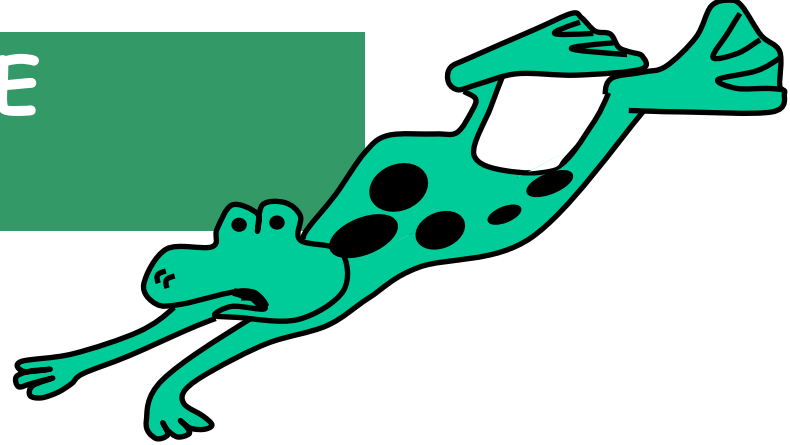


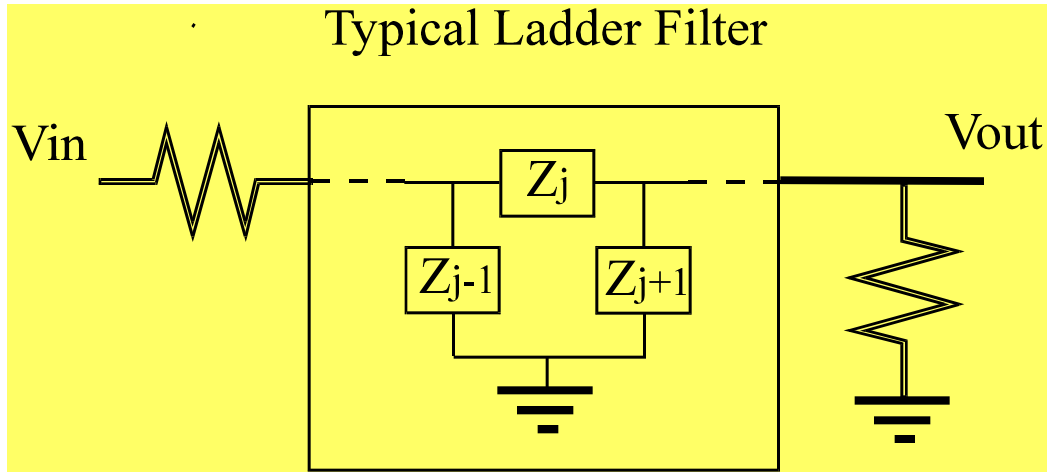
# LEAP FROG TECHNIQUE



## Operational Simulation of LC Ladder Filters

- RLC prototype  $\implies$  low sensitivity
- One form of this technique is called “Leapfrog Technique”
- Fundamental Building Blocks are
  - Integrators
  - Second-order Realizations
- Filters considered
  - LP
  - BP
  - HP
  - BE
  - $j\omega$  Zeros

# Ladder Filters



## Ladder Networks

- ↓ Elements connected in series and in parallel
- ↓ Zeros are easily recognized:  
 $Z_{series} = \text{infinity}$ , or  
 $Z_{shunt} = \text{zero}$

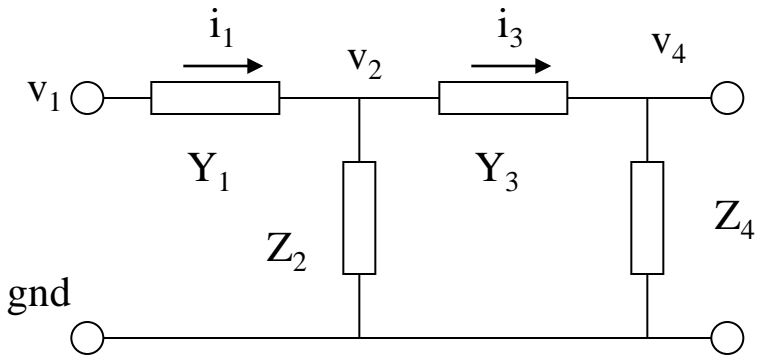
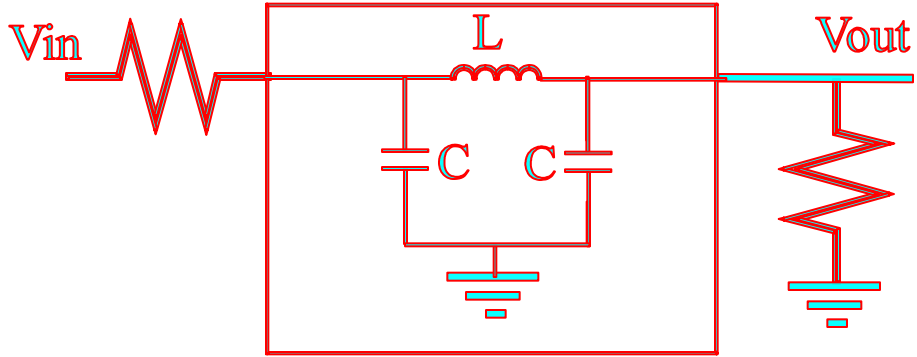
**Problem:**

**How to design active ladder filters?**

**How to design inductors?**

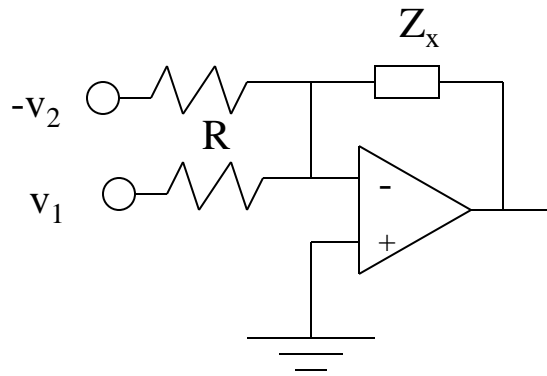
By means of a simple example it is illustrated how to implement an active filter based on the equation of a passive RLC prototype.

Lowpass Filter



$$\begin{aligned}
 i_1 &= (v_1 - v_2)Y_1 \\
 v_2 &= (i_1 - i_3)Z_2 \\
 i_3 &= (v_2 - v_4)Y_3 \\
 v_4 &= (i_3 - 0)Z_4
 \end{aligned}$$

Next we match the equations coefficients with the implementations.



$$V_x' = \dot{i}_1 R_{\text{dummy}}$$

For  $R_{\text{dummy}}=1$  one can make

$$V_x' = \dot{i}_1$$

$$\dot{i}_1 = (v_1 - v_2) Y_1$$

$$v_2 = (\dot{i}_1 - \dot{i}_3) Z_2$$

$$\dot{i}_3 = (v_2 - v_4) Y_3$$

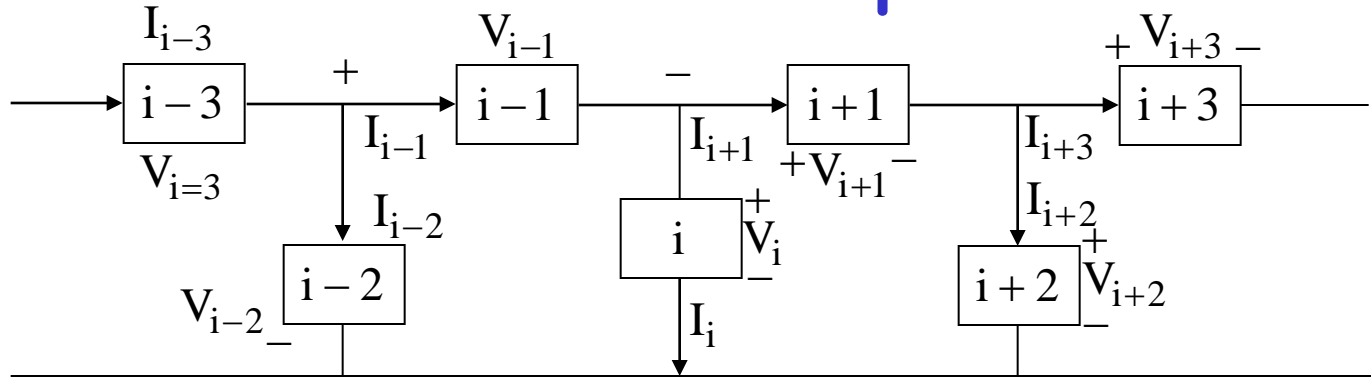
$$v_4 = (\dot{i}_3 - 0) Z_4$$

$$v_x = -(v_1 - v_2) Z_x / R$$

Two key transformations:

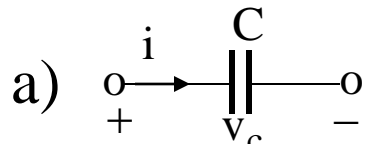
1. If needed convert current equations to voltage equations by multiplying by a dummy (artificial) resistor value of 1.
2. Express the relation of current and voltage of equations always as integrator. The integrator is the basic building block.

# General Principles

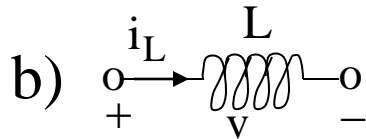


## Interior Portion of a General LC Ladder Network

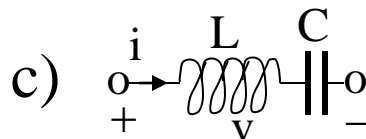
- Interior components are reactive elements only.
- $R_s \neq 0$  or  $R_s = 0$  and  $R_L \neq 0$  (finite)
- Branch voltages are voltage and current
- How to select the proper “**STATE**” variable ?



$$\frac{dv_c}{dt} = \frac{1}{C}i \Rightarrow v_c = \frac{1}{C} \int i(t)dt$$

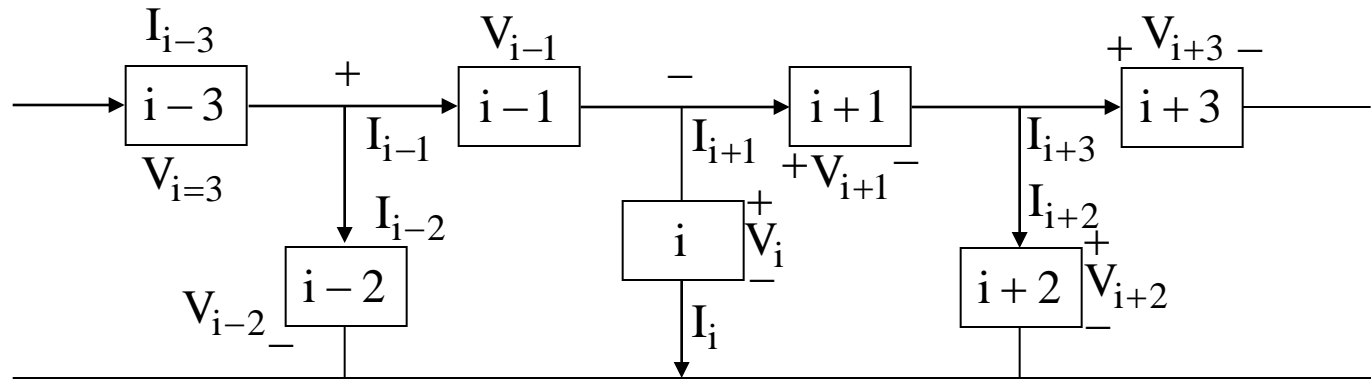


$$\frac{di_L}{dt} = \frac{1}{L}v \Rightarrow i_L = \frac{1}{L} \int v(t)dt$$



$$\frac{dv}{dt} = L \frac{d^2i(t)}{dt^2} + \frac{i}{C} \Rightarrow i = \frac{-1}{LC} \int \int i(t)dt^2 + \frac{1}{L} \int v(t)dt$$

