SIGNAL GENERATORS

- SINUSOIDAL
  - BP (two integrator) based
  - One op amp (Wien-Bridge)

- Triangular and Square Wave

- Saw Tooth

- V-F and F-V Converters
How to make a sinusoidal oscillator based on filters?

Band pass based oscillator

\[ H(s) = \frac{\frac{1}{k} \omega_0 s}{s^2 + \frac{\omega_0}{\alpha} s + \omega_0^2} \]

Assuming small signal conditions, we can write the loop filter

\[ 1 - \beta H(s) = 0 \quad ; \quad K = \frac{1}{k} \frac{\omega_0}{\alpha} \]

\[ 1 - \frac{Ks}{s^2 + \omega_0 \frac{s}{\alpha} + \omega_0^2} = 0 \quad \text{or} \quad \beta H(s) = 1 \]

Thus the characteristic equation yields:

\[ s^2 - \left( k - \frac{\omega_0}{\alpha} \right) s + \omega_0^2 = 0 \]
In order to have the poles in the RHP, it is imposed that
\[ k - \frac{\omega_0}{\alpha} > 0 \]

or
\[ k > \frac{\omega_0}{\alpha} \]

However to move the poles on the \( j\omega \) axis we require a non-linear block implementation of \( B \). There are a number of blocks to accomplish this.

**Implementation 1**

\[ \begin{align*}
V_b & \approx \pm 0.7V \\
& 0.7 \\
& -0.7
\end{align*} \]

**Implementation 2**

Inverting Schmitt Trigger

\[ V_{TH} = \frac{R_{S2}}{R_{S1}} \]
INVERTING SCHMITT TRIGGER

\[
V_{TH} = \frac{R_1}{R_1 + R_2} V_{OH} \quad \text{or} \quad V_{OH} = \left(1 + \frac{R_2}{R_1}\right) V_{TH}
\]

\[
V_{TL} = \frac{R_1}{R_1 + R_2} V_{OL} \quad \text{or} \quad V_{OL} = \left(1 + \frac{R_2}{R_1}\right) V_{TL}
\]

HYSTERESIS WIDTH:
\[
\Delta V_T = V_{TH} - V_{TL} = \frac{R_1}{R_1 + R_2} (V_{OH} - V_{OL})
\]

or
\[
V_{OH} - V_{OL} = \left(1 + \frac{R_2}{R_1}\right) \Delta V_T
\]

(See Section 8.3)
**Numerical Example**

\[ V_{OH} = 13V \quad j \quad V_{OL} = -13V \]

For \( R_2 = 16\, k\Omega \) and \( R_1 = 10\, k\Omega \)

\[ V_{TH} = \frac{13}{26} \times 10 = 5V \]

\[ V_{TL} = \frac{-13}{26} \times 10 = -5V \]

**Let us consider the Non-Inverting Schmitt Trigger**

\[ V_{TH} = -\frac{R_1}{R_2} V_{OL} \iff V_{TH} - 0 = \frac{V_{TH} - 0}{R_1} = 0 - V_{OL} \]

\[ V_{TL} = -\frac{R_1}{R_2} V_{OH} \]

\[ \Delta V_T = \frac{R_1}{R_2} (V_{OH} - V_{OL}) \] **Hysteresis Window**
**Schmitt Trigger in I/O in First Quadrant and Having a Single Supply**

Using Superposition

\[ V_P = \frac{R_{1,3}}{R_{1,3} + R_2} V_{CC} + \frac{R_{1,2}}{R_{1,2} + R_3} V_0 \]

Where \( R_{1,3} = R_1 || R_3 \) and \( R_{1,2} = R_1 || R_2 \)

If \( V_{OH} \leq V_{CC} \) and \( V_{OL} = 0 \)

Then \( R_4 \ll R_3 + R_{1,2} \)

Thus, imposing \( V_P = V_{TL} \) for \( V_0 = V_{OL} = 0 \) and \( V_P = V_{TH} \) for \( V_0 = V_{OH} = V_{CC} \), we get

\[ V_{TL} = \frac{R_{1,3}}{R_{1,3} + R_2} V_{CC} \quad V_{TH} = \frac{R_1}{R_1 + R_{2,3}} V_{CC} \]
Rearranging terms and fix $R_4$ and $R_3 >> R_4$

\[
\frac{1}{R_2} = \frac{V_{TL}}{V_{CC} - V_{TL}} \left( \frac{1}{R_1} + \frac{1}{R_3} \right)
\]

\[
\frac{1}{R_1} = \frac{V_{CC} - V_{TH}}{V_{TH}} \left( \frac{1}{R_2} + \frac{1}{R_3} \right)
\]

Thus the two above equations can be solved.

