Nonlinear Control Techniques for Low-Voltage, Low-Power Applications:

Class D Audio Amplifiers

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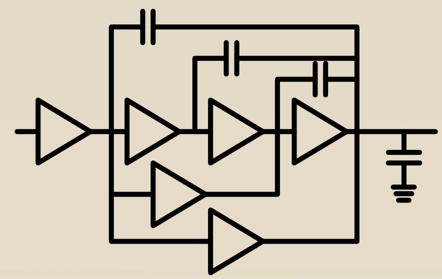

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1. Introduction to audio amplifiers

In a sound system, the power amplifier supplies power to the loudspeaker

The typical speaker input impedance is low, usually in the 4 Ω to 8 Ω range. Thus, the power amplifier must be able to supply the high peak currents required to drive the low impedance.

Standard audible frequency band is from 20 Hz to 20 KHz

For high-fidelity sound system, THD must be < 0.1 %

Least detectable amount of harmonic distortion by humans is ~ 0.3 % in average

Telephone: THD ~ 10 % BW ~ 4 KHz

Main power amplifier classes

Linear

Class A, B, AB

Nonlinear

Class D

 ← Linearity → 

 ← Efficiency → 

Power amplifier classes

Class A amplifier

- Current flows continuously
- High sound quality
- Poor efficiency (25 %)

Class B amplifier

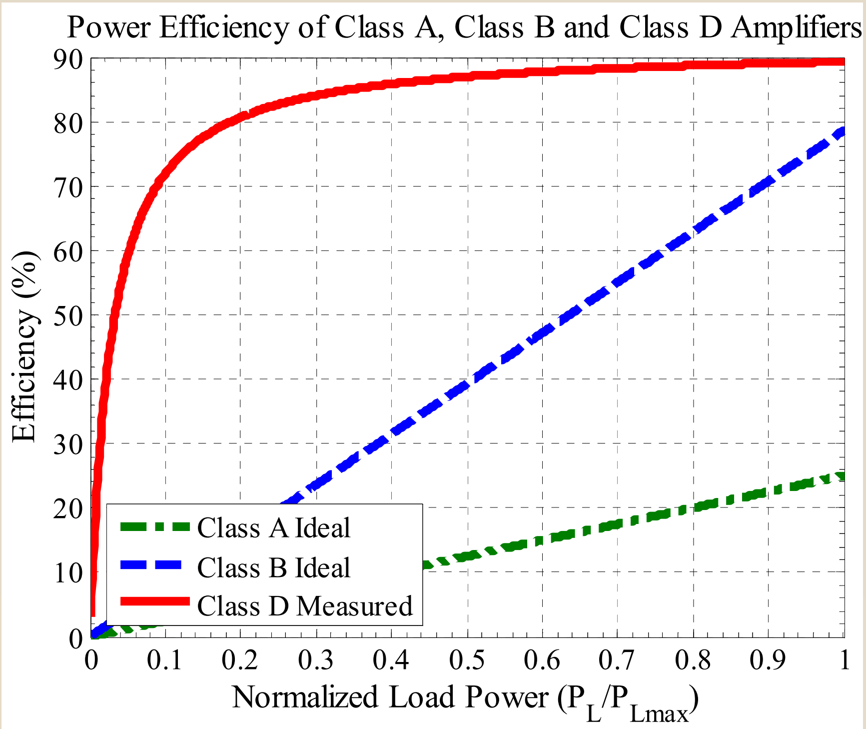
- Current flows half of the period
- Linearity compromised by crossover
- Higher efficiency (78.5 %)

Class AB amplifier

- Hybrid between classes A and B
- Good sound quality
- Higher efficiency (78.5 %)

Class D amplifier

- Switching amplifier
- Ideal THD 0%
- Highest efficiency (100 %)



2. Conventional class D audio amplifier

Continuously switch the output from one rail to another at supersonic frequency (Pulse Width Modulation -PWM-)

There are two main areas of application for class D amplifiers:

1. **Low Power Outputs**
   - From few milliwats to around 5 W
   - Hearing aids, mobile phones, personal stereos, laptop computer audio, etc.
   - Portable products, battery driven
   - High efficiency required
2. **High Power Outputs**
   - From 80 W to 1400 W
   - Home theatre systems, car audio systems, etc.
   - Keeps dissipation and heat sink size minimum

2. Conventional class D audio amplifier

History

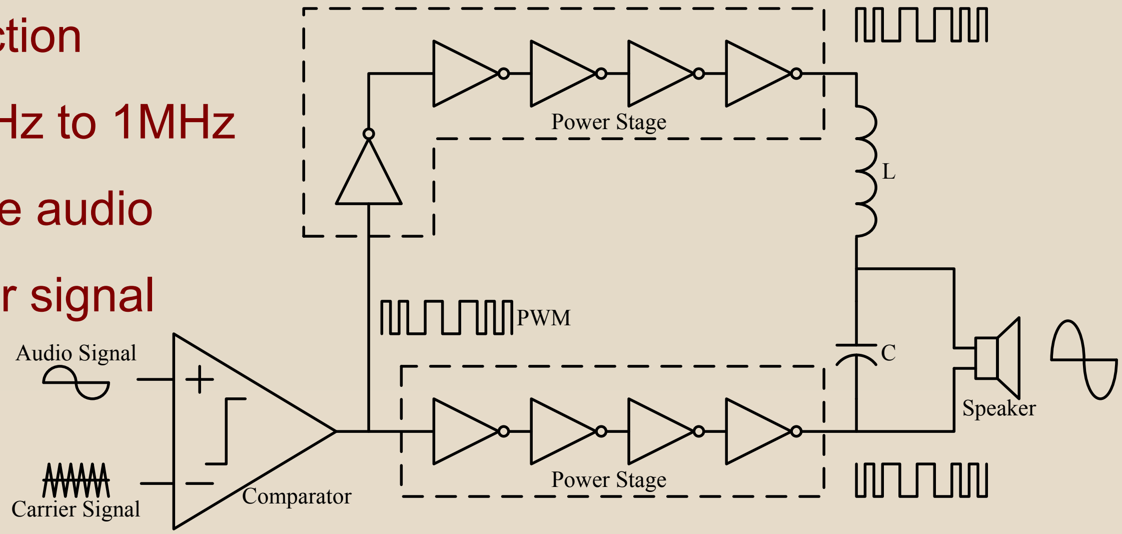
- First proposed around 1950's
- 5 % THD in 1976 state of the art amplifier

Basic Principles

- No output devices operating in the linear mode
- Output stage switches at supersonic frequency
- There is no inherent supply rejection
- Switching frequencies from 50KHz to 1MHz
- PWM is generated comparing the audio signal with a triangle wave carrier signal
- Triangle wave needs to be linear to prevent distortion

Efficiency

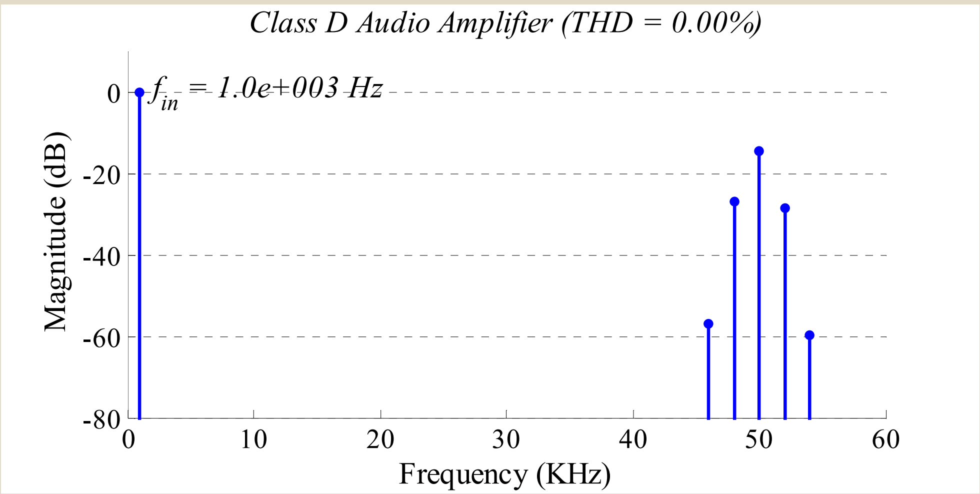
- 100 % at all output levels ideally
- Between 80% and 90% due to parasitic losses in practice



2. Conventional class D audio amplifier

Pulse Width Modulation (PWM)

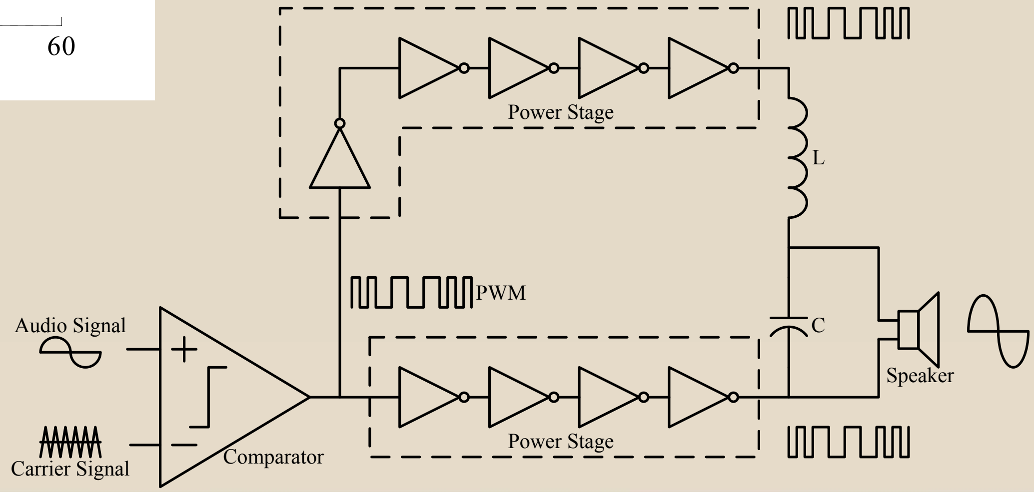
Varies the duty cycle of the converter switches at a high switching frequency (supersonic) to achieve a target average low frequency output voltage.



- Class D amplifier THD is calculated considering all harmonics below 20KHz (max. audible frequency)
- Ideal class D amplifier THD is 0.0 %
- Performance is degraded due to system nonlinearities

Nonlinearity sources

- Passive components nonlinearities
- MOS on-resistance
- Non-ideal triangle carrier signal

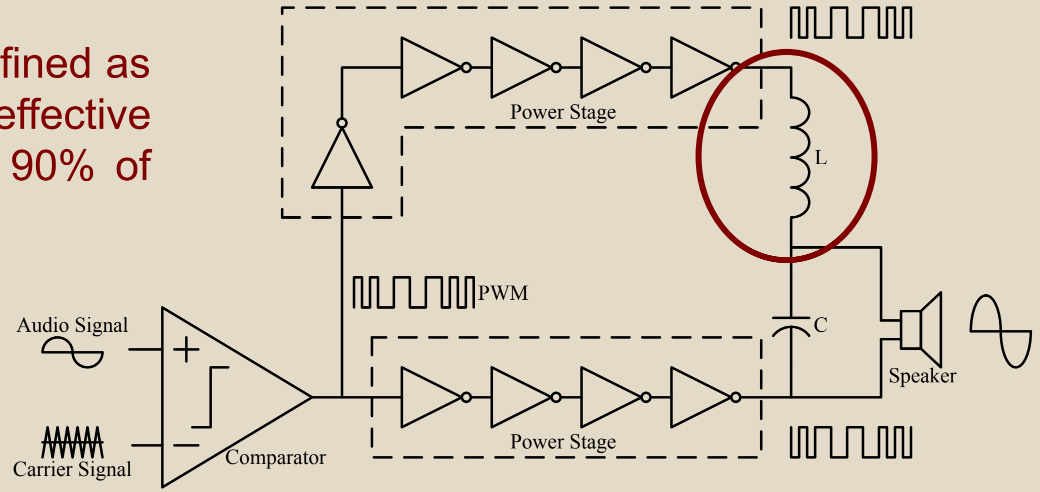


2. Conventional class D audio amplifier

Passive components nonlinearities (†)

The low saturation current of the load inductor causes frequency dependent non-linearity.

Saturation current of inductor is defined as the I_{DC} current level where the effective inductance value is decreased to 90% of its value at zero DC current.



An approximate expression of nonlinear inductance is obtained by series representation

$$THD(\%) = 100\frac{V_{third\ harmonic}}{V_{fundamental}}$$

$$THD(\%) = 100\frac{3 \cdot L \cdot V^2 \cdot 0.1 \cdot 2\pi \cdot f_{in}}{4 \cdot R^3 \cdot I_{sat}^{\ 2}}$$

where L, V, fin, R and I_{sat} are the inductance at zero DC current, the output voltage, the input signal frequency, the speaker resistance and the inductor saturation current respectively

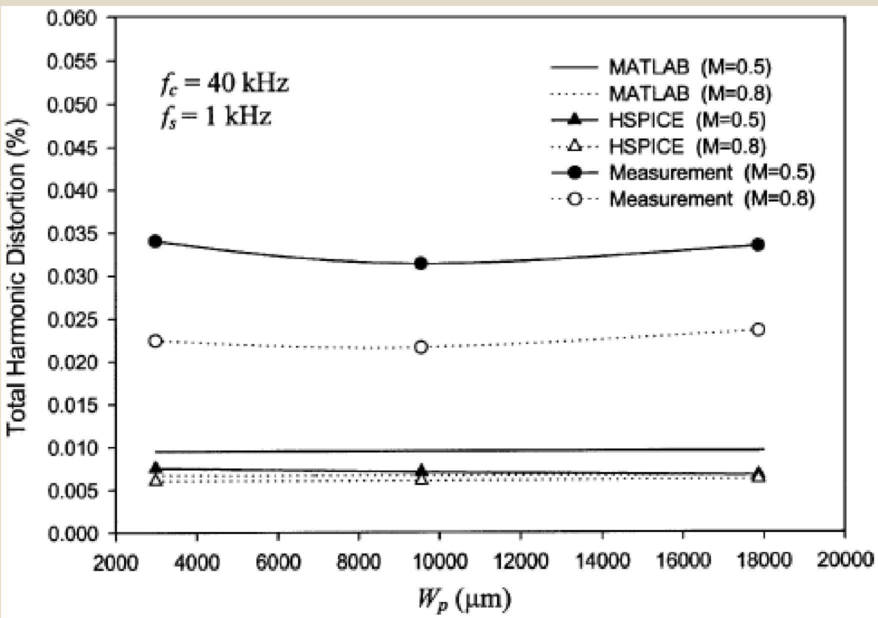
(†) B. Kelleci, E. Sanchez-Sinencio, and A. Karsilayan, "THD+N Estimation in Class-D Amplifiers", IEEE ISCAS, pp. 465-468, 2007

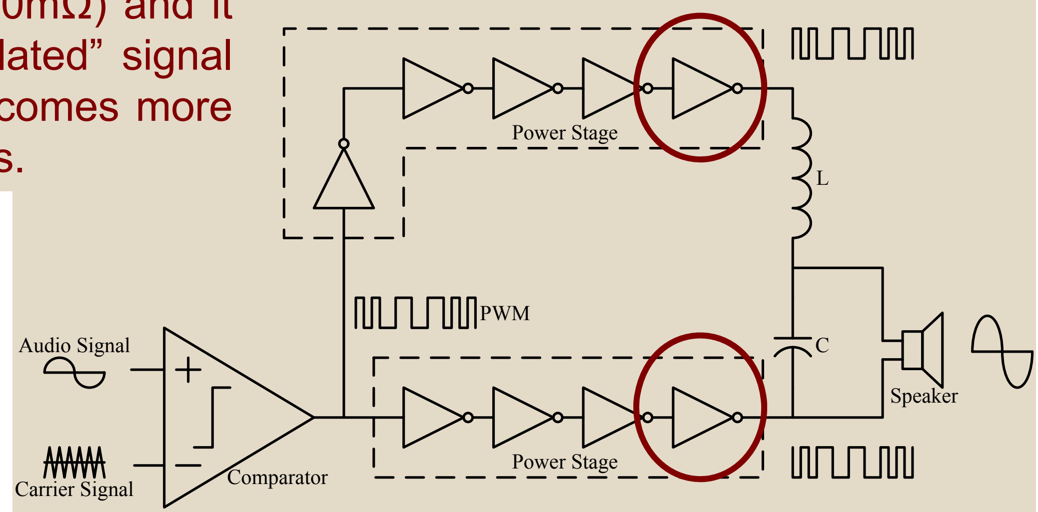
2. Conventional class D audio amplifier

MOS on-resistance (†)

The class D amplifier is more power efficient (ideally 100% efficiency) than linear amplifiers because its output PWM is switch mode.

