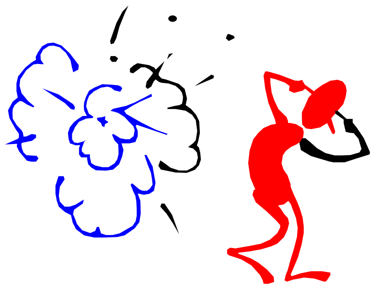
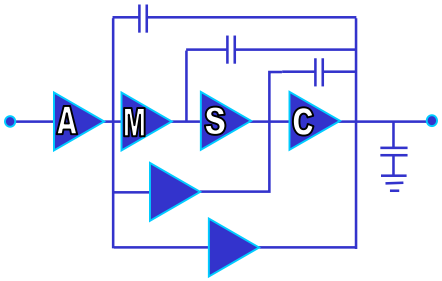
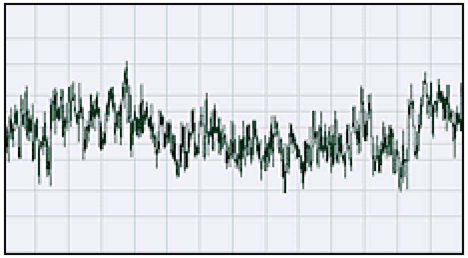




NOISE



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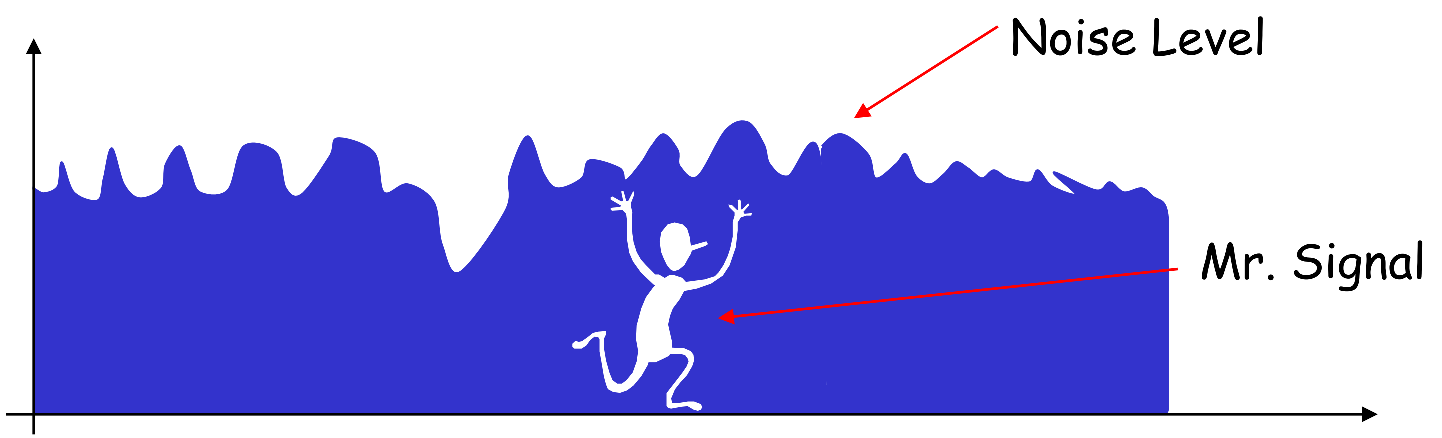
NOISE

- **NOISE** limits the minimum signal level that a circuit can process with acceptable quality.



Can you identify the signal buried in the noise?

How does the minimal signal must be with respect to the noise level?

- Let us consider the street noise, can one predict the (exact) noise at any time?

  No, because it is a random process

- If you decide to blow your car's horn every 5 minutes, then you can say this signal is deterministic.

- So, how can we incorporate noise in circuit design?

  - Observe the noise for a long time

  - Construct a "statistical model"

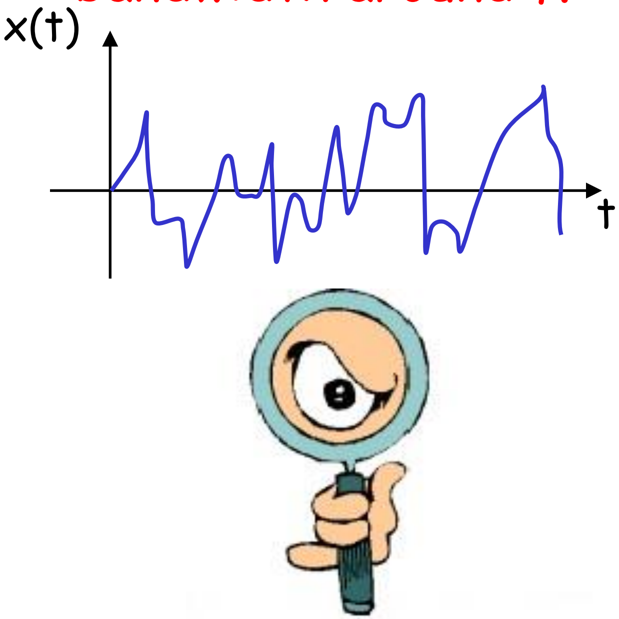
- The average power noise is predictable

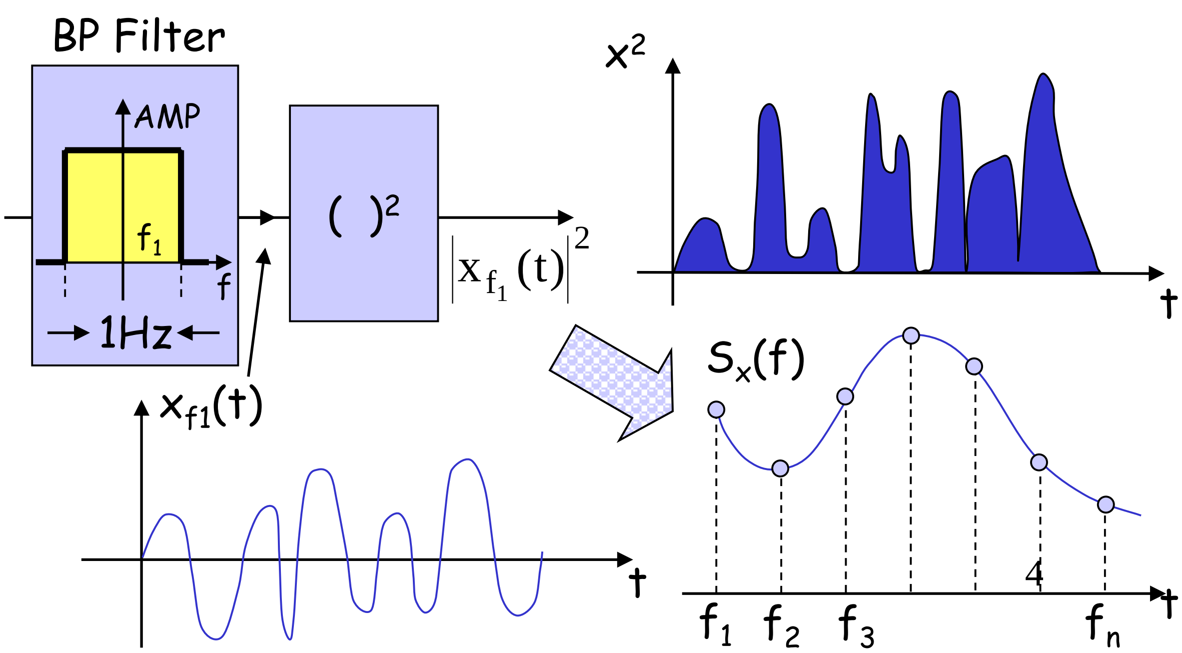
- Most noise sources in circuits exhibit a constant average power

- Average power delivered by a periodic voltage v(t), of period T, to a load resistance R_L is given by

$$P_{av} = \frac{1}{T}\int_{-T/2}^{T/2} \frac{v^2(t)}{R_L}\,dt = \frac{1}{T}\int_{-T/2}^{T/2} v(t)i(t)\,dt$$

- For non-periodic signals, T becomes a large quantity
- How much power a signal carries at each frequency is defined by the "power spectral density" (PSD) ($S_x(f)$)
- $S_x(f)$ is defined as the average power carried by x(t) in a one-hertz bandwidth around f.

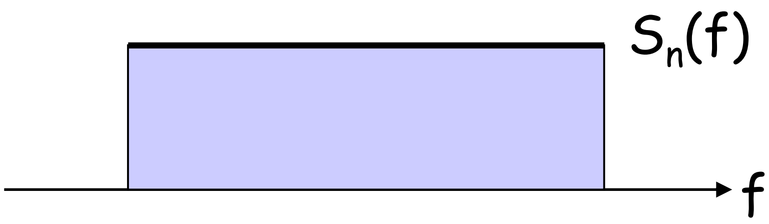




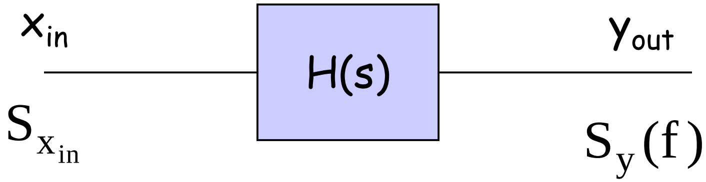
- $S_x(f)$ is expressed in V^2/Hz rather than $VI/Hz = w/Hz$, in fact $\{S_x(f)\}^{1/2} = v/Hz^{-1/2}$ is often used. Say a filter at 1MHz is equal to 1.414 $nV/Hz^{-1/2}$, means an average power in a 1-Hz bandwidth at 1MHz is equal to $(1.414\times10^{-9})^2V^2=2\times10^{-18}V^2$

- Another spectrum is the "White Spectrum" or white noise

In practice is band limited.

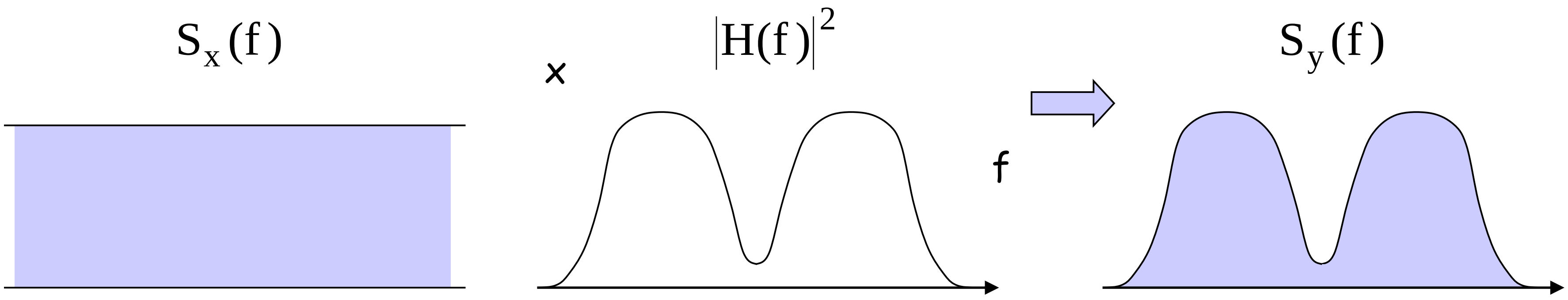


- How do you determine the output spectrum of a linear, time-invariant system with transfer function, H(s)?



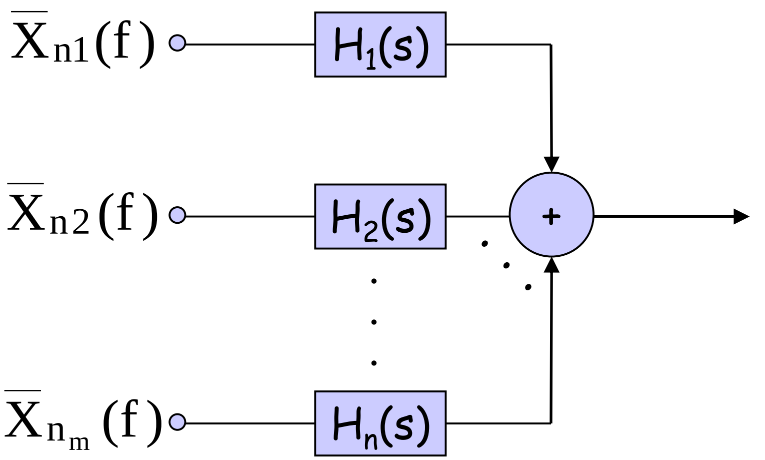
$$S_y(f) = S_x(f) \times |H(f)|^2$$

- How is the shape of the output spectrum?



- A practical example is the telephone bandwidth, where BW of $S_{xin}(f)$ is between 0-20KHz, BW of H(f) is 4KHz and consequently BW of $S_{yout}(f)$ is 4 KHz.

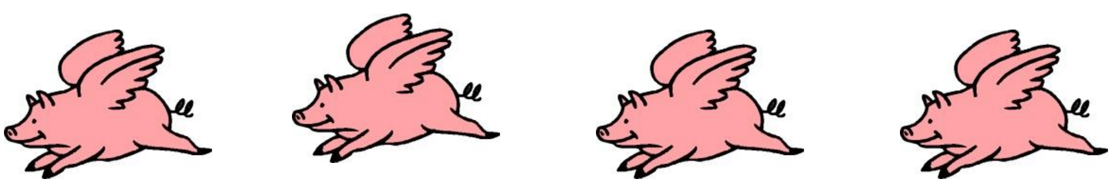
If there are more than two noise sources, how do you compute their total effects?



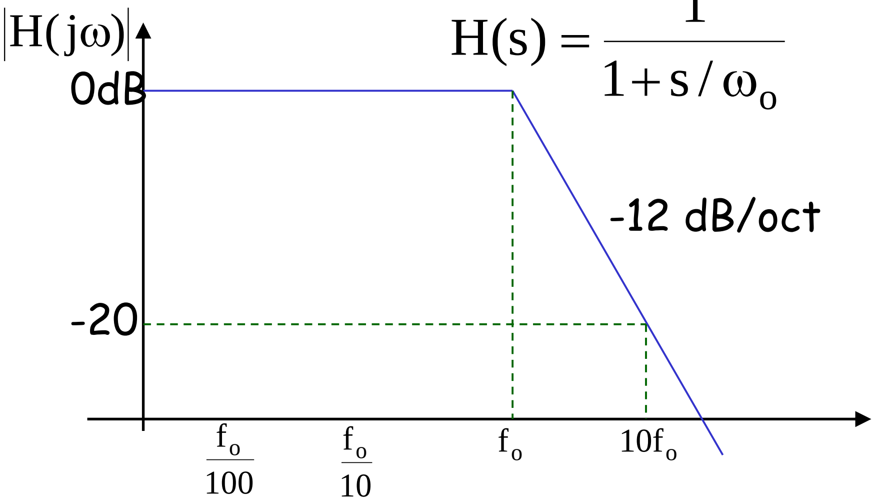
$$S_y(f) = \sum \left|H_i\left(j2\pi f\right)\right|^2 \overline{X}_{ni}^2(f)$$

$\overline{x}_{n1}(f)$, $\overline{x}_{n2}(f)$, ... are uncorrelated noise sources

Superposition is applied to obtain the total output spectrum.

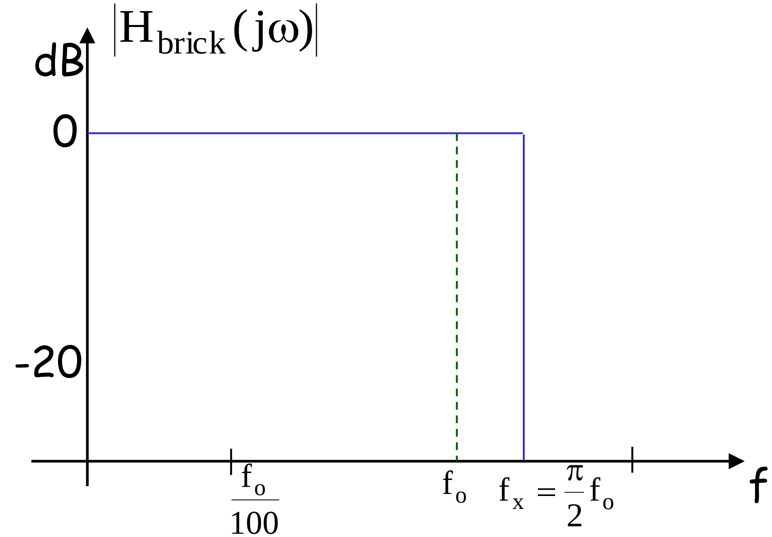


Noise Bandwidth



$$H(s) = \frac{1}{1 + s/\omega_o}$$

First - Order LP



Brick - Wall Filter

Assume an input signal, $V_{ni}(f)=V_{nw}$ (constant)

$$\overline{V}^2_{n,out(rms)} = \int_o^\infty \overline{V}^2_{ni}(f)\left|H(2\pi fj)\right|^2 df = \int_o^\infty \frac{\overline{V}^2_{nw}}{1+\left(f/f_o\right)^2} df = \overline{V}_{nw} f_o \tan^{-1}\left(\frac{f}{f_o}\right)\Bigg|_o^\infty = \frac{\overline{V}_{nw}\pi f_o}{2}$$

If this same input signal, V_{nw}, is applied to the (brick-wall) filter

$$\overline{V}^2_{brick(rms)} = \int_o^{f_x} \overline{V}^2_{nw} df = \overline{V}^2_{nw} f_x \quad , \quad \text{since} \quad V_{n,out} = V_{brick} \quad , \quad \text{then} \quad f_x = \frac{\pi f_o}{2}$$

Equivalent Noise Bandwidth

$$\Delta f = \frac{1}{A_{vo}^2} \int_o^{\infty} \left| A_v(f) \right|^2 df$$

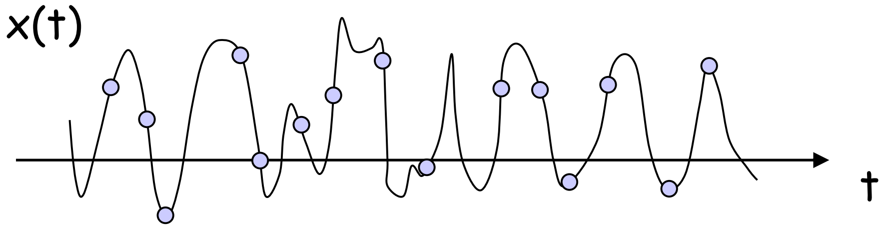
Examples

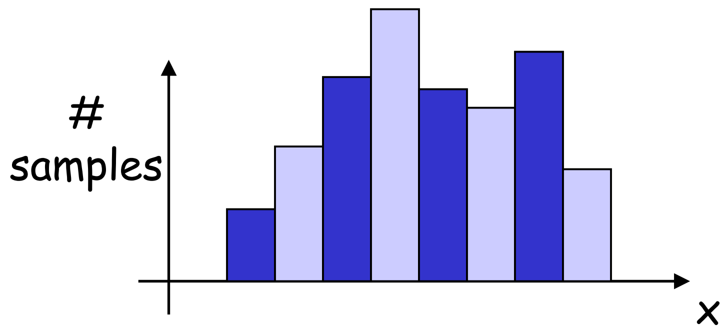
First-order system:

$$A_v(f) = \frac{1}{1 + j\,{}^{f}\!/_{f_{3dB}}} \quad , \quad \Delta f = \int_o^{\infty} \frac{df}{1 + \left({}^{f}\!/_{f_{3dB}} \right)^2} = 1.571\, f_{3dB}$$

$$A_v(f) = \left| \frac{1}{1 + j\,{}^{f}\!/_{f_{3dB}}} \right|^2 \quad , \quad \Delta f = \int_o^{\infty} \left| \frac{1}{1 + \left({}^{f}\!/_{f_{3dB}} \right)^2} \right| df = 1.22\ f_a = 1.22 \times 0.6436\ f_{3dB}$$

- Amplitude Distribution of Noise





Probability Density Function (PDF), the distribution of x(t) is

$$p_x(x)dx = \text{probability of } x < X < x + dx$$

- An important example of PDFs is the Gaussian (or normal) Distribution.

$$p_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2}$$

Where σ and m are the standard deviation and mean of the distribution, respectively.