The BP can be implemented using the Tow–Thomas
Wien-Bridge

\[ A = \frac{V_o}{V_p} = 1 + \frac{R_2}{R_1} \]

Furthermore, we can see as a voltage divider

\[ \frac{Z_p}{Z_p + 2s} V_o = V_p \]

\[ Z_p = \frac{R}{1 + sRC} \]

\[ Z_s = \frac{1 + sRC}{SC} \]

Thus

\[ \frac{V_o}{V_p} = \frac{Z_p + 2s}{Z_p} \]

\[ B = \frac{V_p}{V_o} = \frac{Z_p}{Z_p + 2s} = \frac{1}{1 + \frac{2s}{Z_p}} \]
\[ B = \frac{1}{1 + \frac{2s}{2p}} = \frac{1}{1 + \frac{(1+sec)^2}{sec}} \]

\[ B = \frac{SRC}{(SRC)^2 + 3SRC + 1} \implies \text{Band Pass Filter} \]

\[ 1 - AB = 1 - T \]

\[ T = AB = \frac{(1 + \frac{P_2}{P_1}) \cdot RC}{(SRC)^2 + 3SRC + 1} \]

\[ T = \frac{1 + \frac{P_2}{P_1}}{SRC + 3 + \frac{1}{SRC}} \]

If \( RC = \frac{1}{\omega_0} \)

\[ T(j\omega) = \frac{1 + \frac{P_2}{P_1}}{1 + \frac{P_2}{P_1}} = \frac{1 + \frac{P_2}{P_1}}{3 + \frac{1}{\omega/\omega_0}} \]

\[ T(j\omega_0) = \frac{1 + P_2/P_1}{3} \]

To force oscillation

\[ T(j\omega_0) > 3 \quad \text{i.e.} \quad \frac{P_2}{P_1} > 2 \]

\[ \frac{P_2}{P_1} = 2 + \epsilon \]
The non-linear part of the oscillator must be added.
SQUARE WAVE OSCILLATOR

\[ V_{TH} = \frac{R_1}{R_1 + R_2} \cdot V_{OH} \]

\[ V_{TL} = \frac{R_1}{R_1 + R_2} \cdot V_{OL} \]

\[ \Delta V_T = V_{TH} - V_{TL} = \frac{R_1}{R_1 + R_2} (V_{OH} - V_{OL}) \]

\[ \Delta V_T = \frac{1}{1 + \frac{R_2}{R_1}} (V_{OH} - V_{OL}) \]
Non-Inverting Schmitt Trigger

\[ V_{TH} = -\frac{R_1}{R_2} V_{OL} \]

Tripping voltage \( V_{TL} \) becomes

\[ \frac{V_{OH} - 0}{R_2} = \frac{0 - V_{TL}}{R_1} \]

or

\[ V_{TL} = -\frac{R_1}{R_2} V_{OH} \]

\[ \Delta V_T = \frac{R_1}{R_2} (V_{OH} - V_{OL}) \]

\[ \Delta V_T = \frac{1}{\frac{R_2}{R_1}} (V_{OH} - V_{OL}) \]
Notes on Relaxation Oscillators

\[
\Delta V \text{ or } V(t) = \Delta V \cdot t
\]

\[
\Delta t = \frac{C}{I} \Delta V
\]

Linear

\[
C \Delta V = I \Delta t
\]

Exponential

\[
\frac{\Delta t}{t_0} = \frac{t - t_0}{\tau}
\]

\[
V(t) = V_\infty + (V_0 - V_\infty) e^{-\frac{t-t_0}{\tau}}
\]

\[
\tau = RC
\]

\[
V_1 = V_\infty + (V_0 - V_\infty) e^{-\frac{x_1-t_0}{\tau}}
\]

Thus for the basic-free-running oscillator

\[
\frac{1}{T} = RC \ln \frac{V_{sat} + V_t}{V_{sat} - V_t}
\]

\[
V_t = \frac{V_{sat}}{1 + \frac{R_2}{R_1}}
\]

Then

\[
f_0 = \frac{1}{T} = \frac{1}{2RC \ln (1 + \frac{R_2}{R_1})}
\]
\[ f_0 \neq f(\text{VT}, \text{V}_{\text{SAT}}) \]
SQUARE WAVE GENERATOR

\[ V_{TH} = \frac{R_g}{R_g + R_f} V_{OH} \]

or

\[ V_{TL} = \frac{R_g}{R_g + R_f} V_{OL} \]

HIGH TO LOW:

Assume the output voltage is at \( V_{OH} \), then the non-inverting input is at \( V_{TH} \), and the inverting is in exponential transition from its initial value \( V_{TL} \) toward \( V_{OH} \). Thus

\[ V^{-}(t) = V_{OH} - (V_{OH} - V_{TL}) e^{-t/RC} \]

or equivalent

\[ V^{-}(s) = \frac{V_{OH}}{s} - \frac{(V_{OH} - V_{TL})}{1 + s/RC} \]