The on-resistance is not negligible (~200mΩ) and it will result in a slightly "amplitude modulated" signal at the PWM output. This modulation becomes more pronounced at higher modulation indexes.





where f_c, f_s, M and W_p are the carrier signal frequency, the input signal frequency, the modulation index and the output transistor size respectively.

(†) M. Tan, et al, "An investigation Into The Parameters Affecting THD in Low-Voltage Low-Power Class-D Amplifiers", IEEE TCAS I , Vol. 50, No. 10, pp. 1304-1315, 2003

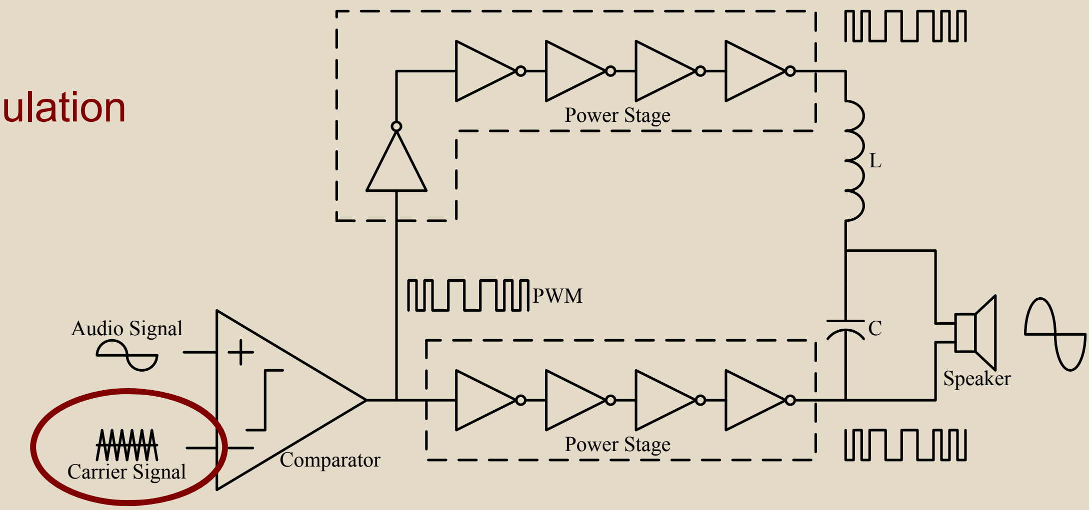
2. Conventional class D audio amplifier

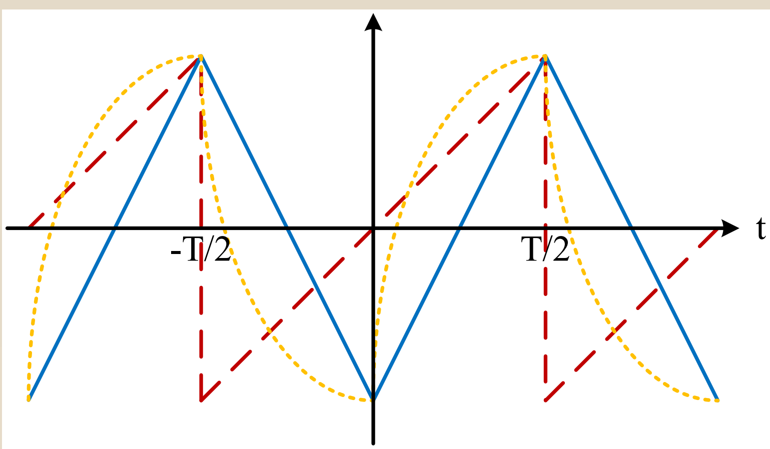
Carrier signal nonlinearity

Carrier signal non-idealities will affect directly the linearity performance in the class D audio amplifier.

There are three main carrier signal modulation schemes:

- Triangle wave modulation
- Sawtooth wave modulation
- Exponential wave modulation





- Harmonic frequency calculation of PWM is complex and is typically done using an FFT analysis of a simulated waveform but it usually leads to errors and miscalculations
- Analytical calculations of harmonic components is usually done by using a Double Fourier Integral Analysis (DFIA). Mathematical expression is quite complex but accurate.

2. Conventional class D audio amplifier

Carrier signal nonlinearity

- A novel mathematical analysis method to model the carrier waveform has been proposed in (†)
    - Assume a exponential carrier signal instead of a triangle wave which may be generated by charging/discharging an RC integrator circuit with square pulses.
    - Shift the nonlinearity of the exponential carrier to the input modulating signal and then apply the Double Fourier Integral Analysis
        1. Remove the nonlinearity of the trailing-edge exponential carrier by transform it to a linearized exponential carrier (linear sawtooth carrier)
        2. Transform the initially-linear modulating signal (audio signal) to a transformed (nonlinear) modulating signal
        3. Repeat (1) and (2) for the leading-edge exponential carrier.
        4. Derive the Double Fourier coefficients of the double-sided PWM output by summing the Fourier coefficients of the trailing-edge and leading-edge PWM outputs.

This mathematical analysis is accurate but its complexity and procedure are extensive

(†) M. Tan, et al, "An investigation Into The Parameters Affecting THD in Low-Voltage Low-Power Class-D Amplifiers", IEEE TCAS I , Vol. 50, No. 10, pp. 1304-1315, 2003

2. Conventional class D audio amplifier

Carrier signal nonlinearity

• We propose to use a simpler method to analyze carrier signal nonlinearity based on the PWM Analysis by Duty Cycle Variation for any kind of periodic carrier signal in the following presentation

Overview of Double Fourier Integral Analysis of a PWM Waveform

• Fourier decomposition is based on the principle that any regular time-varying waveform *f(t)* can be expressed as an infinite series of sinusoidal harmonics:

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos m\omega t + b_m \sin m\omega t \right)$$

where

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos m\omega t \, d(\omega t)$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin m\omega t \, d(\omega t)$$

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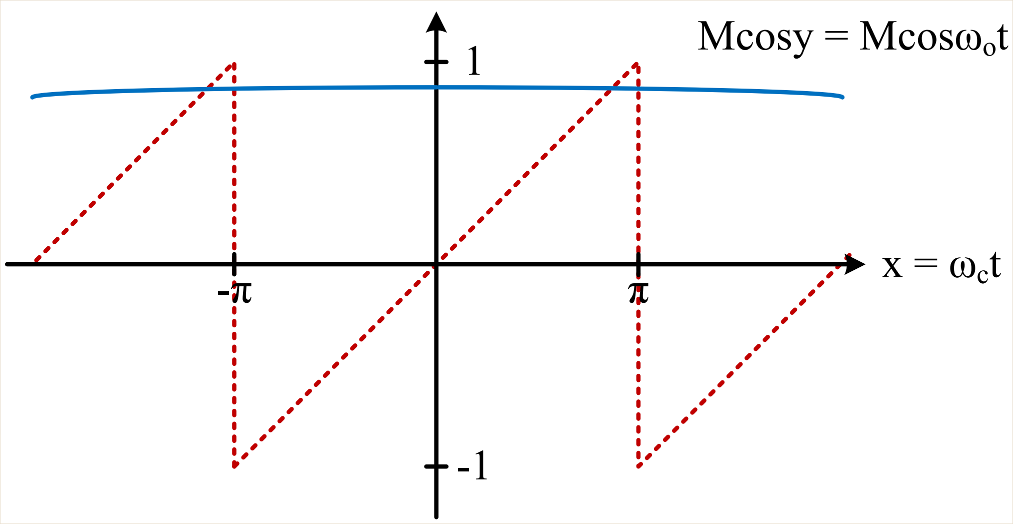
The analytical solution for the harmonic components of a PWM waveform assumes the existence of two time variables

$$x(t) = \omega_c t$$
$$y(t) = \omega_o t$$

where ω_c and ω_o are the carrier angular frequency and the input signal (audio wave) angular frequency respectively.

The objective is to find a function f(t) which describes the PWM signal as a periodic function of x and y by using Double Fourier Integral Analysis (†)



The purpose of the Double Fourier Integral Analysis is to express the PWM waveform as a function of a double variable controlled waveform.

(†) H. S. Black, *Modulation Theory*, Princeton, NJ, Van Nostrand, 1953

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In general, any double-variable time-varying function *f(t)* can be expressed, by using the Double Fourier Integral Analysis, in the following form (†)

$$f(t)=\frac{A_{00}}{2}+\sum_{n=1}^{\infty}\left(A_{0n}\cos(n\omega_o t)+B_{0n}\sin(n\omega_o t)\right)+\sum_{m=1}^{\infty}\left(A_{m0}\cos(m\omega_c t)+B_{m0}\sin(m\omega_0 t)\right)$$

$$+\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\left(A_{mn}\cos(m\omega_c t+n\omega_o t)+B_{mn}\sin(m\omega_c t+n\omega_o t)\right)$$

where *m* is the carrier index variable and *n* is the baseband index variable

- The variables *m* and *n* define the angular frequency of each harmonic component
- The magnitudes of the harmonics components are the A_{mn} and B_{mn} coefficients

This function *f(t)* will provide a exact solution to determine the harmonic components of a PWM opposed to the traditional method of computing an FFT of the waveform, which will always be sensitive to the time resolution of the simulation and the periodicity of the overall waveform.

(†) D. G. Holmes and T. A. Lipo, *PWM For Power Converters*, Wiley Inter-science, USA, 2003

2. Conventional class D audio amplifier

Analyzing the function *f(t)* we have that

$$\frac{A_{00}}{2}$$ where *m* = *n* = *0,* corresponds to the DC offset component of the PWM (if any)

$$\sum_{n=1}^{\infty}\left(A_{0n}\cos(n\omega_o t)+B_{0n}\sin(n\omega_o t)\right)$$ where *m* = *0*, represents the fundamental component and baseband harmonics (if any)

$$\sum_{m=1}^{\infty}\left(A_{m0}\cos(m\omega_c t)+B_{m0}\sin(m\omega_0 t)\right)$$ where *n* = *0*, defines the carrier wave harmonics (high-frequency components)

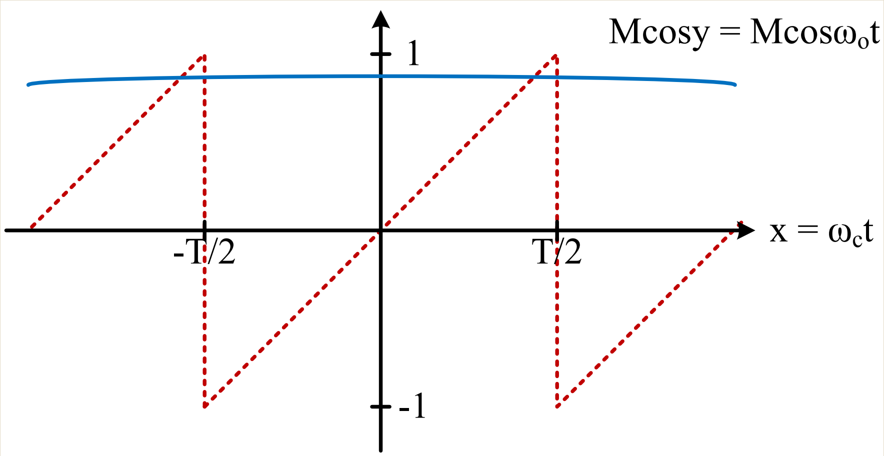
$$\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\left(A_{mn}\cos(m\omega_c t+n\omega_o t)+B_{mn}\sin(m\omega_c t+n\omega_o t)\right)$$ where *m, n ≠ 0*, represents the sideband harmonics

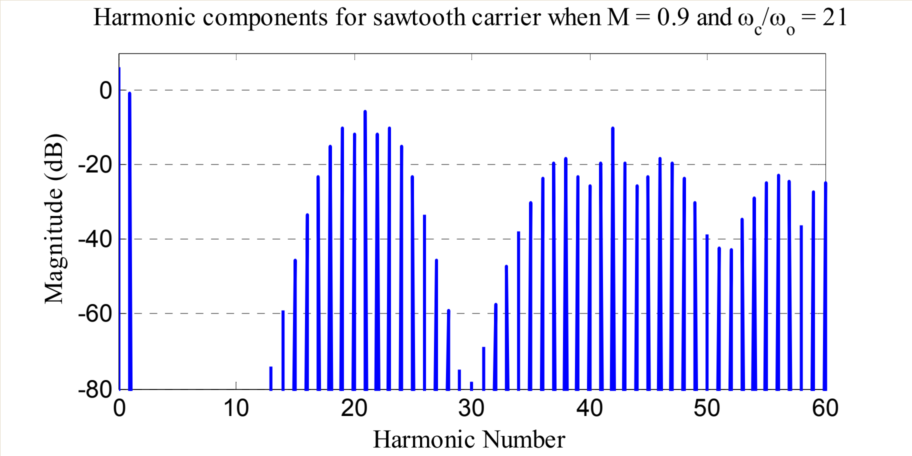
• The different summation terms in the PWM function *f(t)* will depend on the type of carrier wave

• In some cases, it will be easier to express the summations using the Bessel function of the first kind (*J*)

2. Conventional class D audio amplifier

Example 1: Sine-Sawtooth Modulation





The PWM function $v_{an}(t)$ for a sawtooth carrier signal expressed in terms of its harmonics components is

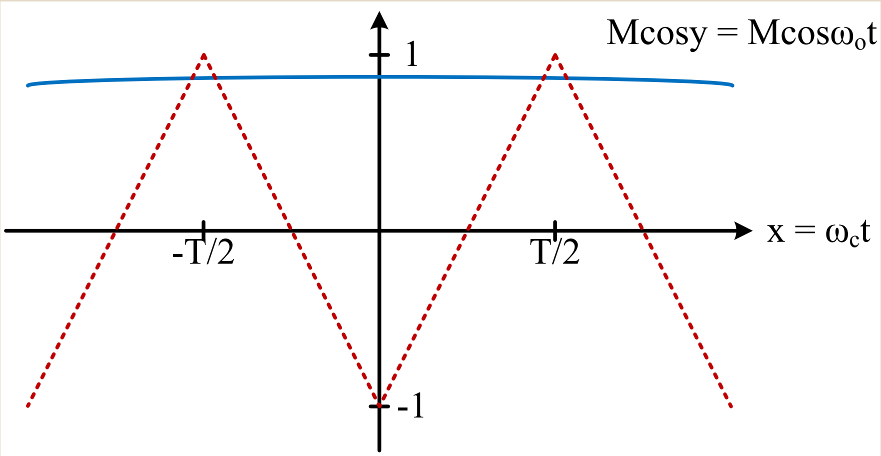
$$v_{an}(t) = V_{DC} + V_{DC} M \cos(\omega_o t) + 2\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\left[\cos m\pi - J_0(m\pi M)\right]\sin m\omega_c t$$

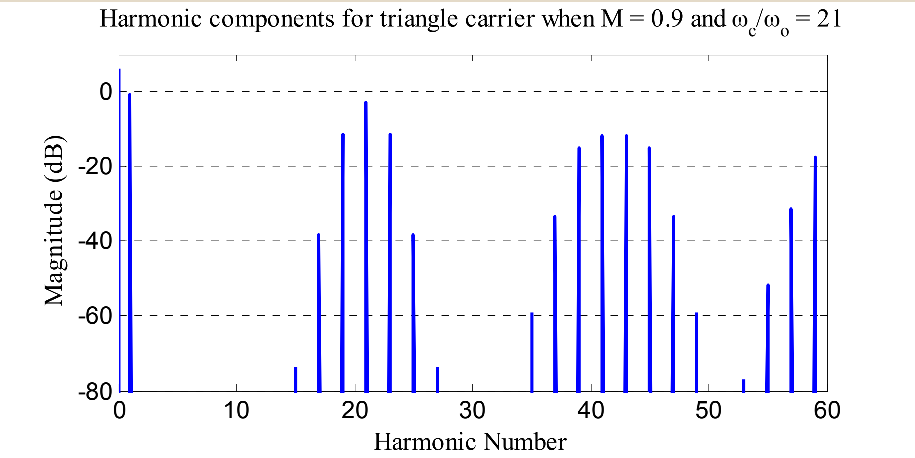
$$+2\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\frac{1}{m}J_n(m\pi M)\left[\sin n\frac{\pi}{2}\cos(m\omega_c t + n\omega_o t) - \cos n\frac{\pi}{2}\sin(m\omega_c t + n\omega_o t)\right]$$

As we expected, the THD of a class D audio amplifier will be 0.0 % since there are no baseband harmonics generated as we only have the fundamental tone.

2. Conventional class D audio amplifier

Example 2: Sine-Triangle Modulation





The PWM function $v_{an}(t)$ for a triangle carrier signal expressed in terms of its harmonics components is

$$v_{an}(t) = V_{DC} + V_{DC} M\cos(\omega_o t) + 4\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(m\frac{\pi}{2}M\right)\sin m\frac{\pi}{2}\cos m\omega_c t$$

$$+4\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\frac{1}{m}J_n\left(m\frac{\pi}{2}M\right)\sin\left([m+n]\frac{\pi}{2}\right)\cos(m\omega_c t + n\omega_o t)$$

As in the previous case, the THD of a class D audio amplifier will be 0.0 % since there are no baseband harmonics generated as we only have the fundamental tone.