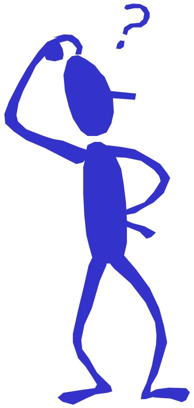
- How do you determine the total average power due to two or more noise sources?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [x_1(t) + x_2(t)]^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (x_1^2(t)dt + x_2^2(t)dt + 2x_1(t)x_2(t)dt$$

$$P_{av} = P_{av_1} + P_{av_2} + \underbrace{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 2x_1(t)x_2(t)dt}_{=0\,?\quad \text{When?}}$$

If noise sources are uncorrelated the third term is zero.

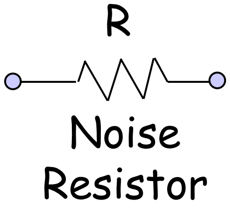
Example of uncorrelated and correlated noises is that of spectators in a sports stadium.

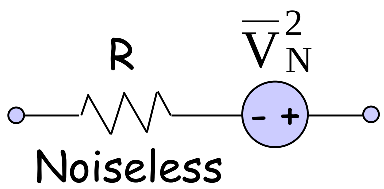


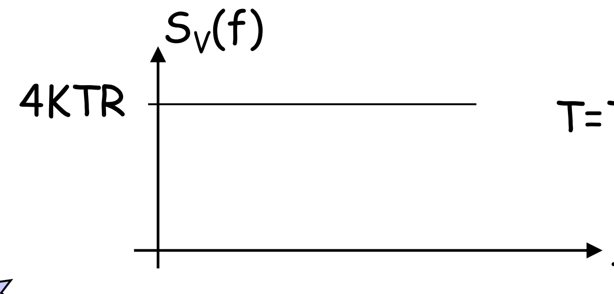
Noise Types

- Device Electronics Noise
- Environmental Noise

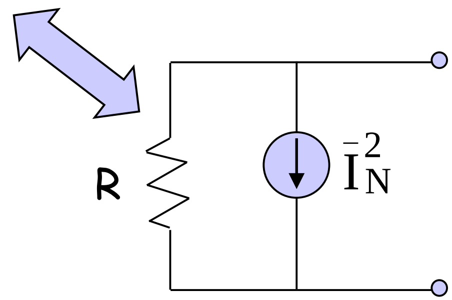
- Thermal Noise







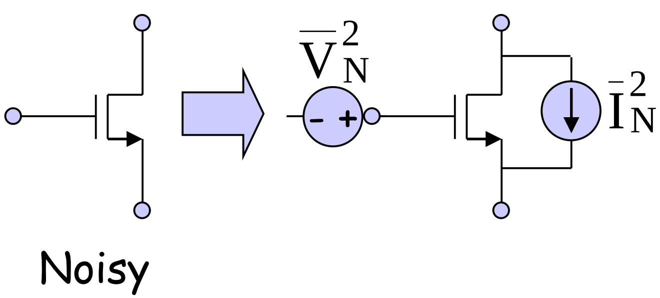
$$\overline{V}_N^2 = 4KTR$$



$$\overline{I}_N^2 = \frac{4KT}{R}$$

Example

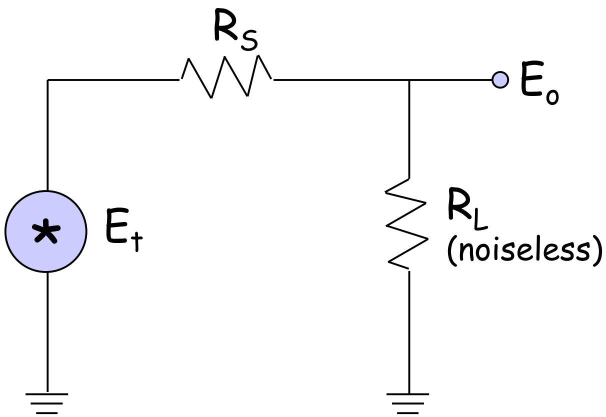
$$R = 50\Omega \quad T = 300^\circ K \quad \Rightarrow \quad \overline{V}_N^2 = 8.28 \times 10^{-19} V^2 / Hz \quad \Rightarrow \quad (0.91 nV / \sqrt{Hz})^2$$



$$\overline{I}_N^2 = 4KT\left(\frac{2}{3}\right)g_m \quad \Rightarrow$$ Thermal (White) Noise

$$\overline{V}_N^2 = \frac{K_{dev}}{WLC_{ox}f} \quad \Rightarrow$$ Flicker Noise

Noise Voltage



Power Supplied to R_L is

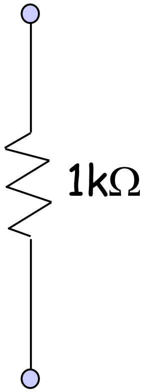
$$P_t = I_o E_o = \frac{E_o^2}{R_L}$$

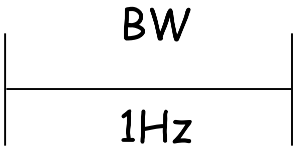
$$P_t \Big|_{R_S = R_L = R} = \frac{E_t^2}{4R} = KT\Delta f \quad , \quad E_o = \frac{E_t}{2R} R = \frac{E_t}{2}$$

Then

$$E_t = \sqrt{4kTR\Delta f} \quad (V_{rms})$$

$$E_t \Big|_{\substack{R = 1K\Omega \\ \Delta f = 1Hz}} = 4n\ V_{rms}$$



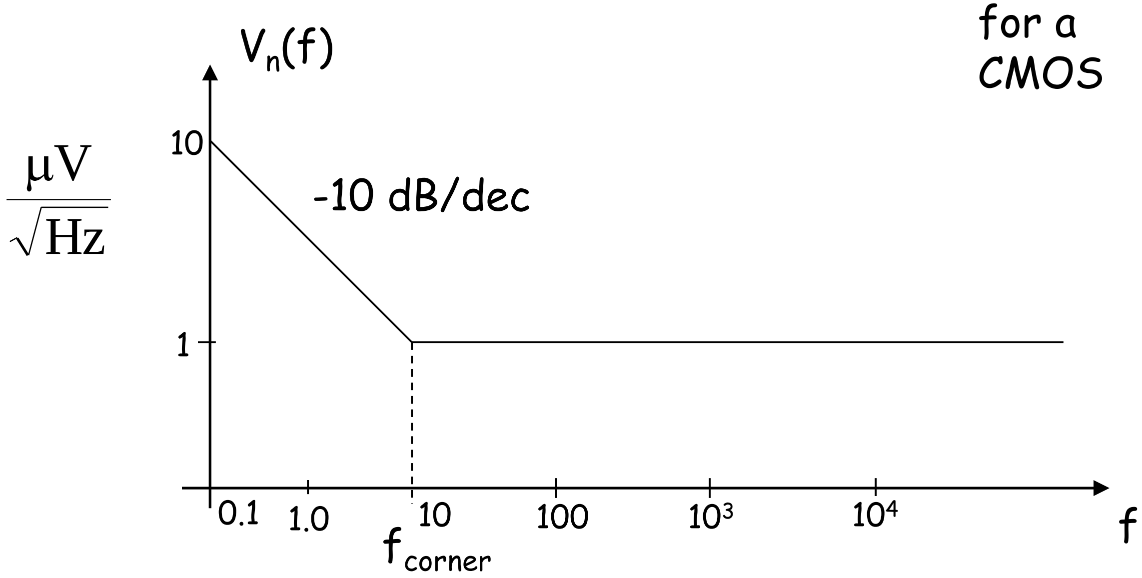


- Flicker or 1/f Noise

PSD is given by

$$\overline{V_n^2}(f) = \frac{K_V^2}{f} = \left(\frac{K_V}{\sqrt{f}}\right)^2 \Bigg| = \frac{K}{C_{ox}WL}\frac{1}{f}$$

for a CMOS



$K \sim 10^{-25} V^2 F$

1/f noise dominates

Low Frequency

white noise dominates

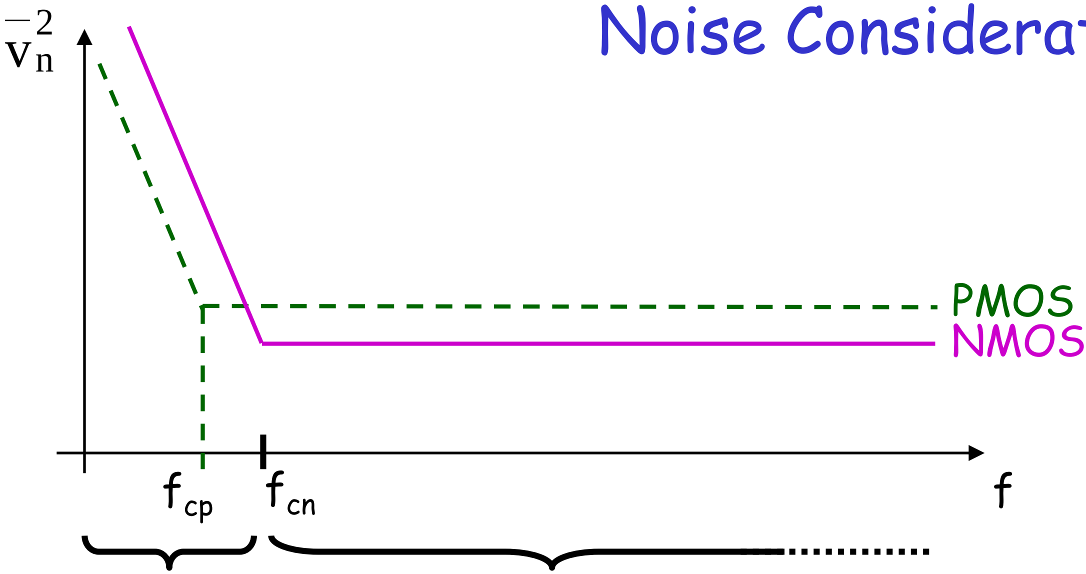
High Frequency

Equating output currents

$$4KT\left(\frac{2}{3}g_m\right) = \frac{K}{C_{ox}WL}\frac{1}{f_c}g_m^2$$

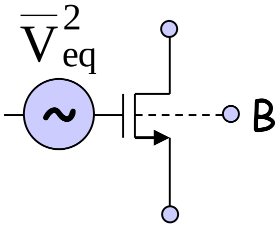
$$f_c = \frac{K}{C_{ox}WL}g_m\frac{3}{8KT}$$

Noise Considerations



Flicker Noise

Thermal or Johnson Noise (White Noise)



$$\eta = \frac{g_{mb}}{g_m}$$

$$\overline{V_N^2} = \overline{V_{eq}^2} = \underbrace{\left\{\frac{KF \cdot \Delta f}{2fC_{ox}K_pWL}\right\}}_{\text{Flicker Noise}} + \underbrace{\frac{8kT(1+\eta)}{3g_m}\Delta f}_{\text{White Noise}}$$

- To reduce Flicker Noise:

  i.e.,

  $$KF_p = 2.7\times10^{-21}\,A$$
  $$KF_n = 4.3\times10^{-21}\,A$$

  Increase W * L (Area)

  To keep same bias point keep same (W/L)

- To reduce White Noise

  Increase g_m. Since $g_m \cong \sqrt{2K_pI_D\left(\frac{W}{L}\right)} = K_p(V_{gs}-V_T)\frac{W}{L}$

  (a) (W/L)↑ and (I_D)↑ will increase g_m, power consumption (I_D), and area

  (b) (W/L)↑ and modify the bias to keep I_D same as before increasing g_m.

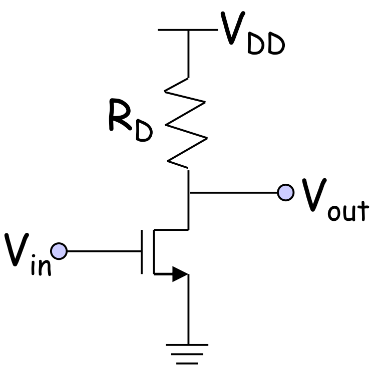
How to compute the total output noise due to individual noise sources?

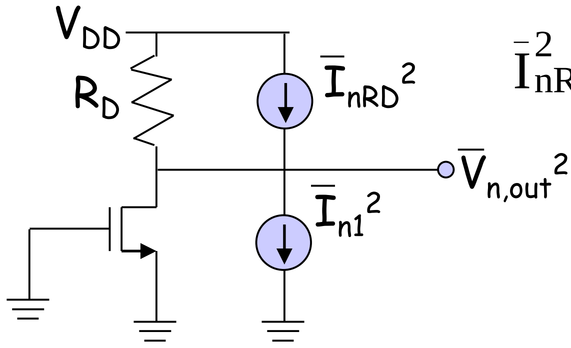
How to compute the input-referred noise?

Example.___

$$\overline{I_{n1}^2} = 4KT\left(2/3\right)g_m + Kg_m^2/(C_{ox}WLf)$$

$$\overline{I_{nRD}^2} = 4KT/R_D$$





$$\overline{V_{n,out}^2} = \left(4KT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\frac{1}{f}g_m^2 + \frac{4KT}{R_D}\right)R_D^2$$

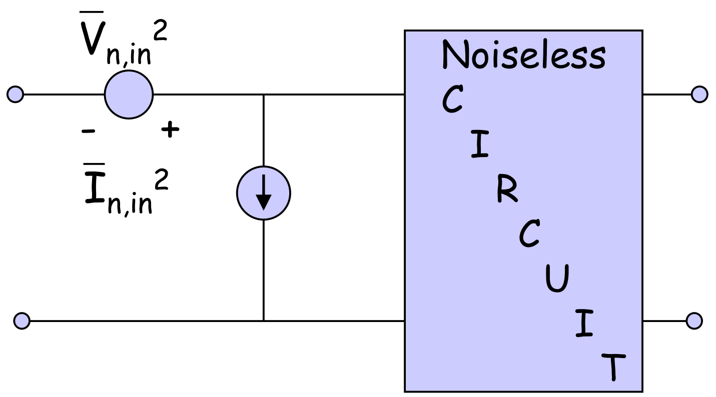
in 1Hz at frequency f

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_V^2} \cong \frac{1}{g_m^2 R_D}\left(\overline{V_{n,out}^2}\right)$$

$$\overline{V_{n,in}^2} = 4KT\frac{2}{3g_m} + \frac{K}{C_{ox}WL}\frac{1}{f} + \frac{4KT}{g_m^2 R_D}$$

⟹ Good performance parameter

- Can we always use the input-referred noise by a single voltage source in series with the input?



A necessary and sufficient representation

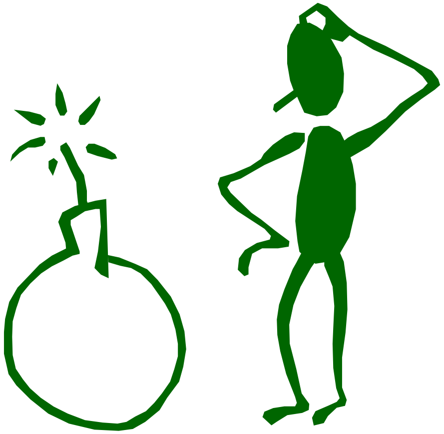
- Simplifications

  - For zero source impedance, $\overline{I^2_{n,in}}$ not affecting the output

  - For infinite source impedance, then $\overline{V^2_{n,\,in}}$ has no effect

- Note that both sources will not count the noise twice.

To compute the rms output noise:



(i) Compute the frequency response bandwidth f_{3dB}

(ii) Compute the noise bandwidth, Δf

$$\Delta f = 1.571 * f_{3dB}$$ (assuming a single pole)

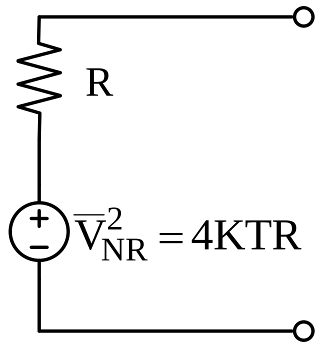
(iii) $$V_{out,noise} = \left\{ \overline{V}^2_{n,out} \cdot \Delta f \right\}^{1/2} V_{rms}$$

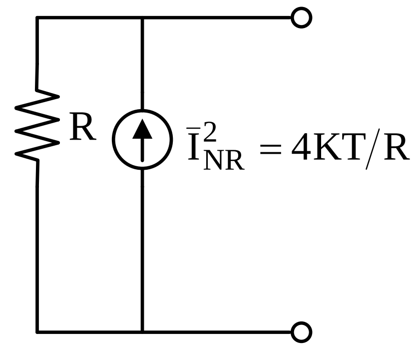
where $\overline{V}^2_{n,out}$ is the spectral density value of the output noise at low frequency.

NOISE CONSIDERATIONS REMARKS

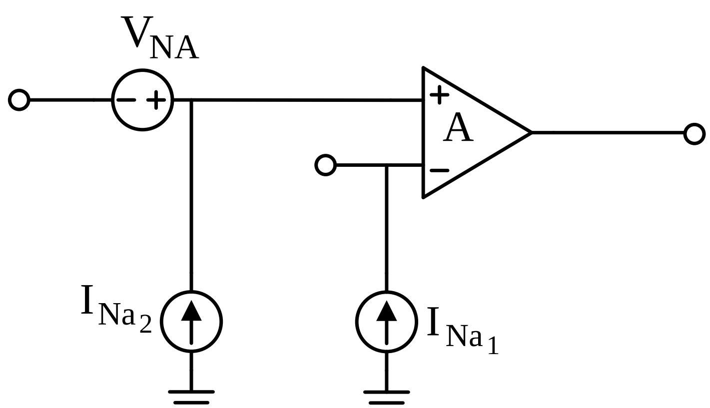
Basic elements and their noise models

Resistor. -

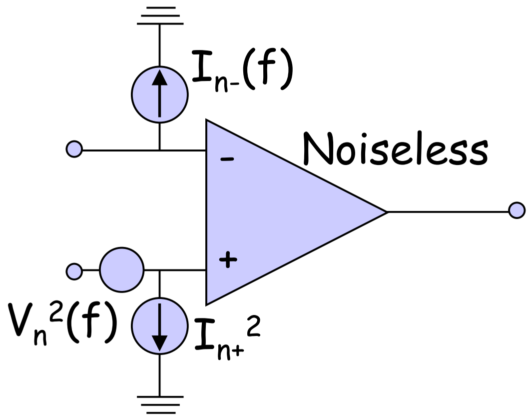




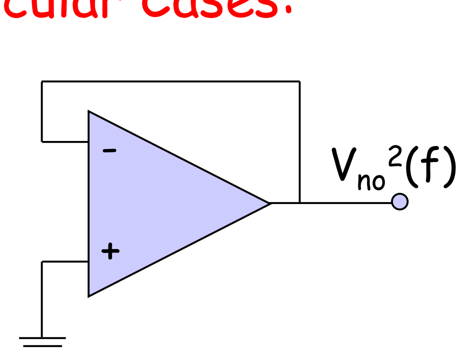
Op Amp



Noise in an op amp macromodel



Particular Cases:

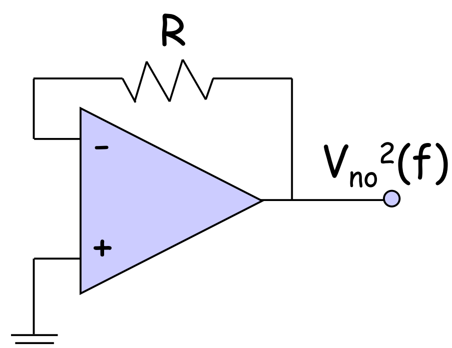


If $\overline{V}_n^2(f)$ ignored:

$\overline{V}_{no}^2 = 0$

Actual:

$\overline{V}_{no}^2 = \overline{V}_n^2$

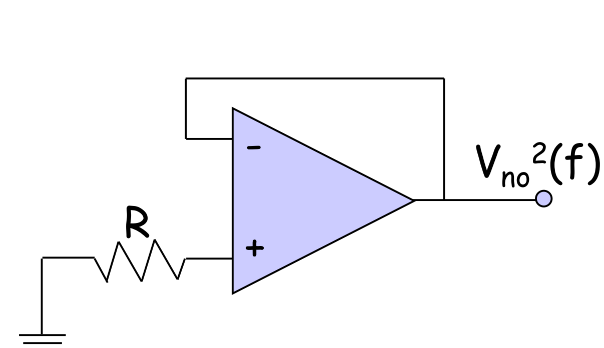


If $I_{n-}(f)$ ignored:

$\overline{V}_{no}^2 = \overline{V}_n^2$

Actual:

$\overline{V}_{no}^2 = \overline{V}_n^2 + (I_{n-}R)^2$



If $I_{n+}(f)$ ignored:

$\overline{V}_{no}^2 = \overline{V}_n^2$

Actual:

$\overline{V}_{no}^2 = \overline{V}_n^2 + (I_{n+}R)^2$

- Ideal capacitors and inductors do not generate noise, but accumulate.

