SIGNAL GENERATORS

• SINUSOIDAL
  – BP (Two integrator) Based
  – One Op Amp (Wien-Bridge)

• EXPONENTIAL, TRIANGULAR AND SQUARE WAVE
How to make a sinusoidal oscillator based on filters?

**Band Pass Based Oscillator**

\[
H(s) = \frac{k \frac{\omega_o}{Q}}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}
\]

Assuming small signal conditions, we can write the loop filter

\[
1 - \beta H(s) = 0 \quad ; \quad K = k \frac{\omega_o}{Q}
\]

\[
1 - \frac{sK'}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2} = 0 \quad \text{or} \quad \beta H(s) = 1
\]

Thus the characteristic equation yields:

\[
s^2 - \left( K' - \frac{\omega_o}{Q} \right) s + \omega_o^2 = 0
\]
In order to have the poles in the RHP, it is imposed that
\[ K' - \frac{\omega_o}{Q} > 0 \quad \text{or} \quad K' \geq \frac{\omega_o}{Q} \]
where \( K' = K_C K \) and \( K_C = \frac{V_{sat}}{\Delta V_{TH}} \)

However to move the poles on the \( j\omega \) axis we require a non-linear block implementation of \( \beta \). There are a number of blocks to accomplish this.

**Implementation 1**

Inverting Schmitt Trigger

**Implementation 2**

\[ K_C = \frac{V_{DD}}{\Delta V_{TH}} \approx \frac{R_1}{R_2} \left( V_{DD} - ( - V_{DD} ) \right) \]
\[ K_C \approx \frac{R_{s2}}{2R_{s1}} \]
Inverting Schmitt Trigger

\[ V_{TH} = \frac{R_1}{R_1 + R_2} V_{OH} \]

\[ V_{TL} = \frac{R_1}{R_1 + R_2} V_{OL} \]
\[ V_{TH} = \frac{R_1}{R_1 + R_2} V_{OH} \quad \text{or} \quad V_{OH} = \left(1 + \frac{R_2}{R_1}\right)V_{TH} \]

\[ V_{TL} = \frac{R_1}{R_1 + R_2} V_{OL} \quad \text{or} \quad V_{OL} = \left(1 + \frac{R_2}{R_1}\right)V_{TL} \]

Hysteresis (window) Width:

\[ \Delta V_T = V_{TH} - V_{TL} = \frac{R_1}{R_1 + R_2} (V_{OH} - V_{OL}) \]

or

\[ V_{OH} - V_{OL} = \left(1 + \frac{R_2}{R_1}\right) \Delta V_T \]
Numerical Example

\[ V_{OH} = 13\text{V} \quad ; \quad V_{OL} = -13\text{V} \]

for \( R_2 = 16\text{K}\Omega \) and \( R_1 = 10\text{K}\Omega \)

\[ V_{TH} = \frac{13}{26} \times 10 = 5\text{V} \]

\[ V_{TL} = -\frac{13}{26} \times 10 = -5\text{V} \]
INVERTING SCHMITT TRIGGER WITH REFERENCE VOLTAGE

\[ V_{\text{TH}} = \frac{V_{\text{OH}} R_1 + V_R R_2}{R_1 + R_2} = \frac{V_R + \frac{R_1}{R_2} V_{\text{OH}}}{1 + \frac{R_1}{R_2}} \]

\[ V_{\text{TL}} = \frac{V_{\text{OL}} R_1 + V_R R_2}{R_1 + R_2} = \frac{V_R + \frac{R_1}{R_2} V_{\text{OL}}}{1 + \frac{R_1}{R_2}} \]

\[ \Delta V_T = \frac{R_1}{1 + \frac{R_1}{R_2}} (V_{\text{OH}} - V_{\text{OL}}) = \frac{R_1}{R_1 + R_2} (V_{\text{OH}} - V_{\text{OL}}) \]
Reader. Exchange input and reference terminals in the figure of Previous page and prove that for the non-inverting comparator

\[
V_{TH} = \frac{R_1}{R_2} V_{OH} + V_R \left( 1 + \frac{R_1}{R_2} \right)
\]