2. Conventional class D audio amplifier

Carrier signal nonlinearity

- Previous examples demonstrated that class D audio amplifier provides 0.0% THD ideally
- In reality, THD > 0.0% because the carrier signals are not ideal
- In fact, for an ideal triangle wave carrier signal *f(x)* we would have

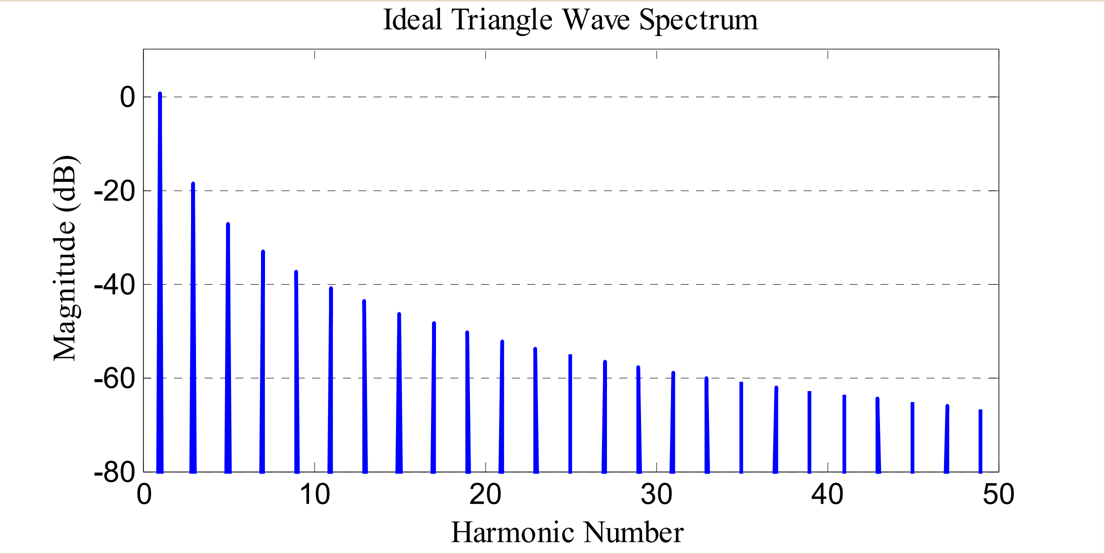
$$f(x)=\frac{8}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{(-1)^{(n-1)/2}}{n^2}\sin\left(\frac{2n\pi x}{T}\right)$$

- And, for an ideal sawtooth wave carrier *f(y)* we would have

$$f(y)=\frac{1}{2}-\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{2n\pi x}{T}\right)$$

We would need an infinite bandwidth system to generate a perfect carrier signal!

Unfortunately, band-limited systems degrade the performance of the overall class D amplifier generating undesired baseband components



2. Conventional class D audio amplifier

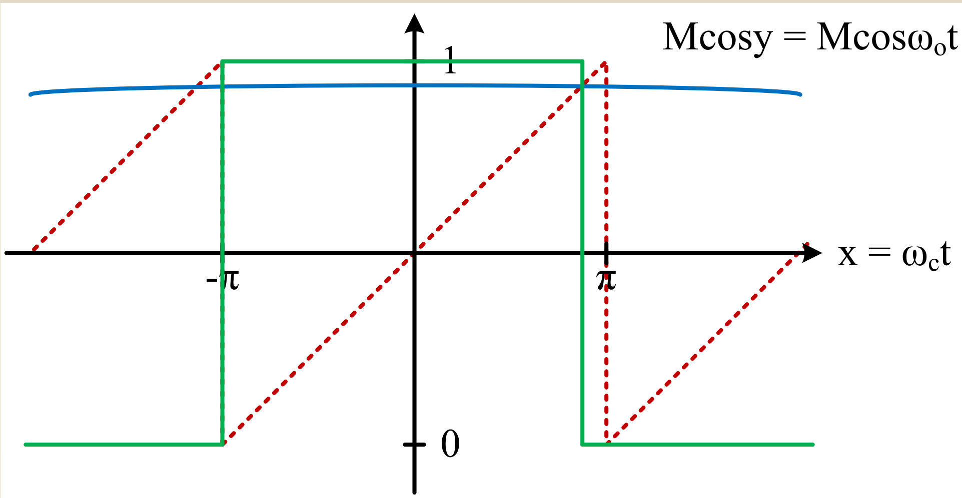
Carrier signal nonlinearity

• Double Fourier Integral Analysis is a complex and tedious mathematical derivation

• Instead, we can use the PWM Analysis by Duty Cycle variation if the input signal (audio wave) is assumed to be constant within each carrier cycle, i.e. $\omega_c >> \omega_o$, which is usually the case.

Example 1a: Sine-Sawtooth Modulation

• Normalizing the period of the sawtooth to 2π and its amplitude to 1, we have



$$v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left(a_m \cos mx + b_m \sin mx\right)$$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\cos mx \, dx$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\sin mx \, dx$$

• We need to calculate the interval where $v_{an}(t)$ switches from 0 to 1

2. Conventional class D audio amplifier

Example 1a: Sine-Sawtooth Modulation (cont.)

- For a_m coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\cos mx\, dx = 2\frac{V_{DC}}{\pi}\int_{-\pi}^{\pi M\cos y}\cos mx\, dx = 2\frac{V_{DC}}{\pi}\left[\sin(m\pi M\cos y)+\sin m\pi\right]$$

- For b_m coefficients, when $m \neq 0$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\sin mx\, dx = 2\frac{V_{DC}}{\pi}\int_{-\pi}^{\pi M\cos y}\sin mx\, dx = 2\frac{V_{DC}}{\pi}\left[\cos m\pi - \cos(m\pi M\cos y)\right]$$

- When $m = 0$ we have

$$a_0 = 2V_{DC}(1+M\cos y) \qquad b_0 = 0$$

- After some mathematical manipulation and applying the Jacobi-Anger expansions, we get the same expression obtained by using the Double Fourier Integral Analysis

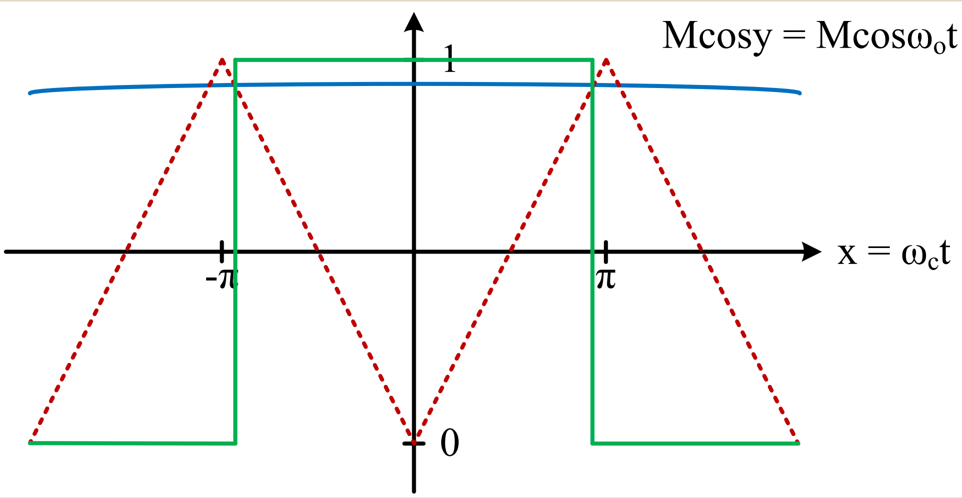
$$v_{an}(t) = V_{DC} + V_{DC}M\cos(\omega_o t) + 2\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\left[\cos m\pi - J_0(m\pi M)\right]\sin m\omega_c t$$

$$+2\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\frac{1}{m}J_n(m\pi M)\left[\sin n\frac{\pi}{2}\cos(m\omega_c t + n\omega_o t) - \cos n\frac{\pi}{2}\sin(m\omega_c t + n\omega_o t)\right]$$

2. Conventional class D audio amplifier

Example 2a: Sine-Triangle Modulation

- Normalizing the period of the sawtooth to 2π and its amplitude to 1, we have



$$v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos mx + b_m \sin mx\right)$$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx$$

- We need to calculate the interval where $v_{an}(t)$ switches from 0 to 1

- For a_m coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx = 2\frac{V_{DC}}{\pi} \int_{-\frac{\pi}{2}(1+M\cos y)}^{\frac{\pi}{2}(1+M\cos y)} \cos mx \, dx = 4\frac{V_{DC}}{\pi}\left[\sin\left(m\frac{\pi}{2}(1+M\cos y)\right)\right]$$

2. Conventional class D audio amplifier

Example 2a: Sine-Triangle Modulation

• For b_m coefficients, since the triangle wave is an even function

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\sin mx\,dx = 2\frac{V_{DC}}{\pi}\int_{-\frac{\pi}{2}(1+M\cos y)}^{\frac{\pi}{2}(1+M\cos y)} \sin mx\,dx = 0$$

• When $m= 0$ we have

$$a_0 = 2V_{DC}(1+M\cos y)$$

• After some mathematical manipulation, as well as in the previous example, and applying the Jacobi-Anger expansions, we get the same expression obtained by using the Double Fourier Integral Analysis

$$v_{an}(t) = V_{DC} + V_{DC}M\cos(\omega_o t) + 4\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}J_0\left(m\frac{\pi}{2}M\right)\sin m\frac{\pi}{2}\cos m\omega_c t$$

$$+4\frac{V_{DC}}{\pi}\sum_{m=1}^{\infty}\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\frac{1}{m}J_n\left(m\frac{\pi}{2}M\right)\sin\left([m+n]\frac{\pi}{2}\right)\cos(m\omega_c t + n\omega_o t)$$

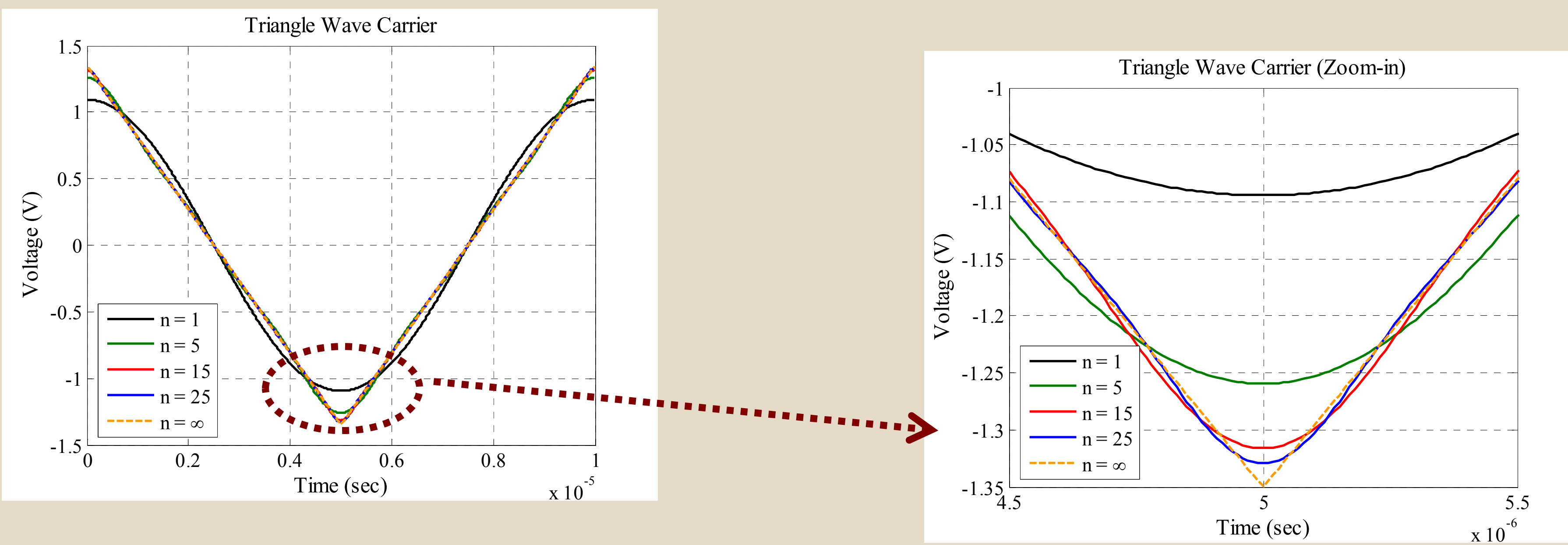
2. Conventional class D audio amplifier

Carrier signal nonlinearity

• *<u>We propose to generalize the PWM Analysis by Duty Cycle variation and apply it to a any carrier waveform to calculate analytically the THD in a class D audio amplifier</u>*

Example 3: Sine-Non-Ideal Triangle Modulation

• Recall that a triangle wave carrier signal is constructed by an infinite sum of sinusoidal functions and since there are not unlimited bandwidth systems, the number of harmonics (n) in a triangle wave carrier signal is finite



2. Conventional class D audio amplifier

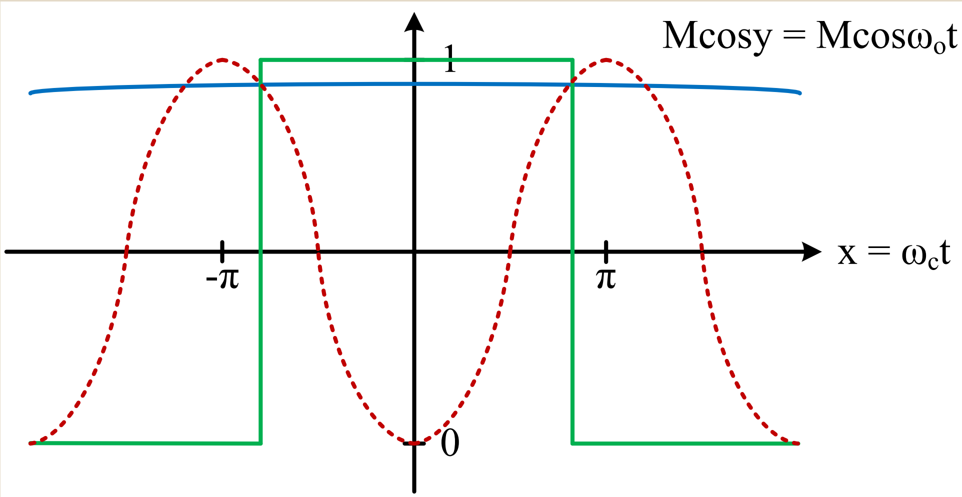
Sine-Non-Ideal Triangle Modulation

- There are two trivial cases in a triangle wave shaped carrier signal

1. When the number of harmonics is infinite we have an ideal triangle wave carrier signal and the THD of the class D amplifier is 0.0%
2. When the number of harmonics is 1 then we have a sine-cosine modulation PWM and the THD of the class D amplifier will depend on the modulation index (M)

- Lets examine the case where the non-ideal triangle wave carrier signal contains one single harmonic component

Example 3: Sine-Cosine Modulation



$$v_{an}(t)=\frac{a_0}{2}+\sum_{m=1}^{\infty}\left(a_m\cos mx+b_m\sin mx\right)$$

$$a_m=\frac{1}{\pi}\int_{-\pi}^{\pi}v_{an}(t)\cos mx\,dx$$

$$b_m=\frac{1}{\pi}\int_{-\pi}^{\pi}v_{an}(t)\sin mx\,dx$$

2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

- For a_m coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\cos mx\,dx = 2\frac{V_{DC}}{\pi}\int_{-\arccos\left(-\frac{\pi^2}{8}M\cos y\right)}^{\arccos\left(-\frac{\pi^2}{8}M\cos y\right)} \cos mx\,dx = 4\frac{V_{DC}}{\pi}\left[\sin\left(m\arccos\left(-\frac{\pi^2}{8}M\cos y\right)\right)\right]$$

- For b_m coefficients, since the cosine is an even function

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{an}(t)\sin mx\,dx = 2\frac{V_{DC}}{\pi}\int_{-\arccos\left(-\frac{\pi^2}{8}M\cos y\right)}^{\arccos\left(-\frac{\pi^2}{8}M\cos y\right)} \sin mx\,dx = 0$$

- When $m = 0$ we have

$$a_0 = 4\frac{V_{DC}}{\pi}\arccos\left(-\frac{\pi^2}{8}M\cos y\right)$$

2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

• Finally, the function $v_{an}(t)$ will be given by

$$v_{an}(t) = 2\frac{V_{DC}}{\pi}\arccos\left(\arcsin\left[2\sum_{k=1}^{\infty}\sin\left(k\frac{\pi}{2}\right)J_k\left(-\frac{\pi^2}{8}M\right)\cos(ky)\right]\right)$$
$$+\sum_{m=1}^{\infty}4\frac{V_{DC}}{\pi}\left[\sin\left(m\arccos\left(-\frac{\pi^2}{8}M\cos y\right)\right)\right]\cos mx$$

• We can see that the fundamental component is not alone in the expression but comes with baseband harmonics product of the cosine shaped-carrier waveform. Such baseband harmonics will produce the harmonic distortion in the class D audio amplifier

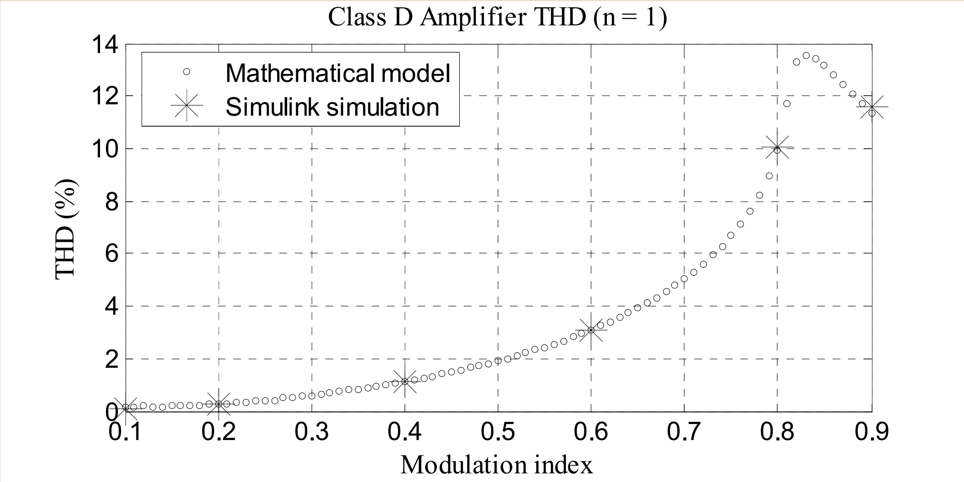
• It can be appreciated that as we increment the modulation index M, the distortion increments exponentially.

