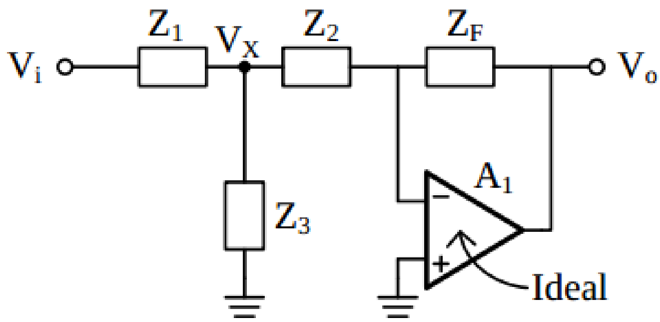
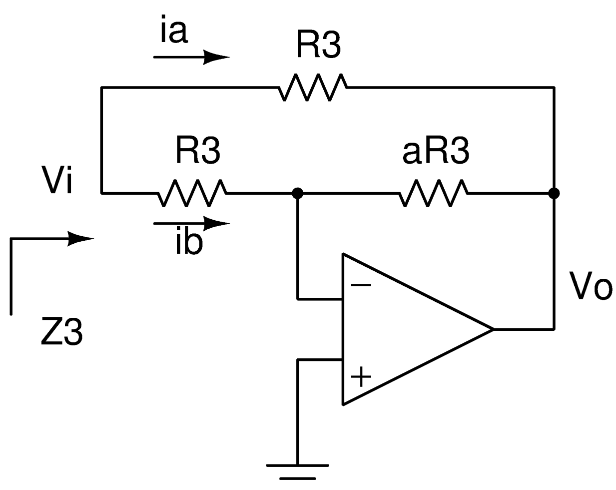
Quiz #4

i) Obtain the $H(s) = \frac{Vo(s)}{Vi(s)}$ of the circuit of fig. 1. Hint: use the concept of T-network.



Where $Z_F - R$, $Z_1 = kR$ and $Z_2 = (1-k)R$

ii) The, obtain $Z_3(s)$ of the circuit (Fig.2) shown and substitute in $H(s)$ obtained before



Solution

Case 1

By using the T-network approximation, we can get the reduced circuit:

$$Z_1' = \frac{(kR)(1-k)R + kRZ_3 + (1-k)RZ_3}{Z_3} \tag{1}$$

$$Z_1' = \frac{R^2k(1-k) + RZ_3}{Z_3} \tag{2}$$

lets define:

$$\frac{ZF}{Z1'} = \frac{RZ_3}{R^2k(1-k) + RZ_3} = \frac{Z_3}{Rk(1-k) + Z_3} \tag{3}$$

Assuming the amplifier behaves as an ideal integrator GB/s, the transfer function is:

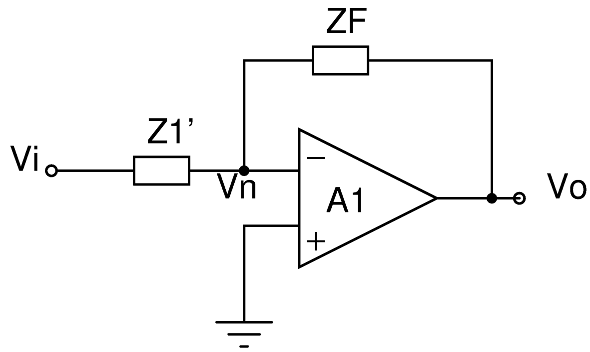


Figure 1: Equivalent circuit

$$H(s) = \frac{\frac{-Z_F}{Z_1'}}{1 + \frac{s}{GB}\left(1 + \frac{Z_F}{Z_1'}\right)} \tag{4}$$

$$H(s) = -\frac{(GB \cdot Z_3)}{(GB(kR - k^2R + Z_3) + s(kR - k^2R + 2Z_3))} \tag{5}$$

