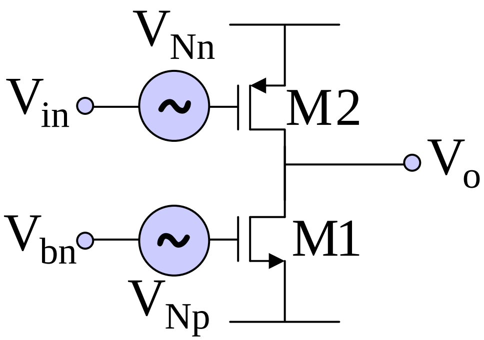
$$A_v = \frac{g_{m1}}{g_{o1}+g_{o2}}$$

$$\overline{V_{n,inp}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2} = 4KT\left(\frac{2}{3g_{m1}} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right)$$

$$\overline{V_{n,inp}^2} \downarrow \; g_{m1} \uparrow \; g_{m2} \downarrow \; (\text{current} - \text{source})$$

Trade-offs of increasing g_{m1}?

Noise Analysis, p driver, n load.



Make V_{in} and V_{bn} null

$$v_{o_N}^2 = \left(\frac{g_{m2}}{g_{o1}+g_{o2}}\right)^2 \overline{V}_{Nn}^2 + \left(\frac{g_{m1}}{g_{o1}+g_{o2}}\right)\overline{V}_{Np}^2$$

$$A_V^2 = \frac{g_{m2}^2}{(g_{o1}+g_{o2})^2}$$

Equivalent input-noise spectral density

$$\overline{V}_{N_{INP}}^2 = \frac{\overline{v}_{oN}^2}{A_V^2} = \frac{(g_{o1}+g_{o2})^2}{g_{m2}^2}\left[\frac{g_{m2}^2\,\overline{V}_{Nn}^2}{(\qquad)^2} + \frac{g_{m2}^2\,\overline{V}_{Np}^2}{(\qquad)^2}\right]$$

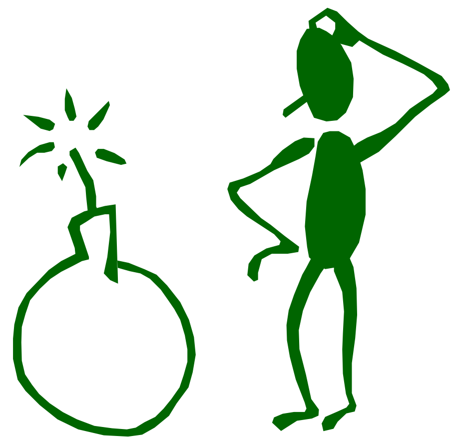
$$\overline{V}_{N_{INP}}^2 = \overline{V}_{Nn}^2 + \frac{g_{m1}^2}{g_{m2}^2}\overline{V}_{Np}^2$$

To reduce the equivalent input-noise spectral

g_{m1} ↓ for bias device

g_{m2} ↑ for signal (driver) device

To compute the rms output noise:



