Op-Amps Stability and Frequency Compensation Techniques

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Stability of Linear Systems

■ Harold S. Black, 1927 → Negative feedback concept

■ Negative feedback provides:
  • Gain stabilization
  • Reduction of nonlinearity
  • Impedance transformation

■ But also brings:
  • Potential stability problems
  • Causes accuracy errors for low dc gain

■ Here we will discuss frequency – compensation techniques
Stability Problem

- Feedback forces $x_d$ to become smaller
- It takes time to detect $x_o$ and feedback to the input
- $x_d$ could be overcorrected (diverge and create instability)
- How to find the optimal (practical) $x_d$ will be based on frequency compensation techniques

$$H_{CL}(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{1/ \beta}{1 + \frac{1}{\beta A(s)}} = \frac{A(s)}{1 + T(s)}$$
**Gain Margin & Phase Margin**

- $G_M$ is the number of dBs by which $|T(j\omega_{-180^\circ})|$ can increase until it becomes 0 dB

  $$G_M = 20 \log \frac{1}{|T(j\omega_{-180^\circ})|}$$

- Phase margin $\phi_m$ is the number of degrees by which $\angle T(j\omega_x)$ can be reduced until it reaches $-\pi (-180^\circ)$

  $$\phi_m = 180^\circ + \angle T(j\omega_x)$$

  or

  $$\angle T(j\omega_x) = \phi_m - 180^\circ$$

* $\omega_x$ is the crossover frequency
Gain Margin & Phase Margin

- At the crossover point,
  \[ T(j\omega_x) = 1 \cdot \angle T(j\omega_x) = 1 \cdot \angle (\phi_m - 180^\circ) = -e^{j\phi_m} \]

- The non-ideal closed loop transfer function becomes

  \[
  H_{CL}(j\omega_x) = \frac{A(j\omega_x)}{1 + \beta A(j\omega_x)} = \frac{A(j\omega_x)}{1 + T(j\omega_x)} = \frac{A_{ideal}}{1 + 1/T(j\omega_x)} = \frac{A_{ideal}}{1 - e^{-j\omega_x}} = \frac{A_{ideal}}{1 - (\cos \phi_m - j \sin \phi_m)}
  \]

  \[
  |H_{CL}(j\omega_x)| = \left| A_{Ideal} \right| \cdot \frac{1}{\sqrt{(1 - \cos \phi_m)^2 + \sin^2 \phi_m}}, A_{Ideal} = \frac{1}{\beta}
  \]
Observe that different $\phi_m$ yield different errors. i.e.

| $\phi_m$ | $|H_{CL}(j\omega_x)|$ |
|----------|---------------------|
| 90°      | 0.707               |
| 60°      | 1.00                |
| 45°      | 1.31                |
| 30°      | 1.93                |
| 15°      | 3.83                |
| 0°       | $\infty$ (oscillatory behavior) |

- In practical systems, $\phi_m = 60^\circ$ is required
- A worst case $\phi_m = 45^\circ$ for a typical lower limit
- For $\phi_m < 60^\circ$, we have $|A(j\omega_x)| > |A_{ideal}|$
  indicating a peaked closed-loop response.
Why a dominant pole is required for a stable amplifier?

A good Op amp design implies:
(i) \(|\beta A(j\omega)| > 1\) over as wide a band of frequencies as possible
(ii) The zeroes of \(\beta A(j\omega) - 1 = 0\) must be all in the left-hand plane

Note that \(\beta A(j\omega) < 0\)

These two conditions often conflict with each other. These trade-offs should be carefully considered. Let’s consider a practical amplifier characterized with these poles,

\[
A(s) = -\frac{A_0}{(1 + s / \alpha_1)(1 + s / \alpha_2)(1 + s / \alpha_3)} = \frac{-A_0\alpha_1\alpha_2\alpha_3}{(s + \alpha_1)(s + \alpha_2)(s + \alpha_3)}
\]

The characteristic equation becomes

\[
\beta A(s) - 1 = \frac{-\beta A_0\alpha_1\alpha_2\alpha_3}{(s + \alpha_1)(s + \alpha_2)(s + \alpha_3)} - 1 = 0
\]

\[
(s + \alpha_1)(s + \alpha_2)(s + \alpha_3) + \beta A_0\alpha_1\alpha_2\alpha_3 = 0
\]
Critical Value of $\beta A_0$

\[ s^3 + s^2(\alpha_1 + \alpha_2 + \alpha_1) + s(\alpha_1 \alpha_2 + \alpha_1 \alpha_3 + \alpha_2 \alpha_3) + \alpha_1 \alpha_2 \alpha_3 (1 + \beta A_0) = 0 \]