Now we need to find Z_3 and plug it in the equation 5.
In order to find the input impedance we need to get the ratio v/i , however since the input current is divided into two branches we can find each independently and calculate the input impedance as the parallel of both. following that procedure, we can neglect the top resistor R_3 and find the transfer function of the inverting amplifier:

$$H1(s) = \frac{Vo}{Vi} = \frac{\frac{-aR_3}{R_3}}{1 + \frac{1}{A}\left(1 + \frac{aR_3}{R_3}\right)} = \frac{-a}{a + \frac{s}{GB}(1 + a)} \tag{6}$$

$$Vo = Vi * H(s) \tag{7}$$

$$i_a = \frac{Vi - Vo}{R_3} = \frac{Vi - ViH1(s)}{R_3} = \frac{Vi(1 = H1(s))}{R_3} \tag{8}$$

$$Z_a = \frac{Vi}{i_a} = \frac{R_3}{1 - H1(s)} \tag{9}$$

$$Z_a = \frac{R_3(GB + (1 + a)s)}{(1 + a)(GB + s)} \tag{10}$$

The impedance of the inverting amplifier is given by:

$$Z_b = R_3 + \frac{aR_3}{1 + GB/s} \tag{11}$$

$$Z_b = \frac{aR_3s + R_3(s + GB)}{(s + GB)} \tag{12}$$

The input impedance is then:

$$Z_3 = Z_a||Z_b = \frac{R3(GB + s + as)}{(2 + a)(GB + s)} \quad (13)$$

Finally, replacing Z_3 into 5, we get the transfer function:

$$H(s) = -\frac{GB \cdot R_3(GB + s + as)}{(GB^2((2 + a)k \cdot R(1 - k) + R_3) + GB(2(2 + a)k \cdot R(1 - k) + (3 + a)R_3)s + ((2 + a)kR(1 - k) + 2(1 + a)R_3)s^2)} \quad (14)$$

Case 2

In contrast with the Case 1, since both amplifiers are approximated by the integrator (GB/s), the T-network approximation is not completely accurate, we need to consider all impedances derived from the $\Delta - Y$ transformation as shown :

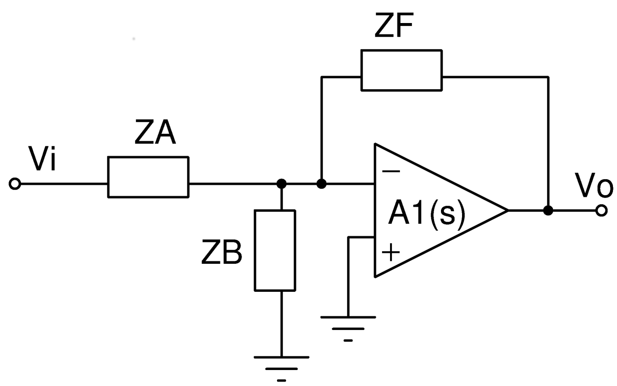


Figure 2: Circuit

$$H(s) = \frac{-\frac{Z_F}{Z_A}}{1 + \frac{1}{A_1}\left(1 + \frac{Z_F}{Z_A} + \frac{Z_F}{Z_B}\right)} \quad (15)$$

where,

$$Z_A = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3} = \frac{k \cdot R(1 - k)R + R \cdot Z_3}{Z_3} \quad (16)$$

$$Z_B = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1} = \frac{k \cdot R(1 - k) + Z_3}{k} \quad (17)$$

Replacing Z_A and Z_B into equation 15.

$$H(s) = \frac{(-GB \cdot Z_3)}{(GB(1 - k)k \cdot R + s((2 - k)kR + 2 * Z_3) + GB \cdot Z_3)} \quad (18)$$

and finally replacing Z_3 in the equation 18, we get the transfer function:

$$H(s) = -\frac{GB \cdot R_3(GB + s + as)}{(GB^2((2 + a)k \cdot R(1 - k) + R_3) + GB(3(2 + a)k \cdot R(1 - k) + (3 + a)R_3)s + (2(2 + a)kR(1 - k) + 2(1 + a)R_3)s^2)} \quad (19)$$

According with the numerator, this circuit implements a lowpass or bandpass (resonator)filter depending on the dominant term.