## Systematic approach by writing voltage (KVL) and current (KCL) equations



$$V_{i-2} = \frac{1}{Y_{i-2}} [I_{i-3} - I_{i-1}]$$

$$I_{i-1} = \frac{1}{Z_{i-1}} [V_{i-2} - V_i]$$

$$V_i = \frac{1}{Y_i} [I_{i-1} - I_{i+1}]$$

$$I_{i+1} = \frac{1}{Z_{i+1}} [V_i - V_{i+2}]$$

$$V_{i+2} = \frac{1}{Y_{i+2}} [I_{i+1} - I_{i+3}]$$

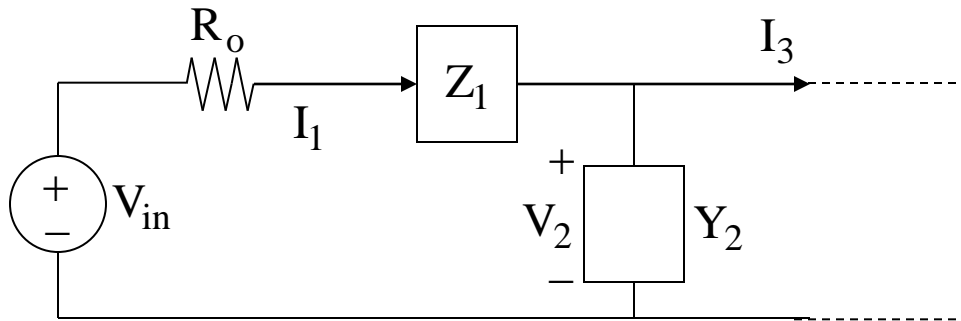
- **Immittance Functions**
- Voltage Transfer Functions, convert I to V functions

i.e.,  $V_k' = I_k R$

# Terminations of LC Ladder Filters

Source

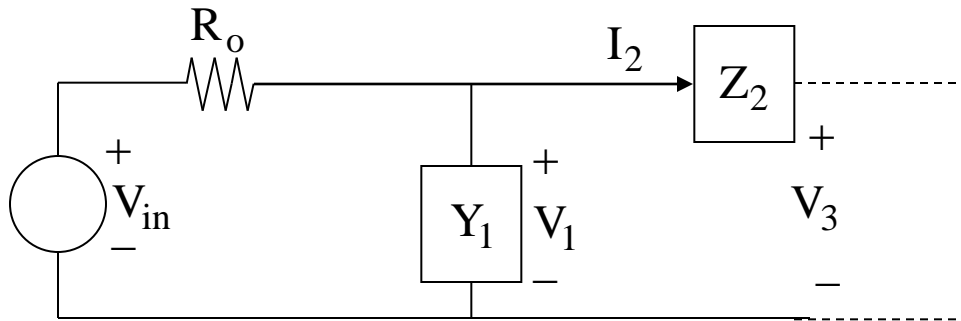
a)



$$V_1' = I_1 R = \frac{R}{Z_1} \left[ V_{in} - \frac{R_o}{R} V_1' - V_2 \right]$$

$$V_2 = \frac{1}{R Y_2} [V_1' - V_3]$$

b)

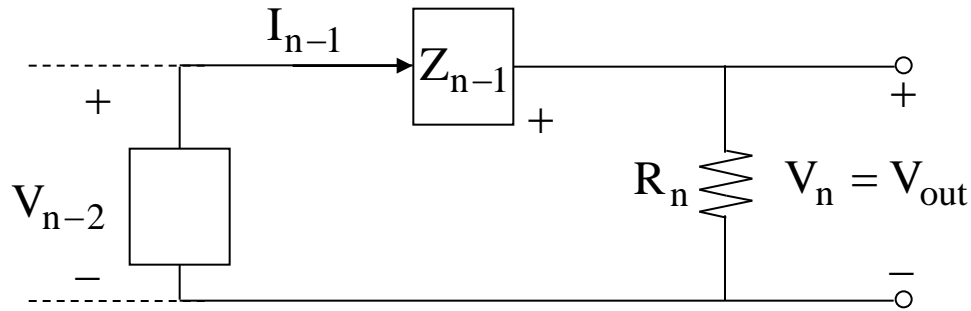


$$Y_1 V_1 = \frac{V_{in} - V_1}{R_o} - I_2 \Rightarrow V_1 = \frac{1}{R Y_1} \left[ \frac{R}{R_o} (V_{in} - V_1) - V_2' \right]$$

$$V_2' = \frac{R}{Z_2} [V_1 - V_3]$$

# Load termination

a)



$$V_{out} = V_n = \frac{R_n}{R} V'_{n-1}$$

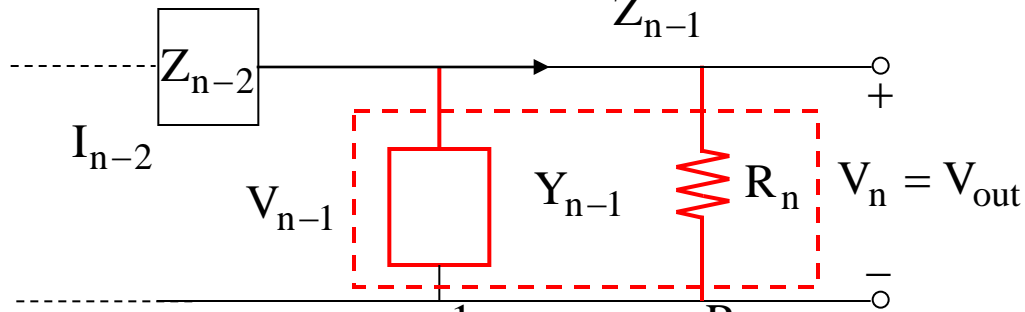
or 
$$V_{out} = R_n [(V_{n-2} - V_{out}) / Z_{n-1}]$$

$$V_{out} = \frac{R}{Z_{n-1}} \frac{R_n}{R} (V_{n-2} - V_{out})$$

$$V_{out} = \frac{\frac{R_n}{R} V_{n-2}}{1 + \frac{R_n}{Z_{n-1}}}$$

Lossy  
Integrator

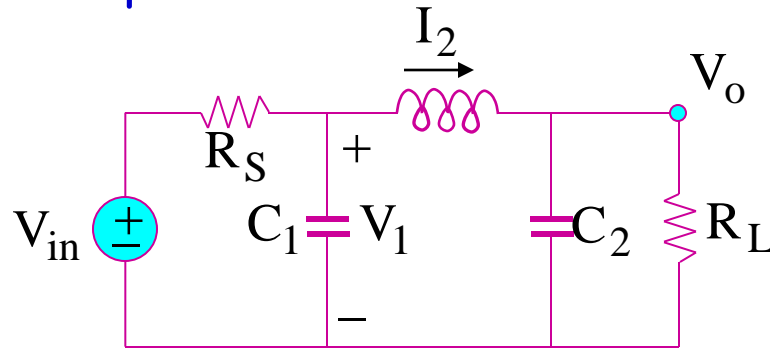
b)



$$V_{out} = \frac{1}{R Y_{n-1}} [V'_{n-2} - \frac{R}{R_n} V_{out}]$$



The approach to map a passive RLC prototype is to pick the state-variables which can be expressed as integrators, since integrators are the basic building block. By applying KCL, KVL, KCL, ..., as many times as the order of the filter, one can write the state-equations that can be implemented by active filters. an example is shown below:



### A Typical Passive RLC Filter

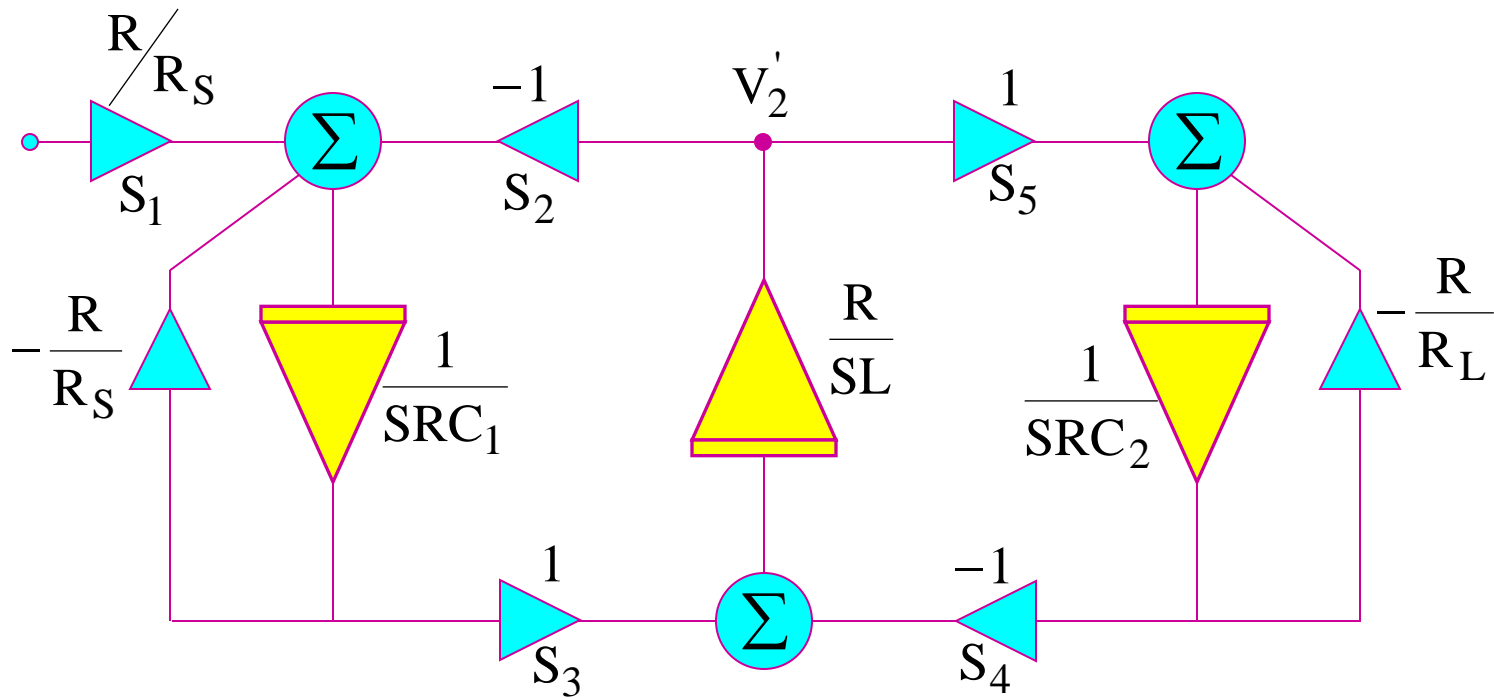
$$\text{KCL} \quad V_1 = \frac{1}{S R C_1} \left[ \frac{R}{R_S} (V_{in} - V_1) - V_2' \right] \quad (15a)$$

where

$$V_2' = I_2 R, \quad R \text{ is an arbitrary value} \quad (15b)$$

$$\text{KVL} \quad V_2' = \frac{R}{S L} (V_1 - V_o) \quad (16)$$

$$\text{KCL} \quad V_o = \frac{1}{S R C_2} \left[ V_2' - \frac{R}{R L} V_o \right] \quad (17)$$



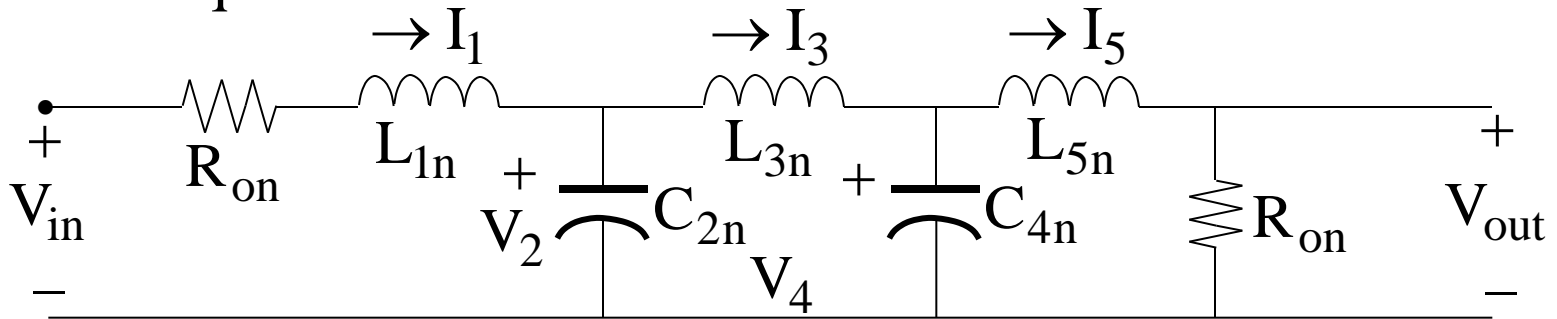
Signal Flow Graph of RLC Prototype of Fig. Shown in previous page.