• Same procedure can be applied for a given number of harmonics components present in the triangle wave carrier, however, the only closed-form solution exists when n = 1 and n = ∞. The solution when 1 < n < ∞ must be calculated numerically.

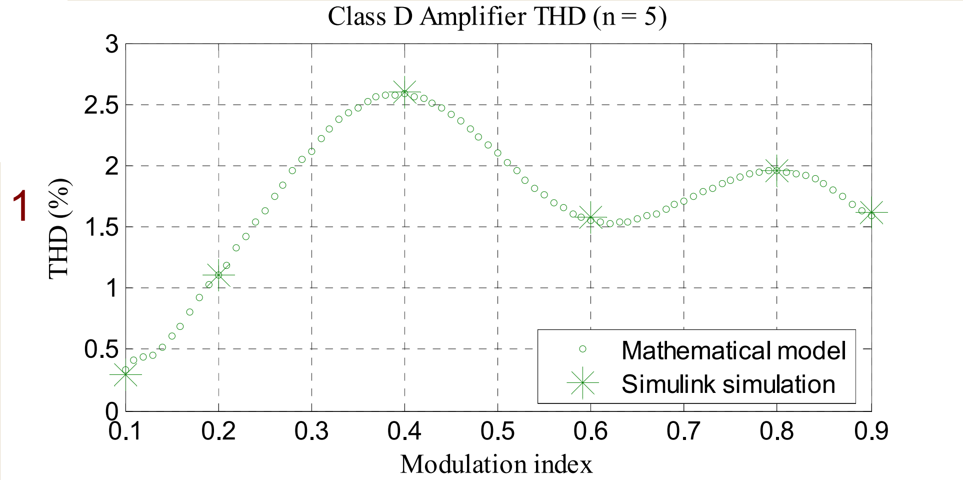
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

• In order to verify the mathematical derivation and its results, we have created a simple SIMULINK model to simulate a class D audio amplifier and compare the traditional FFT method and the analytical solution to find the THD in the amplifier



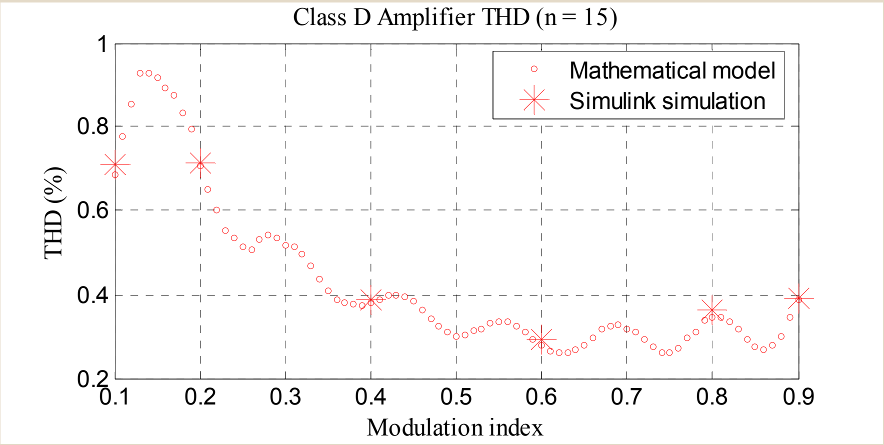
Non-ideal triangle wave carrier signal with n = 1



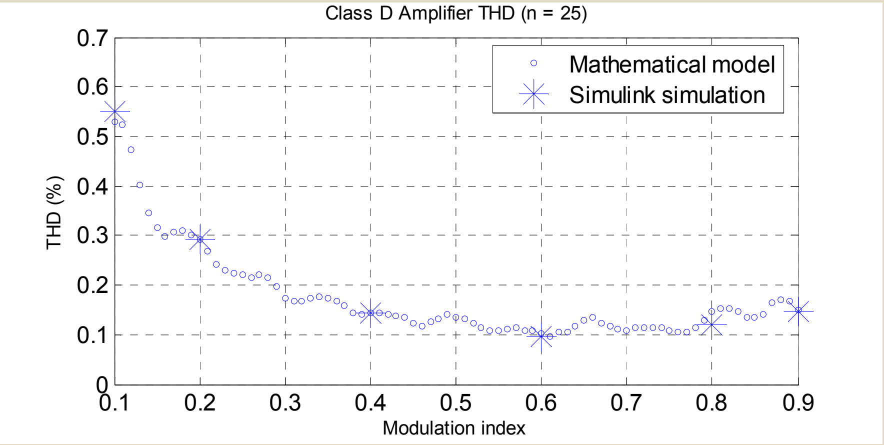
Non-ideal triangle wave carrier signal with n = 5

2. Conventional class D audio amplifier

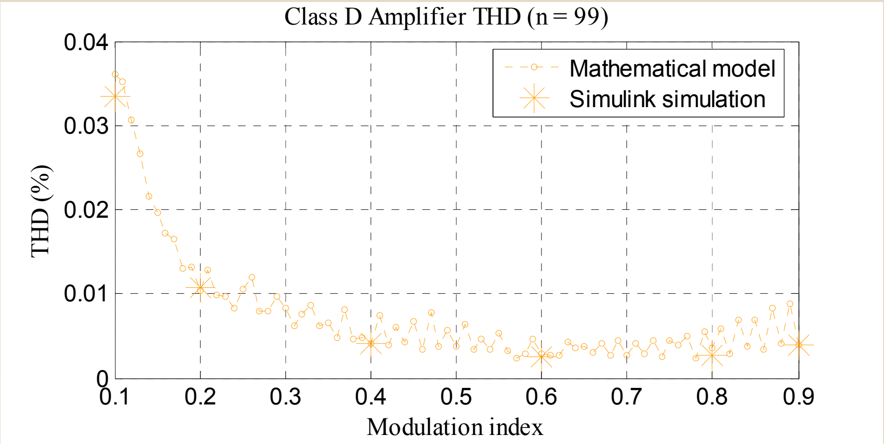
Example 3: Sine-Cosine Modulation (cont.)



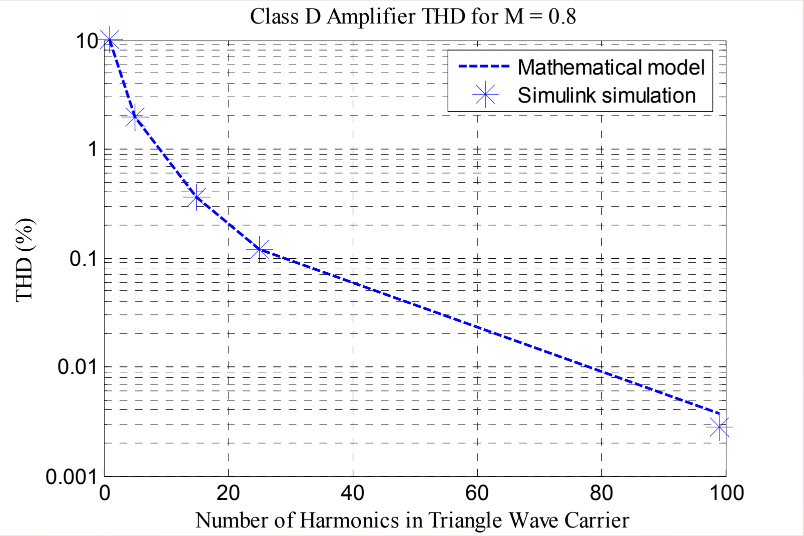
Non-ideal triangle wave carrier signal with n = 15



Non-ideal triangle wave carrier signal with n = 25

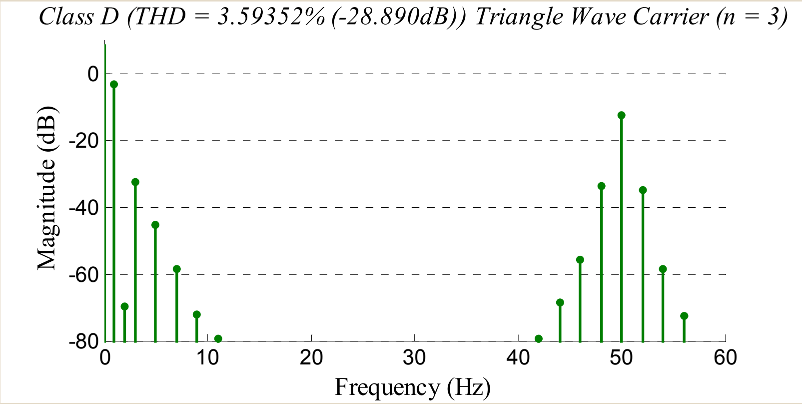


Non-ideal triangle wave carrier signal with n = 99

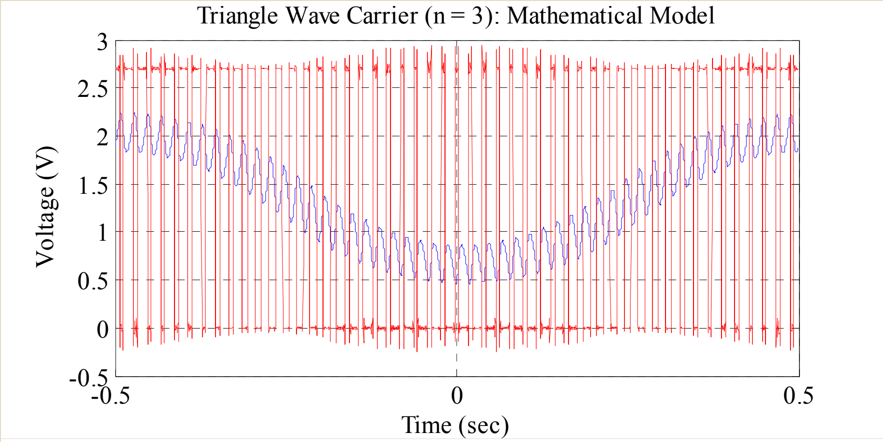


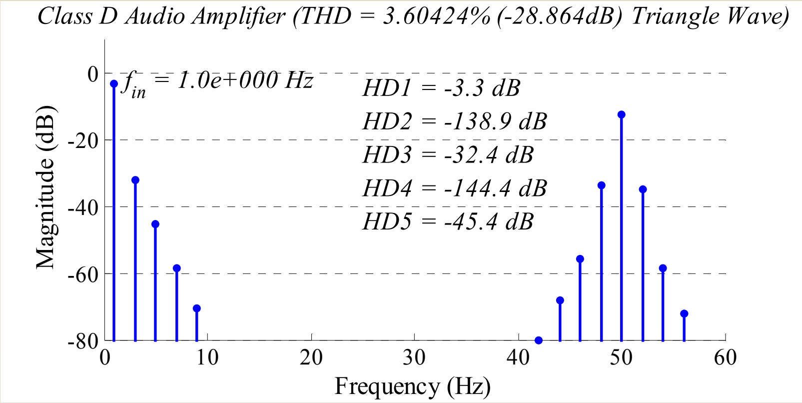
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)



Class D output spectrum when n = 3 (Mathematical Model)





Class D output spectrum when n = 3 (SIMULINK Simulation)

The mathematical model predicts with high accuracy the result in the SIMULINK simulation!

Exercise: Can you provide an analytical expression for the THD output spectrum when a sawtooth carrier signal with only one harmonic is used to PWM an audio signal?

2. Conventional class D audio amplifier

Carrier signal nonlinearity

• Lets now analyze the case when an exponential waveform is used as a carrier signal

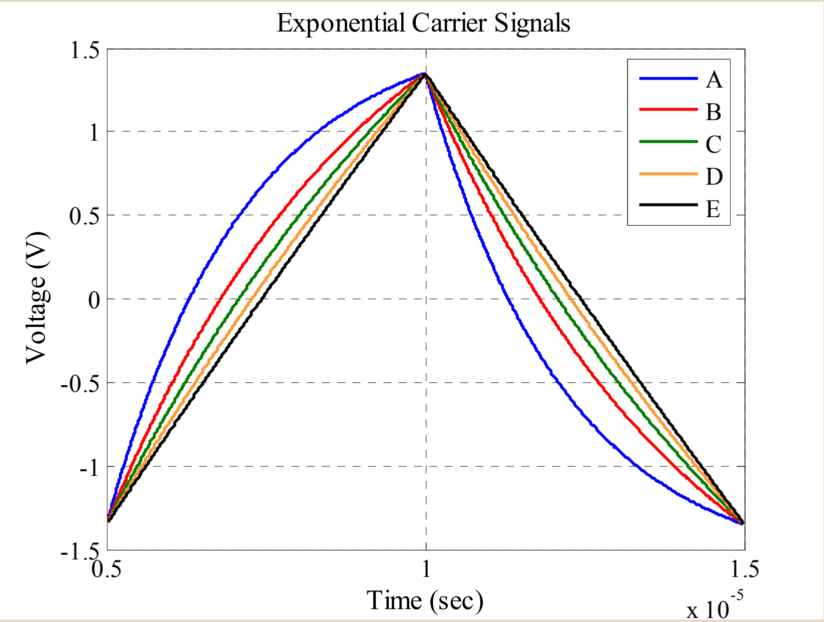
• An exponential waveform is usually employed as a carrier signal due to its simple implementation

Example 4: Sine-Exponential Modulation

• The exponential waveform is generated by charging/discharging a simple RC integrator with square pulses.