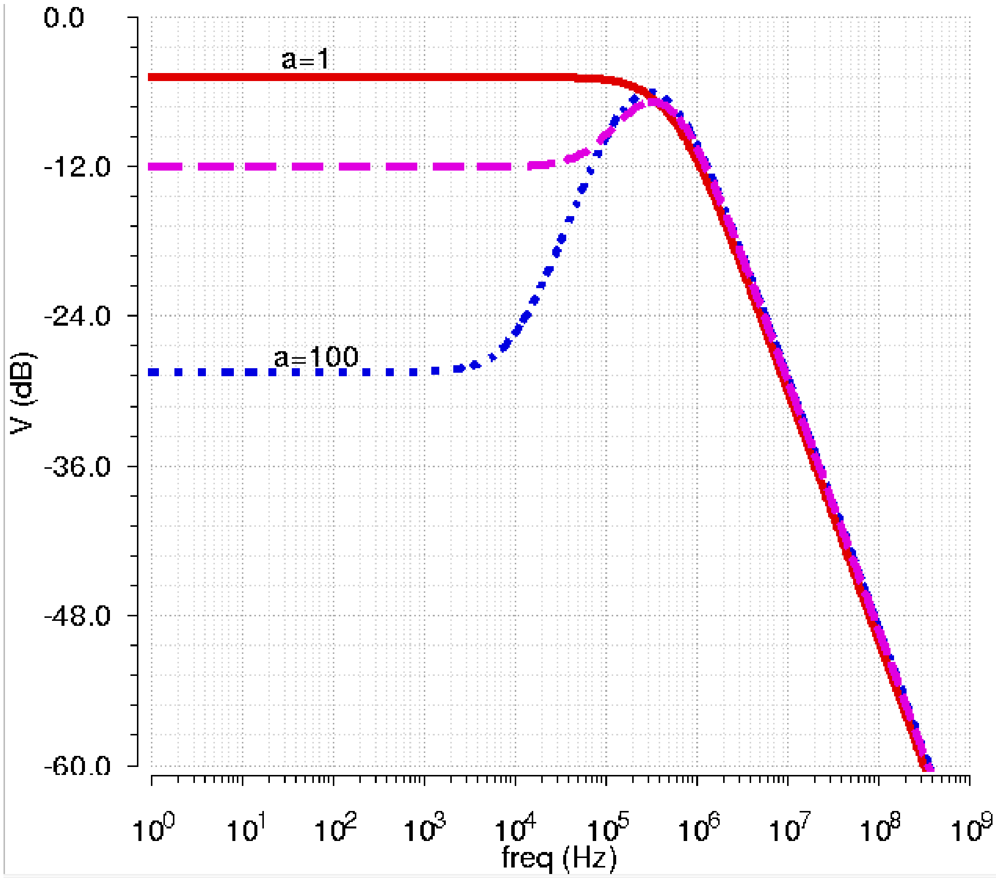


Figure 3: Filter Simulation for different values of a

R_1, R_3	$1k\Omega$
a	10
k	0.5
GB	$2 * pi * 1 \times 10^6$ rad/s

We can verify how by changing a the filter response varies from low pass to resonator
Giving some numbers the expressions were verified by simulations in matlab and cadence.
The transfer function from case 1 and 2 were plotted in matlab and compared.
After running the simulation of the circuit in Cadence we verified that the Case 2 is more accurate.