# Low-Pass Ladder Filters

(Zeros at Infinity) (All Poles)

## Example: A Fifth-Order LP Filter

- 5 State Variables  $[I_1, V_2, I_3, V_4, I_5]$
- 5 State Equations



KVL-KCL-KVL-KCL-KVL (Sequence)

$$* \quad V_{in} - I_1 (R_{on} + sL_{1n}) - V_2 = 0$$

Current Analogs  $V_1' = I_1 R$

$$V_1' = \frac{R}{sL_{in}} \left[ V_{in} - V_2 - \frac{R_{on}}{R} V_1' \right] \quad (1)$$

$$I_1 - I_3 - sC_{2n}V_2 = 0$$

$$V_2 = \frac{1}{sRC_{2n}}[V_1' - V_3'] \quad (2)$$

$$V_2 - sL_{3n}I_3 - V_4 = 0$$

$$V_3' = \frac{R}{sL_{3n}}[V_2 - V_4] \quad (3)$$

$$I_3 - sC_{4n}V_4 - I_5 = 0$$

$$V_4 = \frac{1}{sRC_{4n}}[V_3' - V_5'] \quad (4)$$

$$V_4 - sL_{5n}I_5 - I_5R_{6n} = 0$$

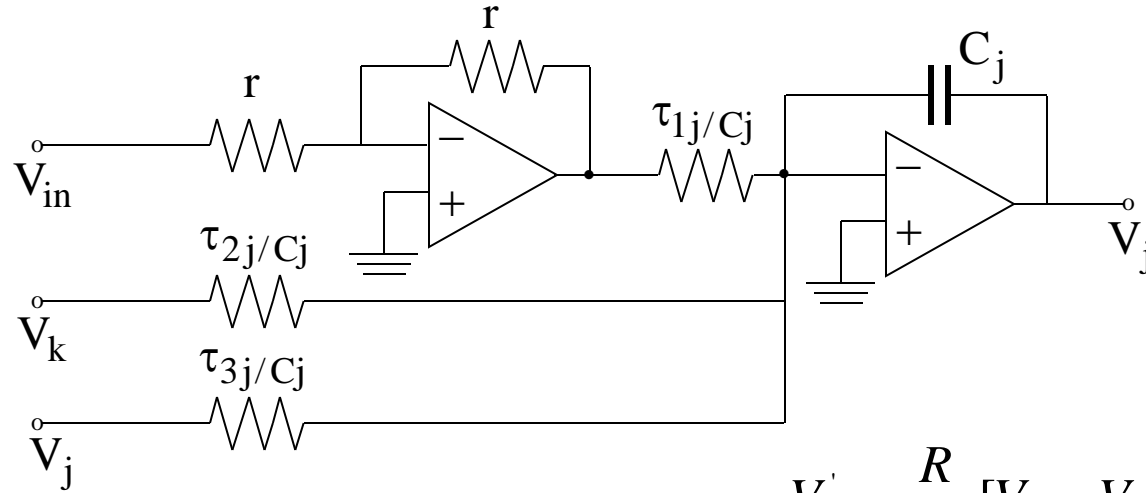
$$V_5' = \frac{R}{sL_{5n}}[V_4 - \frac{R_{6n}}{R}V_5']$$

But

$$V_{out} = I_5R_{6n} = \frac{V_5'}{R}R_{6n}$$

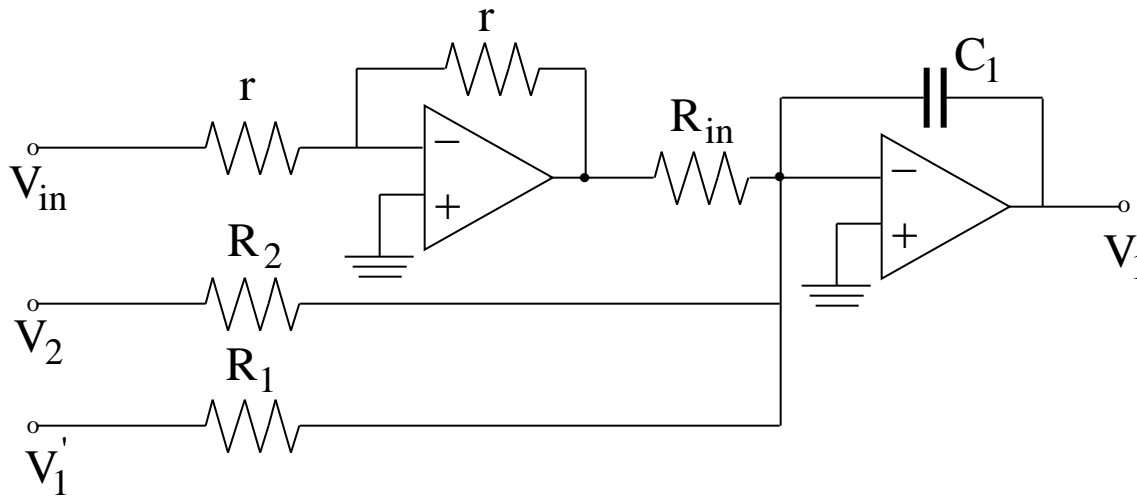
$$V_{out} = \frac{R_{6n}}{R} \frac{R}{sL_{5n}} [V_4 - \frac{R_{6n}}{R} V_{out} \frac{R}{R_{6n}}] \quad (5)$$

# Active RC Building Blocks



$$V_1' = \frac{R}{sL_{in}} [V_{in} - V_2 - \frac{R_{on}}{R} V_1']$$

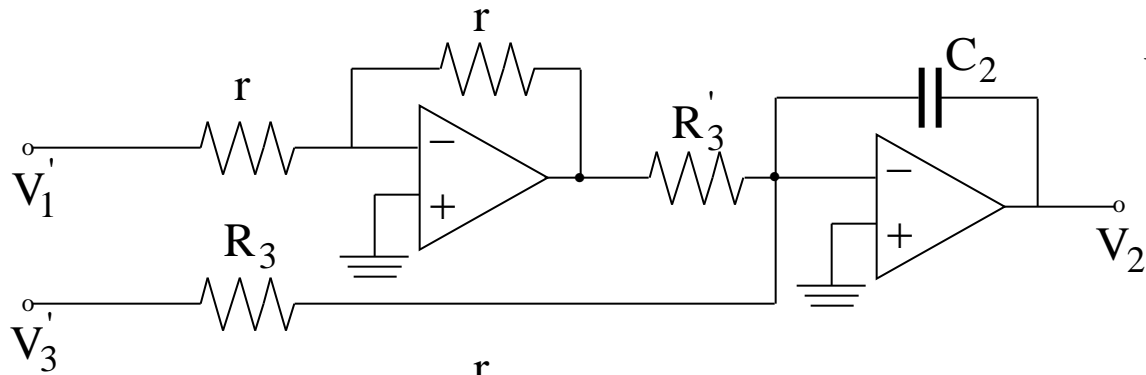
Thus, eq(1) is implemented as follows



$$R_{in} C_1 = \frac{L_{in}}{R}$$

$$R_2 C_1 = \frac{L_{in}}{R}$$

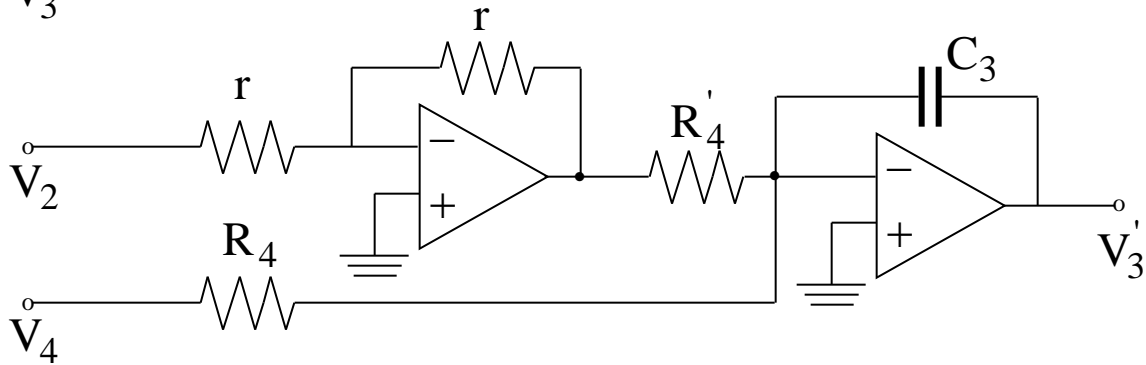
$$R_1 C_1 = \frac{L_{in}}{R_{on}}$$



$$V_2 = \frac{1}{sRC_{2n}} [V_1' - V_3']$$

$$RC_{2n} = R_3' C_2$$

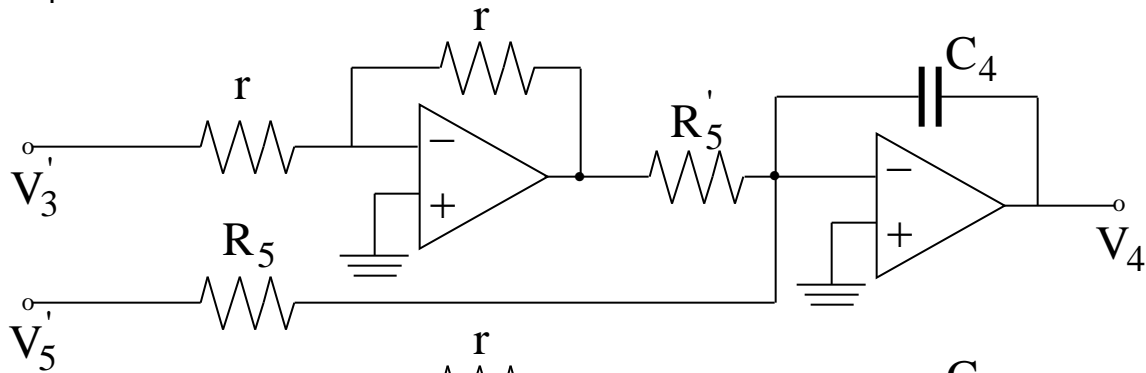
$$RC_{2n} = R_3 C_2$$



$$V_3' = \frac{R}{sL_{3n}} [V_2 - V_4]$$

$$R_4' C_3 = L_{3n} / R$$

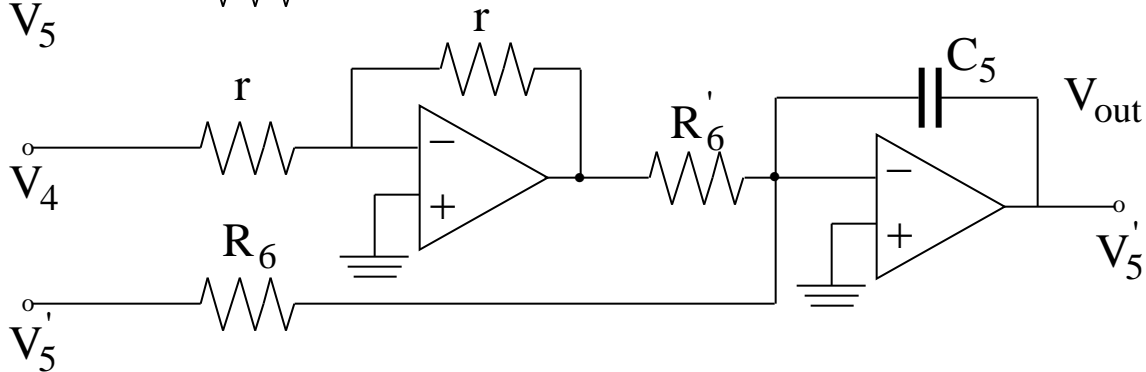
$$R_4 C_3 = L_{3n} / R$$



$$V_4 = \frac{1}{sRC_{4n}} [V_3' - V_5']$$

$$R_5' C_4 = RC_{4n}$$

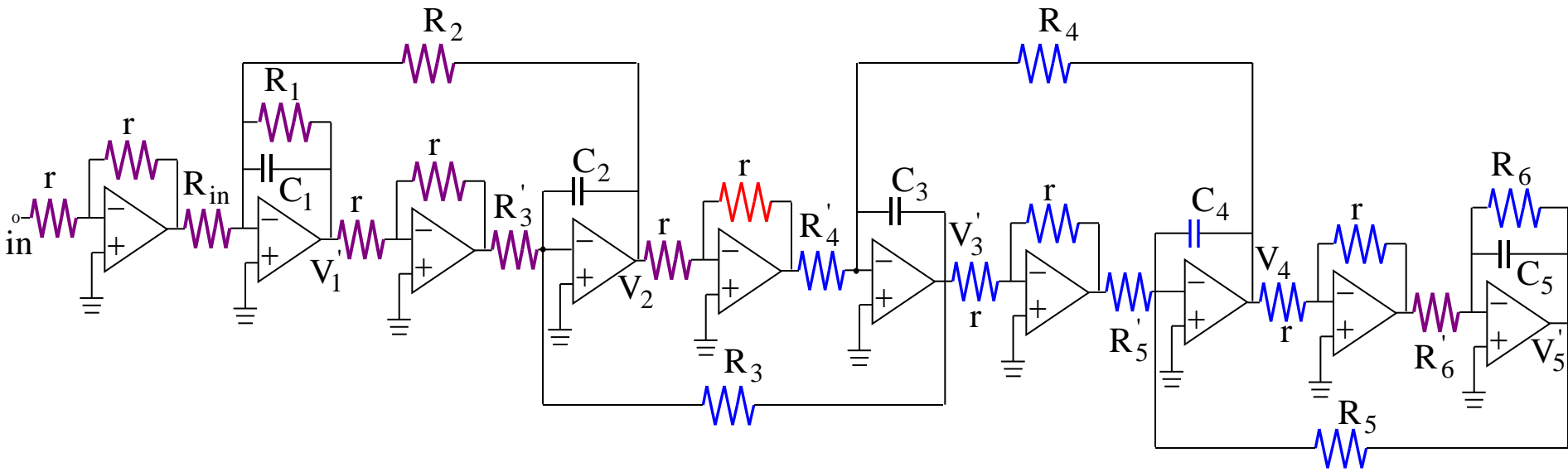
$$R_5 C_4 = RC_{4n}$$



$$V_{out} = \frac{R_{6n}}{R} \frac{R}{sL_{5n}} [V_4 - \frac{R_{6n}}{R} V_{out} \frac{R}{R_{6n}}]$$