Computation of Total RMS Noise Voltage at Filter Output

1. Determine the noise transfer function $T_k(s) = \frac{V_2}{N_k}$ from each equivalent voltage or current noise $n_k^2 = \overline{V}_{n_k}^2$ or $\overline{I}_{n_k}^2$ of the kth element, to the output of the filter.

2. Spectral density from the different noise sources are added:

$$\overline{V}_{no}^2(\omega) = \sum_k \left|T_k(j\omega)\right|^2 \cdot (n_k)^2$$

where $\left|T_k(j\omega)\right|$ is the absolute value of the noise transfer function from source kth.

3. Obtain the total output noise power by integrating the mean square noise spectral density $\overline{V}_{no}^2(\omega)$.

That is:

$$(E_{no})_{rms}^2 = \int_o^\infty \overline{V}_{NO}^2(\omega)d\omega$$

This is often referred as the noise floor.

Dynamic range of an Active-RC is obtained as:

$$DR = 20 \log \frac{(V_{so\ rms})_{max}}{(E_{no})_{rms}} \qquad [dB]$$

Where $V_{so\ rms}$ is the maximum undistorted rms voltage at the output.

How do you simulate noise in SPICE?

- Noise is associated with AC analysis

- Noise V(N) VIN

- AC DEC FI FSTEP FFINAL

Reader.____

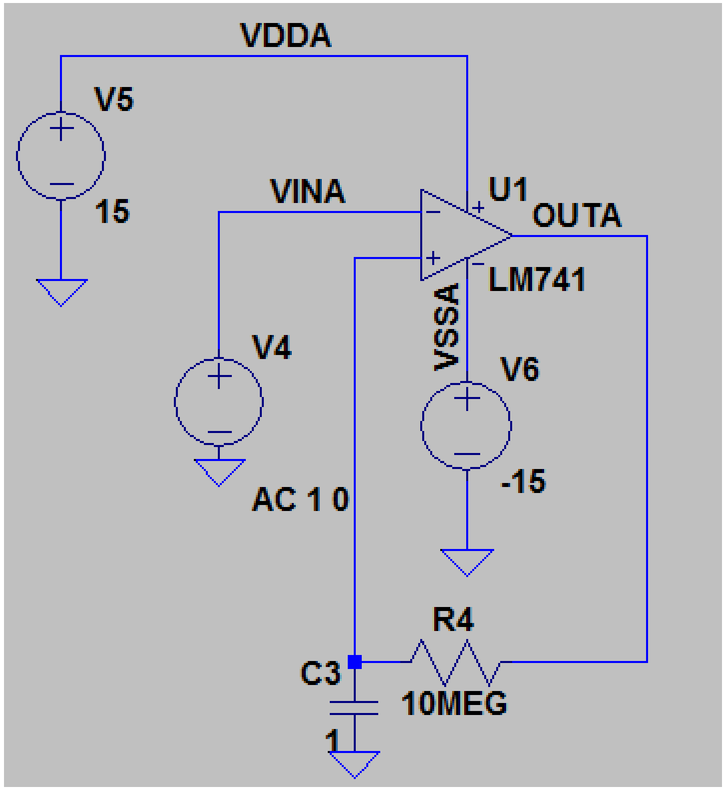
Simulate an example dealing with first-order active RC filter, plot frequency responses and noise spectrum, as well as the total noise at each frequency (total rms noise)
Obtain the signal to noise ratio

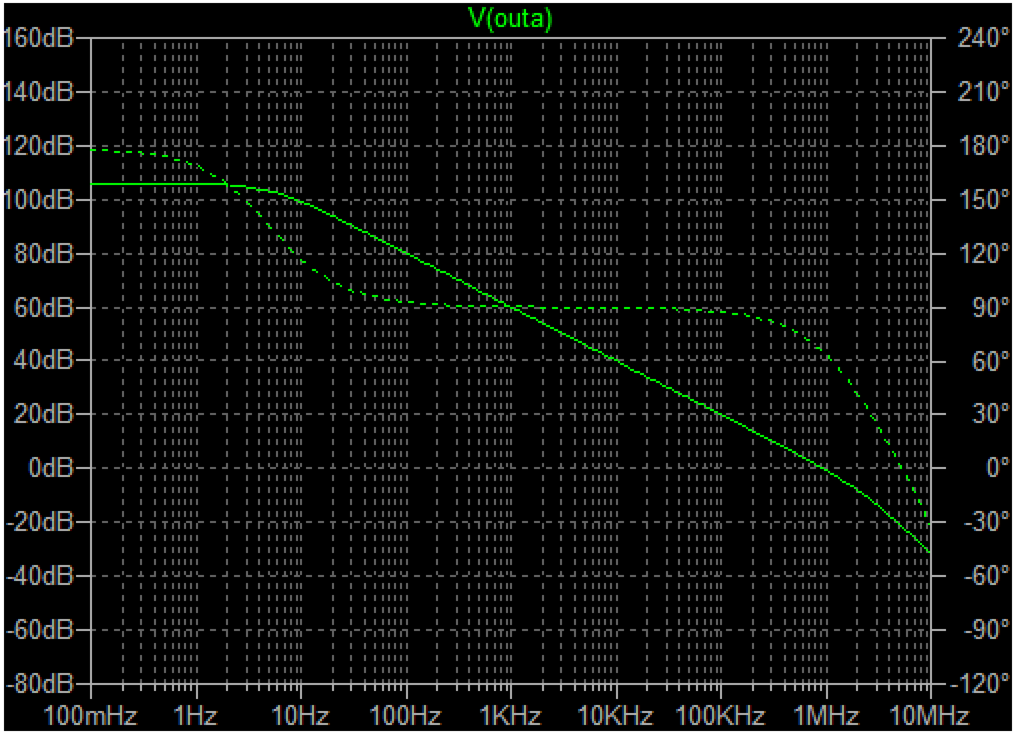
$$S/N = 20\log\frac{\text{signal for \% THD (rms)}}{\text{Total Noise (rms)}}$$

LTSPICE Noise: Analysis & Simulation

Contributed by Kyoohyun Noh

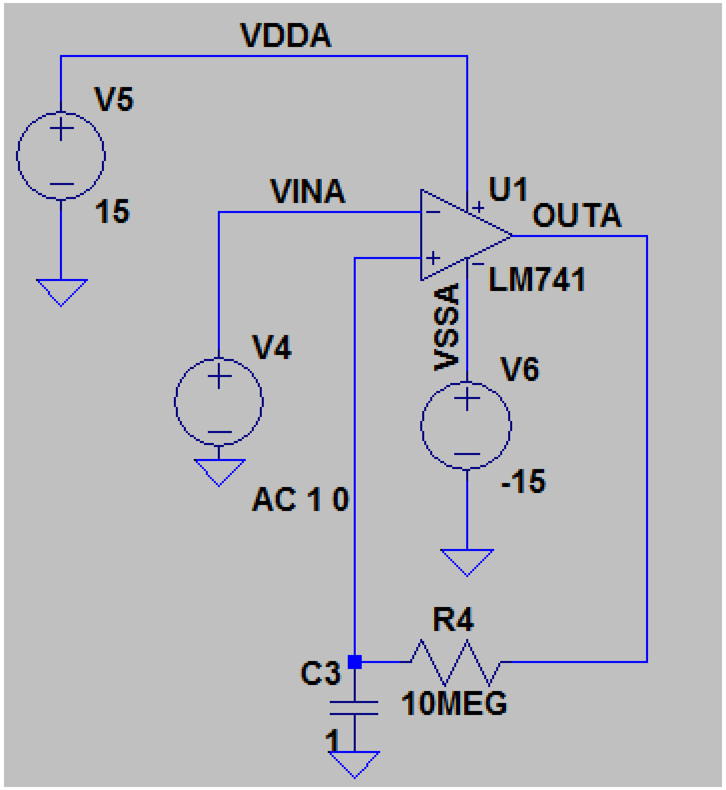
LM741's open loop ac response

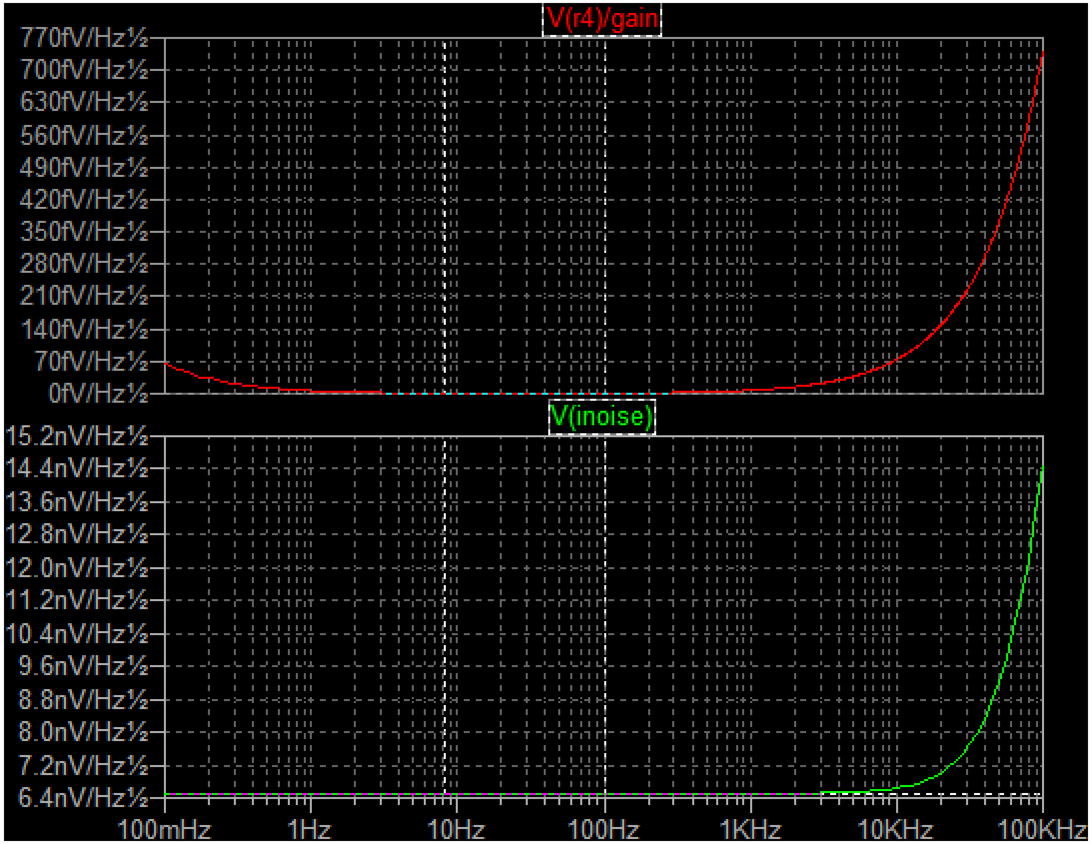




- **LM741's DC gain : 106dB**
- **LM741's -3dB frequency: 5Hz**
- **LM741's unity gain frequency: 1MHz**

LM741's noise simulation

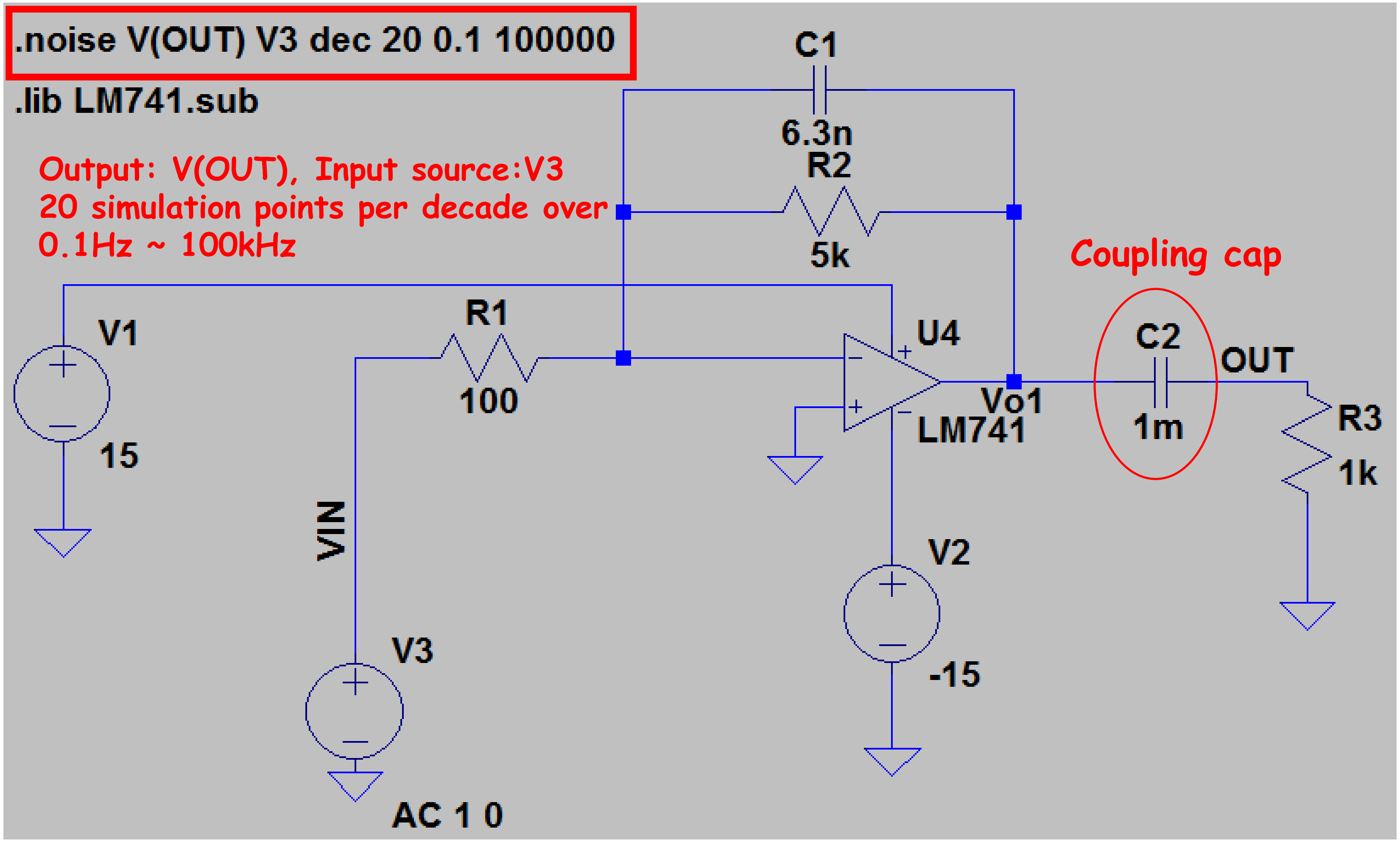




- **LM741's input-referred noise PSD is simulated: R4's noise contribution is small enough to be neglected.**

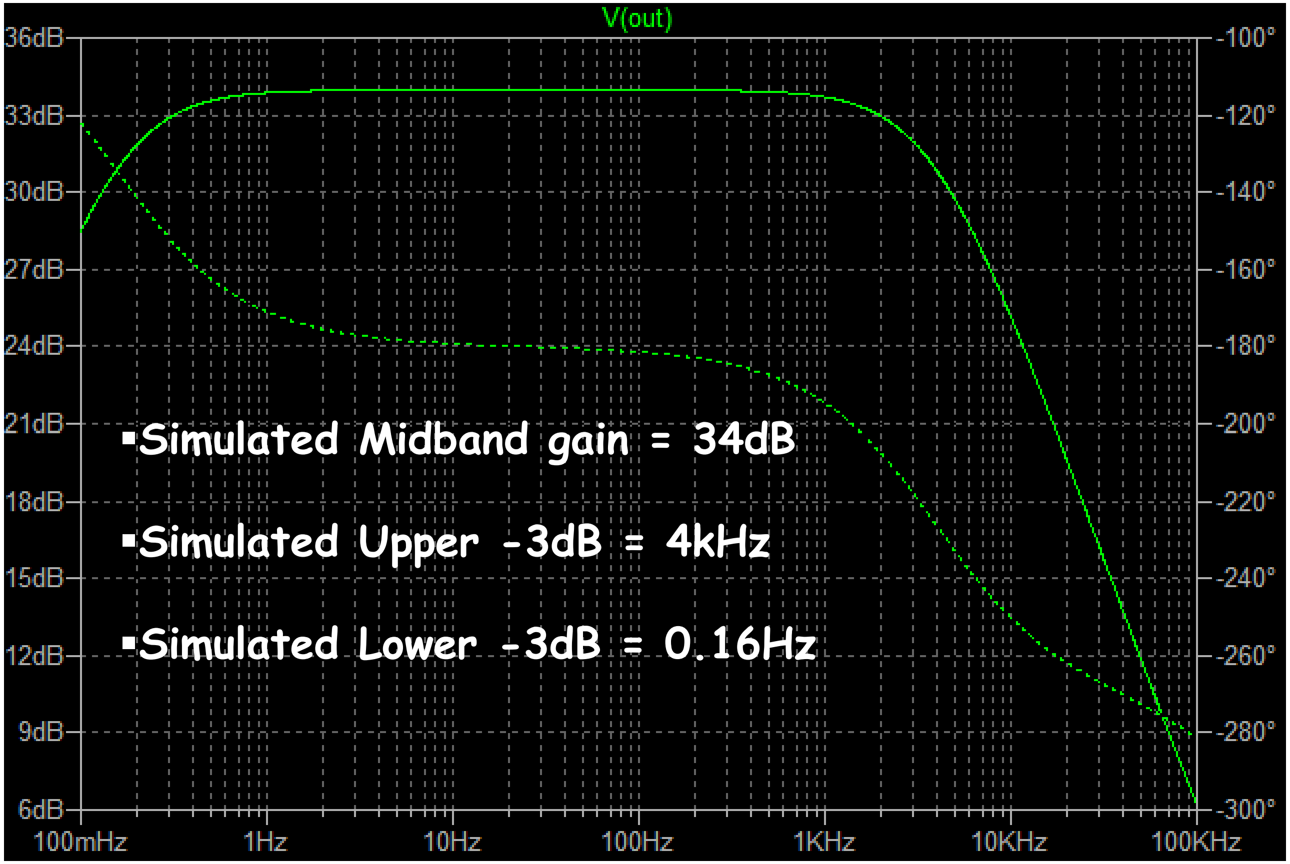
$$\sqrt{\overline{v_{n,LM741,open,input}^2}/\Delta f} = 6.51nV/\sqrt{Hz} \qquad \sqrt{\overline{v_{n,R4,input}^2}/\Delta f} = 0.67\,fV/\sqrt{Hz}$$