Note that $\beta A_0$ is the critical parameter that determines the pole locations for a given $\alpha_1$, $\alpha_2$ and $\alpha_3$ ($0 \leq \beta \leq 1$). Furthermore, when $\beta A_0 = 0$, the roots are at $-\alpha_1$, $-\alpha_2$ and $-\alpha_3$. Therefore, for small $\beta A_0$, the roots should be in the left-hand plane (LHP). However, for $\beta A_0 >> 1$, two of the roots might be forced to move to the right-hand plane (RHP). This can be verified by applying Routh’s stability criterion. Let us write the polynomial as

\[ b_3 s^3 + b_2 s^2 + b_1 s + b_0 = 0 \]

In order to have, in the above equation, left half plane roots, all the coefficients must be positive and satisfy

\[ b_2 b_1 - b_3 b_0 = 0 \]
Critical Value of $\beta A_0$

The condition for imaginary-axis roots become

$$b_3 (j\omega)^3 + b_2 (j\omega)^2 + b_1 (j\omega) + b_0 = 0$$

$$\left(b_0 - b_2 \omega^2\right) + j\omega \left(b_1 - b_3 \omega^2\right) = 0$$

Now, for $s=j\omega$ being a root, both real and imaginary parts must be zero. That is,

$$b_0 - b_2 \omega^2 = 0, \quad b_1 - b_3 \omega^2 = 0 \quad \text{or} \quad b_3 b_0 = b_1 b_2$$

Then the two roots are placed at

$$\omega_{p_{2,3}} = \pm j \sqrt{\frac{b_0}{b_2}} = \pm j \sqrt{\frac{b_1}{b_3}}$$
Critical Value of $\beta A_0$

\[ b_0 = \frac{b_2 b_1}{b_3} \]

Then

\[ \alpha_1 \alpha_2 \alpha_3 [1 + (\beta A_0)_C] = (\alpha_1 + \alpha_2 + \alpha_3)(\alpha_1 \alpha_2 + \alpha_1 \alpha_3 + \alpha_2 \alpha_3) \]

Thus, the critical value of $\beta A_0$ becomes

\[ (\beta A_0)_C = 2 + \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1}{\alpha_3} + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_3} + \frac{\alpha_3}{\alpha_1} + \frac{\alpha_3}{\alpha_2} \]

Thus when $\beta A_0$ becomes $(\beta A_0)_C$, the amplifier will oscillate at

\[ \omega_{OSC} = \sqrt{\frac{b_1}{b_3}} = \sqrt{\alpha_1 \alpha_2 + \alpha_1 \alpha_3 + \alpha_2 \alpha_3} \]

Also when $\beta A_0 > (\beta A_0)_C$, the amplifier has RHP poles, therefore is unstable.
**Critical Value of $\beta A_0$**

Let us consider some numerical examples. Let $A_0 = 10^5$,

(i) Three equal poles $\alpha_1 = \alpha_2 = \alpha_3 = 10^7 \text{rad/s}$.

The amplifier oscillates at $\omega_{osc} = \alpha_1 \sqrt{3} = 10^7 \sqrt{3} \text{rad/s}$

\[
(\beta A_0)_c = 8, \quad \beta_c = 8 / A_0 = 8 \times 10^{-5}
\]

(ii) $\alpha_1 = \frac{\alpha_2}{10^4} = \frac{\alpha_3}{10^4}$, then the critical loop gains yields

\[
(\beta A_0)_c \approx 2 \frac{\alpha_2}{\alpha_1} = 2 \times 10^4, \text{ thus the amplifier is stable if } \beta_c < \frac{2 \times 10^4}{A_0} = 0.2
\]

Since $\beta = R_1 / (R_1 + R_2)$, $\beta < 0.2$ causes $(R_2 / R_1) > 4$, which means that for an inverting (non-inverting) configuration, the gain must be greater than -4 (5) to keep the amplifier stable.
(iii) Let us determine $A_{0c}$ under the most stringent condition $\beta=1$. Then from previous equation,

$$A_0 < 2 + \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1}{\alpha_3} + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_3} + \frac{\alpha_3}{\alpha_1} + \frac{\alpha_3}{\alpha_2}$$