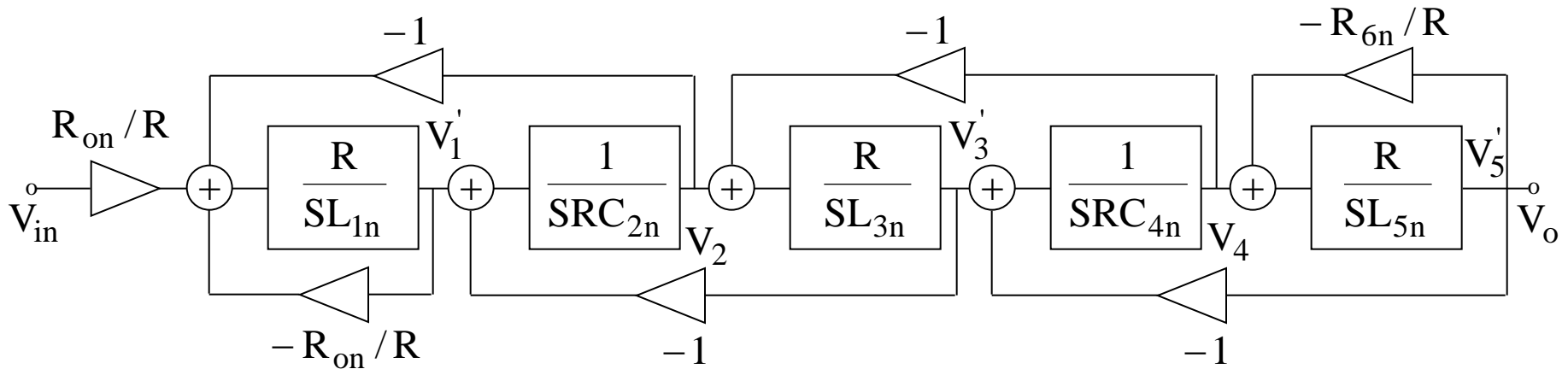
$$R_6' C_5 = L_{5n} / R_{6n}$$

$$R_6 C_5 = L_{5n} / R_{6n}$$

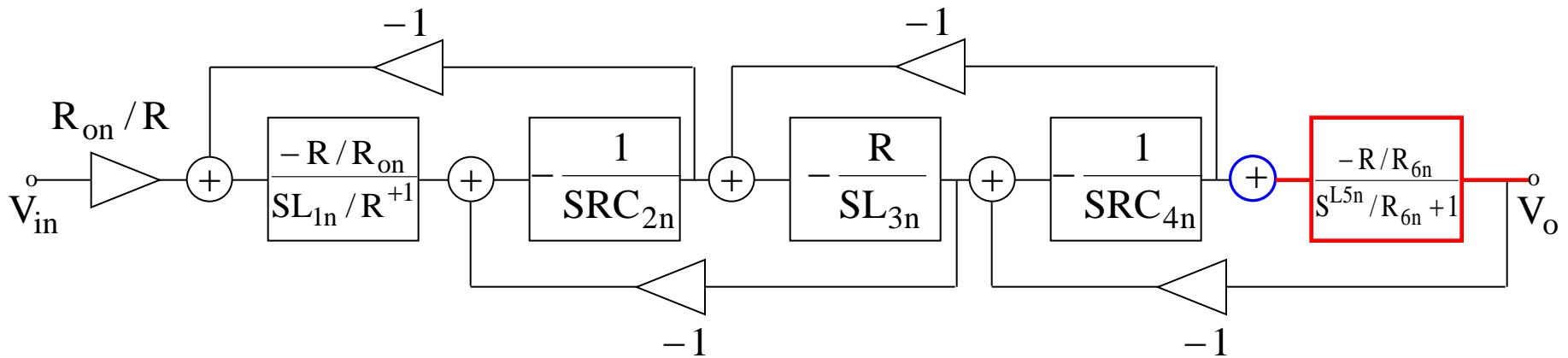


Low-Pass 5<sup>th</sup>-Order Active -RC

Leapfrog Filter



# Leapfrog Building Block Representation

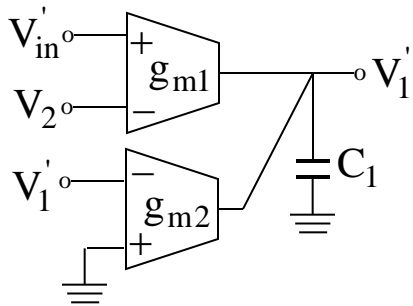




# OTA-C Implementation

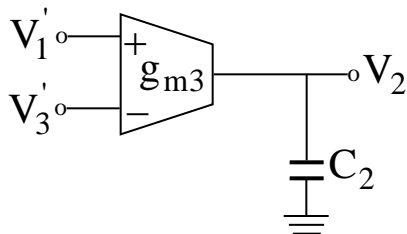
$$V_1' = \frac{R}{sL_{in}} [V_{1n} - V_2 - \frac{R_{on}}{R} V_1']$$

Thus, EQ (1) is implemented as



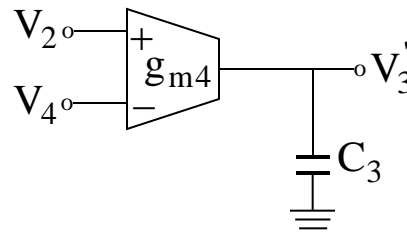
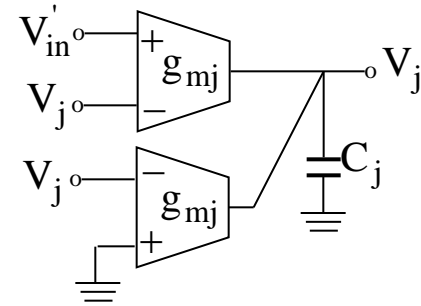
$$\frac{g_{m1}}{C_1} = \frac{L_{1n}}{R}$$

$$\frac{g_{m2}}{C_2} = \frac{L_{1n}}{R_{on}}$$



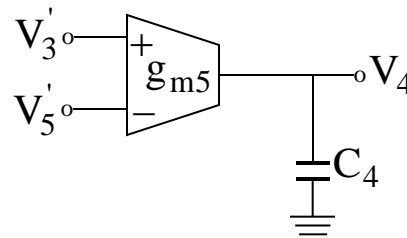
EQ (2)

$$\frac{g_{m3}}{C_2} = RC_{2n}$$



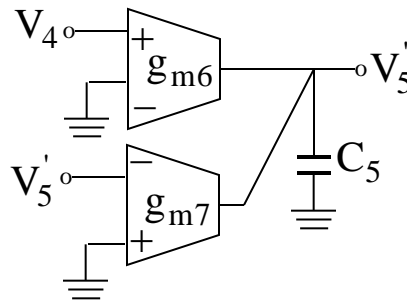
EQ (3)

$$\frac{g_{m4}}{C_3} = \frac{L_{3n}}{R}$$



EQ (4)

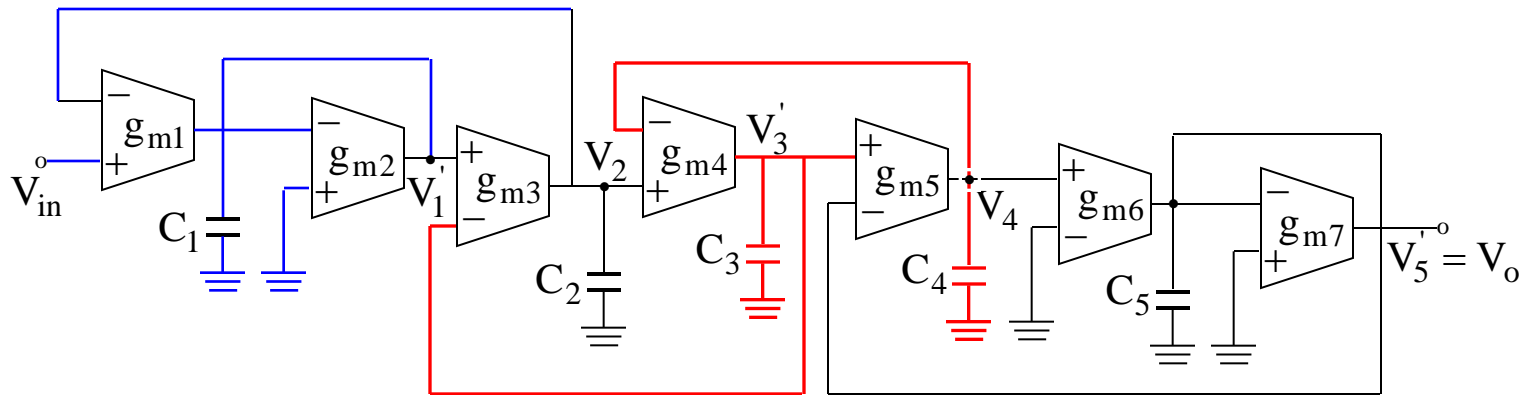
$$\frac{g_{m5}}{C_4} = RC_{4n}$$



EQ (5)

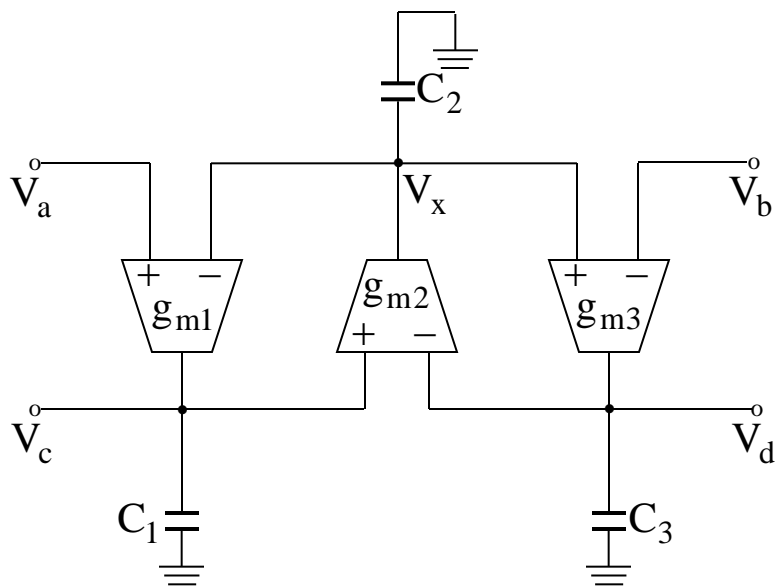
$$\frac{g_{m6}}{C_5} = \frac{L_{5n}}{R_{on}}$$