(i) Compute the frequency response bandwidth f_{3dB}

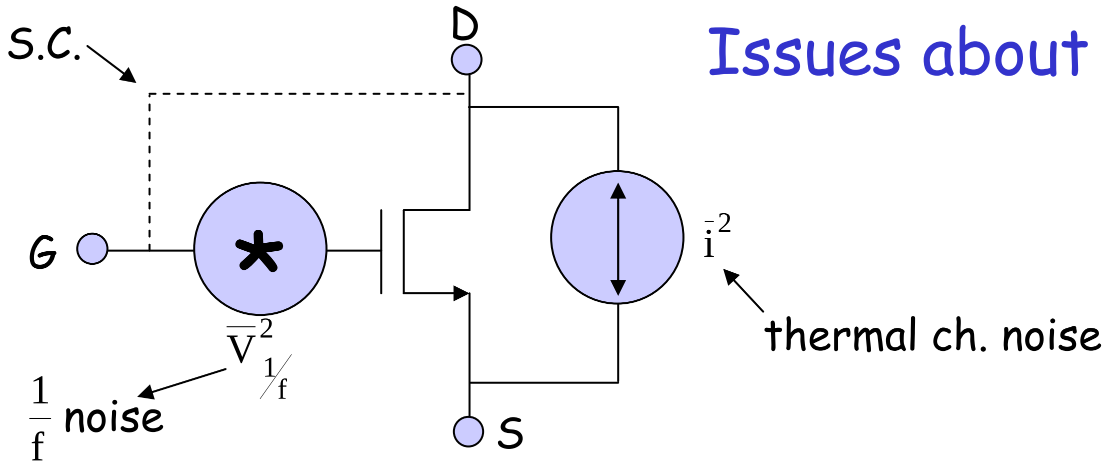
(ii) Compute the noise bandwidth, Δf

$$\Delta f = 1.571 * f_{3dB} \qquad \text{(assuming a single pole)}$$

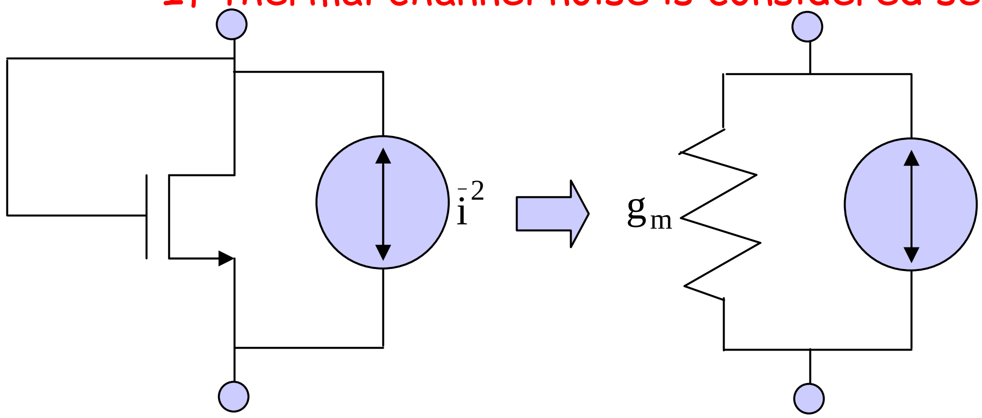
(iii) $$V_{out,noise} = \left\{ \overline{V}^2_{n,out} \cdot \Delta f \right\}^{1/2} V_{rms}$$

where $\overline{V}^2_{n,out}$ is the spectral density value of the output noise at low frequency.

Issues about noise sources



If thermal channel noise is considered seperately



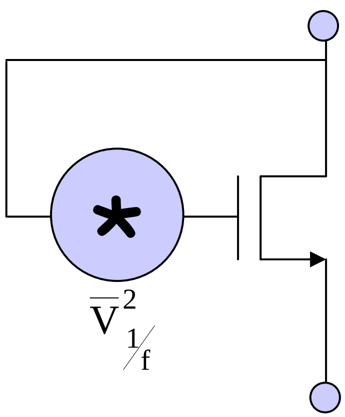
+

$$\overline{i}^2 = \frac{8}{3} KTg_m,$$

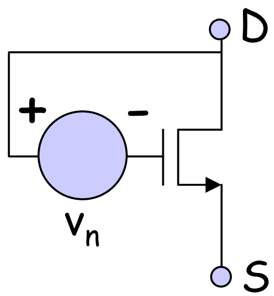
$$\overline{v}^2 = \frac{8}{3} KT \frac{1}{g_m}$$