Defining a set of normalized exponential waves

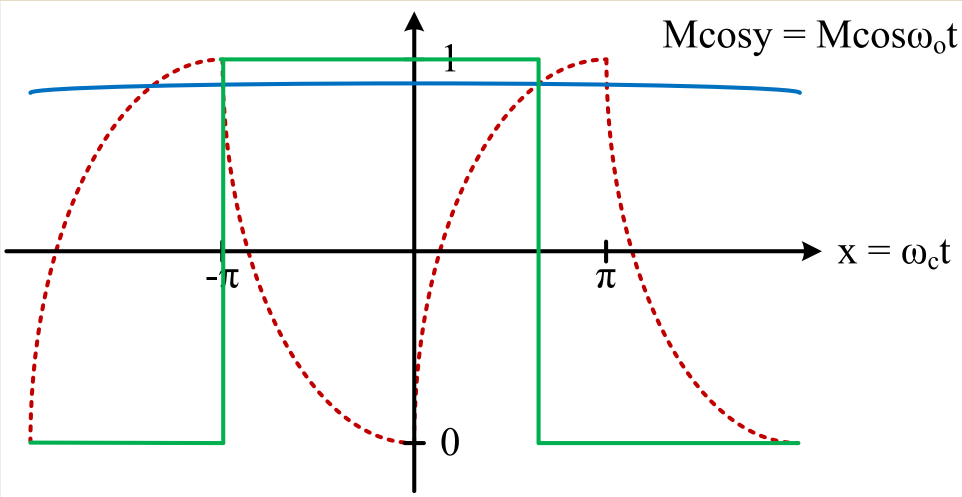
$$f(x)=\begin{cases} V_{DC}\left\{2\dfrac{e^{\left(\frac{-\left(x+\frac{T}{2}\right)}{t_0}\right)}-error}{1-error}-1\right\}, & -\dfrac{T}{2}<x<0 \\ V_{DC}\left\{2\dfrac{\left(1-e^{\left(\frac{-x}{t_0}\right)}\right)}{1-error}-1\right\}, & 0<x<\dfrac{T}{2} \end{cases}$$



2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

Normalizing the exponential carrier to a period of 2π we can calculate the Fourier coefficients based on the PWM Analysis by Duty Cycle Variation



$$v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos mx + b_m \sin mx \right)$$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx$$

2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

- For a_m coefficients, when $m \neq 0$

$$a_m = 2\frac{V_{DC}}{\pi} \int_{-\pi - t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)}^{-t_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)} \cos mx \, dx =$$

$$a_m = 2\frac{V_{DC}}{m\pi}\left[\sin\left(-mt_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)\right) - \sin\left(m\left(-\pi - t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)\right)\right)\right]$$

- For b_m coefficients

$$b_m = 2\frac{V_{DC}}{\pi} \int_{-\pi - t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)}^{-t_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)} \sin mx \, dx =$$

$$b_m = -2\frac{V_{DC}}{m\pi}\left[\cos\left(-mt_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)\right) - \cos\left(m\left(-\pi - t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)\right)\right)\right]$$

2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

• For a_m coefficients, when $m \neq 0$

$$a_0 = 2\frac{V_{DC}}{\pi}\int_{-\pi - t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)}^{-t_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)} dx$$

$$a_0 = 2\frac{V_{DC}}{\pi}\left[-t_0 \ln\left(1-\frac{1}{2}(1-error)\left(\frac{M}{V_{DC}}\cos y+1\right)\right)+\left(\pi + t_0 \ln\left(\frac{M}{2V_{DC}}\cos y(1-error)+\frac{(1+error)}{2}\right)\right)\right]$$

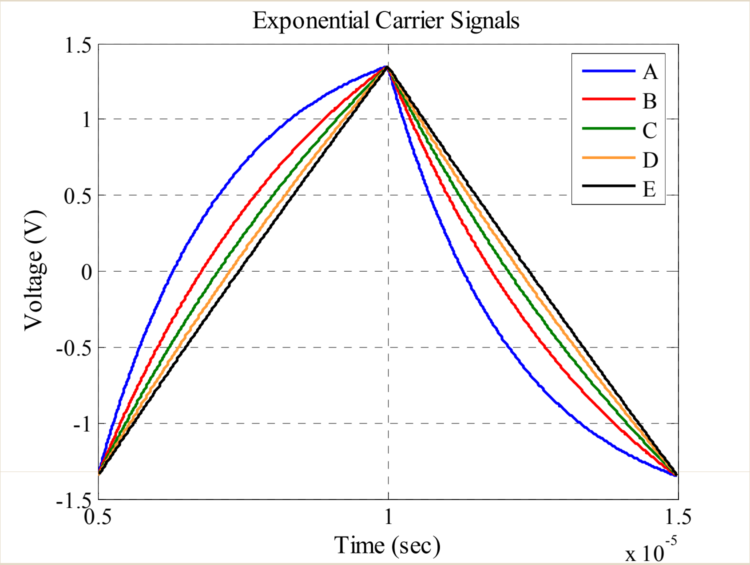
• As in the previous example, the coefficient a_0 will have the fundamental tone as well as baseband harmonics that will degrade the class D audio amplifier THD

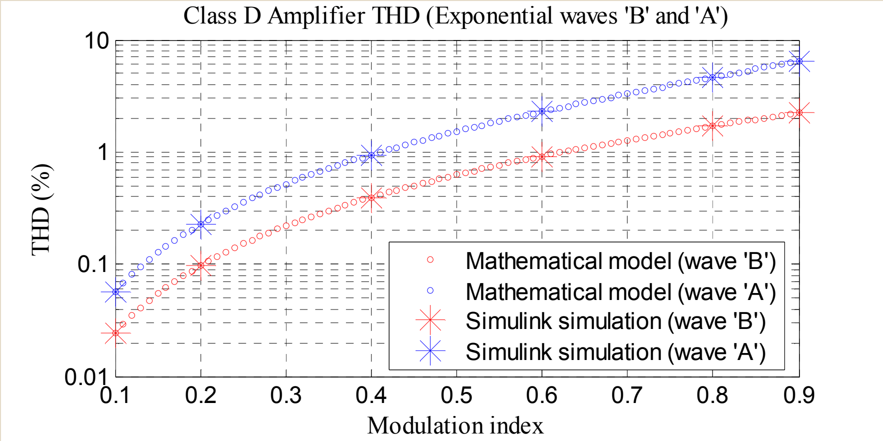
• As the 'error' parameter increases, the exponential wave behaves in a quasi-triangular way and the baseband harmonics magnitude decrease.

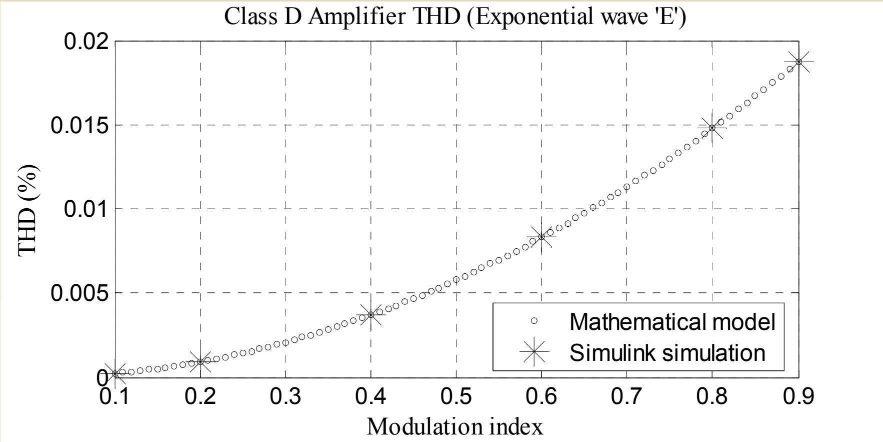
• Like in the previous example, a SIMULINK model was created and simulated in order to compare the results of both procedures.

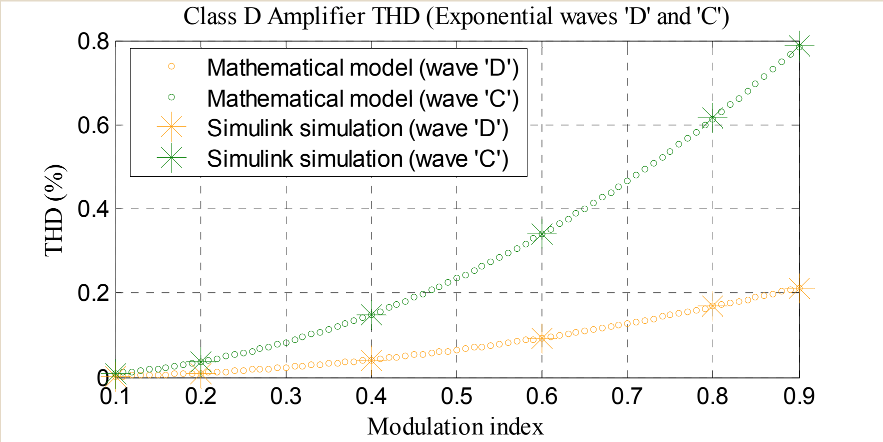
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)



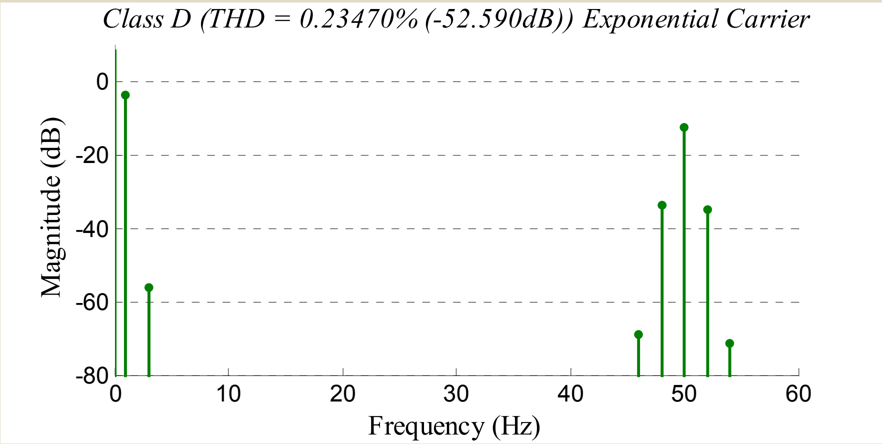




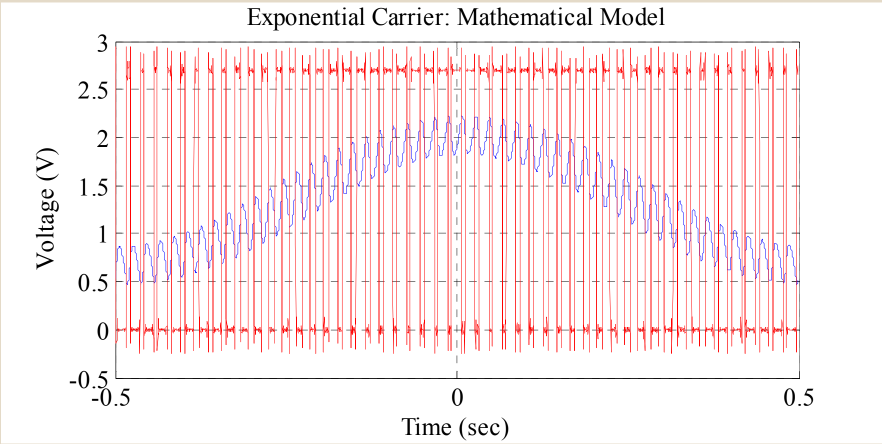


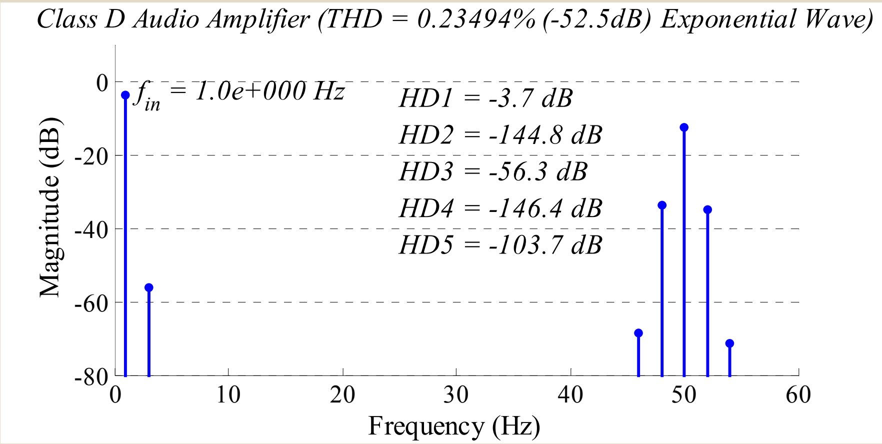
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)



Class D output spectrum (mathematical model)



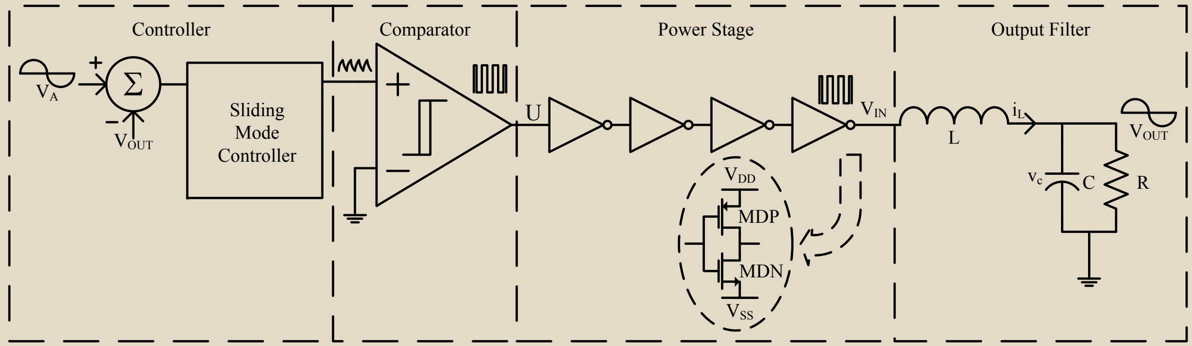


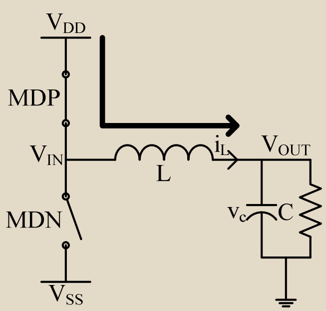
Class D output spectrum (SIMULINK simulation)

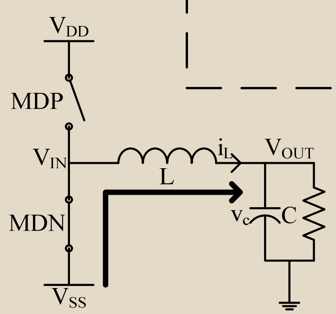
The mathematical model predicts with high accuracy the result in the SIMULINK simulation!

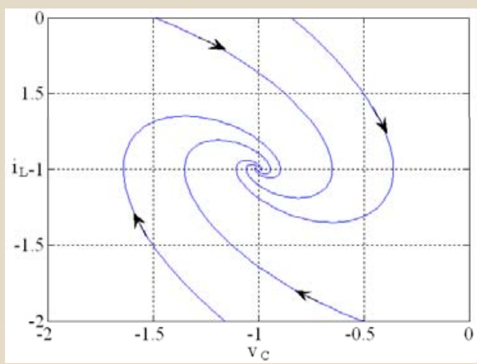
This analysis method can be further applied to any carrier wave or even multiphase systems where multilevel PWM is generated.

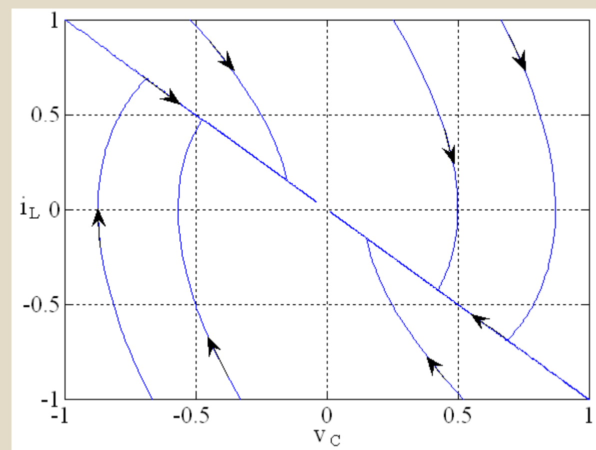
3. Proposed class D audio amplifier (†) (single ended architecture)

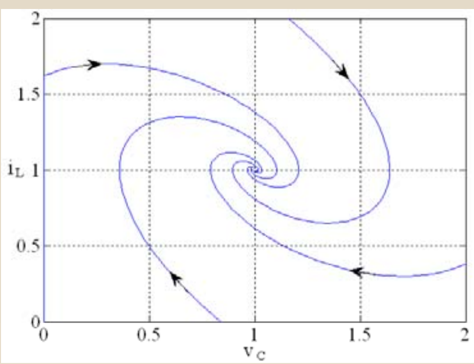










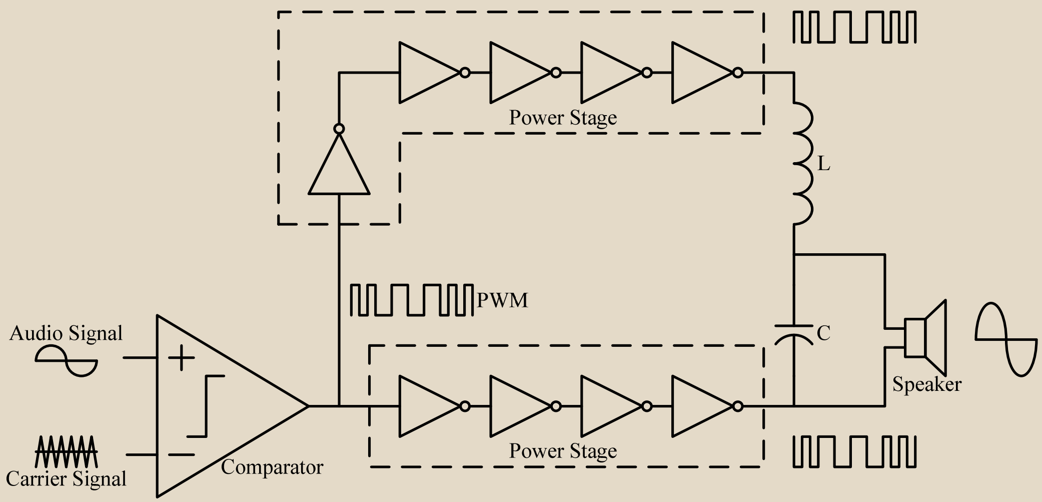


Class D amplifier with sliding surface

(†) M. Rojas-Gonzalez, E. Sanchez-Sinencio, “Design of a Class D Audio Amplifier IC Using Sliding Mode Control and Negative Feedback”, IEEE Transactions on Consumer Electronics, Vol. 53, No. 2, May 2007.