Filter Schematic for .noise simulation

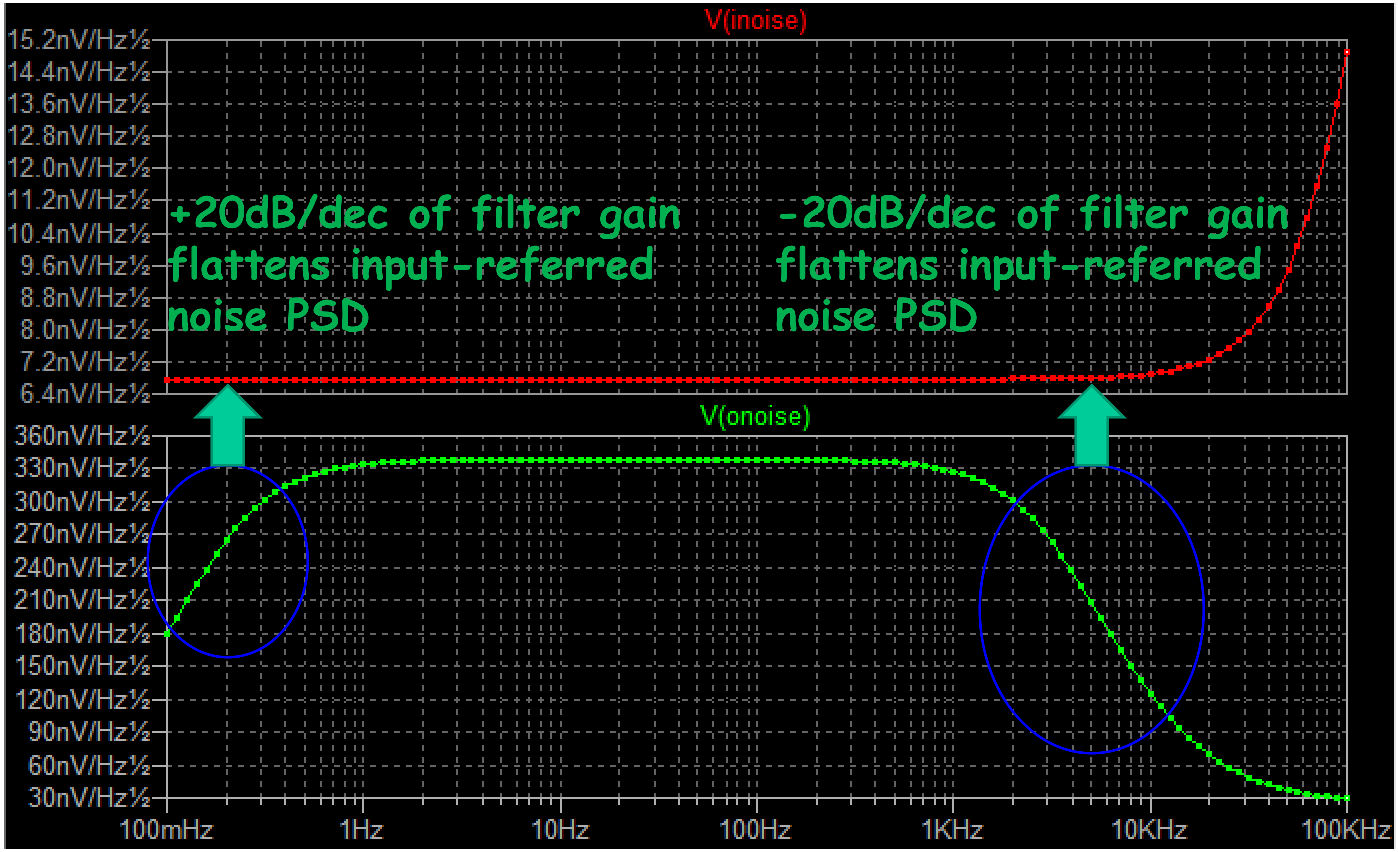


- Filter's midband gain : 50 (34dB) from R_2/R_1
- Expected upper -3dB bandwidth : about 5kHz (from $1/R_2C_1$)
- Expected lower -3dB bandwidth : about 0.16Hz (from $1/R_3C_2$)

Filter Transfer Function



Input/Output-referred noise PSD



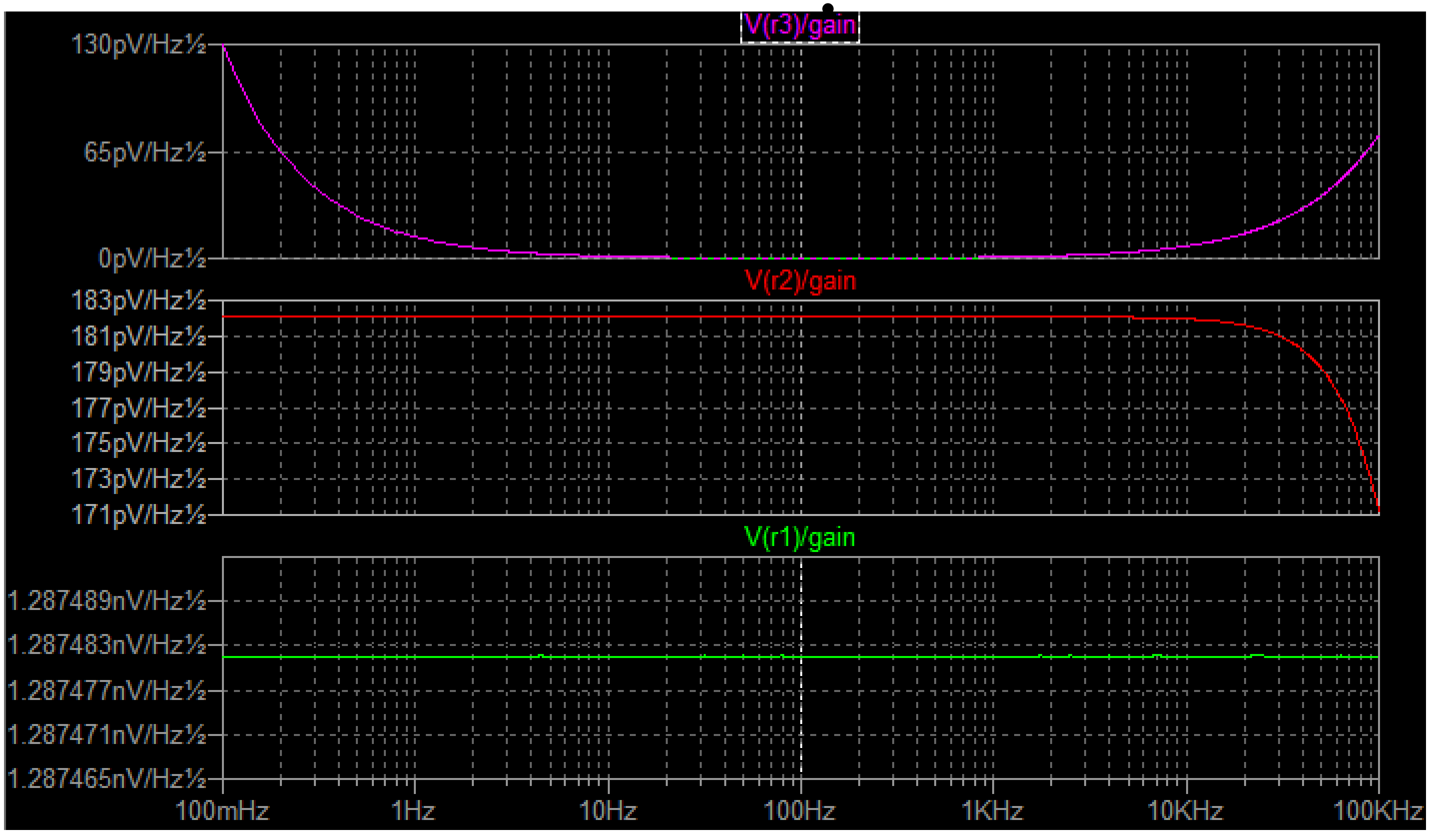
- **Midband input-referred noise PSD**

$$\sqrt{\overline{v_{n,input}^2}/\Delta f} = 6.77nV/\sqrt{Hz}$$

- **Midband output-referred noise PSD**

$$\sqrt{\overline{v_{n,output}^2}/\Delta f} = 338nV/\sqrt{Hz}$$

Noise contribution to Input-referred



Output Resistance R3

Feedback Resistance R2

Input Resistance R1

Noise Component	Theoretical input-referred noise voltage	Simulation
R1	$\sqrt{4kTR_1} = 1.29nV/\sqrt{Hz}$	$1.29nV/\sqrt{Hz}$
R2	$(R_1/R_2)\cdot\sqrt{4kTR_2} = 182pV/\sqrt{Hz}$	$182pV/\sqrt{Hz}$
R3	0	$90fV/\sqrt{Hz}$

Assume T=300K

Input-referred noise contribution in the filter

- LM741's noise contribution cannot be obtained directly in the filter simulation. Instead, it should be estimated from the other noise values

$$\sqrt{\overline{v_{n,LM741,filter,input}^2}/\Delta f} = \sqrt{(6.77nV)^2 - (1.29nV)^2 - (182pV)^2 - (90fV)^2} = 6.64nV/\sqrt{Hz}$$

- The input-referred noise voltage of the LM741 was obtained in the open loop simulation.

$$\sqrt{\overline{v_{n,LM741,open,input}^2}/\Delta f} = 6.51nV/\sqrt{Hz}$$

- Theoretical LM741's input-referred noise contribution agrees well with the simulation result

$$\sqrt{\overline{v_{n,LM741,filter,input}^2}/\Delta f} = (1+\frac{R_1}{R_2})\cdot\sqrt{\overline{v_{n,LM741,open,input}^2}/\Delta f}$$

$$= 1.02*6.51nV/\sqrt{Hz} = 6.64nV/\sqrt{Hz}$$

Summary

Noise components	Midband input-referred noise voltage [nV/sqrt(Hz)]	Midband input-referred noise PSD [V^2/Hz]	Contribution [%]
R1	1.29	1.66×10^{-18}	3.63
R2	0.182	3.31×10^{-20}	0.07
R3	0.00009	8.10×10^{-27}	~0
LM741	6.64	4.41×10^{-17}	96.3
Total		4.58×10^{-17}	100

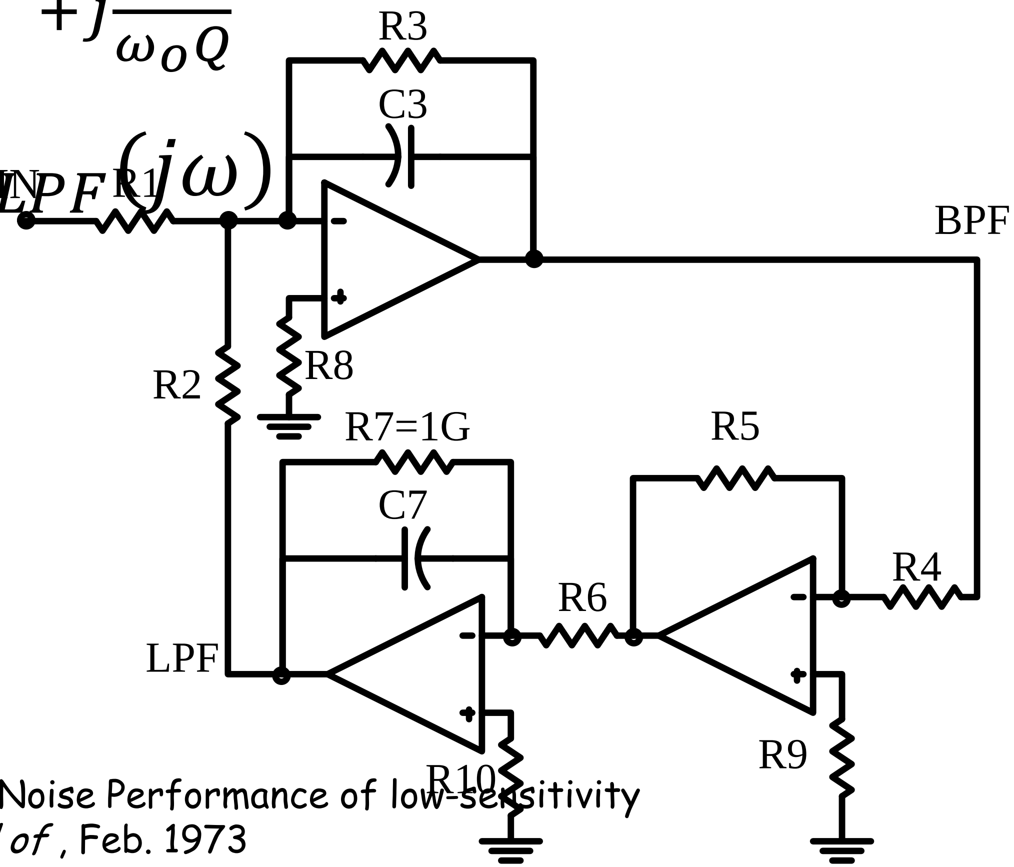
- A dominant contributor to the filter noise is LM741
- Theoretical noise analysis agrees well with simulation results

Biquad Filter Noise

Courtesy of Mohamed Abuzaid

Biquad Filter

- $H_{LPF}(j\omega) = \dfrac{1}{1-\left(\dfrac{\omega}{\omega_o}\right)^2 + j\dfrac{\omega}{\omega_o Q}}$
- $H_{BPF}(j\omega) = j\dfrac{\omega}{\omega_o} H_{LPF}(j\omega)$
- R2=R4=R5=R6=R
- C3=C7=C
- $\omega_o = 1/CR$
- $Q = R_3/R$



Bruton, L.T.; Trofimenkoff, F.N.; Treleaven, D.H., "Noise Performance of low-sensitivity active filters," *Solid-State Circuits, IEEE Journal of*, Feb. 1973

Noise Analysis

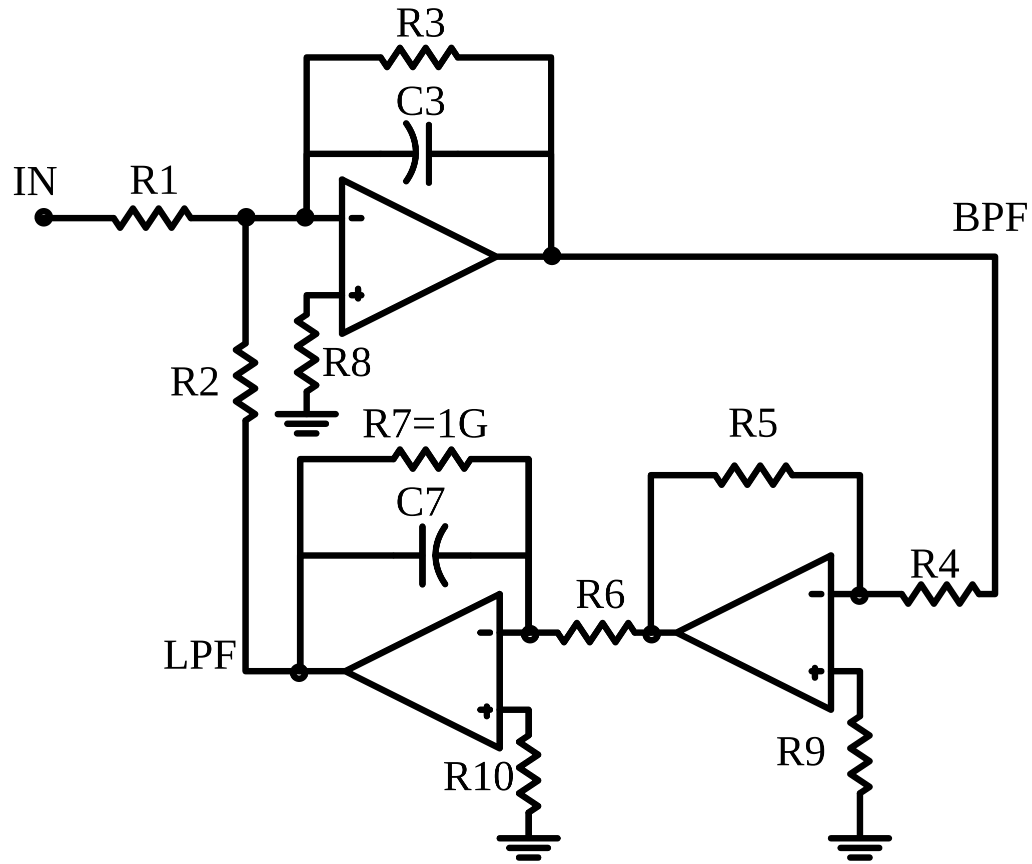
- Power Spectral Density of output noise:
- $\epsilon_{OR}^2(f) = 4KT\sum_{i=1}^{6} R_i\,|H_{io}(jf)|^2\ (\frac{V^2}{Hz})$ Divide by: 2π
- Power Spectral Density of output noise:
- $\epsilon_{OR}^2(\omega) = \frac{2KT}{\pi}\sum_{i=1}^{6} R_i\,|H_{io}(j\omega)|^2\ (\frac{V^2}{rad/s})$
- Integrated output noise:
- $E_{OR} = KTR(5Q+1)\omega_o\ (V^2)$

Continue Noise Analysis

- Assumptions for next results:
  - In band, $H_{BPF}(j\omega) = H_{LPF}(j\omega)$
  - Assume $Q >> 1$
- Power Spectral Density of output noise:
- $\epsilon_{OR}^2(f) \approx 20KT.H_{BPF}(j\omega)\ (\frac{V^2}{Hz})$
- At the center frequency
- $\epsilon_{OR}^2(f) \approx 20KT.Q\ (\frac{V^2}{Hz})$
  - "this is not correct in paper (12), they put an extra ω term"