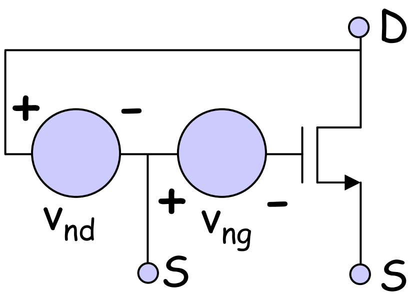
-

If the 1/f noise is considered seperately

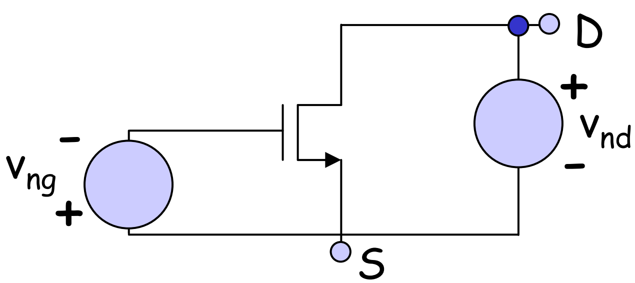


The noise can be seen as
a drain-source noise voltage
source equal to $v^2_{1/f}$
(see next page)





$v_n = v_{nd} + v_{ng}$



$v_{nd} = g_m r_o v_{ng}$ (must be satisfied)

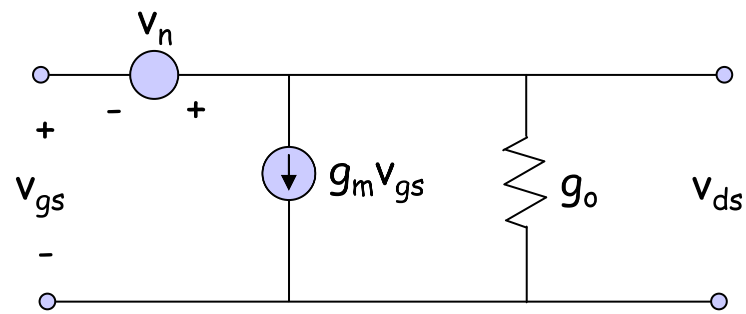
$$v_n = v_{nd} + v_{ng}$$

$$= v_{ng}\,(1 + g_m r_o)$$

or

$$v_n = v_{nd}\,(1 + 1/g_m r_o)$$

$$v_n \simeq v_{nd} = v_{ds}$$



$$v_{gs} = v_d + v_n - v_s$$

$$= v_{ds} + v_n$$

$$v_{ds} = -\frac{g_m v_{gs}}{g_o}$$

$$\therefore \quad v_{ds} = \frac{v_n}{(1 + g_m r_o)}$$

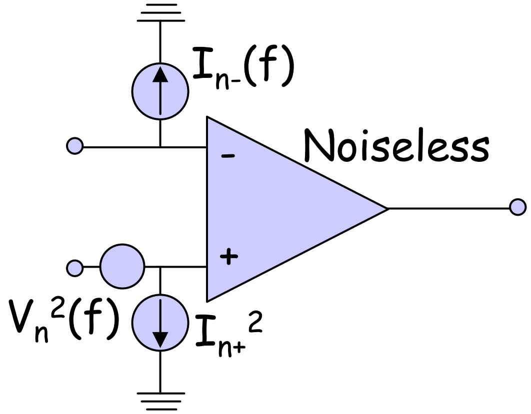
$$\& \quad v_{ds} = -\frac{g_m r_o}{1 + g_m r_o} v_n$$

$$\text{or} \quad \overline{v}_{gs}^{2} = \frac{\overline{v}_n^{2}}{(1 + g_m r_o)^2} \cong \frac{\overline{v}_n^{2}}{(g_m r_o)^2}$$

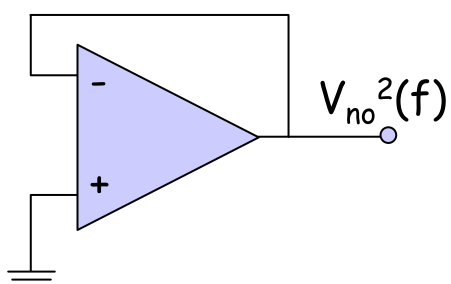
$$\& \quad \overline{v}_{ds}^{2} = \frac{\overline{v}_n^{2} \times (g_m r_o)^2}{(1 + g_m r_o)^2} \cong \overline{v}_n^{2}$$