\[
V_{LH} = \frac{R_1}{R_2} V_{OL} + V_R \left( 1 + \frac{R_1}{R_2} \right)
\]

\[
\Delta V_T = \frac{R_1}{R_2} (V_{OH} - V_{OL})
\]
Non-Inverting Schmitt Trigger

\[ V_{TH} = -\frac{R_1}{R_2} V_{OL} \quad \iff \quad \frac{V_{TH} - 0}{R_1} = \frac{0 - V_{OL}}{R_2} \]

\[ V_{TL} = -\frac{R_1}{R_2} V_{OH} \]

\[ \Delta V_T = \frac{R_1}{R_2} (V_{OH} - V_{OL}) \quad \text{Hysteresis Window} \]
Schmitt Trigger in I/O in first quadrant and having a single supply.

Using Superposition

\[
V_p = \frac{R_{1,3}}{R_{1,3} + R_2} V_{CC} + \frac{R_{1,2}}{R_{1,2} + R_3} V_o
\]

where \( R_{1,3} = R_1 \parallel R_3 \) and \( R_{1,2} = R_1 \parallel R_2 \)

If \( V_{OH} \approx V_{CC} \) and \( V_{OL} = 0 \)
then \( R_4 \ll R_3 + R_{1,2} \)
thus, imposing \( V_p = V_{TL} \) for \( V_o = V_{OL} = 0 \)
and \( V_p = V_{TH} \) for \( V_o = V_{OH} = V_{CC}, \) we get

\[
V_{TL} = \frac{R_{1,3}}{R_{1,3} + R_2} V_{CC} \quad ; \quad V_{TH} = \frac{R_1}{R_1 + R_{2,3}} V_{CC}
\]
Rearranging terms and fix $R_4$ and $R_3 \ll R_4$

\[
\frac{1}{R_2} = \frac{V_{TL}}{V_{CC} - V_{TL}} \left( \frac{1}{R_1} + \frac{1}{R_3} \right)
\]

\[
\frac{1}{R_1} = \frac{V_{CC} - V_{TH}}{V_{TH}} \left( \frac{1}{R_2} + \frac{1}{R_3} \right)
\]

Thus the two above equations can be solved.

The BP in page 1 can be implemented using the Tow-Thomas

---

![Circuit Diagram](image-url)
- Seen as non-inverter

\[ A = \frac{V_o}{V_p} = 1 + \frac{R_2}{R_1} \]

Furthermore we can see \( V_p \) a voltage divider

\[ \frac{Z_p}{Z_p + Z_s} \]

\[ V_o = V_p \]

Thus

\[ Z_p = \frac{R}{1 + sRC} \]

\[ Z_s = \frac{1 + sRC}{sC} \]

\[ \frac{V_o}{V_p} = \frac{Z_p + Z_s}{Z_p} = 1 + \frac{Z_s}{Z_p} \]

\[ B = \frac{V_p}{V_o} = \frac{Z_p}{Z_p + Z_s} = \frac{1}{1 + \frac{Z_s}{Z_p}} \]
B = \frac{1}{1 + \frac{Z_s}{Z_p}} = \frac{1}{1 + (1 + sRC)^2} \frac{sRC}{sRC} \\
B = \frac{sRC}{(sRC)^2 + 3sRC + 1} \Rightarrow \text{Band Pass Filter}

Characteristic Equation

1 - AB = 1 - T

T = AB = \frac{\left(1 + \frac{R_2}{R_1}\right)sRC}{(sRC)^2 + 3sRC + 1}

T = \frac{1 + \frac{R_2}{R_1}}{sRC + 3 + \frac{1}{sRC^2}}^2

If \quad RC = \frac{1}{\omega_o}

T(j\omega) = \frac{1 + \frac{R_2}{R_1}}{j \frac{\omega}{\omega_o} - \frac{j}{\omega/\omega_o} + 3} = \frac{1 + \frac{R_2}{R_1}}{3 + j \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}

T(j\omega_o) = \frac{1 + \frac{R_2}{R_1}}{3}
To force oscillation

\[ T(j\omega_o) > 1 \quad \text{i.e. } \frac{R_2}{R_1} > 2 \]

\[ \frac{R_2}{R_1} = 2 + \varepsilon \]