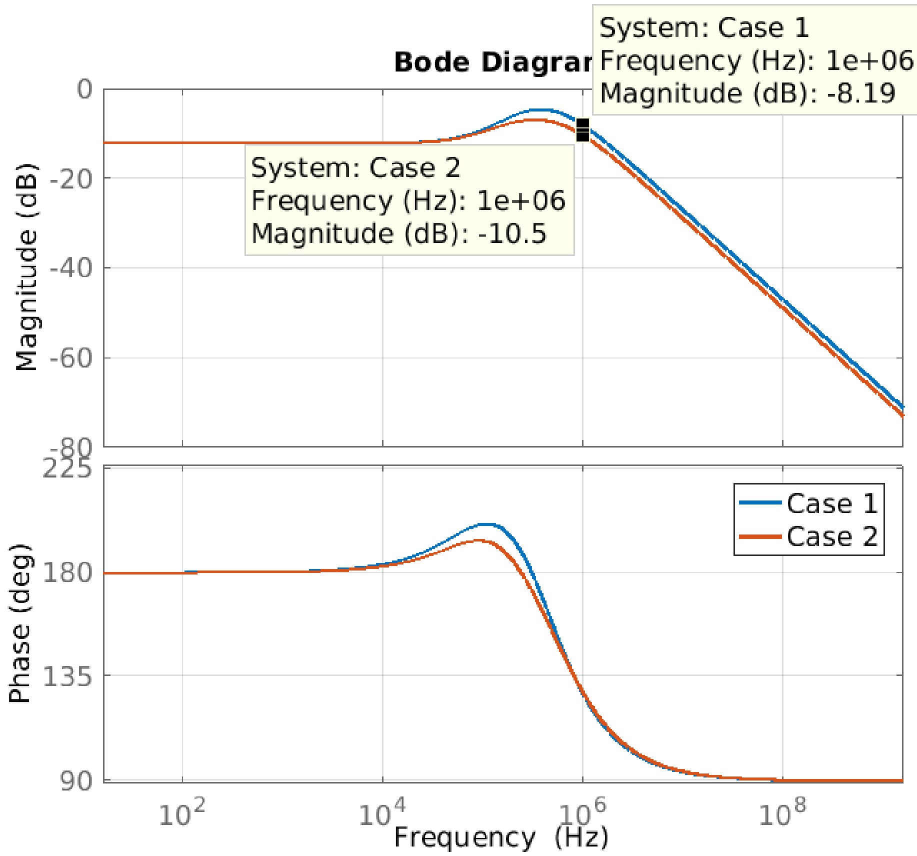


Figure 4: Matlab Simulation

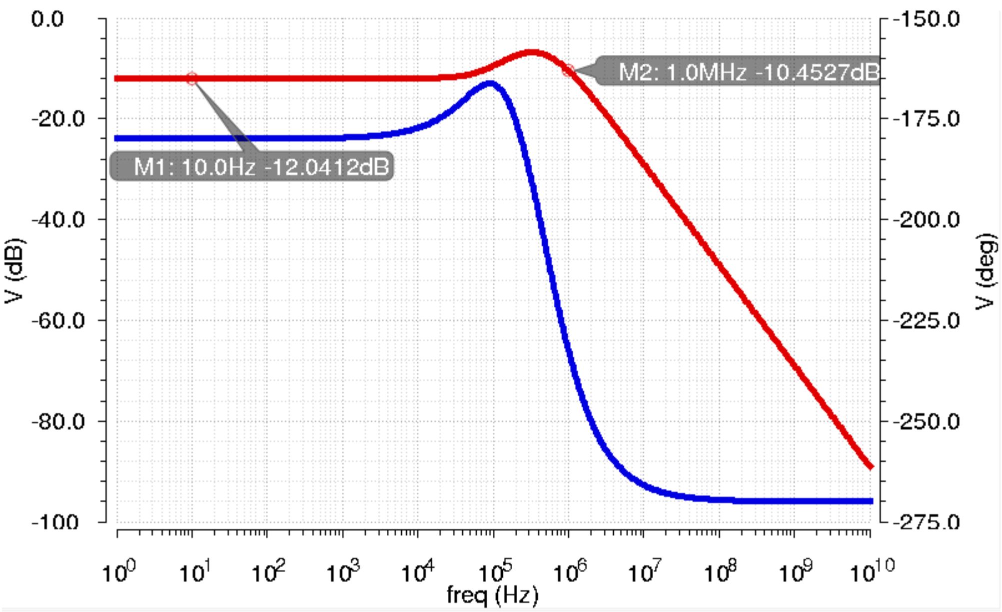


Figure 5: Cadence Simulation