In order to have a large $A_0$, the poles must be widely separated. i.e. $\alpha_1 << \alpha_2 << \alpha_3$, then the $A_0$ inequality can be approximated as

$$A_0 < \frac{\alpha_2}{\alpha_1} + \frac{\alpha_3}{\alpha_1}$$

To obtain a conservative $A_0$, let $\alpha_2 = \alpha_3$, which yields

$$A_0 < 2 \frac{\alpha_2}{\alpha_1}$$

This inequality bounds the DC gain to provide a stable closed loop configuration.
Peaking and Ringing

- Peaking in the frequency domain usually implies ringing in the time domain
- Normalized second-order all-pole (low pass) system

\[
H(s)\bigg|_{s=j\omega} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2} + j \frac{1}{Q} \cdot \frac{\omega}{\omega_0}}
\]
Peaking and Ringing

**GP:** Peak gain – We have the error function,

\[
|E(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \frac{1}{Q^2} \cdot \frac{\omega^2}{\omega_0^2}}} = \frac{1}{\sqrt{D(\omega)}}
\]  

(1)

To find out the maximum value of \(|E(j\omega)|\), calculate the derivative of \(D(\omega)\) and make it equal to zero

\[
\frac{d}{d\omega} D(\omega) = 2\left(\frac{1}{Q^2} - 2\right) \frac{\omega}{\omega_0^2} + 4\frac{\omega^3}{\omega_0^4} = 0 \quad \Rightarrow \quad \omega_*^2 = \left(1 - \frac{1}{2Q^2}\right) \omega_0^2 \quad \text{OR} \quad \omega = 0
\]

For \(Q > 1/\sqrt{2}\), use \(\omega_*\) in Eq. (1) and we get the peak gain

\[
GP = \frac{2Q^2}{\sqrt{4Q^2 - 1}} \geq |E(j0)| = 1
\]  

(2)
Peaking and Ringing

**OS: Overshoot – Inverse Laplace transform**

\[
\begin{align*}
\text{s-domain: } & \quad \frac{b}{(s + a)^2 + b^2} \xrightarrow{\text{Inverse Laplace}} \text{time-domain: } e^{-at} \sin(bt)u(t) \\
\end{align*}
\]

For a 2\textsuperscript{nd} order all pole error function, the \textbf{impulse response} is

\[
H(s)\bigg|_{s=j\omega} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{\omega_0^2}{b} \cdot \frac{b}{(s + a)^2 + b^2}
\]

\[
v_{in}(t) = \delta(t)
\]

\[
a = \frac{\omega_0}{2Q}
\]

\[
b = \sqrt{1 - \frac{1}{4Q^2}\omega_0}
\]

Consider the \textbf{normalized step response} of this system,

\[
y(t) = \int_0^t h(t)dt = 1 - \frac{\omega_0}{b} e^{-at} \sin(bt + \phi), \phi = \tan^{-1} \frac{b}{a} \quad \text{for } v_{in}(t) = u(t) = \int \delta(t)dt \quad (3)
\]

Use the damping factor \(\xi\) to represent, \( a = \xi\omega_0, b = \sqrt{1 - \xi^2\omega_0} \)
Peaking and Ringing

Usually, the damping factor $\xi=1/2Q$ is used to characterize a physical 2\textsuperscript{nd} order system. Thus, we rewrite the normalized time-domain equation (3)

$$y(t) = 1 - \frac{\omega_0}{b} e^{-at} \sin(bt + \phi) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right)$$

$$\phi = \tan^{-1} \frac{b}{a} = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

For under damped case $\xi < 1$ ($Q > 0.5$), the overshoot is the peak value of $y(t)$. To find the peak value, we first calculate the derivative of $y(t)$, and make it equal to 0.

$$\frac{d}{d(\omega_0 t)} y(t) = -\frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right)$$

$$+ e^{-\xi\omega_0 t} \cos\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right)$$
Peaking and Ringing

\[
\frac{d}{d(\omega_0 t)} y(t) = 0
\]

\[
\Rightarrow \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi \omega_0 t} \sin\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right) = e^{-\xi \omega_0 t} \cos\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right)
\]

\[
\Rightarrow \tan\left(\sqrt{1-\xi^2} \omega_0 t + \phi\right) = \frac{\sqrt{1-\xi^2}}{\xi} = \tan \phi
\]

To satisfy the equality above, we have

\[
\omega_0 t = \frac{n\pi}{\sqrt{1-\xi^2}}, n = 0, 1, 2, ...
\]