The non-linear part of the oscillator must be added if we want the comparator not to do the non-linear operation.
SQUARE WAVE OSCILLATOR

Let us look closer to the Schmitt Trigger

\[ V_{TH} = \frac{R_1}{R_1 + R_2} V_{OH} \]

\[ V_{TL} = \frac{R_1}{R_1 + R_2} V_{OL} \]

\[ \Delta V_T = V_{TH} - V_{TL} = \frac{R_1}{R_1 + R_2} (V_{OH} - V_{OL}) \]

\[ \Delta V_T = \frac{1}{1 + \frac{R_2}{R_1}} (V_{OH} - V_{OL}) \]
Non-Inverting Schmitt Trigger

\[ V_{\text{TH}} = -\frac{R_1}{R_2} V_{\text{OL}} \]

Tripping voltage \( V_{\text{TL}} \) becomes

\[ \frac{V_{\text{OH}} - 0}{R_2} = \frac{0 - V_{\text{TTL}}}{R_1} \]

or

\[ V_{\text{TTL}} = -\frac{R_1}{R_2} V_{\text{OH}} \]

\[ \Delta V_T = \frac{R_1}{R_2} (V_{\text{OH}} - V_{\text{OL}}) \]

\[ \Delta V_T = \frac{1}{R_2} (V_{\text{OH}} - V_{\text{OL}}) \]
Notes on Relaxation Oscillators

\[ \Delta V \begin{cases} V_1 \\ V_0 \end{cases} \]
\[ \Delta t \]
\[ \text{Linear} \]

t

t_0 

t_1

\[ C \Delta V = I \Delta t \]

\[ \Delta t = \frac{C}{I} \Delta v \]

Thus for the basic-free-running exponential oscillator

\[ \frac{T}{2} = RC \ln \frac{V_{sat} + V_T}{V_{sat} - V_T} \]

\[ V_T = \frac{V_{sat}}{1 + \frac{R_2}{R_1}} \]

then

\[ f_o = \frac{1}{T} = \frac{1}{2RC \ln(1 + 2R_1/R_2)} \]

Exponential

\[ v(t) = V_\infty + (V_0 - V_\infty)e^{\frac{t-t_0}{\tau}} \]

\[ \tau = RC \]

\[ V_1 = V_\infty + (V_0 - V_\infty)e^{\frac{t_1-t_0}{\tau}} \]

\[ \Delta t = t_1 - t_0 = \tau \ln \frac{V_\infty - V_0}{V_\infty - V_1} \]

\[ f_o \neq f(V_T, V_{sat}) \]
SQUARE WAVE GENERATOR

\[ V_{TH} = \frac{R_g}{R_g + R_f} V_{OH} \]

or

\[ V_{TL} = \frac{R_g}{R_g + R_f} V_{OL} \]

High to Low.

Assume the output voltage is at \( V_{OH} \), then the non-inverting input is at \( V_{TH} \), and the inverting is in exponential transition form its initial value \( V_{TL} \) toward \( V_{OH} \). Thus

\[ v^- (t) = V_{OH} - \left( V_{OH} - V_{TL} \right) e^{-t/RC} \]

or equivalent in the s-domain

\[ V^- (s) = \frac{V_{OH}}{s} - \frac{\left( V_{OH} - V_{TL} \right)}{1 + s/RC} \]

The total exponential transition time is the solution to the equality

\[ v^- (t) = v^+_{TH} \]

or

\[ V_{OH} - \left( V_{OH} - \frac{R_g}{R_g + R_f} V_{OL} \right) e^{-t/RC} = \frac{R_g}{R_g + R_f} V_{OH} \]

Note.- See Section 10.2 Text
Then

\[ V_{OH} \left(1 - \frac{R_g}{R_g + R_f}\right) = \left(V_{OH} - \frac{R_g}{R_g + R_f} V_{OL}\right) e^{-t/RC} \]

The transition time is given by

\[ t_{HL} = RC \ln \left[ \frac{(R_g + R_f) V_{OH} - R_g V_{OL}}{R_f V_{OH}} \right] \]

\[ t_{HL} = RC \ln \left(1 + \frac{R_g (V_{OH} - V_{OL})}{R_f V_{OH}}\right) \]

\[ t_{HL} = RC \ln \left(1 + \frac{\Delta V_T}{V_{OH}}\right) \]
Low to High

Same as before by interchanging $V_{OH}$ and $V_{OL}$. Then the duty cycle is determined by