3. Proposed class D audio amplifier: PWM generation

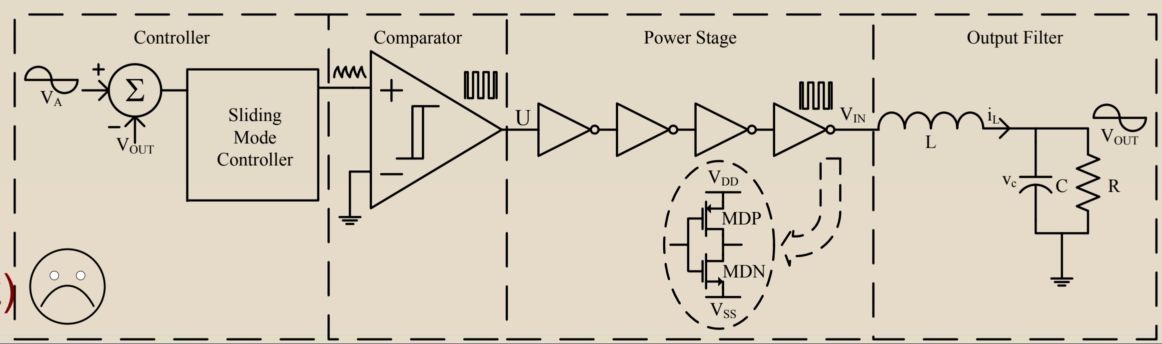
Traditional architecture



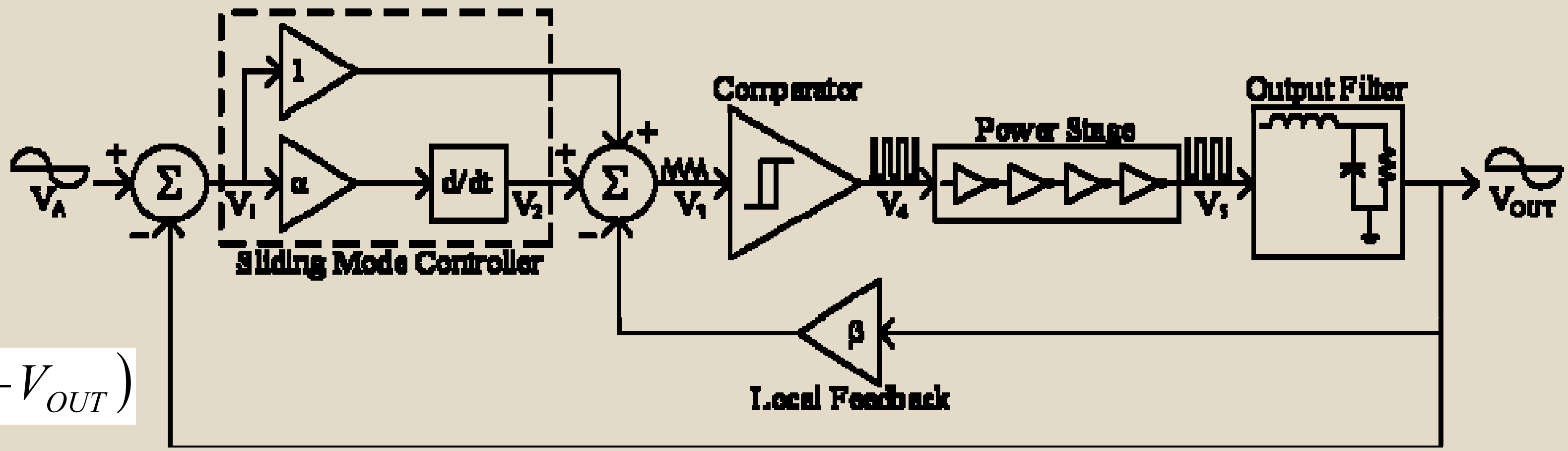
- Ideally fixed frequency
- Jitter (degrades linearity)
- Non-linear circuit
- Carrier generator adds complexity
- Non-ideal triangle wave

Proposed architecture

- No dedicated triangle wave
- Linearity no compromised
- Good transient response
- Variable frequency (450KHz-600KHz)



3. Proposed class D audio amplifier

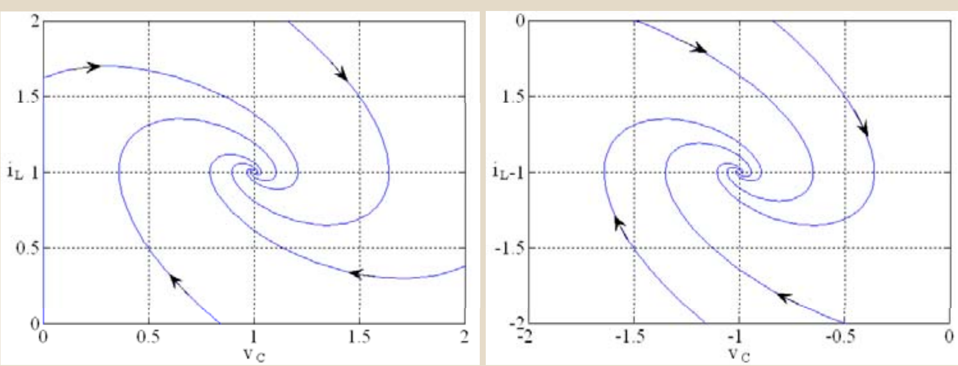


Error function

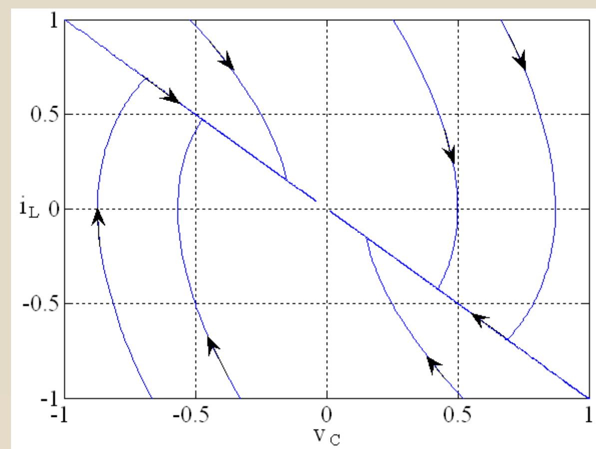
$$V_1 = e_1 = (V_A - V_{OUT})$$

Switching function

$$V_3 = s(e_1, e_2) = e_1 + \alpha e_2 = e_1 + \alpha \dot{e}_1 = (1 + \alpha s) e_1(s)$$

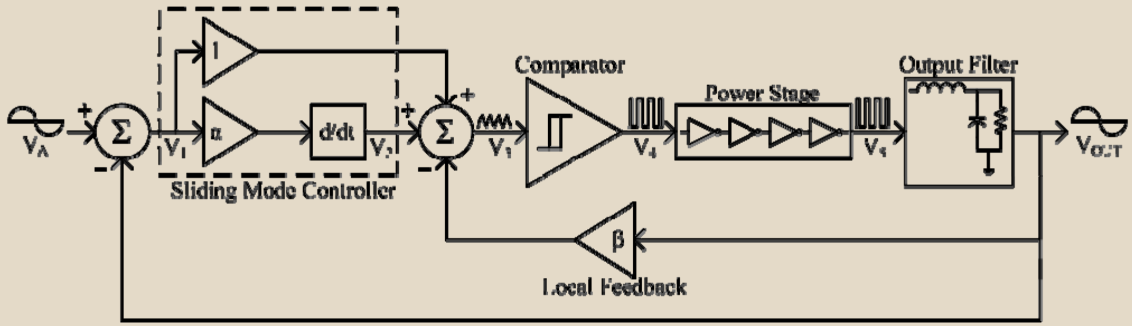


Phase portraits in a class D audio amplifier



Class D Amplifier with sliding surface

3. Proposed class D audio amplifier

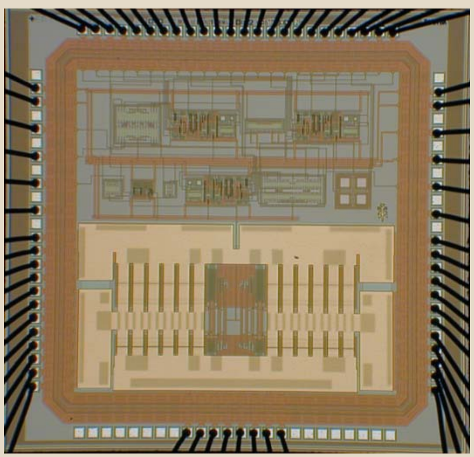


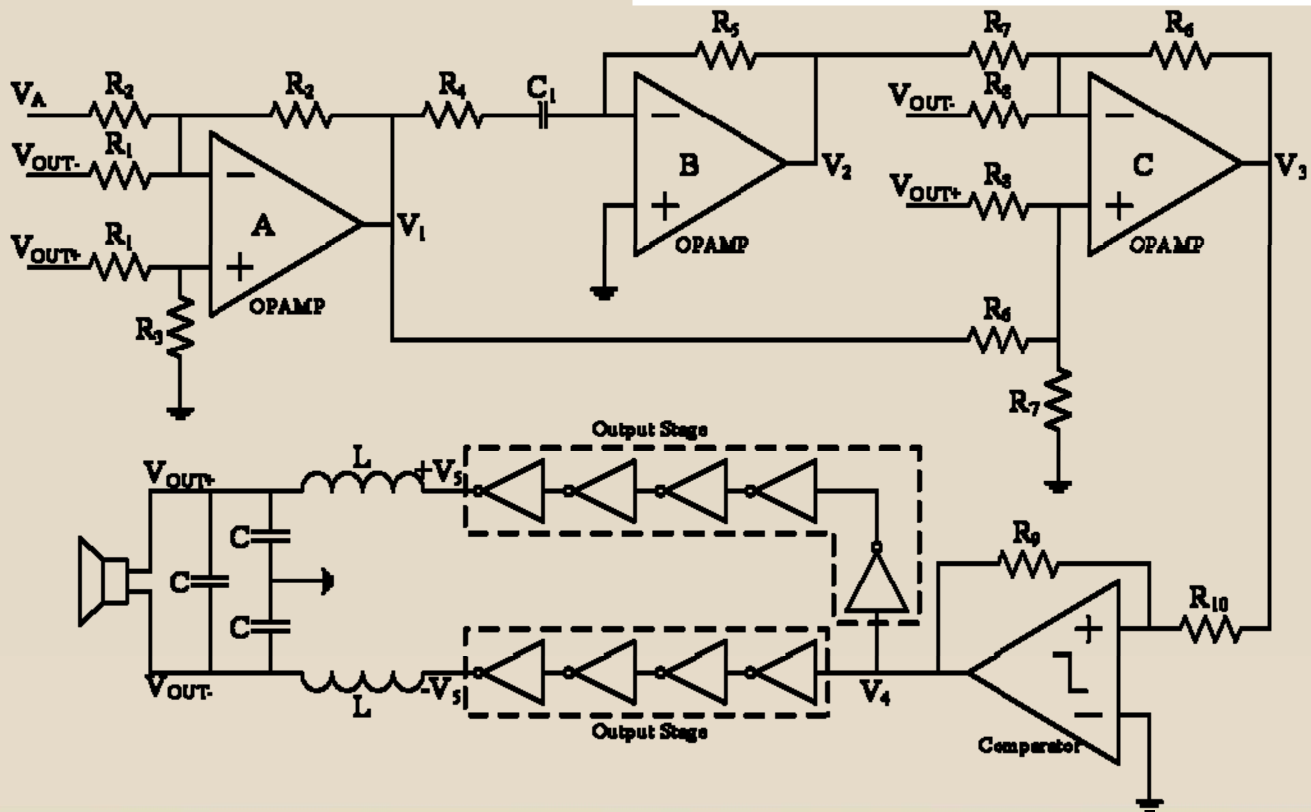
Error function

$$V_1 = e_1 = (V_A - V_{OUT})$$

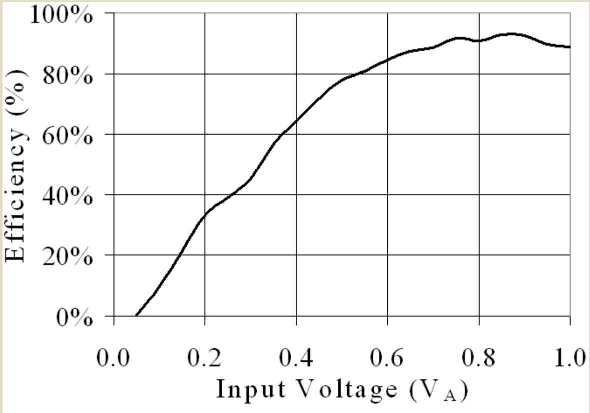
Switching function

$$V_3 = s(e_1) = e_1 + \alpha\dot{e}_1 = (1+\alpha s)e_1(s)$$

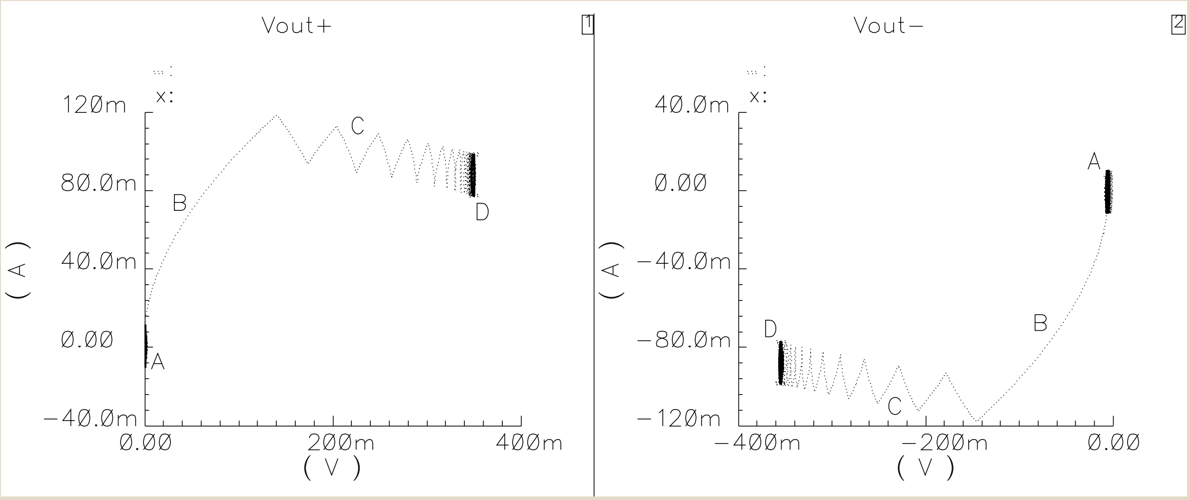


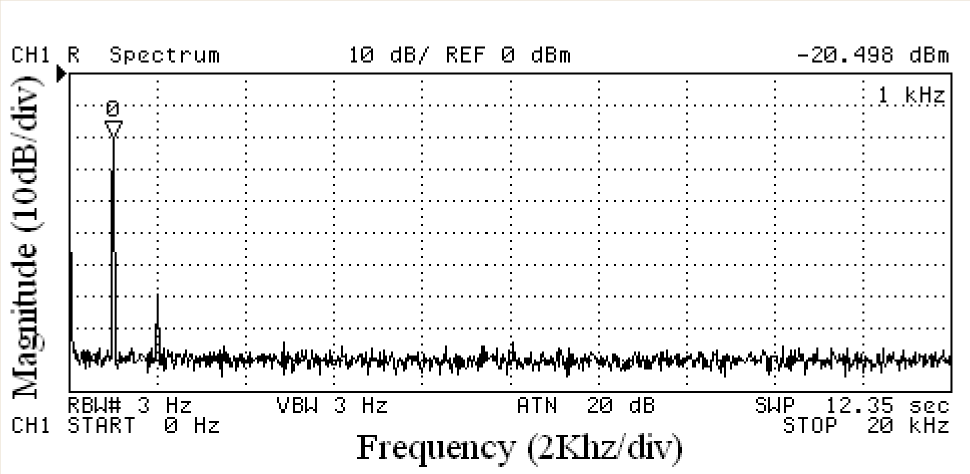


3. Proposed class D audio amplifier



Efficiency vs. input signal





Output spectrum (300 mVpp)