The total exponential transition time is the solution to the equality

\[ V^{-}(t) = \frac{V_{OH}}{R_g} \]

or

\[ V_{OH} - (V_{OH} - \frac{R_g}{R_g + R_f} V_{OL}) e^{-t/RC} = \frac{R_g}{R_g + R_f} V_{OH} \]

Note: See Section 10.2 Text
Then

\[ V_{OH} \left( 1 - \frac{R_f}{R_g + R_f} \right) = \left( V_{OH} - \frac{R_g}{R_g + R_f} V_{OL} \right) e^{-\frac{t}{RC}} \]

The transition time is given by

\[ t_{thL} = RC \ln \left( \frac{(R_g + R_f) V_{OH} - R_g V_{OL}}{R_f V_{OH}} \right) \]

\[ t_{thL} = RC \ln \left( 1 + \frac{R_g}{R_f} \frac{V_{TH} - V_{OL}}{V_{OH}} \right) \]

\[ t_{thL} = RC \ln \left( 1 + \frac{\Delta V_T}{V_{OH}} \right) \]
Low to High
Same as before by interchanging \( V_{OH} \) and \( V_{OL} \).

Then the duty cycle is determined by the sum of the transition times:

\[
\tau = RC \left[ \ln \left( 1 + \frac{R_S (V_{DH} - V_{OL})}{R_f V_{OH}} \right) + \ln \left( 1 + \frac{R_S (V_{OL} - V_{OH})}{R_f V_{OL}} \right) \right]
\]

\[
\tau = RC \left[ \ln \left( 1 + \frac{\Delta V_I}{V_{OH}} \right) + \ln \left( 1 - \frac{\Delta V_I}{V_{OL}} \right) \right]
\]

For 50% duty cycle \( V_{OL} = -V_{OH} \) yields

\[
t_{HL} = t_{LH} = RC \ln \left( 1 + 2 \frac{R_S}{R_f} \right)
\]

Then the period is given by

\[
T = 2t_{HL} = 2RC \ln \left( 1 + 2 \frac{R_S}{R_f} \right)
\]

\( T \) will be affected by the SR of the comparator. Also the parasitic capacitance associated to the "+" terminal of the comparator.

Note that \( V_{OL} \) and \( V_{OH} \) cannot be exactly predicted thus a clamping circuit is placed usually at the output.
A stable, nonsymmetric square-wave generator

- One option is by making \(-V_{OL} \neq V_{OH}\)
- Another option is by changing the equality of rising and decaying times.

Then one can show that

\[
T = \frac{\pi}{2} t_h + t_l
\]

\[
T = RC \ln \left(1 + \frac{2R_{S}}{R_f}\right) + RC \ln \left(1 + \frac{2R_{S}}{R_f}\right)
\]

Duty cycle \(\Delta = \frac{t_h}{T} = \frac{R}{R + R'}\)
BASIC TRIANGULAR/SQUARE WAVE GENERATOR

- Two quasi-static states: 
  - For Vol and Voh

- The Schmitt Trigger toggles between output states when VP is at ground potential.

Transition occurs

\[ V_{TP} = -\frac{R_f}{R_f} \cdot V_{SP} = \frac{R_f}{R_f} \cdot V_{OH} \]

\[ V_{TP} = -\frac{1}{RC} \int \left[ V_{SP}(t) - V_S \right] dt \]

When, \( V_{SP} \) is in the high state, \( V_{TP} \) is linearly decreasing between its toggle values:

\[ V_{TP} = -\frac{1}{RC} \int (V_{OH} - V_S) dt = \frac{V_S - V_{OH}}{RC} \cdot t + V_{TP}(0) \]

\( \Rightarrow \) Initial condition
Thus, the transition is the differences in toggle values divided by the slope of the linear transition.

\[ 2 \frac{R_g}{R_f} V_{OH} = \frac{V_{OH} - V_s}{RC} \]

Thereforē, \[ t_- = \frac{2R_g V_{OH} \cdot RC}{R_f (V_{OH} - V_s)} \]

Now when the positive transition occurs for \( V_s > 0 \)

\[ V_{SR} = -\frac{1}{RC} \int -(V_{OH} + V_s) dt = \frac{V_{OH} + V_s}{RC} t + V_s \]

\[ 2 \frac{R_g}{R_f} V_{OH} = \frac{V_{OH} + V_s}{RC} t_+ \Rightarrow t_+ = \frac{2R_g RC V_{OH}}{R_f (V_{OH} + V_s)} \]

Thus the period becomes:

\[ \tau = t_- + t_+ = 4R_g RC \frac{V_{OH}^2}{R_f (V_{OH}^2 - V_s^2)} \]

Duty cycle \( + \) \[ = \frac{t_+}{\tau} = \frac{1}{2} \left( 1 - \frac{V_s}{V_{OH}} \right) \]