$$\frac{g_{m7}}{C_5} = \frac{L_{5n}}{R_{on}}$$

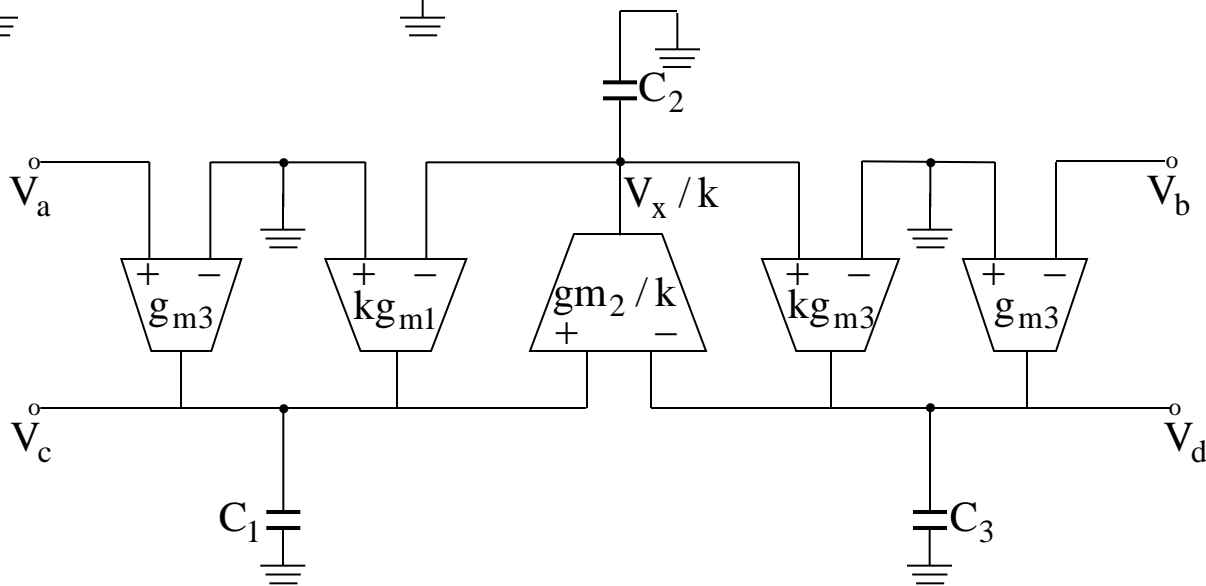


5<sup>th</sup>-Order OTA-C Leapfrog Implementation

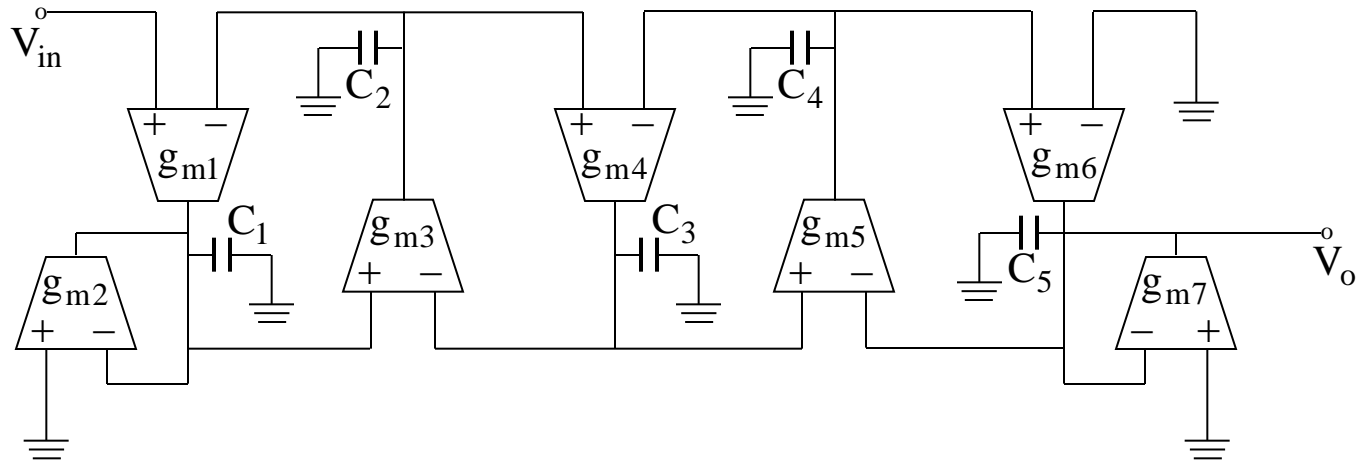
# Voltage Scaling of OTA-C All Pole Leapfrog Filters.

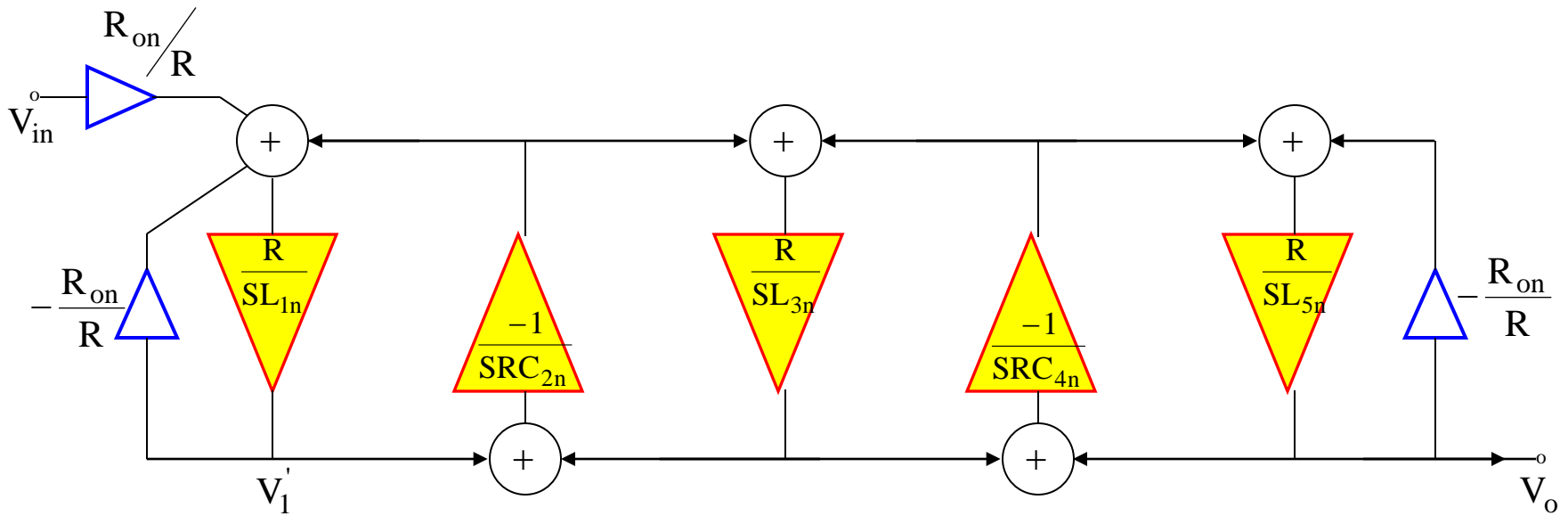


$$V_x = g_{m2}(V_c - V_d)/sC_2$$



Changing  $V_x \rightarrow \frac{V_x}{k}$  without changing  $V_a$ ,  $V_b$ ,  $V_c$ , and  $V_d$





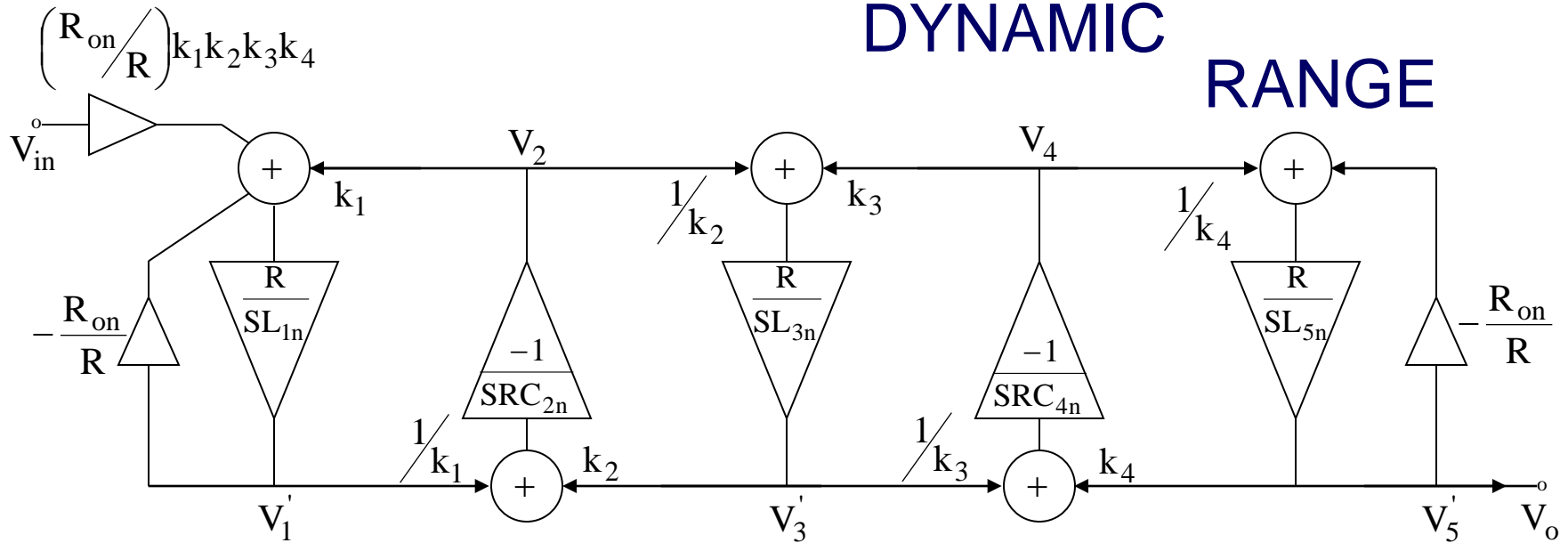
**System Level Representation: 5<sup>th</sup>-Order LP Leapfrog Filter**

# SCALING

## MAXIMUM

## DYNAMIC

## RANGE



$$V_{in} = 1$$

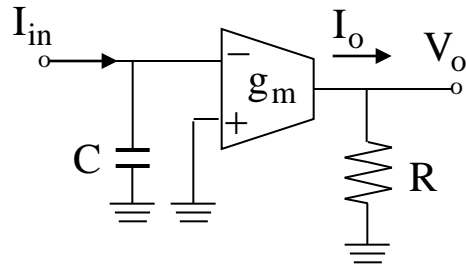
$$V_4 = V_o \left( s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_o \max}{V_3 \max} = k_3 \quad ; \quad \frac{V_o \max}{V_1 \max} = k_1$$

$$V_4 = T(s) \left( s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_o \max}{V_2 \max} = k_2$$

$$\frac{V_o \max}{V_4 \max} = k_4$$

# Current Level Scaling Technique

## Case 1



$R$  represents input resistance of a current-mirror

$$I_o = -g_m / sC$$

$$V_o = I_o R = -\frac{g_m R}{sC}$$

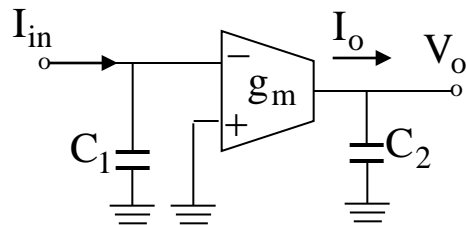
$$kI_o = -k \frac{g_m}{sC} \begin{cases} \text{option 1} & g_m \rightarrow kg_m \\ \text{option 2} & C \rightarrow C/k \end{cases}$$

$$V_o = kI_o R$$

If  $V_o$  needs to be constant, then  $R$  must be decreased by  $k$ .

In summary  $\begin{cases} \text{option 1} & g_m \rightarrow kg_m & \& & R \rightarrow R/k \\ \text{option 2} & C \rightarrow C/k & \& & R \rightarrow R/k \end{cases}$

## Case 2

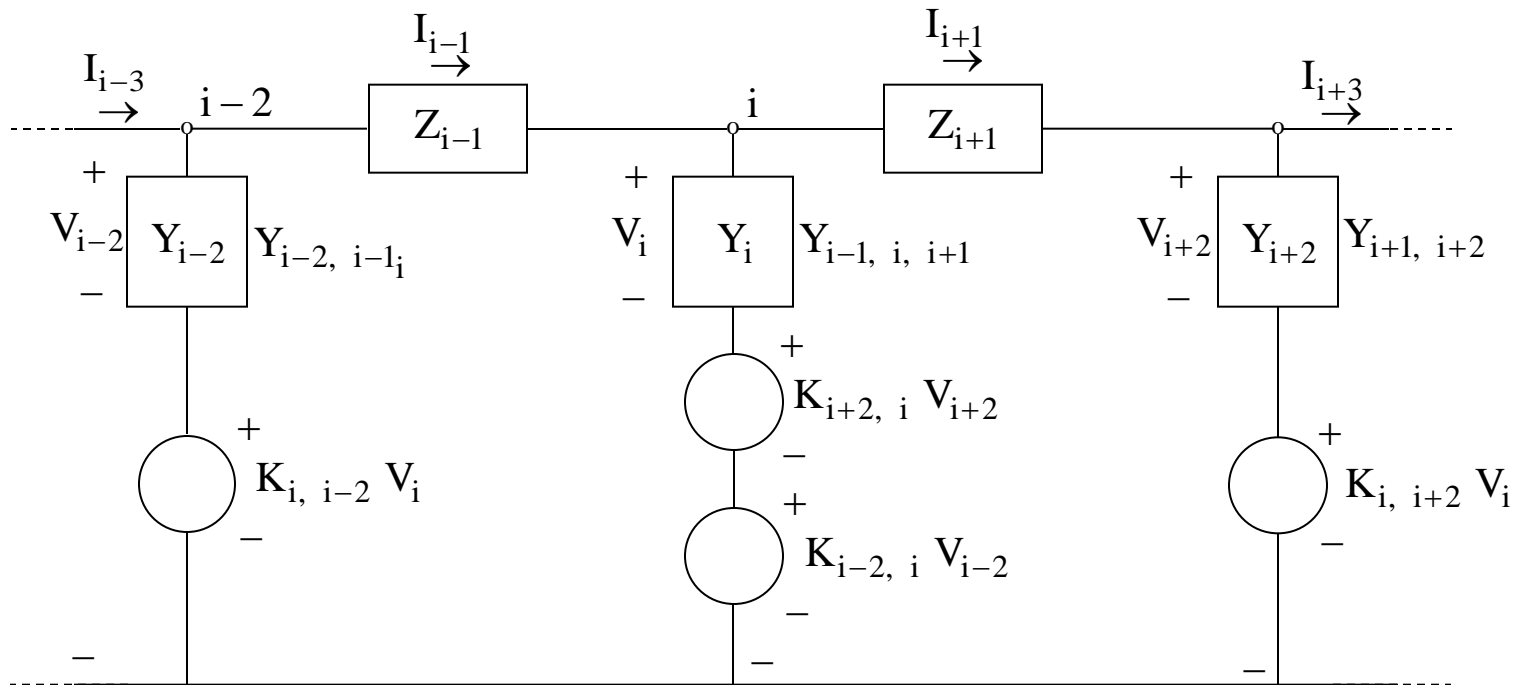
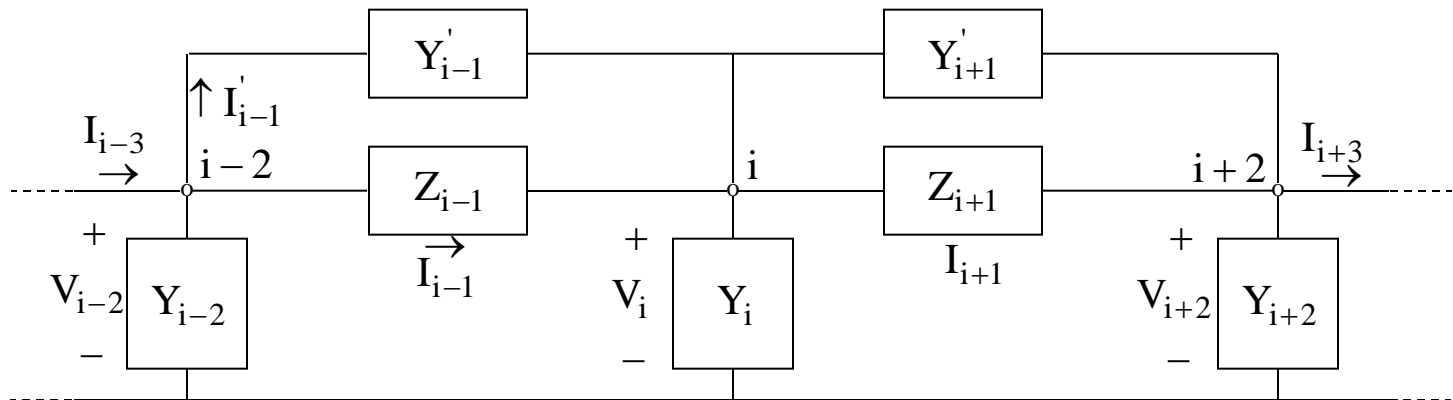