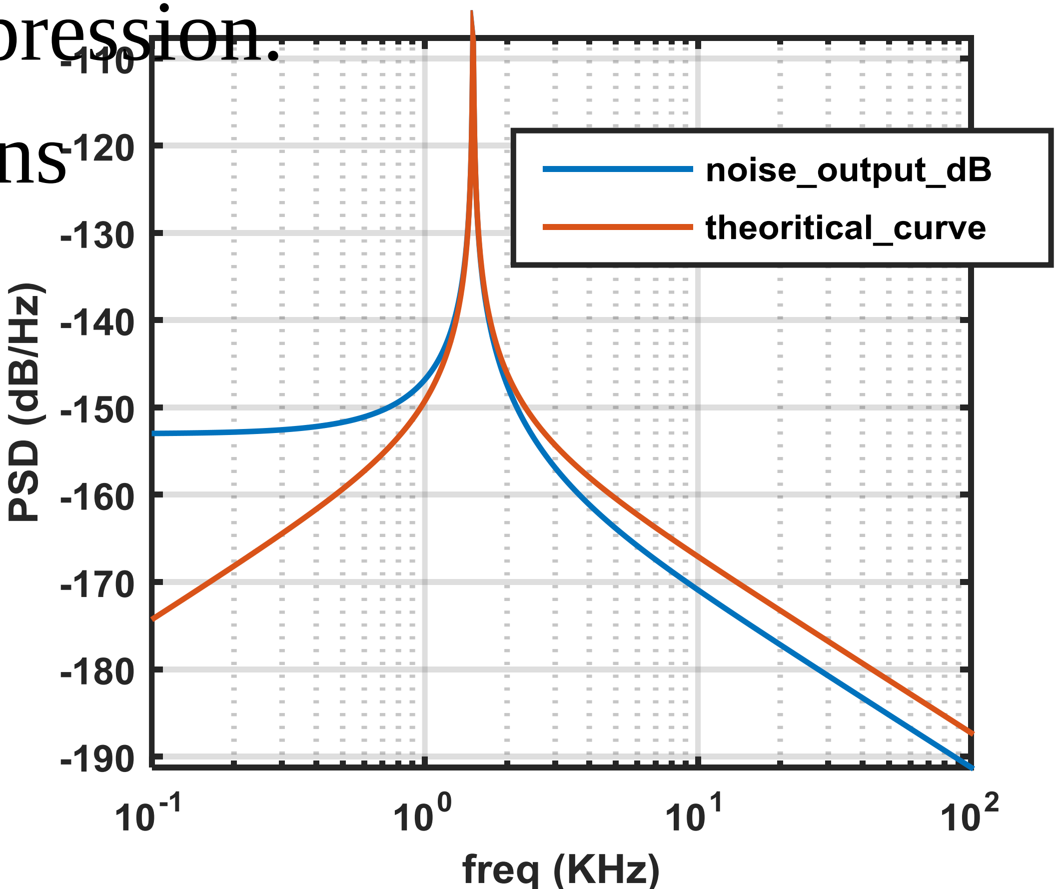
Simulation Ideal Opamp

- Biquad with the specs:
- $f_o = 1.5\, kHz$
- $Q = 150$
- So, the values:
- $R = 10\ K\Omega$
- $R_3 = 1.5\ M\Omega$
- $C = 10.6\, nF$



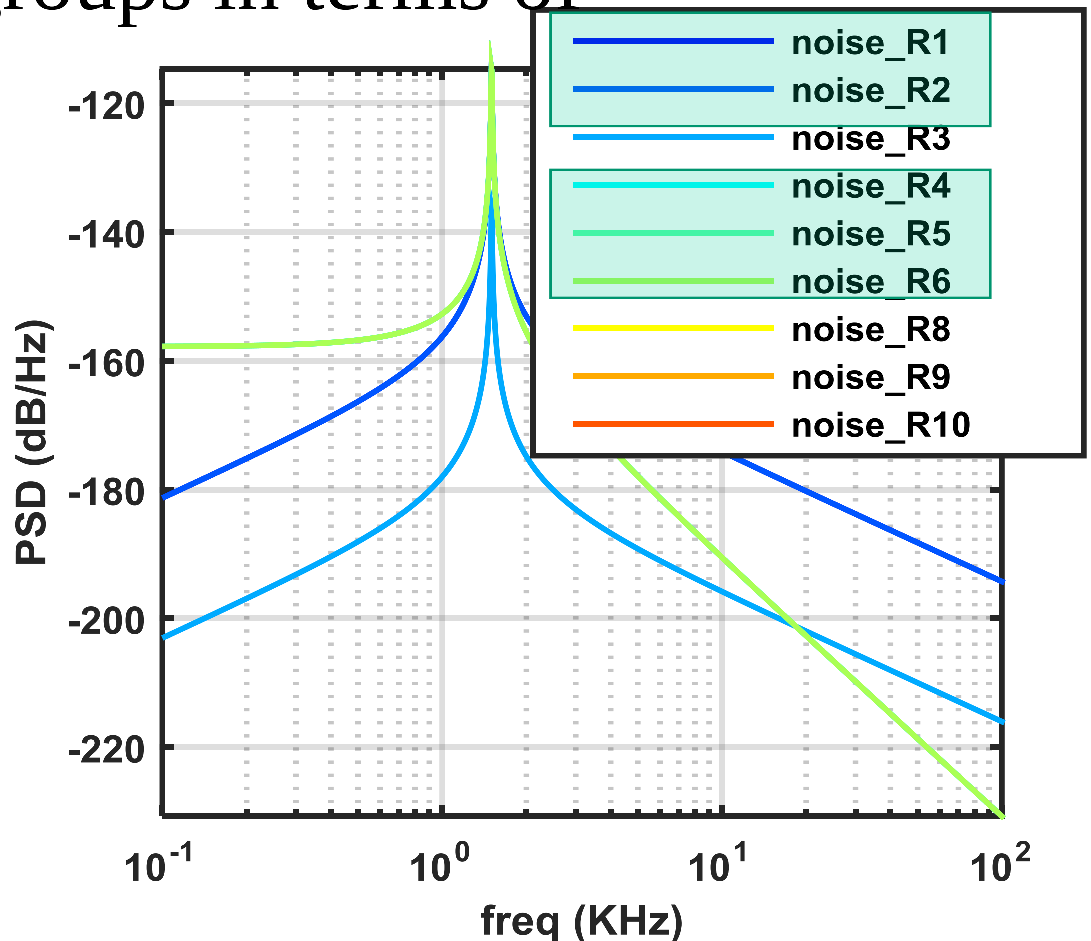
Simulation Ideal Opamp

- Compare the PSD of the output noise and the theoretical expression.
- Inband, expressions are identical.



Noise Contribution

- There are three groups in terms of contribution



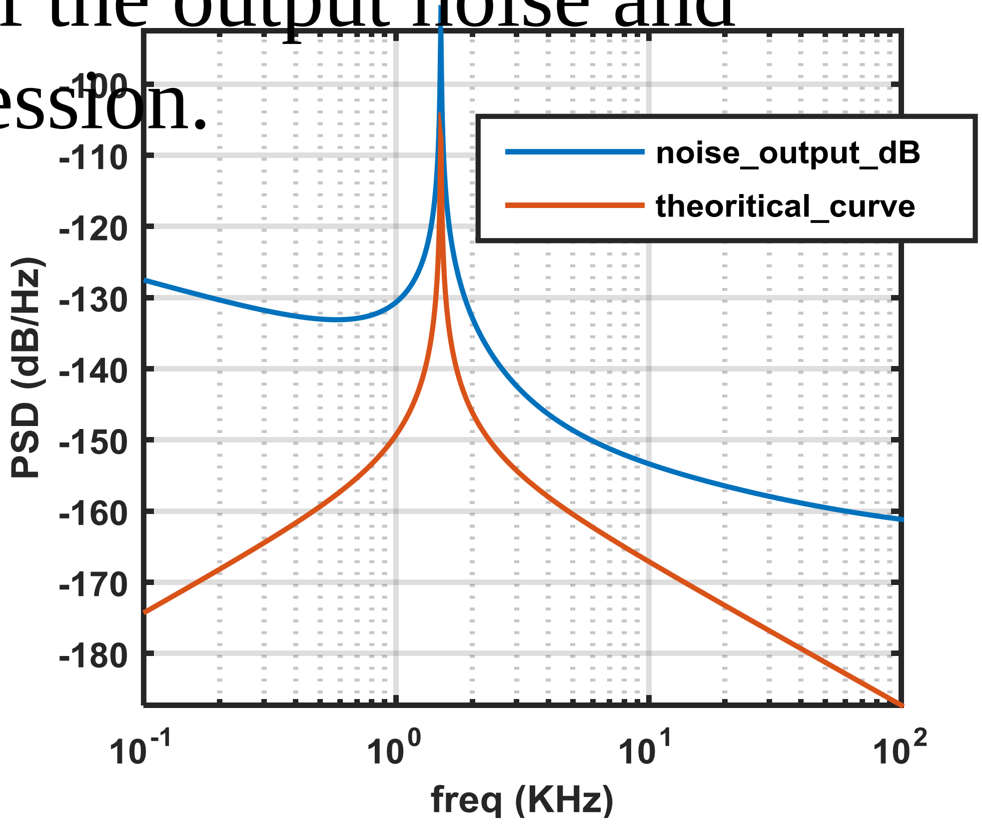
Noise Comparison

	Theoretical	Spice Simulation
Integrated Noise	-95.33 dB	-95.33 dB
Noise at peak	-107 dB/Hz	-107 dB/Hz

Using Actual Opamp

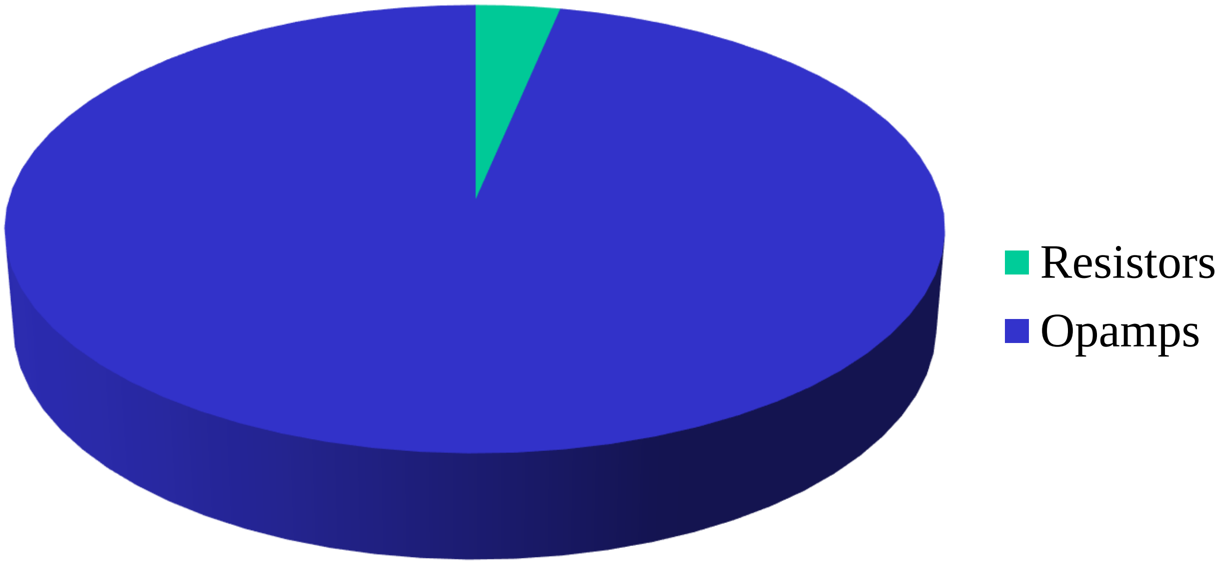
- Opamp noise: 6.3 $nV/\sqrt{Hz}$
- Compare the PSD of the output noise and the theoretical expression.
- In band, there is extra contribution due to opamp

(15 dB higher)



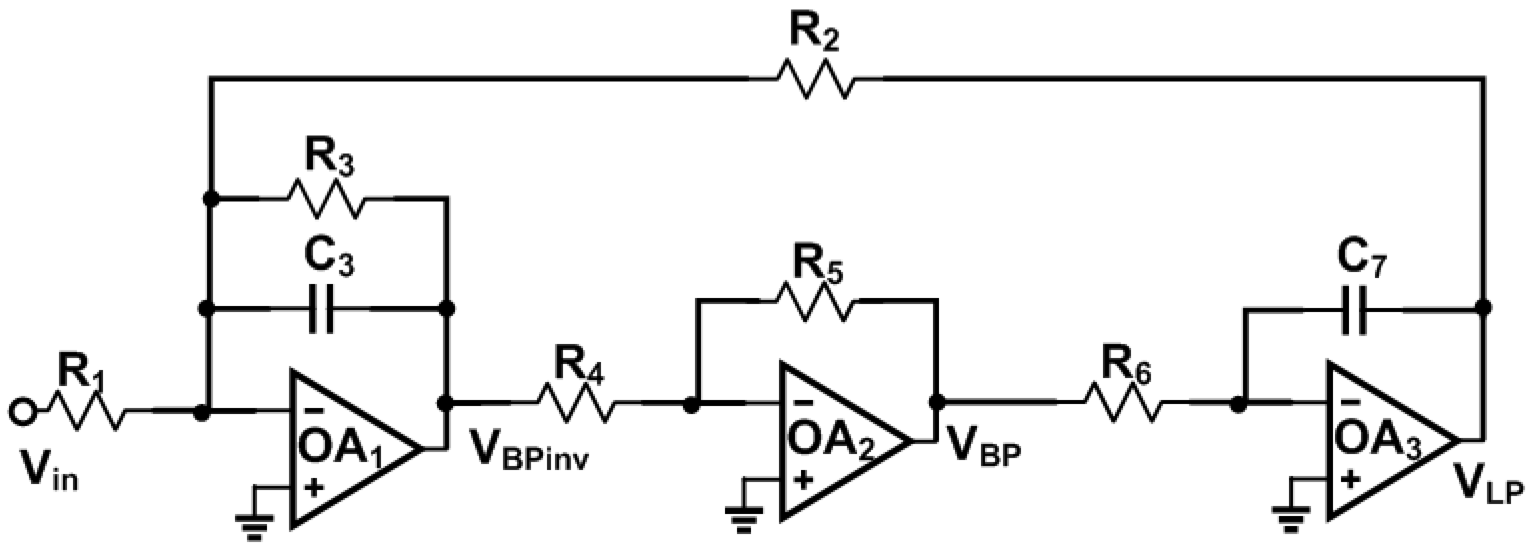
Noise Comparison

	Theoretical	Spice Simulation
Integrated Noise	-95.33 dB	-80.17 dB
Noise at peak	-107 dB/Hz	-92 dB/Hz

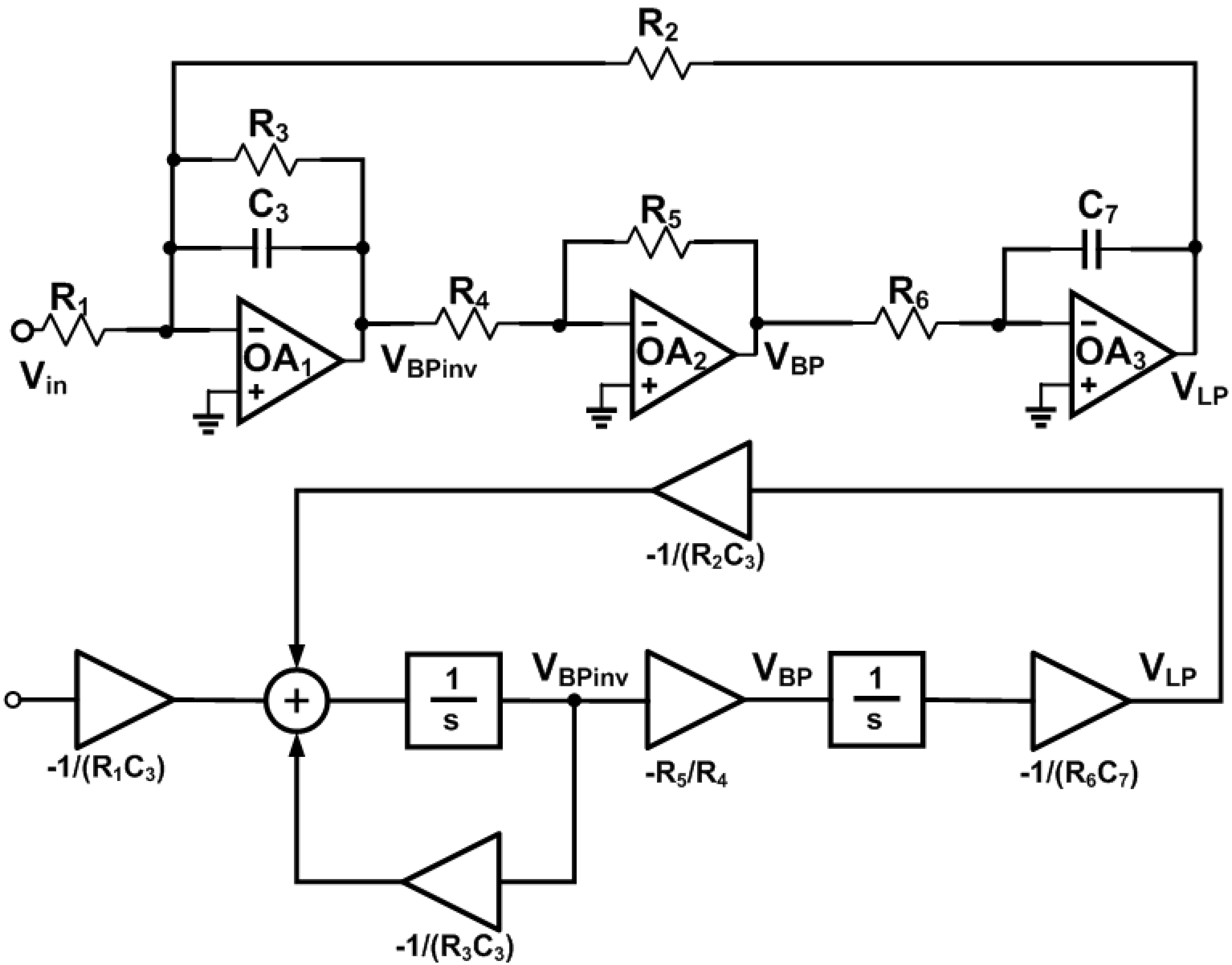


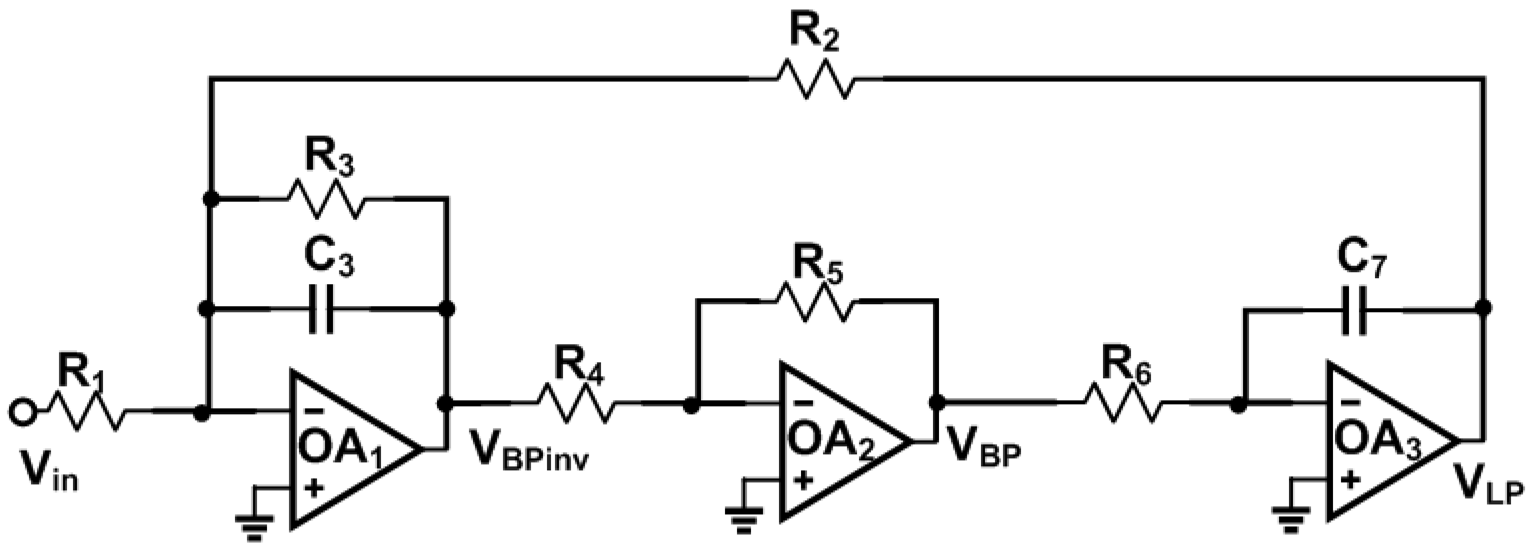
Biquad Tow-Thomas Filter Noise Analysis & Simulation