The sum of the transition times:

$$\tau = RC \left[ \ln \left( 1 + \frac{R_g (V_{OH} - V_{OL})}{R_f V_{OH}} \right) + \ln \left( 1 + \frac{R_g (V_{OL} - V_{OH})}{R_f V_{OL}} \right) \right]$$

$$\tau = RC \left[ \ln \left( 1 + \frac{\Delta V_T}{V_{OH}} \right) + \ln \left( 1 - \frac{\Delta V_T}{V_{OL}} \right) \right]$$

For 50% duty cycle $V_{OL} = -V_{OH}$ yields

$$t_{HL} = t_{LH} = RC \ln \left( 1 + \frac{2R_g}{R_f} \right)$$

Then the period is given by

$$T = 2t_{HL} = 2RC \ln \left( 1 + 2 \frac{R_g}{R_f} \right)$$

$T$ will be affected by the SR of the comparator. Also the parasitic capacitance associated to the “+” terminal of this comparator.

Note that $V_{OL}$ and $V_{OH}$ cannot be exactly predicted thus often a clamping circuit is placed usually at the output.
A Stable, Nonsymmetric Square-Wave Generator

— One option is by making $-V_{OL} \approx V_{OH}$

— Another option is by changing the equality of rising and decaying times.

Then one can show that

$$T = t_H + t_L$$

$$T = RC \ln \left(1 + \frac{2R_g}{R_f}\right) + R'C \ln \left(1 + \frac{2R_g}{R_f}\right)$$

Duty cycle $\Delta \frac{t_H}{T} = \frac{R}{R + R'}$
Basic Triangular/Square Wave Generator

- Two Quasi-static state.
  - For outputs $V_{OH}$ and $V_{OL}$
- The Schmitt Trigger Toggles between output states.

Transition occurs

$$V_{TR(\text{trans})} = -\frac{R_g}{R_f} V_{SQ} = \frac{R_g}{R_f} V_{OH}$$

$$V_{TR} = -\frac{1}{RC} \int [V_{SQ}(t) - V_s] dt$$

When, $v_{SQ}$ is in the high state, $V_{TR}$ is linearly decreasing between its toggle values.

$$v_{TR} = -\frac{1}{RC} \int (V_{OH} - V_s) dt = \frac{V_s - V_{OH}}{RC} t + v_{TR}^{(o^+)}$$

Initial condition
Thus, the voltage transition is the differences in toggle values divided by the slope of the linear transition.

\[ 2 \frac{R_g}{R_f} V_{OH} = \frac{V_{OH} - V_s}{RC} t_- \]

Therefore

\[ t_- = \frac{2R_g V_{OH} \cdot RC}{R_f(V_{OH} - V_s)} \]

Now when the positive transition occurs for \( V_{s>0} \)
\[ v_{TR} = -\frac{1}{RC} \int - (V_{OH} + V_s) \, dt = \frac{V_{OH} + V_s}{RC} t + v_{ti}(o^-) \]

\[ 2 \frac{R_g}{R_f} V_{OH} = \frac{V_{OH} + V_s}{RC} t_+ \Rightarrow t_+ = \frac{2R_gRCV_{OH}}{R_f(V_{OH} + V_s)} \]

Thus the period becomes:

\[ T = t_- + t_+ = 4R_gRC \frac{V_{OH}^2}{R_f(V_{OH}^2 - V_s^2)} \]

\[ \approx \frac{4RC}{R_g/R_f} \quad \text{for } V_s = 0 \]

\[ \frac{R_g}{R_f} \]

Duty cycle 
\[ D = \text{Duty cycle} + = \frac{t_+}{T} = \frac{1}{2} \left( 1 - \frac{V_s}{V_{OH}} \right) \]

for \( V_s = 0 \), \( D = 0.5 \)

for \( V_s = \frac{1}{4} V_{OH} \), then \( D = \frac{1}{2} (1 - 0.25) = \frac{3}{8} \)