$C_2$  represents input impedance of next block

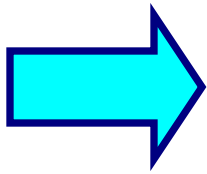
$$I_o = -\frac{g_m}{sC_1}$$

$$V_o = I_o \frac{1}{sC_2}$$

# LC Ladder Filters with Finite Zeros







## Dependent Sources to Replace the Bridging Admittances

KCL at  $i-2$

$$I_{i-3} - I_{i-1} = Y_{i-2}V_{i-2} + Y'_{i-1}[V_{i-2} - V_i]$$

or

$$V_{i-2} = \left[ \frac{Y'_{i-1}}{Y_{i-2} + Y'_{i-1}} \right] V_i + \frac{I_{i-3} - I_{i-1}}{Y_{i-2} + Y'_{i-1}}$$

$$V_{i-2} = K_{i,i-2}V_i + \frac{(I_{i-3} - I_{i-1})}{Y_{i-2,i-1}}$$

### KCL @ NODE $i$

$$I_{i-1} - I_{i+1} = Y_i V_i - Y'_{i-1}(V_{i-2} - V_i) + Y'_{i+1}(V_i - V_{i+2})$$

or

$$V_i = \frac{Y'_{i-1}}{Y'_{i-1} + Y_i + Y'_{i+1}} V_{i-2} + \frac{Y'_{i+1}}{Y'_{i-1} + Y_i + Y'_{i+1}} V_{i+2} + \frac{I_{i+1} - I_{i-1}}{Y'_{i-1} + Y_i + Y'_{i+1}}$$

$$V_i = K_{i-2,i}V_{i-2} + K_{i+2,i}V_{i+2} + \frac{V_{i+1} - V_{i-1}}{RY_{i-1,i,i+1}}$$

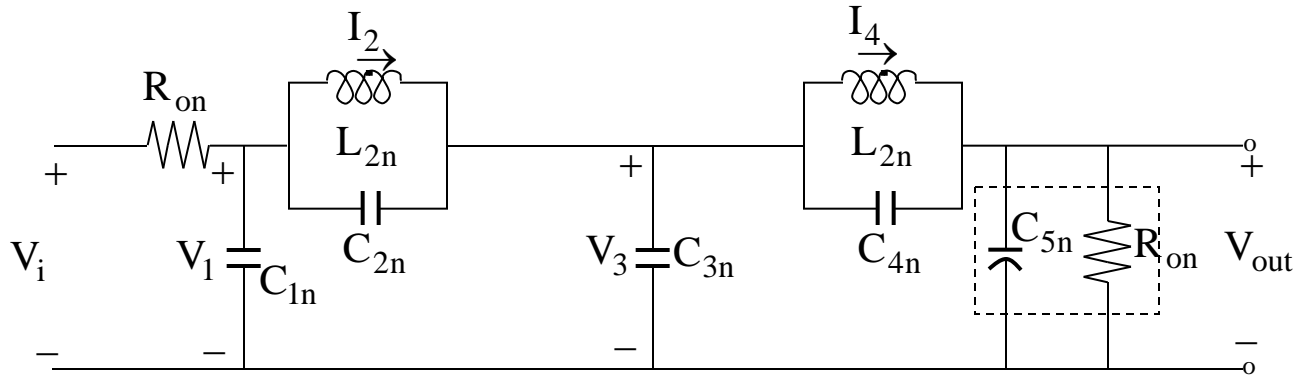
In a similar for KCL for node  $i+2$  and solving for  $V_{i+2}$  results

$$V_{i+2} = K_{i,i+2}V_i + (V_{i+1} - V_{i+3})/RY'_{i,i+2}$$

# LC Ladder Simulation

1. Start from the desired LC LP prototype network satisfying SPECS.
2. Eliminate shunt elements in the series branches or series element in the shunt branches.
3. Normalize and transform the network if the realization is not low-pass.
4. Select the state variables such that  $\frac{dx}{dt} = f(x, \mu s)$
5. Solve for each state variable in terms of the other states and output variable including the proper scaling.
6. Synthesize each equation with its corresponding building block.

# Example: Low-Pass With Finite $j\omega$ Zeros



$$C_{12n} = C_{1n} + C_{2n}$$

$$C_{234n} = C_{2n} + C_{3n} + C_{4n}$$

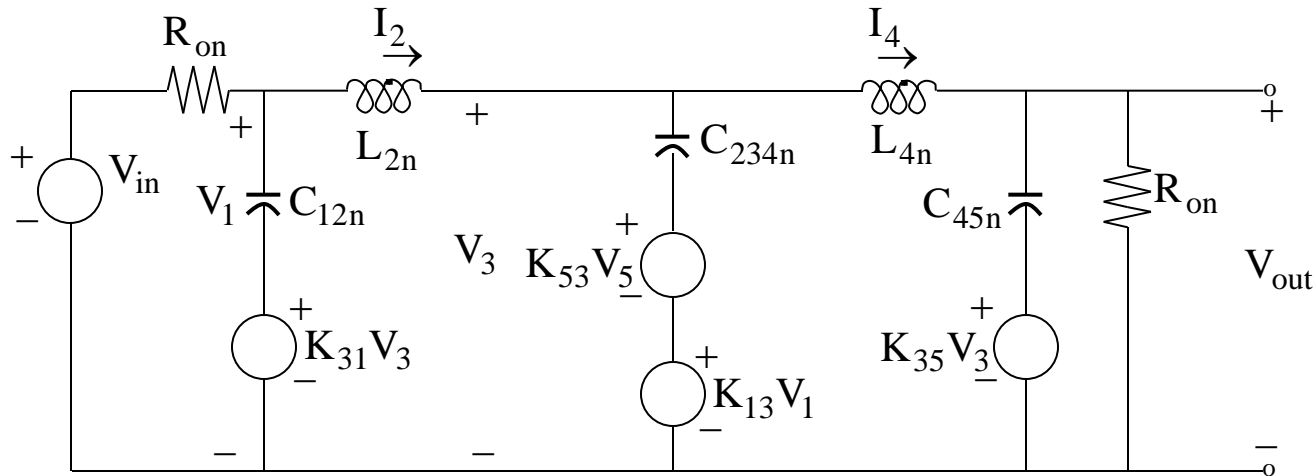
$$C_{45n} = C_{4n} + C_{5n}$$

$$K_{31} = \frac{C_{2n}}{C_{12n}}$$

$$K_{53} = \frac{C_{4n}}{C_{234n}}$$

$$K_{13} = \frac{C_{2n}}{C_{234n}}$$

$$K_{35} = \frac{C_{4n}}{C_{45n}}$$



$$C_{12n} = C_{1n} + C_{2n}$$

$$C_{234n} = C_{2n} + C_{3n} + C_{4n}$$

$$C_{45n} = C_{4n} + C_{5n}$$

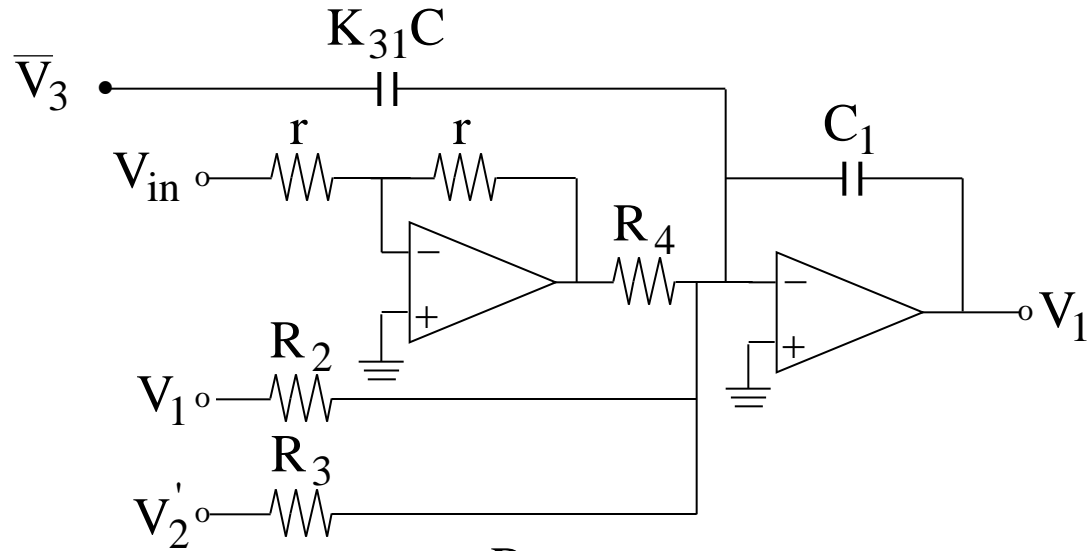
$$K_{31} = \frac{C_{2n}}{C_{12n}}$$

$$K_{53} = \frac{C_{4n}}{C_{234n}}$$

$$K_{13} = \frac{C_{2n}}{C_{234n}}$$

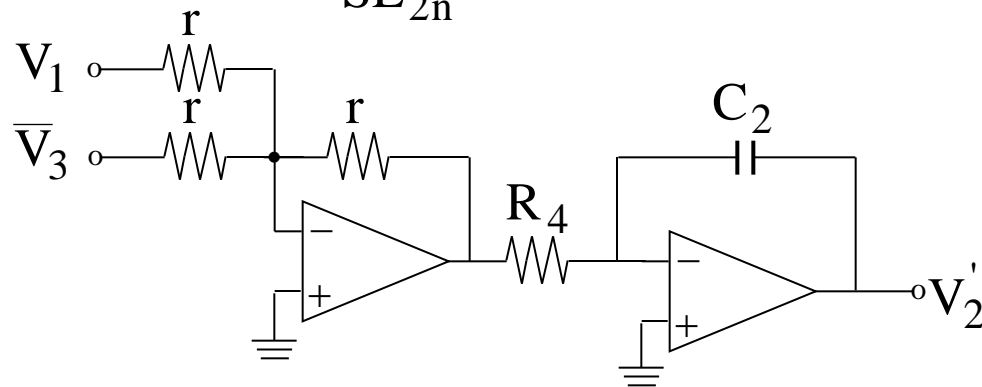
$$K_{35} = \frac{C_{4n}}{C_{45n}}$$

$$V_1 = \frac{1}{SC_{12n}} \left[ \frac{V_{in}}{R_{on}} - \frac{V_1}{R_{on}} - \frac{V_2'}{R} \right] + K_{31} V_3 \quad (1)$$