Courtesy of Kyoohyun Noh



- **Simultaneous Biquad Filter Implementations**
  - Low-pass(LP), Band-pass(BP) output
- **Independent tuning of Q and filter frequency**

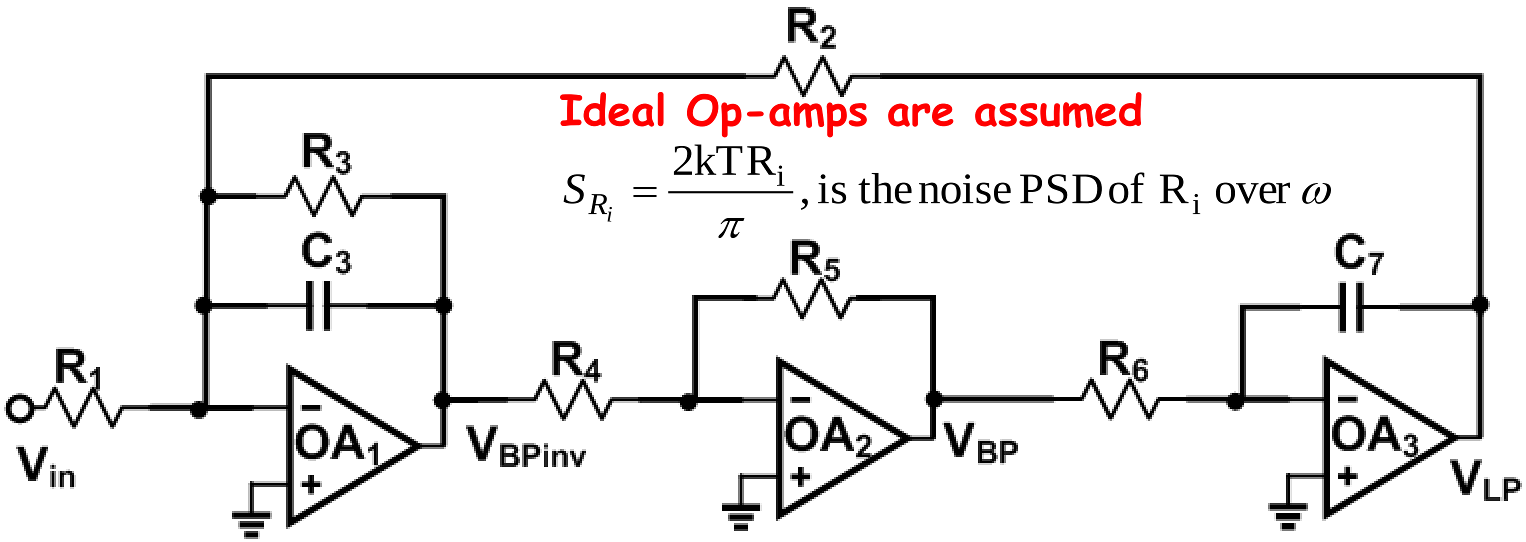

R2
R3
C3
R5
C7
R1
R4
R6
OA1
OA2
OA3
Vin
VBPinv
VBP
VLP
-1/(R2C3)
1
s
VBPinv
VBP
1
s
VLP
-1/(R1C3)
-R5/R4
-1/(R6C7)
-1/(R3C3)



$$T_{BPinv}(s) = -\frac{R_3}{R_1} \cdot \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0{}^2}$$

$$\omega_0 = \sqrt{\frac{R_5}{C_3 C_7 R_2 R_4 R_6}} \qquad Q = \omega_0 R_3 C_3 = \sqrt{\frac{C_3 R_3 \cdot R_3 R_5}{C_7 R_6 \cdot R_2 R_4}}$$

Output Noise PSD from passive components



Ideal Op-amps are assumed

$S_{R_i} = \dfrac{2kTR_i}{\pi}$, is the noise PSD of R_i over ω

$$S_{R_1,BPinv} = S_{R_1} \cdot \left(\frac{1}{\omega_0 R_1 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_2,BPinv} = S_{R_2} \cdot \left(\frac{1}{\omega_0 R_2 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_3,BPinv} = S_{R_3} \cdot \left(\frac{1}{\omega_0 R_3 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_4,BPinv} = S_{R_4} \cdot \left|H_{LP}(s)\right|^2$$

$$S_{R_5,BPinv} = S_{R_5} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left|H_{LP}(s)\right|^2$$

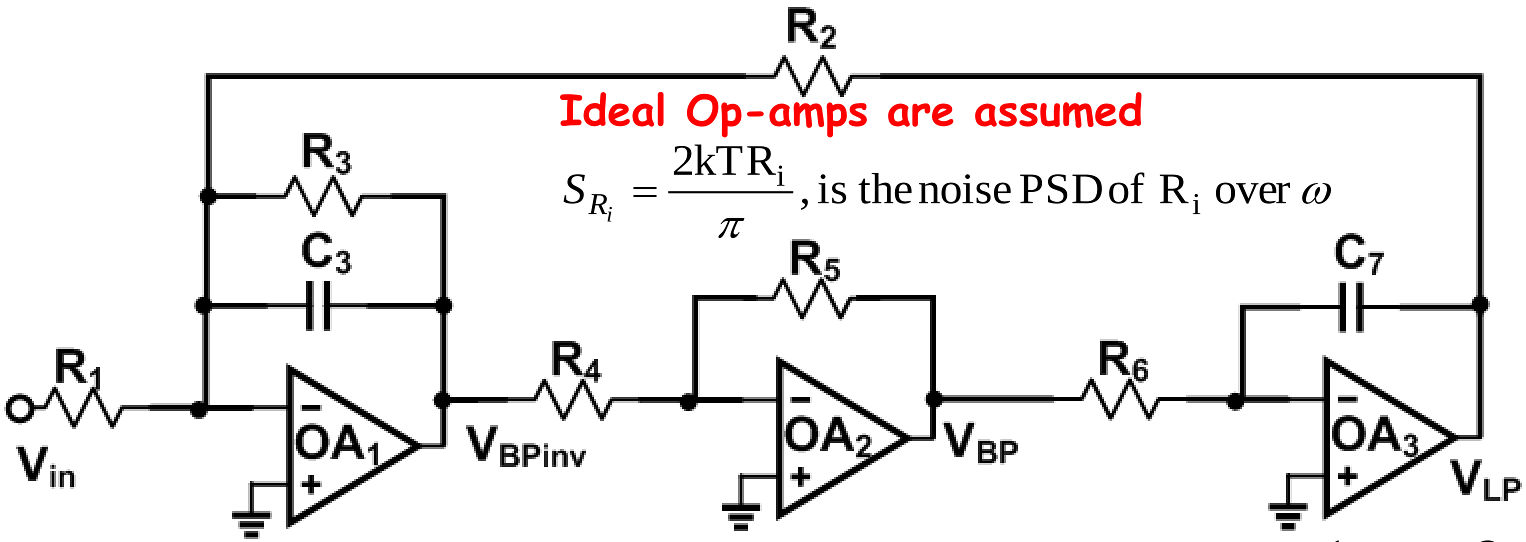
$$S_{R_6,BPinv} = S_{R_6} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left|H_{LP}(s)\right|^2$$

$$H_{LP}(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

$$H_{BP}(s) = \frac{\frac{s}{\omega_0}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

L.T. Bruton, *et. al.*, “Noise Performance of Low-Sensitivity Active Filters,” *JSSC*. pp.85-91,

Integrated output noise components



Ideal Op-amps are assumed

$$S_{R_i} = \frac{2kTR_i}{\pi}$$, is the noise PSD of R_i over ω

$$S_{R_1,BPinv} = S_{R_1} \cdot \left(\frac{1}{\omega_0 R_1 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_2,BPinv} = S_{R_2} \cdot \left(\frac{1}{\omega_0 R_2 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_3,BPinv} = S_{R_3} \cdot \left(\frac{1}{\omega_0 R_3 C_3}\right)^2 \cdot \left|H_{BP}(s)\right|^2$$

$$S_{R_4,BPinv} = S_{R_4} \cdot \left|H_{LP}(s)\right|^2$$

$$S_{R_5,BPinv} = S_{R_5} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left|H_{LP}(s)\right|^2$$

$$S_{R_6,BPinv} = S_{R_6} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left|H_{LP}(s)\right|^2$$

Angular freq. integration

$$\overline{v_{R_1,BPinv}}^2 = S_{R_1} \cdot \left(\frac{1}{R_1 C_3}\right)^2 \cdot \left(\frac{Q\pi}{2\omega_0}\right)$$

$$\overline{v_{R_2,BPinv}}^2 = S_{R_2} \cdot \left(\frac{1}{R_2 C_3}\right)^2 \cdot \left(\frac{Q\pi}{2\omega_0}\right)$$

$$\overline{v_{R_3,BPinv}}^2 = S_{R_3} \cdot \left(\frac{1}{R_3 C_3}\right)^2 \cdot \left(\frac{Q\pi}{2\omega_0}\right)$$

$$\overline{v_{R_4,BPinv}}^2 = S_{R_4} \cdot \left(\frac{Q\pi\omega_0}{2}\right)$$

$$\overline{v_{R_5,BPinv}}^2 = S_{R_5} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left(\frac{Q\pi\omega_0}{2}\right)$$

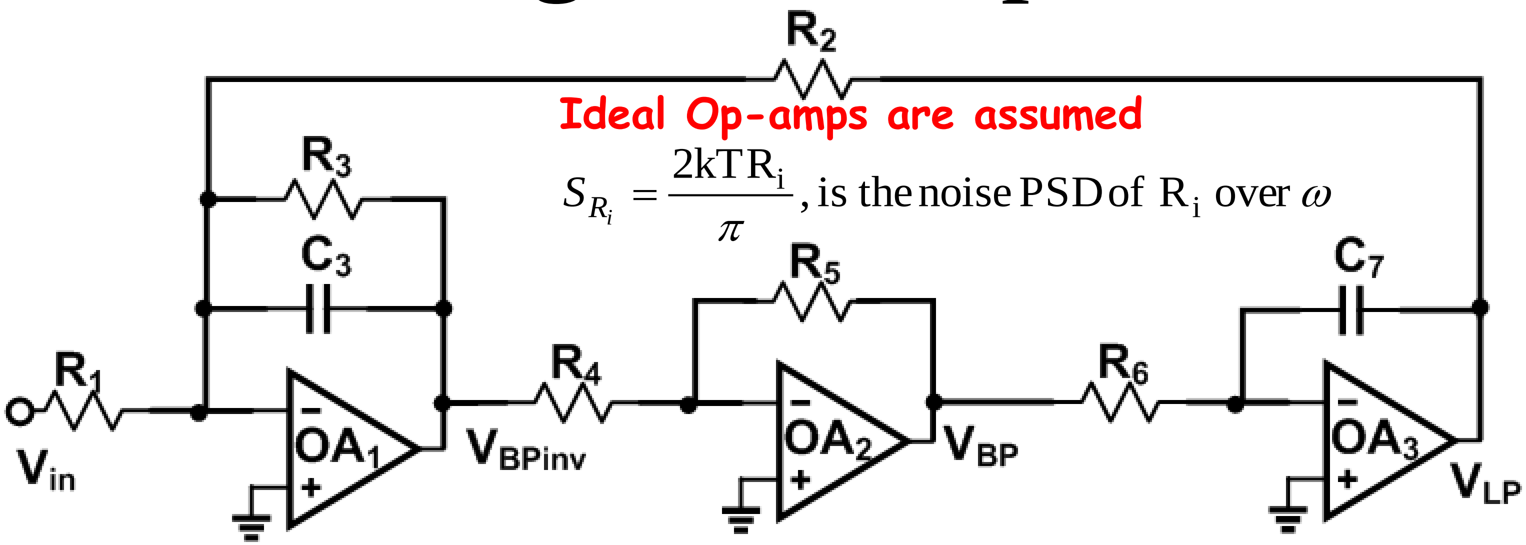
$$\overline{v_{R_6,BPinv}}^2 = S_{R_6} \cdot \left(\frac{R_4}{R_5}\right)^2 \cdot \left(\frac{Q\pi\omega_0}{2}\right)$$

L.T. Bruton, *et. al.*, "Noise Performance of Low-Sensitivity Active Filters," *JSSC*. pp.85-91, Feb. 1973

Total integrated output noise

Ideal Op-amps are assumed

$$S_{R_i} = \frac{2kTR_i}{\pi}, \text{ is the noise PSD of } R_i \text{ over } \omega$$



$$S_{total,BPinv,peak}\Big|_{\omega=\omega_0,Q>>1} = \sum_{i=1}^{6} S_{R_i,BPinv}\Big|_{\omega=\omega_0,Q>>1}$$

$$\overline{v_{noise,total,BPinv}^{\,2}} = \overline{v_{R_1,BPinv}^{\,2}} + \overline{v_{R_2,BPinv}^{\,2}} + \overline{v_{R_3,BPinv}^{\,2}} + \overline{v_{R_4,BPinv}^{\,2}} + \overline{v_{R_5,BPinv}^{\,2}} + \overline{v_{R_6,BPinv}^{\,2}}$$

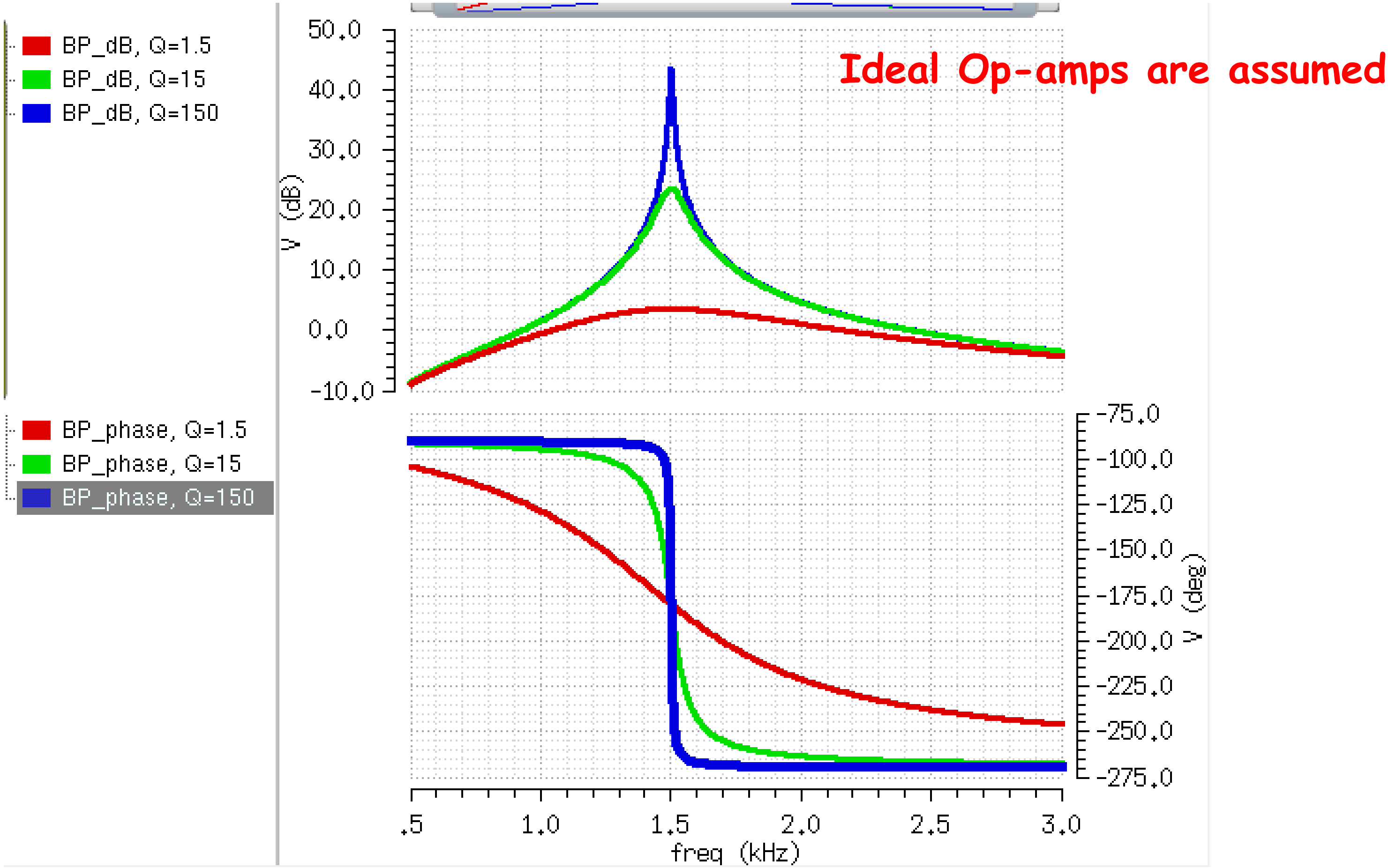
- If $R=R_1=R_2=R_4=R_5=R_6$, $C=C_3=C_7$

$$\omega_0 = \frac{1}{RC} \qquad Q = \omega_0 R_3 C = \frac{R_3}{R}$$

$$\therefore \overline{v_{noise,total,BPinv}^{\,2}} = kTR(5Q+1)\omega_0 \qquad S_{total,BPinv,peak}\Big|_{\omega=\omega_0,Q>>1} \approx 10kTR \cdot \frac{Q^2}{\pi} \quad [V^2/(rad/s)]$$

L.T. Bruton, *et. al.*, "Noise Performance of Low-Sensitivity Active Filters," *JSSC*. pp.85-91, Feb. 1973

BPF Simulation



- BPF with different Qs are implemented with ideal op-amps
    - C=10.6nF is assumed
    - Each R is listed in the next slide

Simulated total integrated output

	Q=150				Q=15				Q=1.5			
	Resistance [Ohm]	Theory [V^2]	Simulation [V^2]	Contribution [%]	Resistance [Ohm]	Theory [V^2]	Simulation [V^2]	Contribution [%]	Resistance [Ohm]	Theory [V^2]	Simulation [V^2]	Contribution [%]
R1	1.00E+04	5.85E-11	5.84E-11	19.96	1.00E+04	5.85E-12	5.71E-12	19.63	1.00E+04	5.85E-13	5.50E-13	17.3
R2	1.00E+04	5.85E-11	5.84E-11	19.96	1.00E+04	5.85E-12	5.71E-12	19.63	1.00E+04	5.85E-13	5.50E-13	17.3
R3	1.50E+06	3.90E-13	3.90E-13	0.13	1.50E+05	3.90E-13	3.81E-13	1.31	1.50E+04	3.90E-13	3.66E-13	11.53
R4	1.00E+04	5.85E-11	5.85E-11	19.99	1.00E+04	5.85E-12	5.76E-12	19.81	1.00E+04	5.85E-13	5.70E-13	17.96
R5	1.00E+04	5.85E-11	5.85E-11	19.99	1.00E+04	5.85E-12	5.76E-12	19.81	1.00E+04	5.85E-13	5.70E-13	17.96
R6	1.00E+04	5.85E-11	5.85E-11	19.99	1.00E+04	5.85E-12	5.76E-12	19.81	1.00E+04	5.85E-13	5.70E-13	17.96
Total		2.93E-10	2.93E-10	100		2.97E-11	2.91E-11	100		3.32E-12	3.17E-12	100

$$\overline{v_{R_i,BPinv}}^2 = 4kTR \cdot (\frac{Q\omega_0}{4}) \text{ for i} = 1,2,4,5,6$$