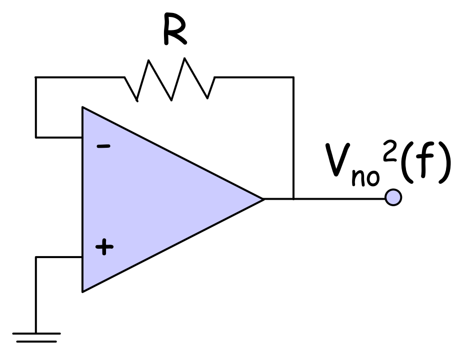
So for a diode the 1/f noise shows up as a noise voltage (shrinking the diode) of value equal to $\bar{v}_n^{2}$.

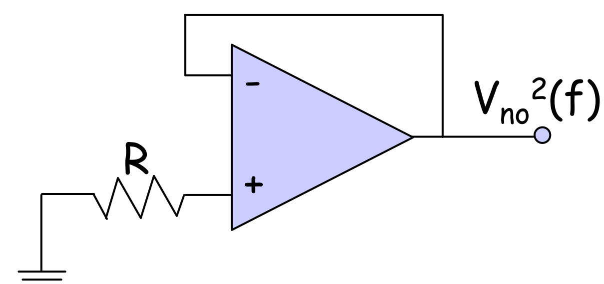
Noise in an op amp macromodel



Particular Cases:







If $\overline{V_n}^2(f)$ ignored:

$$\overline{V_{no}}^2 = 0$$

Actual:

$$\overline{V_{no}}^2 = \overline{V_n}^2$$

If $I_{n-}(f)$ ignored:

$$\overline{V_{no}}^2 = \overline{V_n}^2$$

Actual:

$$\overline{V_{no}}^2 = \overline{V_n}^2 + (I_{n-}R)^2$$

If $I_{n+}(f)$ ignored:

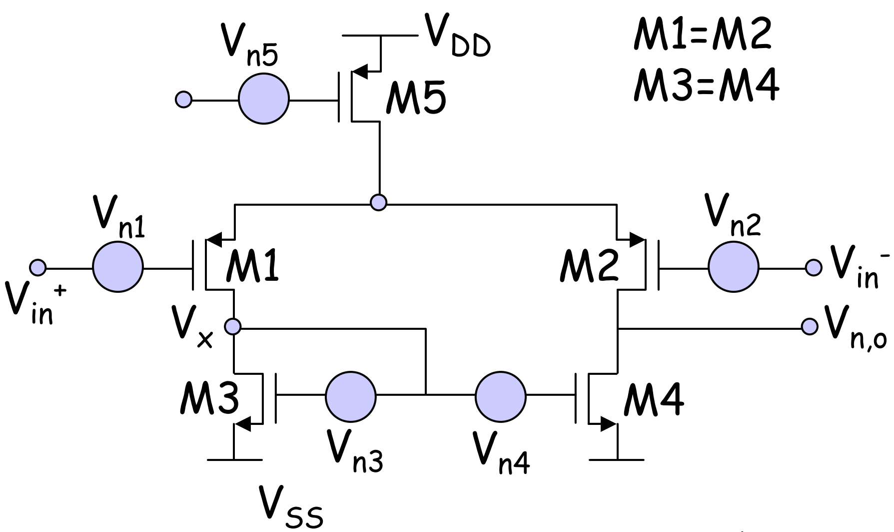
$$\overline{V_{no}}^2 = \overline{V_n}^2$$

Actual:

$$\overline{V_{no}}^2 = \overline{V_n}^2 + (I_{n+}R)^2$$

- Ideal capacitors and inductors do not generate noise, but accumulate.