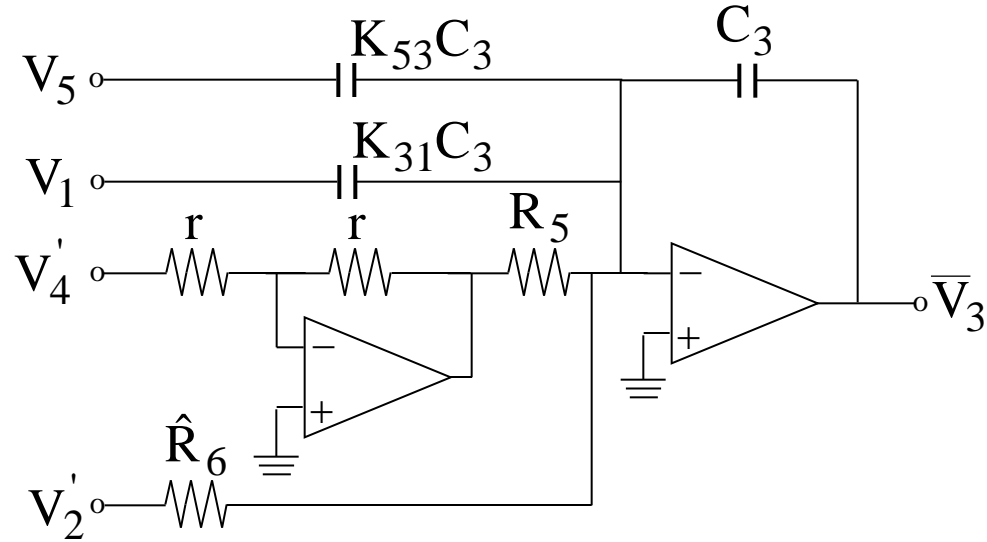
$$V_2' = \frac{R}{SL_{2n}} [V_1 - V_3] \quad (2)$$

$$V_2' = \frac{R}{SL_{2n}} [V_1 + \bar{V}_3] \quad (2')$$

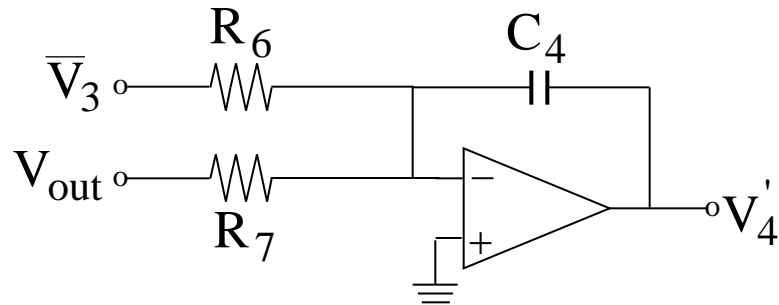


$$V_3 = \frac{1}{\text{SRC}_{234n}} [V_2' - V_4'] + K_{53} V_5 + K_{31} V_1 \quad (3)$$

$$\bar{V}_3 = \frac{1}{\text{SRC}_{234n}} [V_4' - V_2'] - K_{53} V_5 - K_{31} V_1 \quad (3')$$



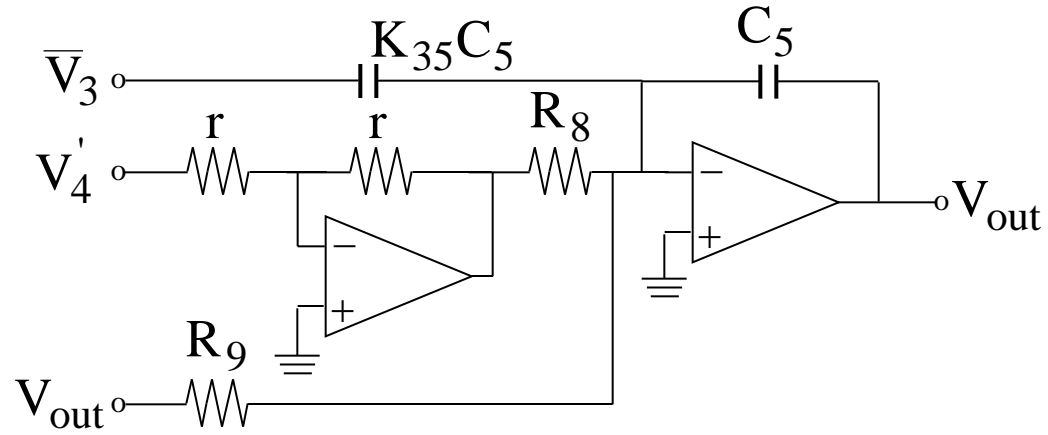
$$V_4' = \frac{R}{\text{SL}_{4n}} [-\bar{V}_3 - V_{\text{out}}] \quad (4)$$

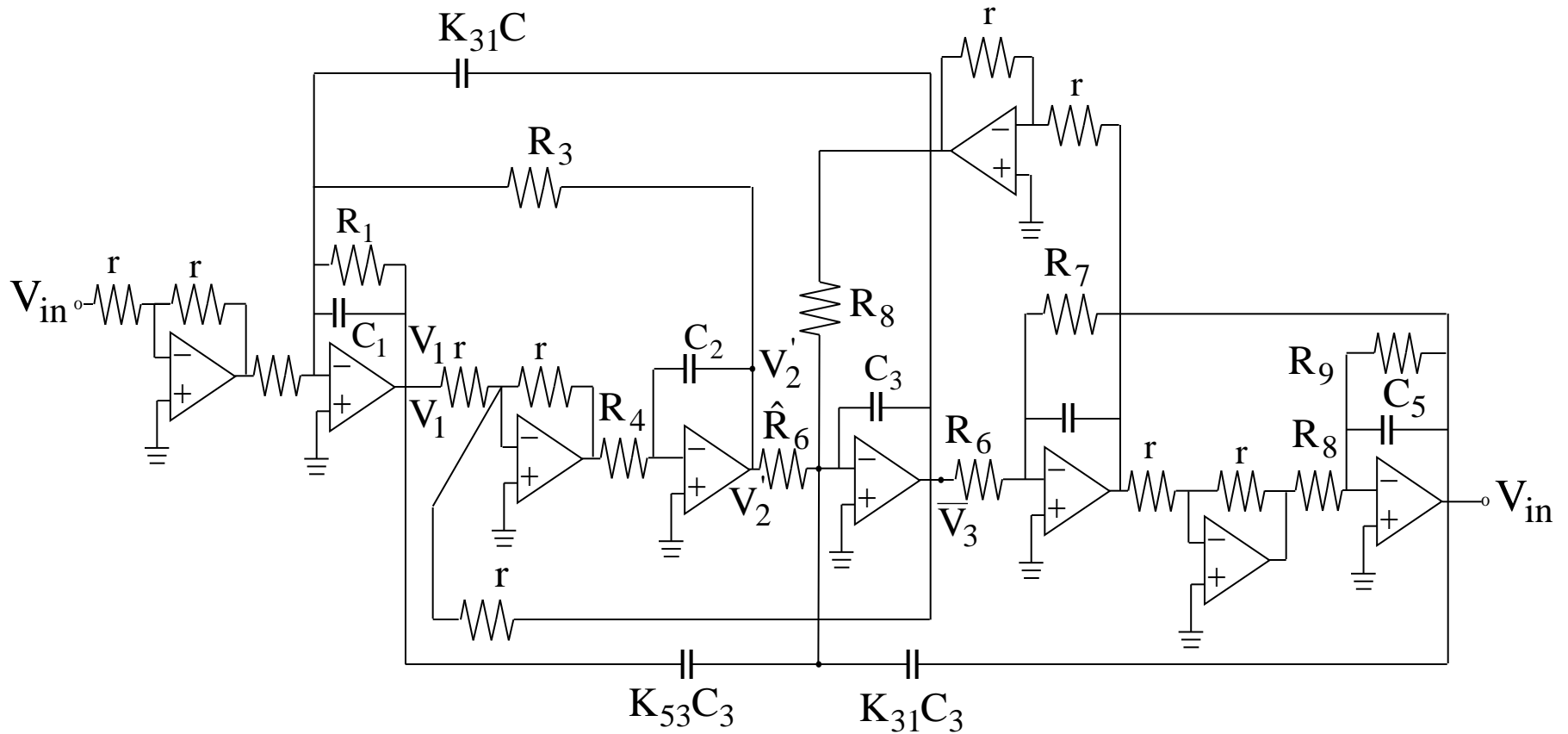


$$R_6 C_4 = L_{4n} / R$$

$$R_7 C_4 = L_{4n} / R$$

$$V_{\text{out}} = V_6 = \frac{1}{SC_{45n}} \left[ \frac{V_4'}{R} - \frac{V_{\text{out}}}{R_{6n}} \right] - K_{35} \bar{V}_3 \quad (5)$$



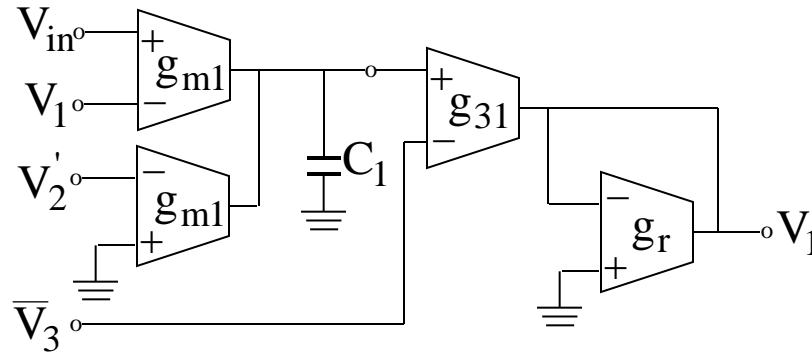


Active RC Fifth-Order Elliptic Filter



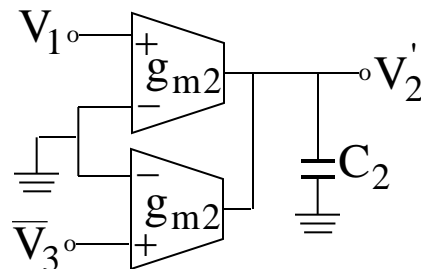
# OTA-C Elliptic Implementation

$$V_1 = \frac{1}{SC_{12n}} \left[ \frac{V_{in}}{R_{on}} - \frac{V_1}{R_{on}} - \frac{V_2'}{R} \right] + K_{31} V_3 \quad (1)$$



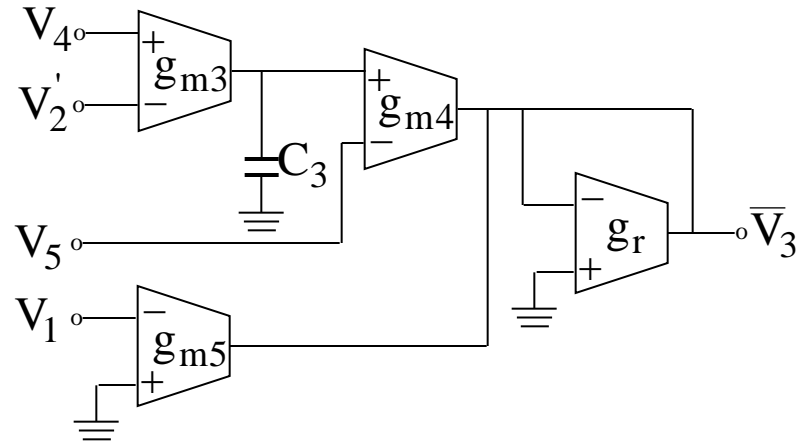
$$\left. \begin{aligned} \frac{g_{31}g_{m1}}{g_r C_1} &= C_{12n} R_{on} \\ \frac{g_{31}}{g_r} &= K_{31} \end{aligned} \right\} \begin{aligned} g_{31} &= K_{31} g_r \\ g_{m1} &= \frac{C_{12n} R_{on}}{K_{31}} C_1 \end{aligned}$$