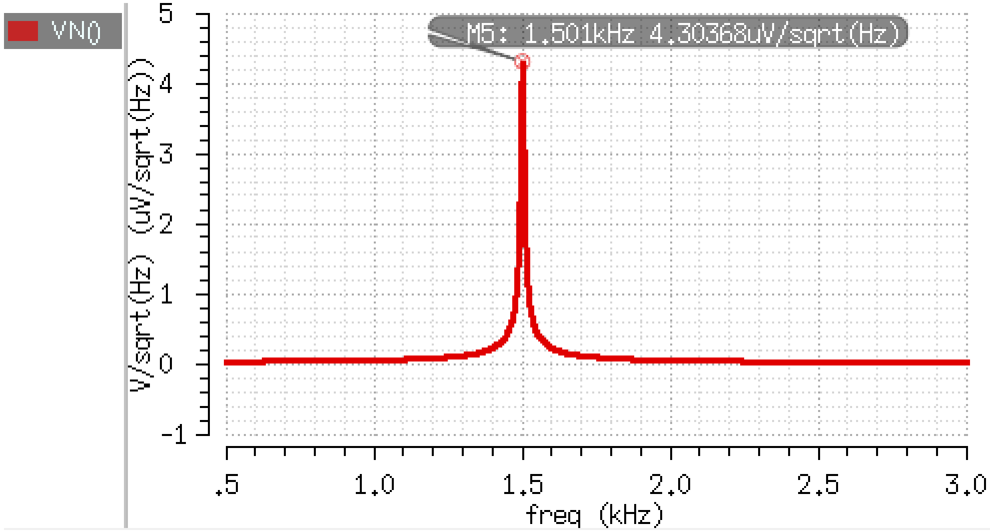
$$\overline{v_{R_3,BPinv}}^2 = 4kTR \cdot \frac{R}{R_3} \cdot (\frac{Q\omega_0}{4})$$

$$\overline{v_{noise,total,BPinv}}^2 = kTR(5Q+1)\omega_0$$

Ideal Op-amps are assumed

- Theoretical estimation agrees well with the simulation results
- Feedback resistor R_3 of the lossy integrator makes the least contribution to the High-Q High-gain BPF output noise among passive components

BPF Output Noise PSD



	Theory [uV/sqrt(Hz)]	Simulation [uV/sqrt(Hz)]
Peak output noise PSD [V/sqrt(Hz)]	4.31	4.3

Q=150, R=10k

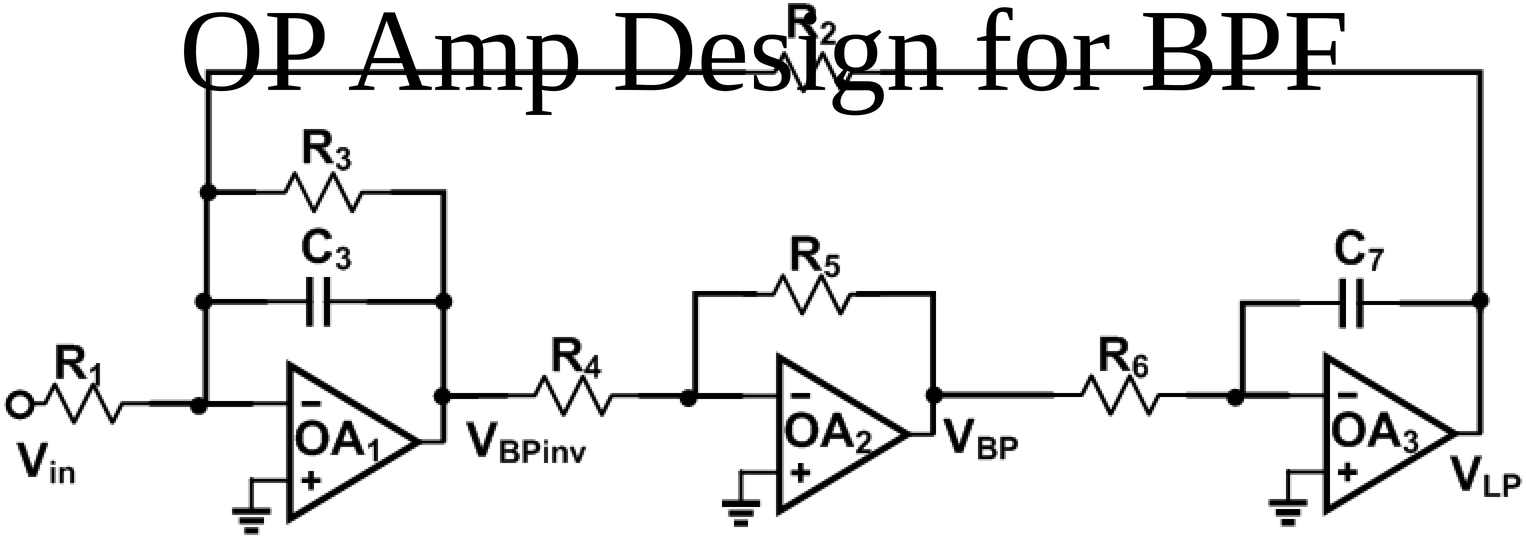
Ideal opamps are assumed

$$S_{total,BPinv,peak}\Big|_{\omega=\omega_0,Q>>1} \approx 10kTR \cdot \frac{Q^2}{\pi} \quad [V^2/(rad/s)]$$

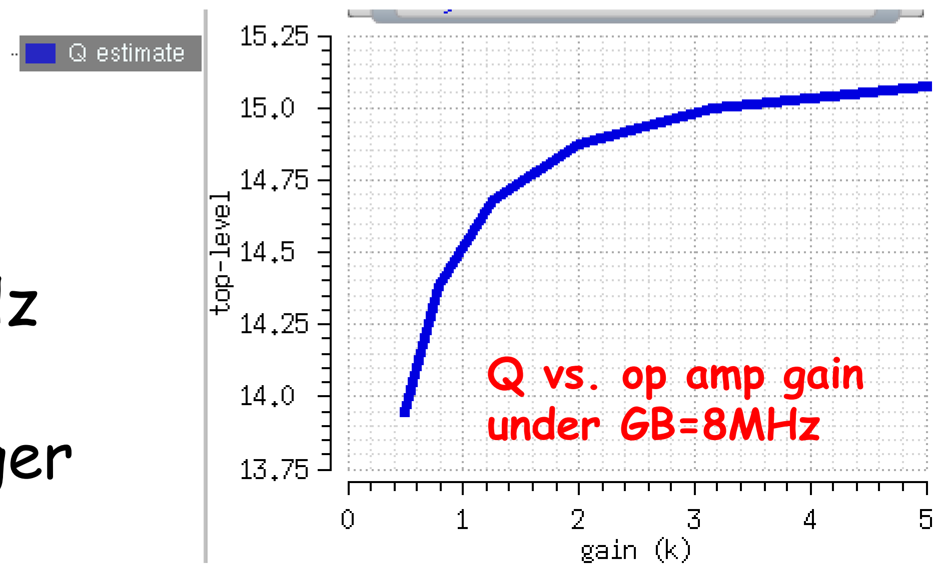
$$\Leftrightarrow \quad \sqrt{S_{total,BPinv,peak}}\Big|_{\omega=\omega_0,Q>>1} \approx Q \cdot \sqrt{20kTR} \quad [V/\sqrt{Hz}]$$

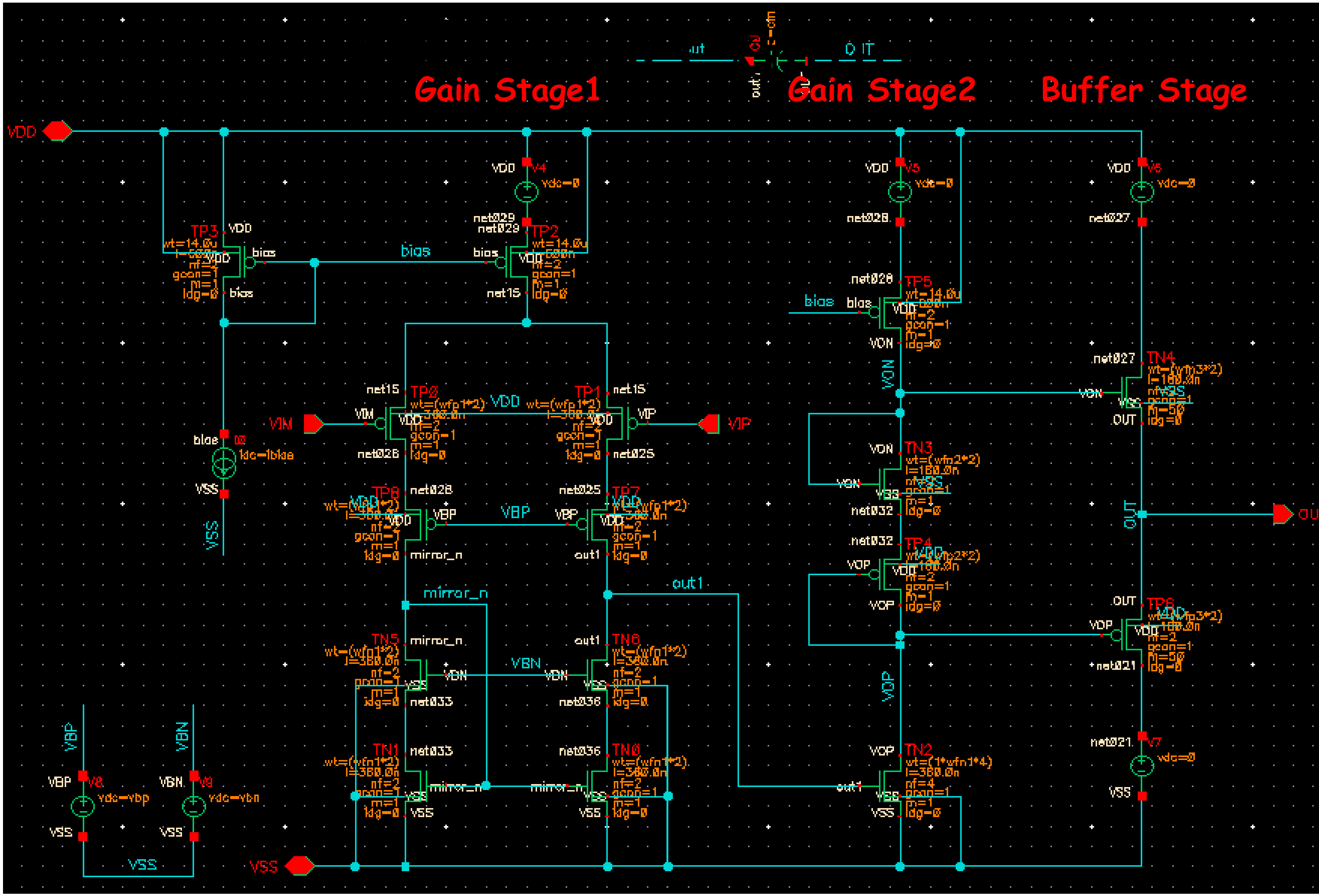
- Theoretical estimation agrees well with the simulation results

OP Amp Design for BPF



- BPF Q =15, f_0=1.5kHz
- Assume GB is large enough
  - GB=8MHz >>Qf_0=22.5kHz
- Amplifier gain is set to larger than 3000 from behavioral simulation




Gain Stage1
Gain Stage2
Buffer Stage

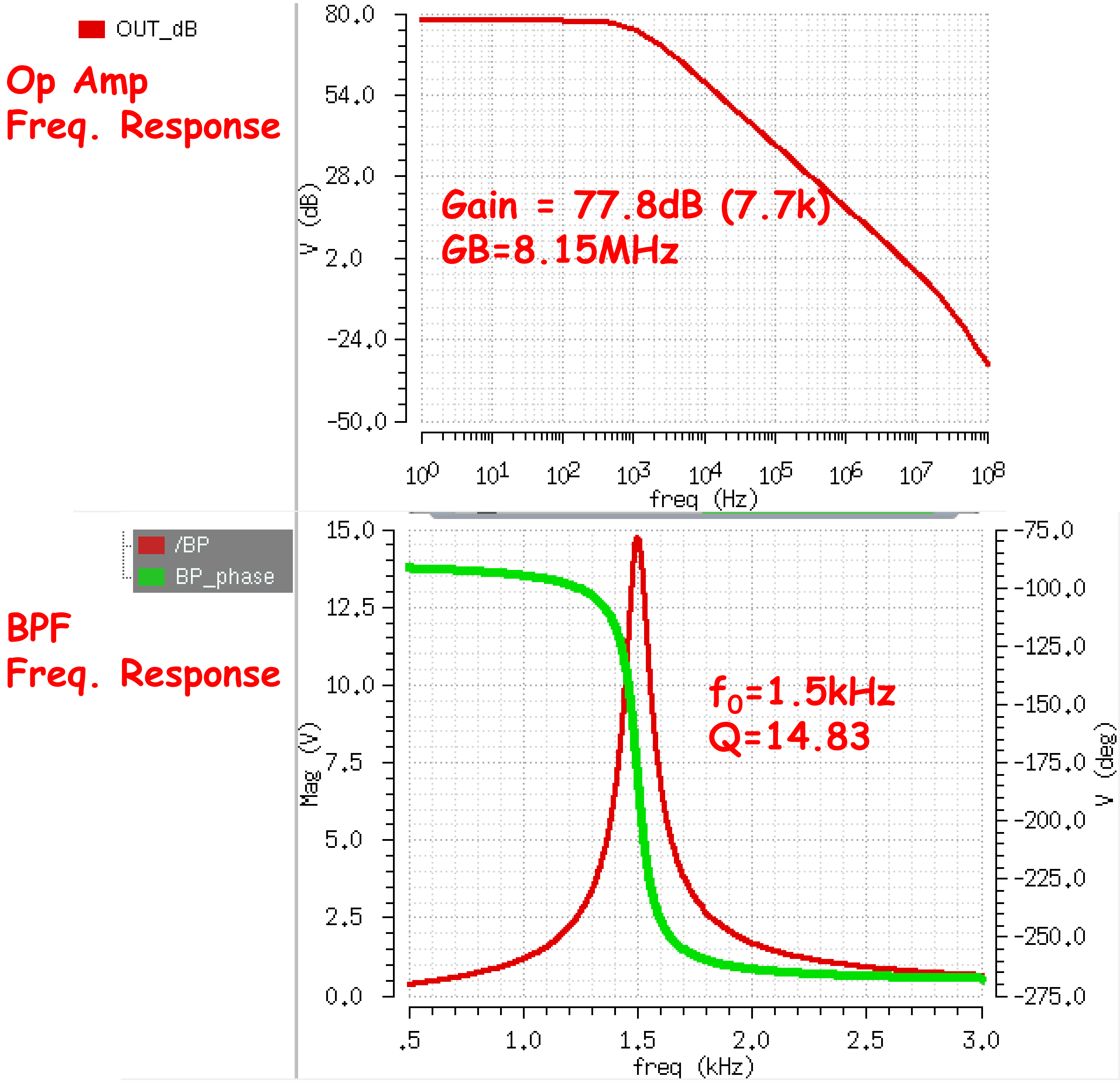
OP Amp details

	Specification
Process	CMOS 180nm
VDD [V]	1.8

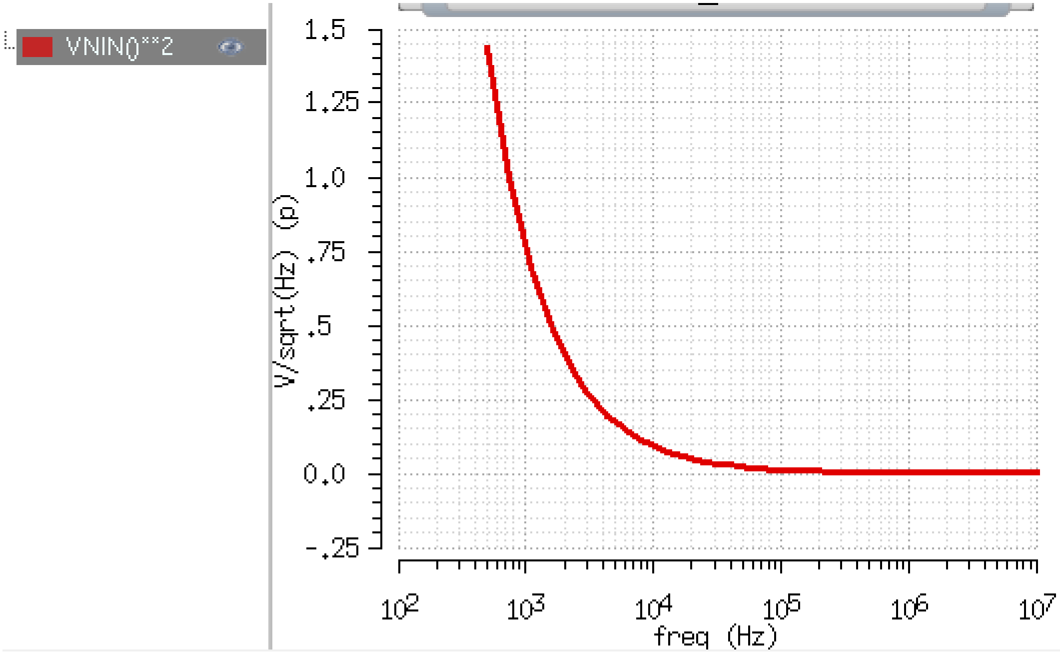
Bias Current Generation ibias = 20uA	W/L
Diode connected PMOS	14/0.6
Current source	14/0.6

1st Stage VBP=0.85, VBN=0.7	W/L	2nd Stage	W/L	Buffer Stage Cm=2pF	W/L
PMOS input	10/0.36	PMOS current source	14/0.6	NMOS	20/0.18
PMOS cascode	10/0.36	NMOS battery	20/0.18	PMOS	20/0.18
NMOS cascode	2/0.36	PMOS battery	80/0.18		
NMOS Mirror	2/0.36	NMOS Gm	40/0.36		

OP Amp/BPF AC response

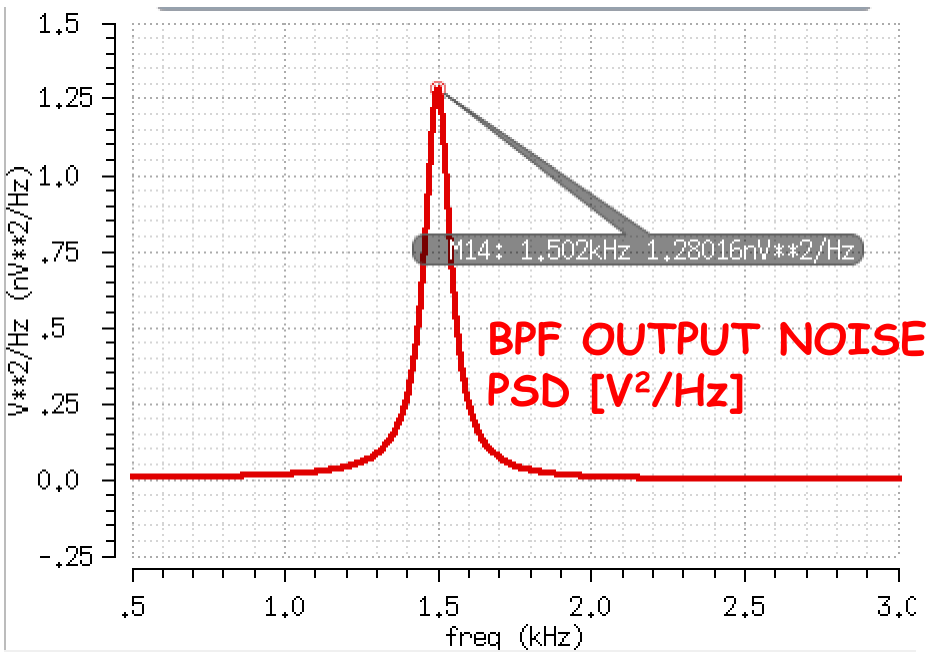


Op Amp input-referred noise PSD



- Op Amp's flicker noise is dominant below about 100kHz
  - BPF output noise is expected to be dominated by flicker noise of the Op amps