Sliding mode phases

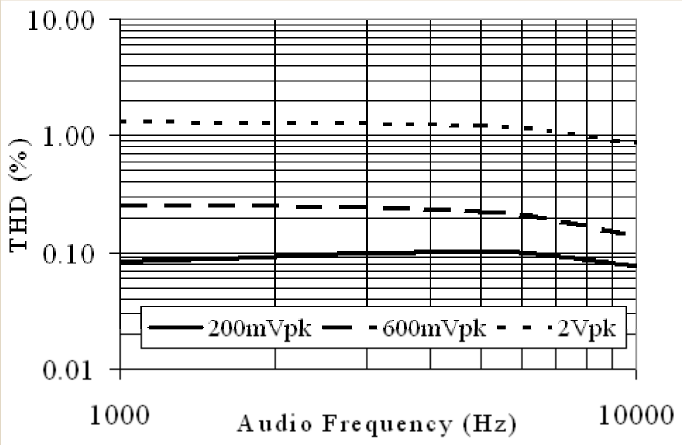
A – Initial condition

B – Reaching mode

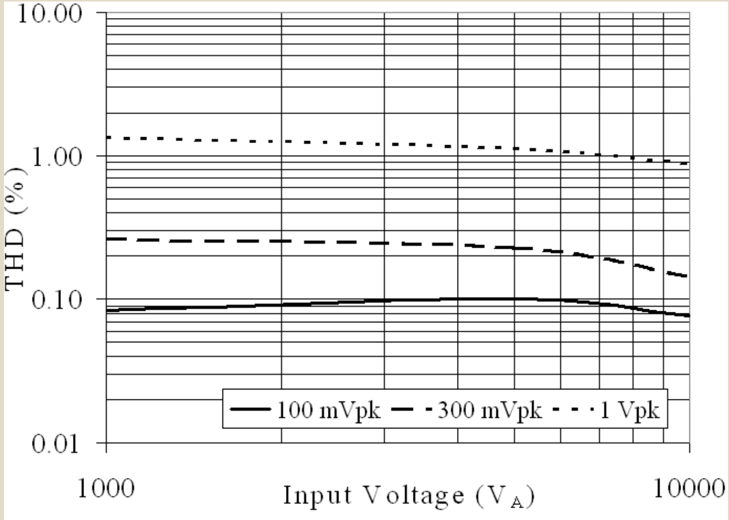
C – Sliding surface

D – Sliding equilibrium point

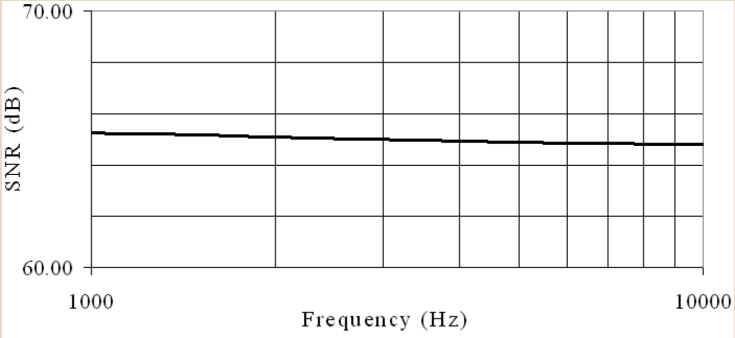
3. Proposed class D audio amplifier



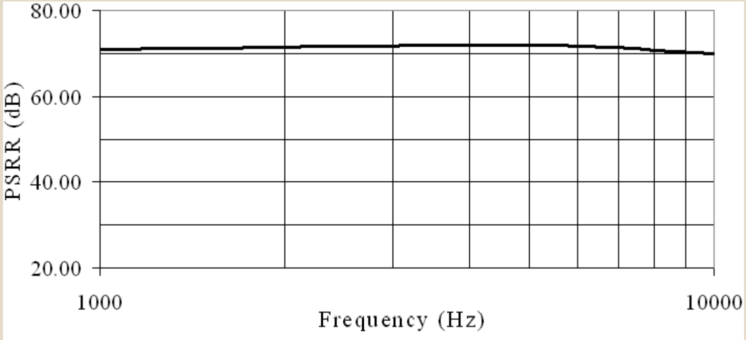
THD versus audio frequency input



THD versus audio voltage input



SNR versus audio frequency input



PSRR versus audio frequency input

3. Proposed class D audio amplifier

Design	THD	η	Supply	Load	I_0
[1]	0.28%	92%	2.5 V	8 Ω	25.2 μA
[2]	0.11%	70%	5.0 V	8 Ω	-
[3]	0.03%	76%	4.2 V	8 Ω	4.7 mA
[4]	0.20%	90%	5.0 V	4 Ω	-
[5]*	0.08%	85%	5.0 V	4 Ω	8.0 mA
[6]*	0.40%	87%	2.7 V	4 Ω	2.8 mA
[7]	0.04%	79%	3.6 V	8 Ω	2.5 mA
[8]	0.10%	92%	12 V	8 Ω	-
This work	**0.08%**	**91%**	**2.7 V**	**8 Ω**	**2.0 mA**

[1] S. C. Li, V. C. Lin, K. Nandhasri and J. Ngarmnil, “New high-efficiency 2.5V/0.45W RWDM class D audio amplifier for portable consumer electronics”, *IEEE Trans. on Circuits and Systems I*, Vol. 52, No. 9, pp. 1767-1774, September 2005.

[2] K. Philips, J. Van Der Homber and C. Dijkmas, “Power DAC: a single-chip audio DAC with 70% efficient power stage in 0.5um CMOS”, *IEEE International Solid-State Circuits Conference*, pp. 154-155, February 1999.

[3] B. Forejt, V. Rentala, J. D. Arteaga and G. Burra, “A 700+-mW class D design with direct battery hookup in a 90nm process”, *IEEE Journal of Solid-State Circuits*, Vol. 40, No. 9, pp. 1880-1887, September 2005.

[4] J. Lee, J. Lee, G. Lee and S. Kim, “A 2W BTL single-chip class D power amplifier with very high efficiency for audio applications”, *IEEE International Symposium on Circuits and Systems*, Vol. 5, pp. 493-496, May 2000.

[5] TPA2000D2 2W Filterless Stereo class D Audio Power Amplifier Datasheet, Texas Instruments Inc., Publication Number SLOS291E, May 2003.

[6] MAX4295 Mono, 2W Switch-Mode (class D) Audio Power Amplifier Datasheet, Maxim Integrated Products Inc., January 2001.

[7] P. Muggler, W. Chen, C. Jones, P. Dagli and N. Yazdi, “A filter free class D audio amplifier with 86% power efficiency”, *Proceedings of the 2004 International Symposium on Circuits and Systems*, Vol. 1, pp. I-1036-1039, May 2004.

[8] S. Choi, J. Lee, W. Jin and J. So, “A design of a 10W single-chip class D audio amplifier with very high efficiency using CMOS technology”, *IEEE Trans. on Consumer Electronics*, Vol. 45, No. 3, pp. 465-473, August 1999.

4. Class D audio amplifiers: two design approaches

Motivation

• Single ended architecture generates even-order distortion tones which degrades linearity

• Single ended version generates “quasi” differential output by adding an extra inverter (causes delay and distortion)

Advantages of fully-differential version

• Even-order cancellation enhances linearity

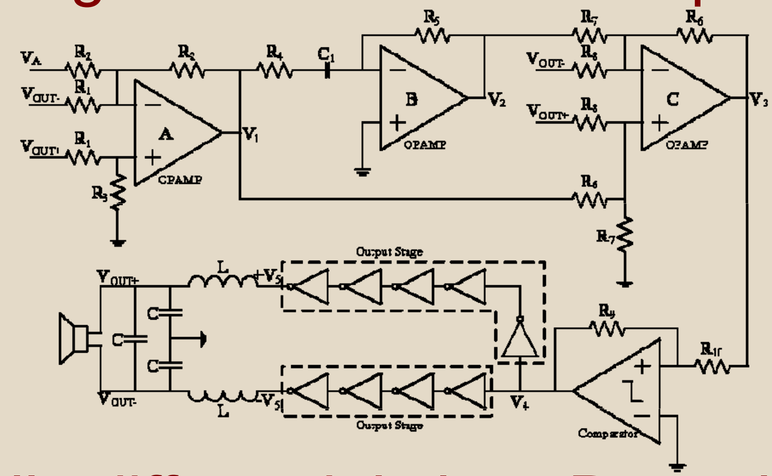
• No delay in signal paths

Improvements from single-ended version

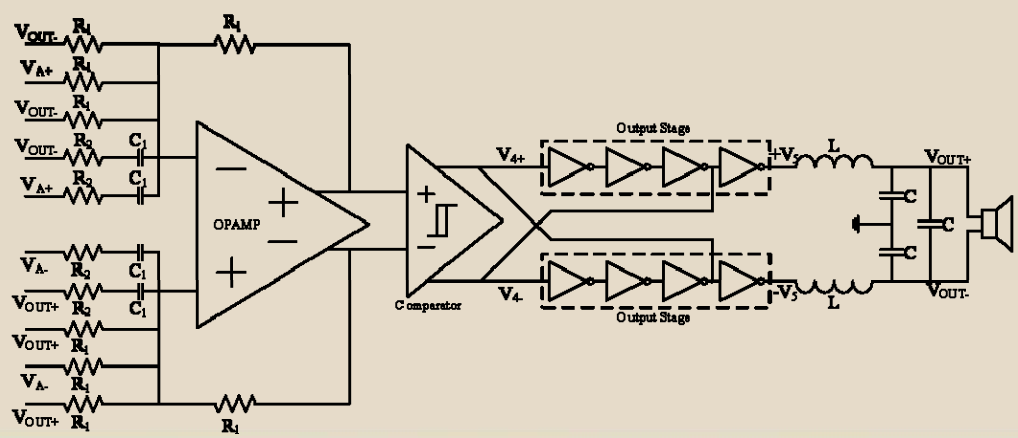
• Reduction of building blocks (operations are done in a single OPAMP)

• Comparator design is done with internal positive feedback instead of poly-resistors

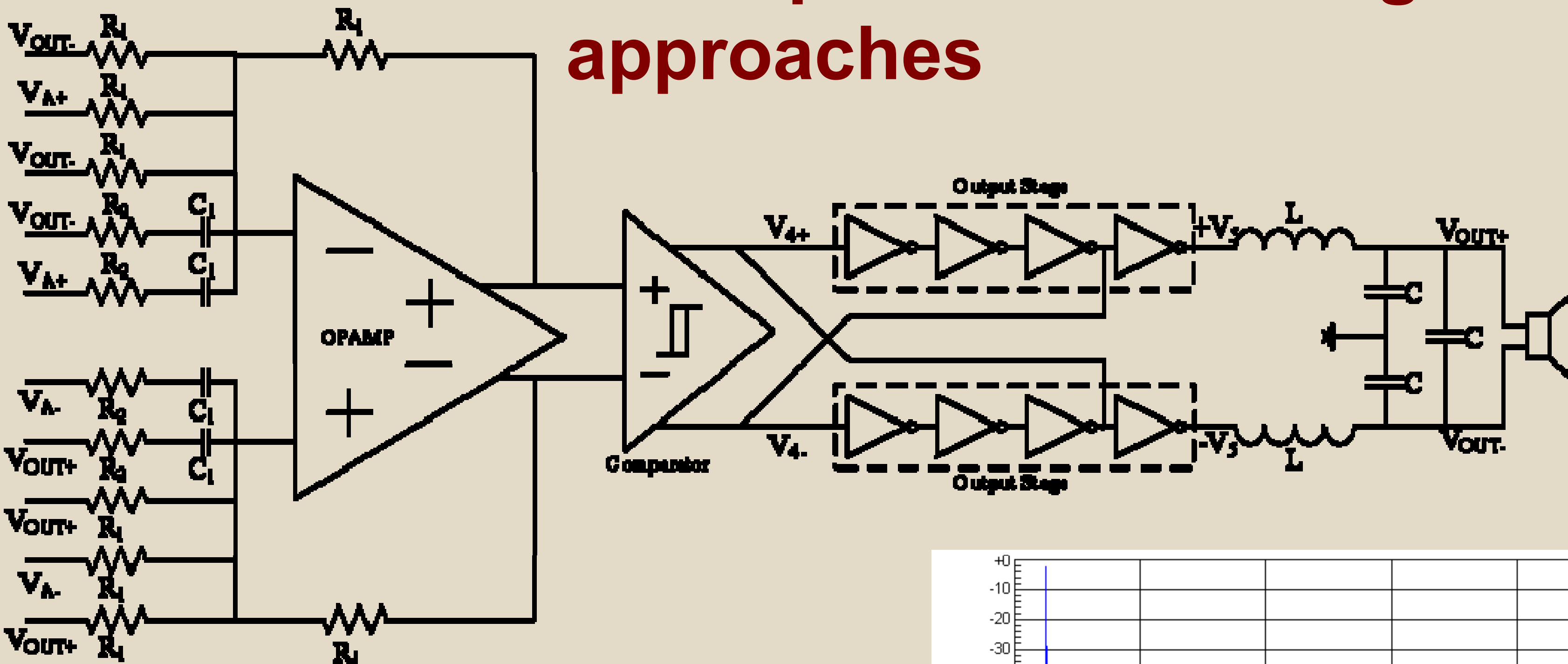
Single-ended class D amplifier



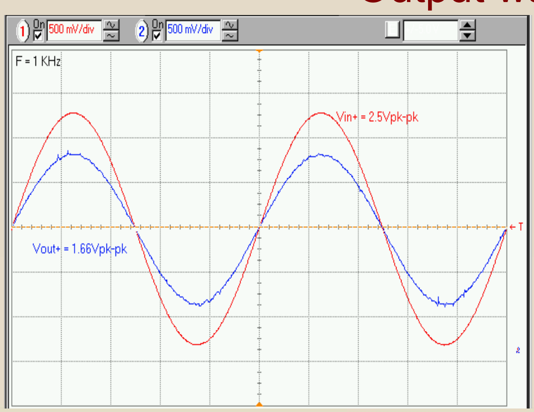
Fully-differential class D amplifier

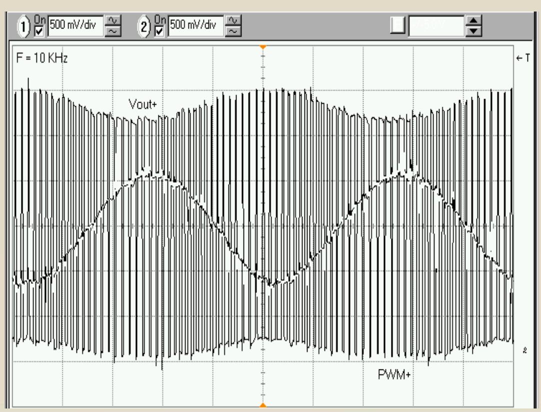


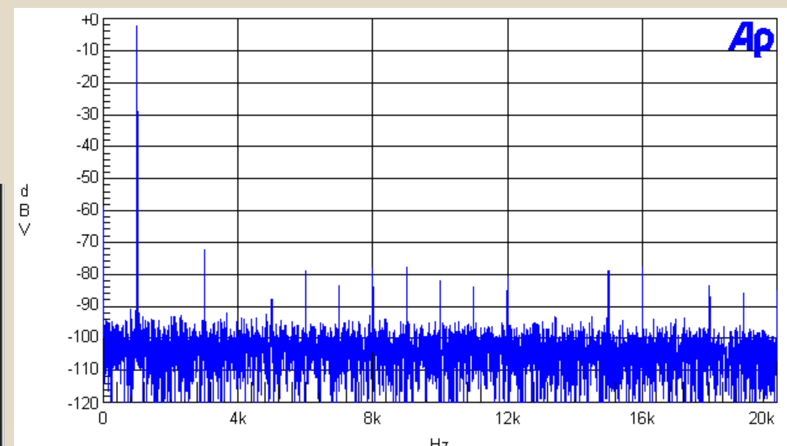
4. Class D audio amplifiers: two design approaches