CMOS Simple OTA: Noise Analysis Example



- First determine the gain from the noise sources to the output

$$\left|\frac{V_{n,o}}{V_{n1}}\right| = \left|\frac{V_{n,o}}{V_{n2}}\right| = g_{m1}R_o \overset{?}{=} \left|\frac{V_n}{V_{in}^{+} - V_{in}^{-}}\right| = A_V$$

$$\left|\frac{V_{n,o}}{V_{n4}}\right| = g_{m4}R_o$$

since $V_{gs3} = V_{gs4} \Rightarrow V_{n3} = V_{n4}$, then

$$\left|\frac{V_{n,o}}{V_{n3}}\right| = g_{m4}R_o$$

Assuming $V_x = V_{n,o}$; $R_{ox} \cong \dfrac{1}{g_{m3}}$

$$\left|\frac{V_{n,o}}{V_{n5}}\right| = \frac{g_{m5}}{2g_{m3}} \qquad \text{(small)}$$

$$V_{n,o}^2(f) = 2(g_{m1}R_o)^2 V_{n1}^2(f) + 2(g_{m3}R_o)^2 V_{n3}^2(f)$$

$$V_{n,input}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2\left(\frac{g_{m3}}{g_{m1}}\right)^2$$

$$V_{n,input}^2(f)\Big|_{\text{white part}} = \left(\frac{16}{3}\right)kT\left(\frac{1}{g_{m1}}\right) + \left(\frac{16}{3}\right)kT\left(\frac{g_{m3}}{g_{m1}}\right)^2\frac{1}{g_{m3}}$$

; $V_{n,input}^2(f)\downarrow \quad g_{m1}\uparrow$

$1 < g_{m1}/g_{m3} < 4 \quad ?$

For the white noise portion of $V_{ni}(f)$ and $V_{n3}(f)$

$$V_{ni}^2 = 4KT\left(\frac{2}{3}\right)\left(\frac{1}{g_{m1}}\right) \qquad i = 1, 3$$

Yielding

$$V_{n,input}^2(f) = \left(\frac{16}{3}\right)KT\left(\frac{1}{g_{m1}}\right) + \left(\frac{16}{3}\right)KT\left(\frac{g_{m3}}{g_{m1}}\right)\left(\frac{1}{g_{m3}}\right) \quad ; \quad g_{m1}\uparrow ?$$

On the other hand, for the 1/f noise

$$V_{n,input}^2(f)\Bigg| = 2V_{n1}^2 + 2V_{n3}^2(f)\frac{\mu_n\left(W/L\right)_3}{\mu_p\left(W/L\right)_1}$$

$$g_m = \{2\mu_j C_{ox}\left(\frac{W}{L}\right)_j I_{Dj}\}$$

Yielding

$$V_{n,input}^2(f) = \frac{2}{C_{ox}f}\left[\frac{K_1}{W_1L_1} + \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{K_3L_1}{W_1L_3^2}\right)\right]$$

For $V_{ni}^2(f) = \dfrac{K_i}{W_iL_iC_{ox}f}$

$L_3\uparrow$ $V^2_{n,input}\downarrow$ but signal swing $\downarrow$

By integrating from f_1 to f_2

$$V_{n,input}^2(rms) = 2\left[\frac{a_p}{W_1L_1} + a_n\left(\frac{\mu_n}{\mu_p}\right)\frac{L_1}{L_3^2W_1}\right] \quad ; \quad a_i = \frac{K_i}{C_{ox}}\ln\left(\frac{f_2}{f_1}\right)$$

How do you simulate noise in SPICE?

- Noise is associated with AC analysis

- Noise V(N) VIN
- AC DEC FI FSTEP FFINAL

Reader.___

Simulate both examples dealing with two transistors, plot frequency responses and noise spectrum, as well as the total noise at each frequency (total rms noise)

Obtain the signal to noise ratio

$$S/N = 20\log\frac{\text{signal for \% THD (rms)}}{\text{Total Noise (rms)}}$$

References

1. B. Razavi, "*Design of Analog CMOS Integrated Circuits*", Preview Edition, Mcgraw Hill, 2000.
2. D.A. Johns and K. Martin, *Analog Integrated Circuit Design*, New York; Wiley, 1997.
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