$$V_2' = \frac{R}{SL_{2n}} [V_1 + \bar{V}_3] \quad (2)$$

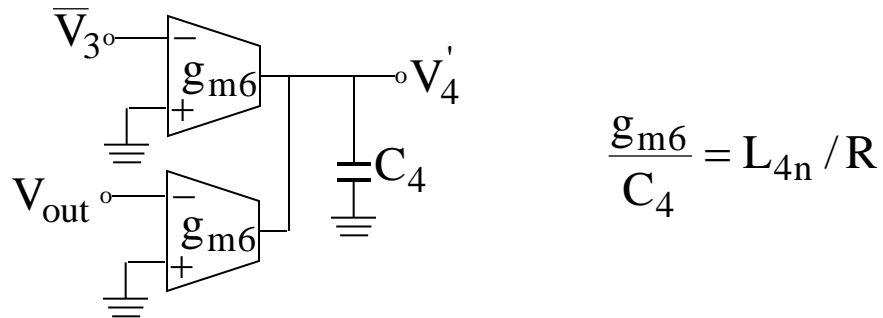


$$\frac{g_{m2}}{C_2} = \frac{L_{2n}}{R}$$

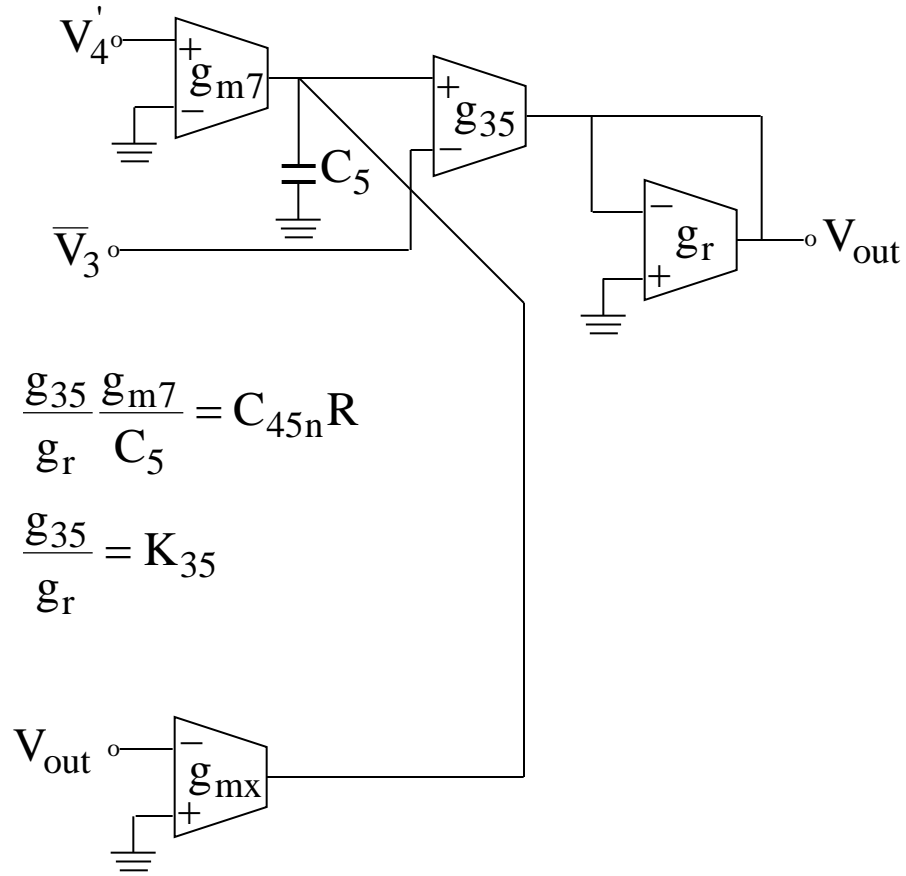
$$\bar{V}_3 = \frac{1}{sRC_{234n}} [V_4' - V_2'] - K_{53}V_5 - K_{31}V_1 \quad (3)$$

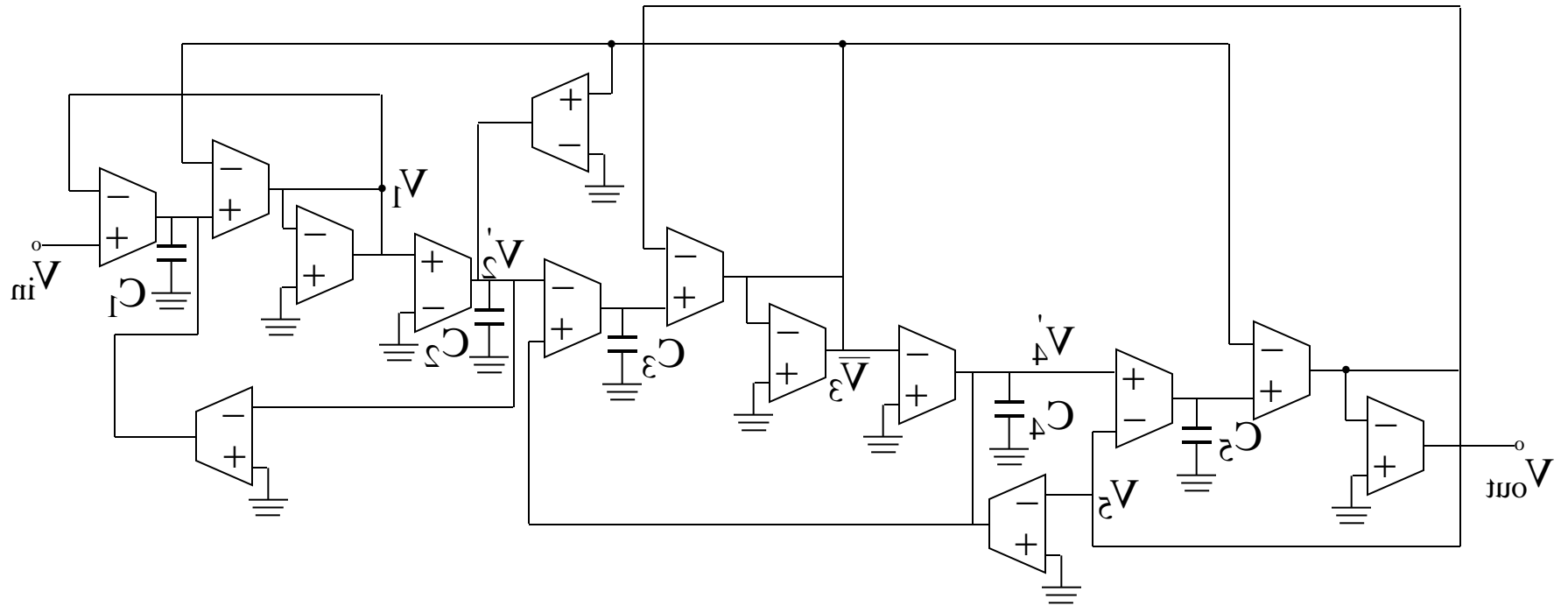


$$V_4' = \frac{R}{sL_{4n}} [-\bar{V}_3 - V_{out}] \quad (4)$$



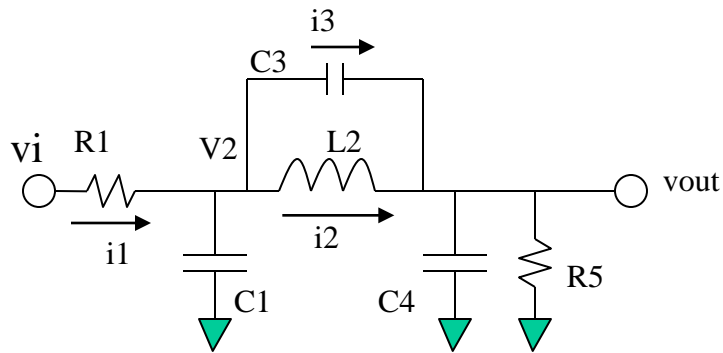
$$V_{\text{out}} = V_5 = \frac{1}{sC_{45n}} \left[ \frac{V_4'}{R} - \frac{V_{\text{out}}}{R_{\text{on}}} \right] - K_{35} \bar{V}_3 \quad (5)$$



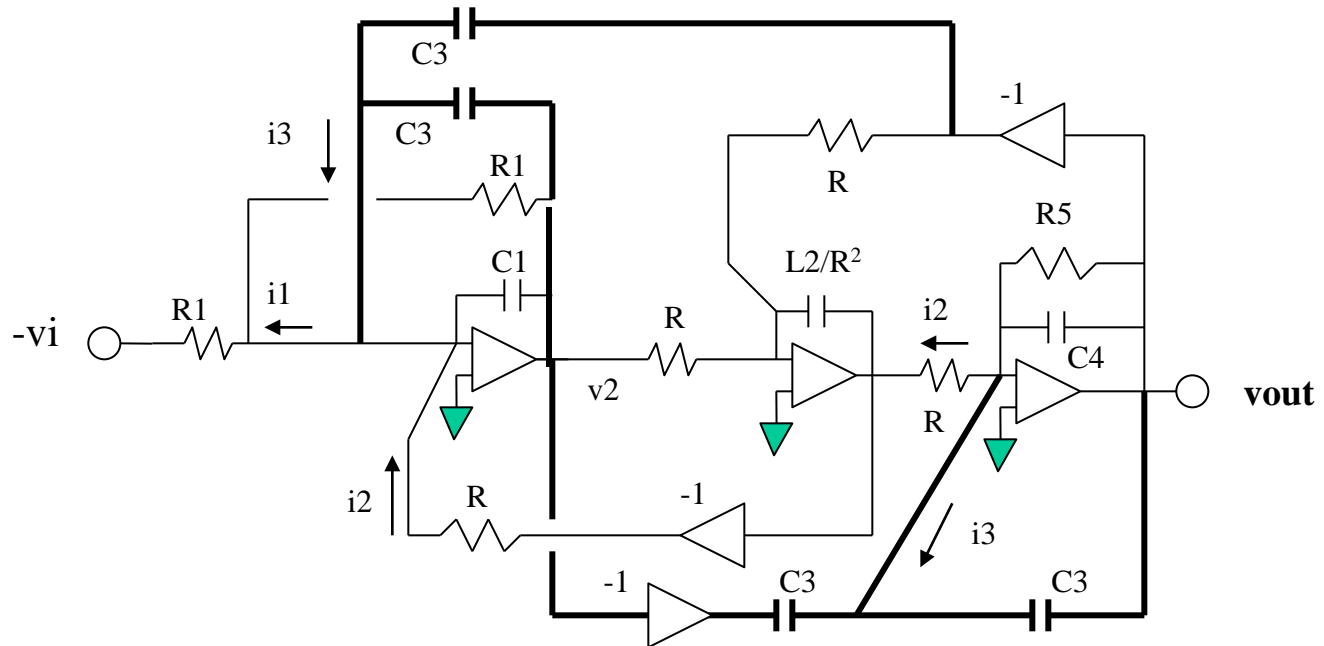


Fifth-Order OTA-C Elliptic Filter

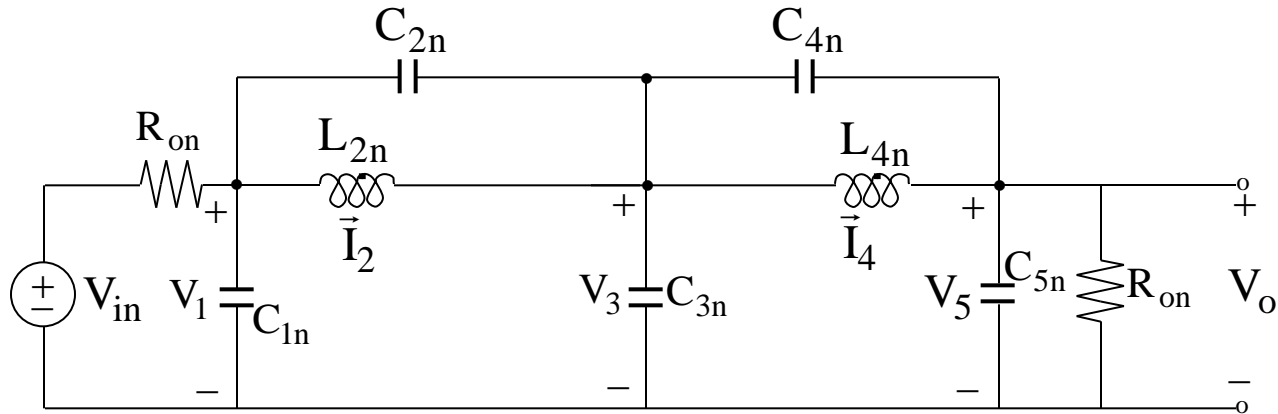
## Simulating Floating Capacitors (Elliptic Filters)



$$i_3 = (v_2)sC_3 - (v_{out})sC_3$$



# Finite $j\omega$ Zeros: An Alternative Approach



$$sC_{1n} V_1 = \frac{V_{in}}{R_{on}} - I_2 - \frac{V_1}{R_{on}} - sC_{2n} (V_1 - V_3) \quad (1)$$

$$V_2' = I_2 R \quad ; \quad I_2 = \frac{V_2'}{R}$$

$$sC_{1n} V_1 = \frac{V_{in}}{R_{on}} - \frac{V_2'}{R} - \frac{V_1}{R_{on}} - sC_{2n} (V_1 - V_3) \quad (1')$$

$$sL_{2n} \frac{V_2'}{R} = V_1 - V_3 \quad (2)$$

$$sC_{3n} V_3 = \frac{V_2'}{R} - \frac{V_4'}{R} + sC_{2n} (V_1 - V_3) - sC_{4n} (V_3 - V_5) \quad (3)$$

$$s \frac{L_{4n}}{R} V_4' = V_3 - V_5 \quad (4)$$

$$sC_{5n} V_5 = \frac{V_4'}{R} - \frac{V_5}{R_{6n}} + sC_{4n} (V_3 - V_5) \quad (5)$$

## Component Calculations for an OTA-C implementation

$$\frac{C_1}{g_{m1}} = C_{1n} R_{on}$$

$$R = R_{on}$$

$$g_{m2} = \frac{1}{R_{on}}$$

$$\frac{C_6}{g_{m3}} = \frac{L_{2n}}{R}$$

$$\frac{C_3}{g_{m4}} = RC_{3n}$$

$$\frac{C_7}{g_{m5}} = \frac{L_{4n}}{R}$$

$$g_{m6} = \frac{1}{R}$$

$$g_{m7} = \frac{1}{R_{6n}}$$

Note that if  $R_{6n} = R$  then the OTA # 7 can be eliminated and the output of OTA # 6 can be simply connected to its negative terminal.