Output waveforms

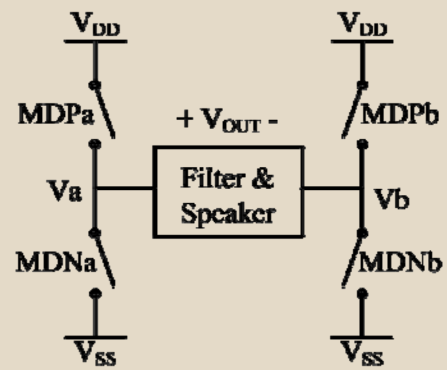






Output spectrum

4. Class D audio amplifiers: two design approaches



Motivation

• Multilevel converters present better linearity as number of level increases

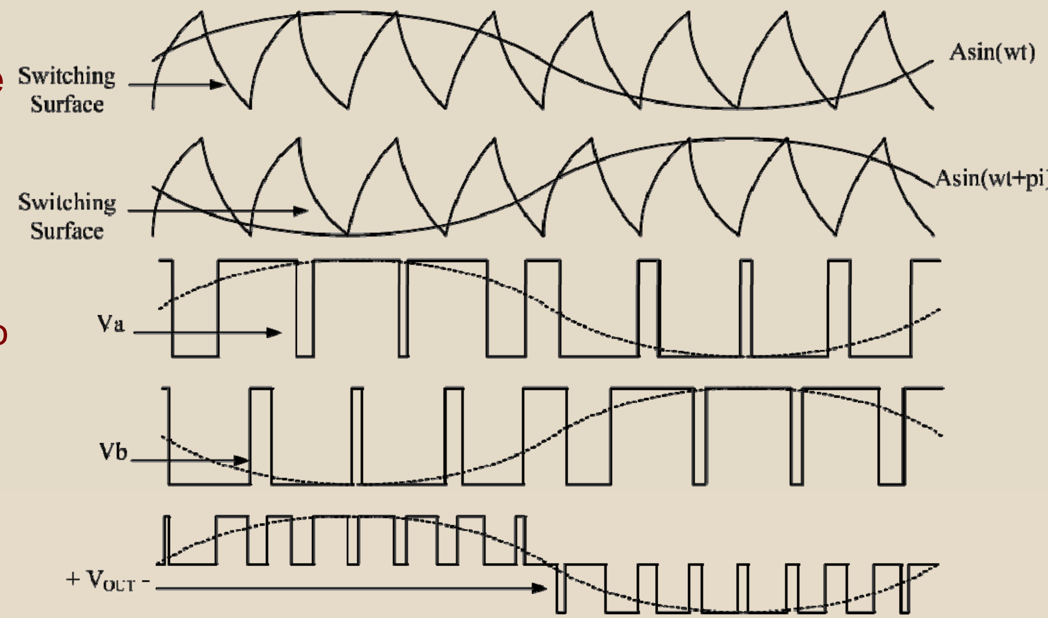
• High frequency components are pushed to higher frequencies (for three level modulation, carrier fs is pushed to 2xfs)

• Possibility of adding an extra-level by using H-bridge already present in class D output stage

Advantages of three-level architecture

• Multi-level modulation = linearity improvement

• No additional hardware cost



Characteristics

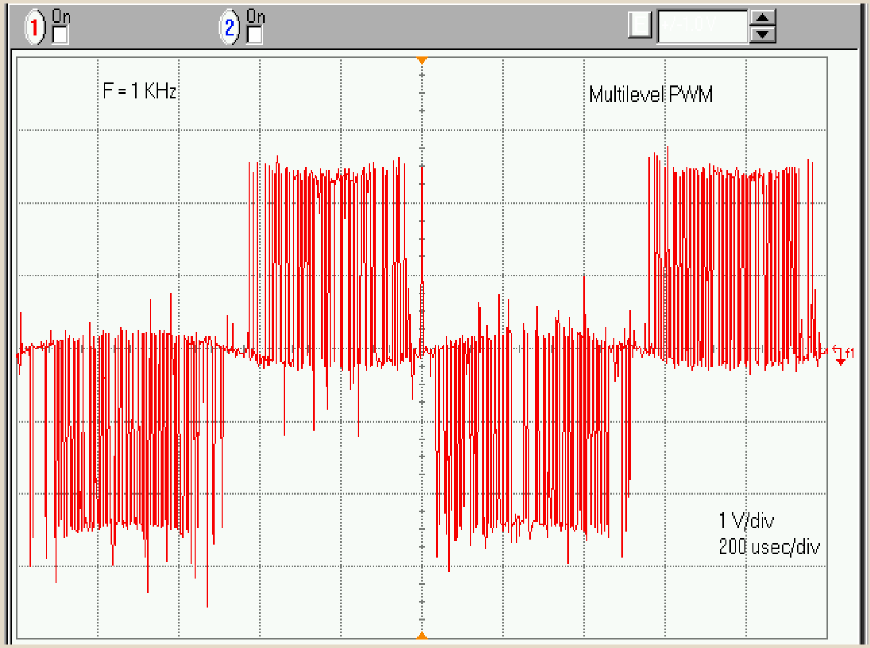
• Two identical switching surfaces are created (Two binary comparators are used)

• Each switching surface is fed by the audio signal shifted 180 degrees from each other

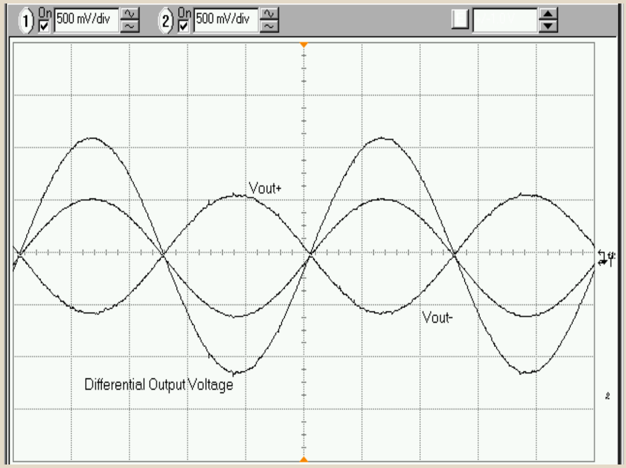
• Each output stage operates at two different levels

• Differential output becomes multi-level!!!

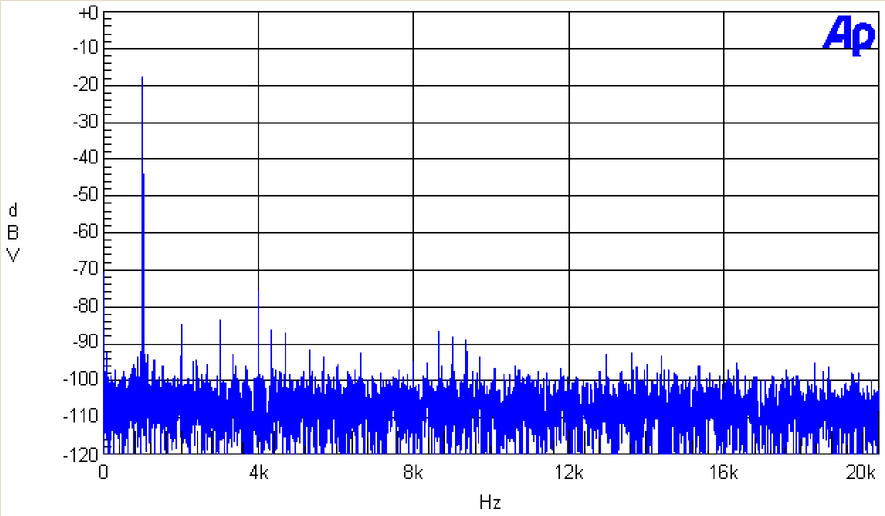
4. Class D audio amplifiers: two design approaches



Three-level PWM

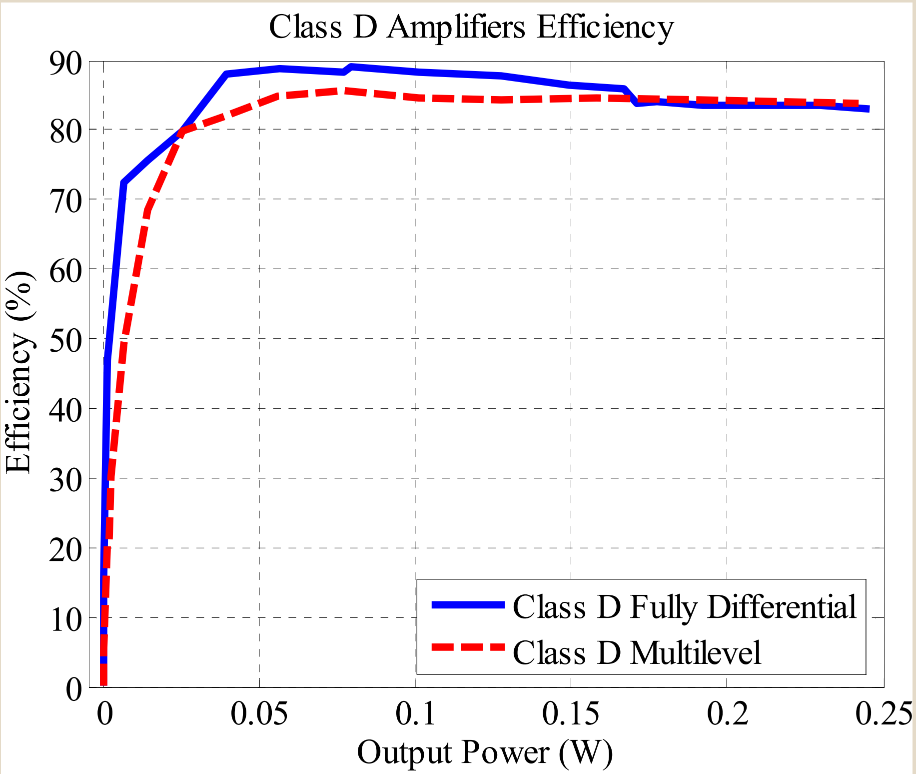


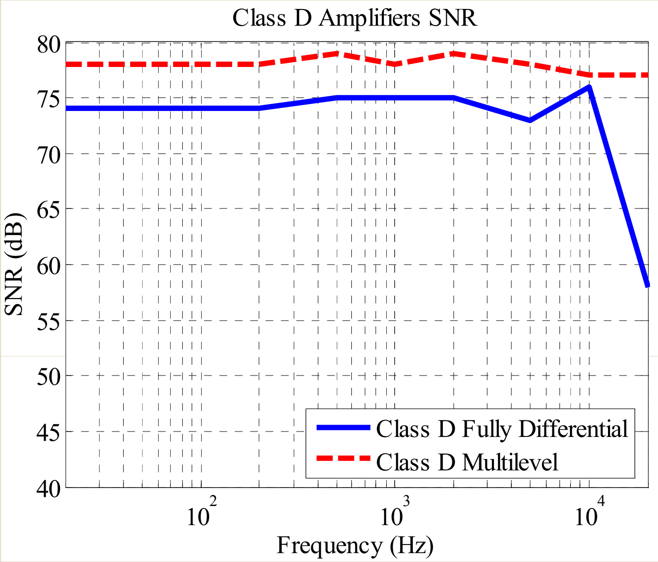
Output waveform

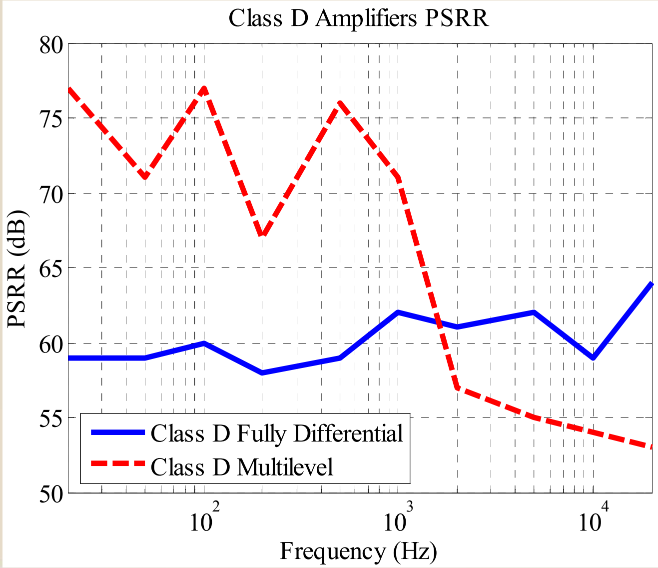


Output spectrum

4. Class D audio amplifiers: two design approaches







Class D fully-differential and multilevel have similar efficiencies and linearity but multilevel modulation gives better SNR and PSRR due to the extra level of quantization.

Table of comparison

$$FM = \frac{\eta}{I_0 \times THD \times 100e3}$$

Design	THD	η	Supply	Load	I_0	SNR	PSRR	f_s	$P_{O,\,max}$	Area	Process	FM
[3]	0.20%	-	3.0V	8Ω	-	81dB	-	1.5MHz	381 mW	1.20 mm^2	0.35um CMOS	-
[4]	0.07%	92%	2.5V	8Ω	25.2uA	80dB	85dB	200KHz	330mW	0.60 mm^2	0.50um CMOS	5
[5]	0.50%	85%	-	-	-	85dB	40dB	-	-	-	-	-
[6]	0.11%	70%	5.0V	8Ω	-	-	90dB	1.0MHz	250mW	12.5 mm^2	0.50um CMOS	-
[7]	0.03%	76%	4.2V	8Ω	4.7mA	98dB	70dB	410KHz	700mW	0.44 mm^2	90nm DCMOS	6
[8]	0.20%	90%	5.0V	4Ω	-	-	-	450KHz	1250mW	12.3 mm^2	0.65um CMOS	-
[9]*	0.08%	70%	5.0V	4Ω	4.0mA	87dB	77dB	250KHz	1000mW	-	-	2
[10]*	0.40%	87%	2.7V	4Ω	2.8mA	-	-	125KHz	700mW	-	-	1
[11]	0.04%	79%	3.6V	8Ω	2.5mA	-	84dB	250KHz	500mW	2.25 mm^2	1.2um BiCMOS	8
[12]	0.10%	92%	12.0V	8Ω	-	-	-	180KHz	10.0W	25.0 mm^2	4.00um CMOS	-
[13]	0.19%	-	3.0V	8Ω	-	95dB	-	700KHz	400mW	2.25 mm^2	0.35um CMOS	-
[14]	0.04%	80%	3.3V	4Ω	-	-	-	20MHz	-	-	0.35um CMOS	-
SE	*0.08%*	*91%*	*2.7V*	*8Ω*	*2.0mA*	*65dB*	*70dB*	*500KHz*	*200mW*	*4.70 mm^2*	*0.50um CMOS*	*6*
FD	*0.04%*	*89%*	*2.7V*	*8Ω*	*1.3mA*	*75dB*	*62dB*	*450KHz*	*250mW*	*1.88 mm^2*	*0.50um CMOS*	*17*
ML	*0.10%*	*85%*	*2.7V*	*8Ω*	*836uA*	*78dB*	*75dB*	*450KHz*	*250mW*	*2.48 mm^2*	*0.50um CMOS*	*10*

* Commercial product.

Thank you

References

[3] A. Yasuda, T. Kimura, K. Ochiai and T. Hamasaki, “A class D amplifier using a spectrum shaping technique”, *Proceedings of the IEEE 2004 Custom Integrated Circuits Conference*, pp. 173-176, October 2004.

[4] S. C. Li, V. C. Lin, K. Nandhasri and J. Ngarmnil, “New high-efficiency 2.5V/0.45W RWDM class D audio amplifier for portable consumer electronics”, *IEEE Trans. on Circuits and Systems I*, Vol. 52, No. 9, pp. 1767-1774, September 2005.

[5] M. Score and D. Dapkus, “Optimized modulation scheme eliminates output filter”, *Proceedings of the 109th AES Convention*, pp. 22-25, September 2000.

[6] K. Philips, J. Van Der Homber and C. Dijkmas, “Power DAC: a single-chip audio DAC with 70% efficient power stage in 0.5um CMOS”, *IEEE International Solid-State Circuits Conference*, pp. 154-155, February 1999.

[7] B. Forejt, V. Rentala, J. D. Arteaga and G. Burra, “A 700+-mW class D design with direct battery hookup in a 90nm process”, *IEEE Journal of Solid-State Circuits*, Vol. 40, No. 9, pp. 1880-1887, September 2005.

[8] J. Lee, J. Lee, G. Lee and S. Kim, “A 2W BTL single-chip class D power amplifier with very high efficiency for audio applications”, *IEEE International Symposium on Circuits and Systems*, Vol. 5, pp. 493-496, May 2000.

[9] TPA2000D2 2W Filterless Stereo class D Audio Power Amplifier Datasheet, Texas Instruments Inc., Publication Number SLOS291E, May 2003.

[10] MAX4295 Mono, 2W Switch-Mode (class D) Audio Power Amplifier Datasheet, Maxim Integrated Products Inc., January 2001.

[11] P. Muggler, W. Chen, C. Jones, P. Dagli and N. Yazdi, “A filter free class D audio amplifier with 86% power efficiency”, *Proceedings of the 2004 International Symposium on Circuits and Systems*, Vol. 1, pp. I-1036-1039, May 2004.

[12] S. Choi, J. Lee, W. Jin and J. So, “A design of a 10W single-chip class D audio amplifier with very high efficiency using CMOS technology”, *IEEE Trans. on Consumer Electronics*, Vol. 45, No. 3, pp. 465-473, August 1999.