BPF output noise



BPF OUTPUT Integrated NOISE

Noise Components	Simulation [V^2]	Contribution [%]
OA1	9.35e-8	45.9
OA2	7.39e-8	36.3
OA3	3.61e-8	17.8
Total	2.04e-7	100

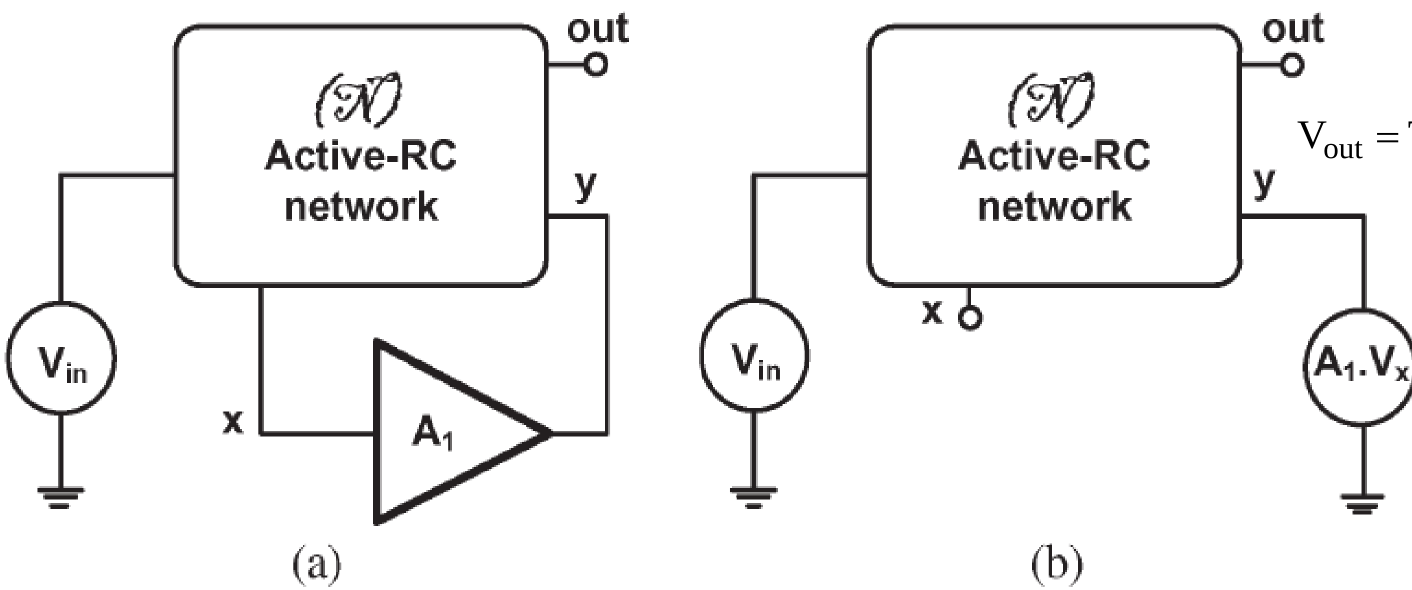
Major noise contributors

Device	Param	Noise Contribution	% Of Total
/I20/TN0	Sfl	4.27968e-08	21.03
/I20/TN1	Sfl	4.26169e-08	20.94
/I22/TN0	Sfl	3.37571e-08	16.59
/I22/TN1	Sfl	3.36152e-08	16.52
/I21/TN0	Sfl	1.65274e-08	8.12
/I21/TN1	Sfl	1.64579e-08	8.09
/I20/TP1	Sfl	3.76278e-09	1.85
/I20/TP0	Sfl	3.7604e-09	1.85
/I22/TP1	Sfl	2.99979e-09	1.47
/I22/TP0	Sfl	2.9979e-09	1.47

/I20: lossy Integ., /I22: Inverting amp, /I21: Integ.

- Op Amp's noise is dominant in this example
  - Flicker noise of each amp's 1st stage current mirror is dominant in this design due to low BPF frequency
- Noise from passive components are negligible

Noise Reduction In Active-RC Filters*



$$V_{out} = T_{out,\,in}\ V_{in} + T_{out,\,y} V_y$$

Fig. 1. (a) General active-RC network. (b) Equivalent circuit of (a).

* K.Gharib Doust and M. S. Bakhiar, "A Method for Noise Reduction in Active-RC Circuits", *IEEE Trans. on Circuits and Systems – II,* Vol. 58, pp. 906-910, December 2011.

Introduce A_f to reduce noise without modifying the original transfer function.

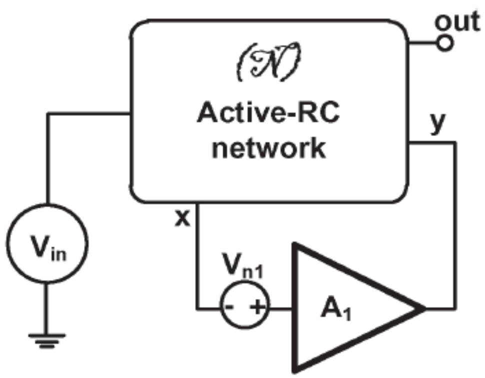


Fig. 2. Circuit with the equivalent input noise source of A_1.

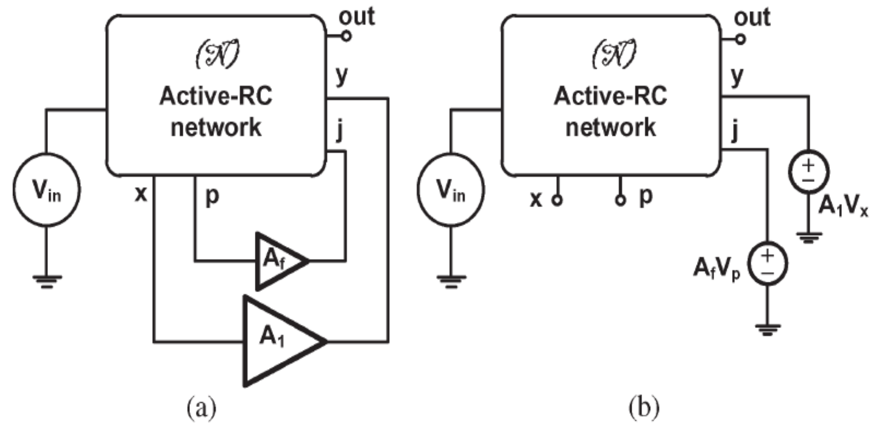


Fig. 3. (a) General active-RC circuit with new inserted amplifier A_f. (b) Equivalent circuit of (a).

$$T_{out,in} = \left.\frac{V_{out}}{V_{in}}\right|_{V_y=0}$$

$$T_{out,y} = \left.\frac{V_{out}}{V_y}\right|_{V_{in}=0}$$

$$\frac{V_{out}}{V_{in}} = \frac{T_{out,in} - A_1\left(T_{out,in}T_{x,y}T_{out,y}\right)}{1 - A_1T_{x,y}}$$

$$\overline{V}_{out}^2 = \frac{A_1^2\left|T_{out,y}\right|^2}{\left|1 - A_1T_{x,y}\right|^2}\overline{V}_{n_1}^2$$

Node j must be a ground before inserting A_j

Make $V_p = k\,V_x$ and

$A_1T_{x,y} >> k\,A_f\,T_{v,j}$

$A_1T_{out,y} >> kA_f\,T_{out,j}$

That is $A_1 >> A_f$

To keep original frequency response

$T_{out,in}T_{xy} \cong T_{x,in}T_{out,j}$

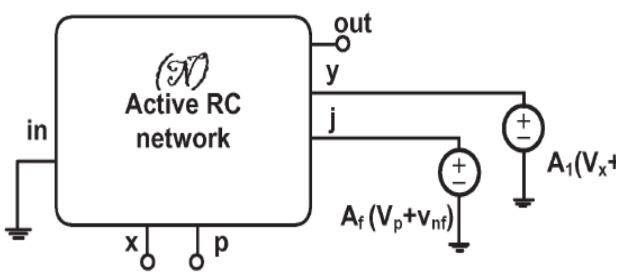
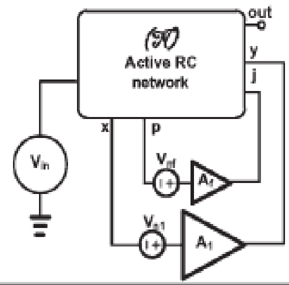
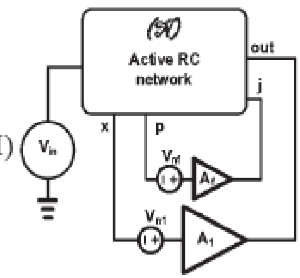
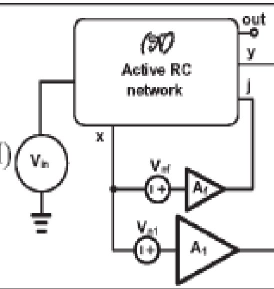
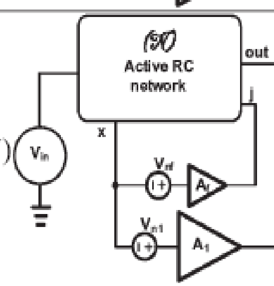


Fig. 4. Equivalent circuit of Fig. 3(a) for noise estimation.

TABLE I

DESIGN SUMMARY FOR VARIOUS CIRCUIT CONFIGURATIONS

General circuit	Conditions	Optimum TF	NIF
(I) 	$A_fT_{p,j} \ll A_1T_{x,y}$ $V_p = kV_x$ $kA_fT_{out,j} \ll A_1T_{out,y}$	$A_f = \frac{-kT_{out,y}}{(\psi^2+k^2)Q(x)}$	$\frac{(\psi^2+k^2)}{\psi^2}$
(II) 	$A_fT_{p,j} \ll A_1T_{x,out}$ $V_p = kV_x$ OR $out \equiv j$ when $in = 0$ and $in \equiv j$ when $out = 0$	$A_f = \frac{k}{(\psi^2+k^2)T_{x,j}}$	$\frac{(\psi^2+k^2)}{\psi^2}$
(III) 	$A_fT_{x,j} \ll A_1T_{x,out}$ $A_fT_{out,j} \ll A_1T_{out,y}$	$A_f = \frac{-T_{out,y}}{(\psi^2+1)Q(x)}$	$\frac{(\psi^2+1)}{\psi^2}$
(IV) 	$A_fT_{x,j} \ll A_1T_{x,out}$	$A_f = \frac{1}{(\psi^2+1)T_{x,j}}$	$\frac{(\psi^2+1)}{\psi^2}$

Design Example: $f_o = 2.5\,\mathrm{MHz}$, $Q = 20$, $\overline{V}_{out,n}$ @ $f_o = 1.34\mu V_{rms}$, $\overline{V}_{out,n}$ integrated noise $= 524\mu V_{rm}$

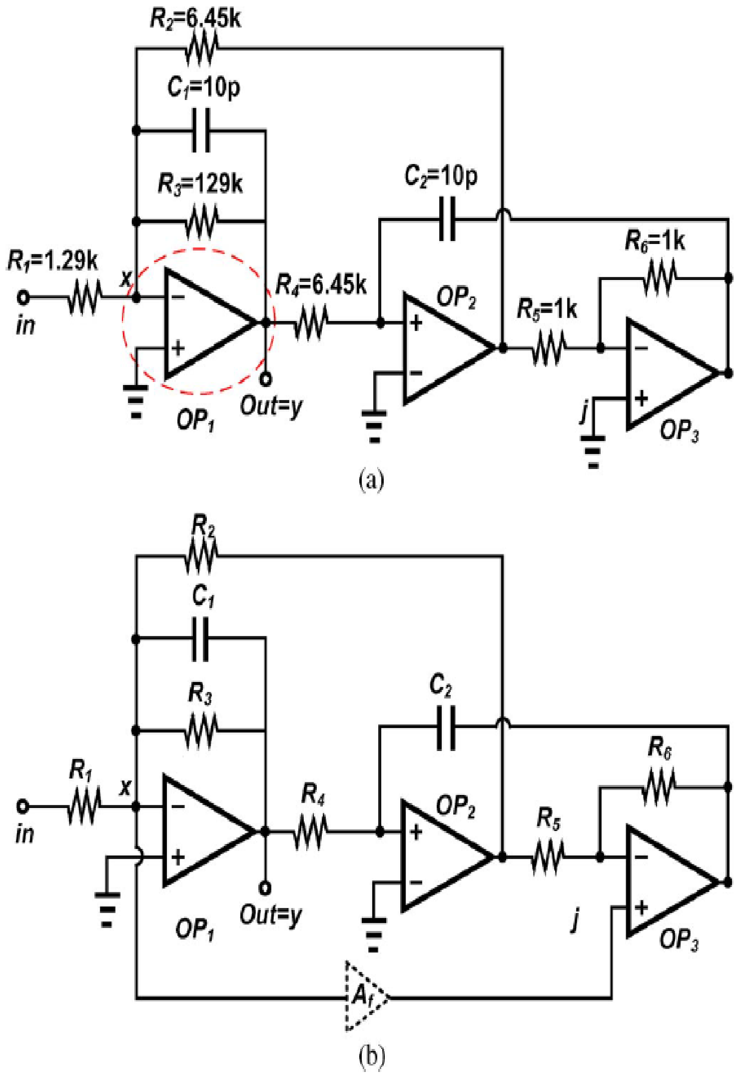


Fig. 5. (a) Ackerberg–Mossberg active-*RC* bandpass filter. (b) Bandpass filter with the noise cancellation path.

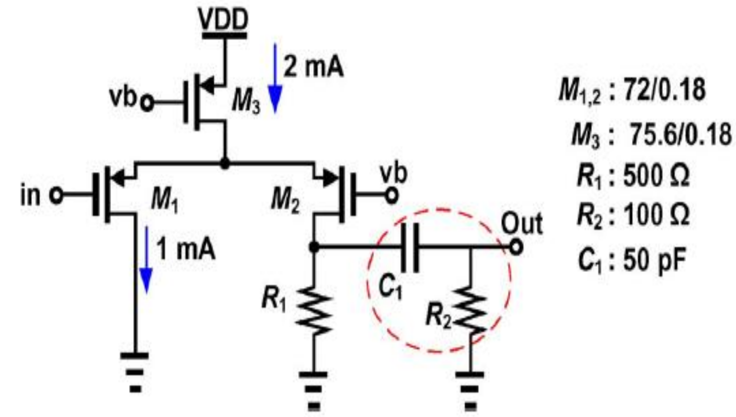


Fig. 6. Schematic of inserted amplifier A_f.

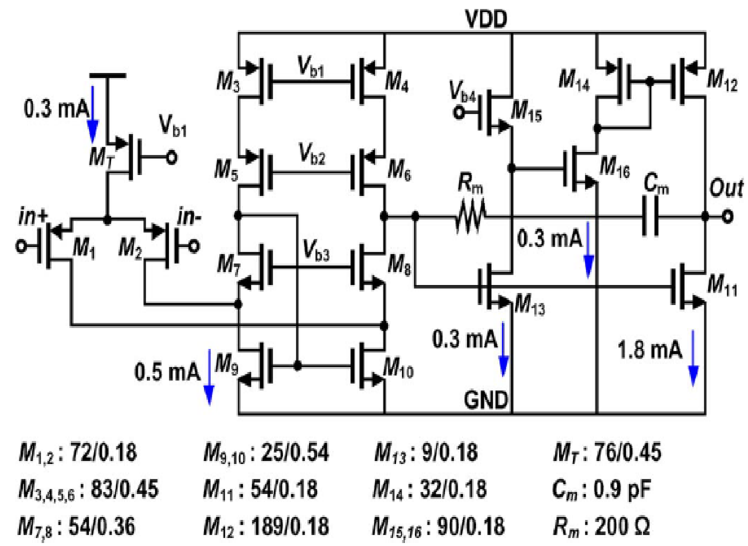


Fig. 9. Schematic of the opamp.

Class - AB

$$V_p = V_x \quad , \quad k = 1$$

$$T_{out,y} = 1$$

$$Q(x) = \frac{-2}{1/Q + R_2/R_1 + 1 + s/\omega_o} = \frac{-2}{6.05 + s/\omega_o}$$

Then

$$A_f \cong \frac{-kT_{out,y}}{Q(x)\left(\psi^2 + k^2\right)} = \frac{6.05 + s/\omega_o}{2\left(1 + \psi^2\right)}$$

$$A_f\Big|_{\psi^2 << 1} \cong 3.025 + j0.5 \cong 3.025$$

Note that in order to obtain the noise reduction without A_f , A_1 would have to increase its power by 150%. The A_f power added is about 60% of A_1. Also note that a simple short circuit of x to node j also reduce the output noise by 2.4dB.

Noise Reduction Comparison Plots

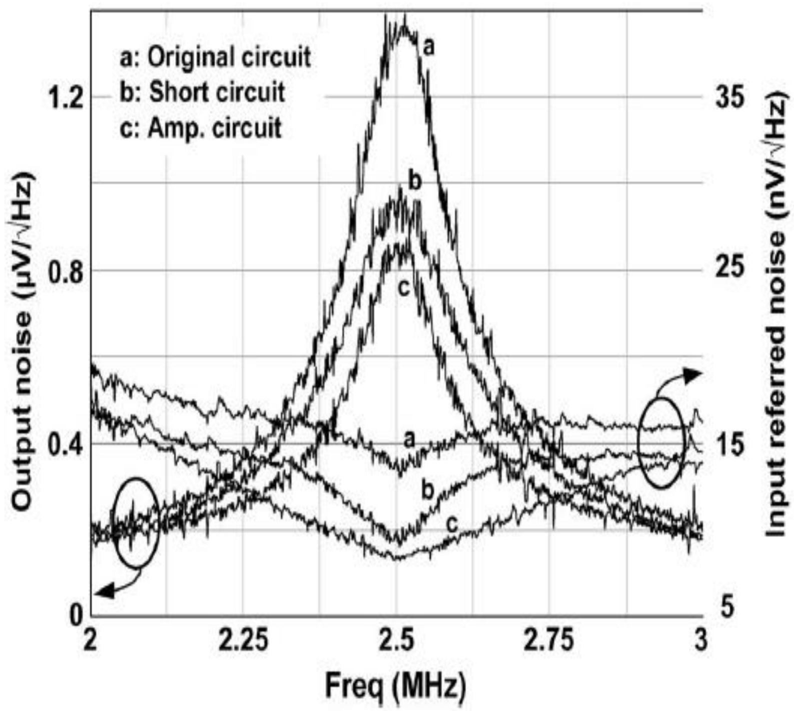


Fig. 8. Measured output noise and input referred noise of the original and the modified circuits.

Conclusions

Noise reduction can be done by inserting one or more reduction paths without affecting original transfer function. These paths can be active or passive when power is limited.

TABLE II
COMPARISON OF THE ORIGINAL CIRCUIT AND THE MODIFIED CIRCUITS

	Original circuit	Amp circuit	Short circuit
f_0 (MHz)	2.5	2.5	2.5
Q	20	20.2	20.1
$V_{\text{out}_n,tot}$(in BW) (μV)	432	272	325
$V_{\text{out}_n,\text{tot}}$ (in 2×BW) (μV)	524	322	370
MAX (V_{out_n}) (μV)	1.34	0.85	1.06
Gain (dB)	40	40	40

References

1. B. Razavi, "*Design of Analog CMOS Integrated Circuits*", Preview Edition, Mcgraw Hill, 2000.

2. D.A. Johns and K. Martin, *Analog Integrated Circuit Design*, New York; Wiley, 1997.

3. M.S. Gupta, "Selected papers on *Noise in Circuits and Systems*", IEEE Press 1998.