In other words, the **normalized step response** \( y(t) \) in Eq.(3) achieves **extreme values** at time steps of \( n=0, 1, 2, \ldots \)
Peaking and Ringing

\[ n=0 \quad n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \quad n=6 \]

\[ \frac{\pi}{\omega_0 \sqrt{1-\xi^2}} \quad \frac{2\pi}{\omega_0 \sqrt{1-\xi^2}} \quad \frac{3\pi}{\omega_0 \sqrt{1-\xi^2}} \quad \frac{4\pi}{\omega_0 \sqrt{1-\xi^2}} \quad \frac{5\pi}{\omega_0 \sqrt{1-\xi^2}} \quad \frac{6\pi}{\omega_0 \sqrt{1-\xi^2}} \]

Amplitude vs. Time

- Peak Value
- \( Q=5 \)


Peaking and Ringing

\[
n = 0, \quad \omega_0 t = 0, \quad y(t) = 0
\]

\[
n = 1, \quad \omega_0 t = \frac{\pi}{\sqrt{1 - \xi^2}}, \quad y(t) = 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\frac{\pi \xi}{\sqrt{1 - \xi^2}}} \sin(\pi + \phi) = 1 + e^{-\frac{\pi \xi}{\sqrt{1 - \xi^2}}}
\]

\[
n = 2, \quad \omega_0 t = \frac{2\pi}{\sqrt{1 - \xi^2}}, \quad y(t) = 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\frac{2\pi \xi}{\sqrt{1 - \xi^2}}} \sin(2\pi + \phi) = 1 - e^{-\frac{2\pi \xi}{\sqrt{1 - \xi^2}}}
\]

\[
n = 3, \quad \omega_0 t = \frac{3\pi}{\sqrt{1 - \xi^2}}, \quad y(t) = 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\frac{3\pi \xi}{\sqrt{1 - \xi^2}}} \sin(3\pi + \phi) = 1 + e^{-\frac{3\pi \xi}{\sqrt{1 - \xi^2}}}
\]

\[\ldots \ldots \ldots \ldots\]

Therefore, the global peak value of \(y(t)\) is achieved when \(n=1\).

Thus, the overshoot is defined as

\[
OS(\%) = 100 \times \frac{\text{Peak Value} - \text{Final Value}}{\text{Final Value}} = 100 \times e^{-\frac{\pi \xi}{\sqrt{1 - \xi^2}}} \quad (5)
\]
Peaking and Ringing

- $\phi_m$: Phase Margin

For a 2nd order all-pole error function

$$E(s) = \frac{T(s)}{1+T(s)} = \frac{1}{1+1/T(s)} = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{1}{Q}\frac{s}{\omega_0} + 1}$$

Therefore, the loop gain $T(s)$ is given by

$$T(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{1}{Q}\frac{s}{\omega_0}}$$

The cross-over frequency thus can be obtained

$$|T(j\omega_x)| = \left(\sqrt{\left(\frac{\omega_x}{\omega_0}\right)^2 + \left(\frac{\omega_x}{\omega_0Q}\right)^2}\right)^{-1} = 1$$
Peaking and Ringing

Solve the equation and get the crossover frequency,

\[ \omega_x = \omega_0 \left( \sqrt{\frac{1}{4Q^4} + 1} - \frac{1}{2Q^2} \right)^{1/2} = \omega_0 \left( \sqrt{4\xi^4 + 1} - 2\xi^2 \right)^{1/2} \]

And thus the phase margin is

\[ \phi_m = 180^\circ + \angle T(j\omega_x) = \cos^{-1}\left( \sqrt{\frac{1}{4Q^4} - \frac{1}{2Q^2}} \right) = \cos^{-1}\left( \sqrt{4\xi^4 + 1} - 2\xi^2 \right) \]

Study the relationship between phase margin and gain peaking (Eq.2) or overshoot (Eq.5), we have

\[
\begin{align*}
GP(60^\circ) & \approx 0.3dB & OS(60^\circ) & \approx 8.8\% & Q & \approx 0.82 \\
GP(45^\circ) & \approx 2.4dB & OS(45^\circ) & \approx 23\% & Q & \approx 1.18
\end{align*}
\]
Peaking and Ringing
One effective method of assessing stability for minimum phase systems from the magnitude Bode plots is by determining the ROC.

**The Rate of Closure (ROC)**

Determining the ROC is done by observing the slopes of $|A|$ and $|1/\beta|$ at their intersection point (cross-over frequency $f_x$) and deciding the magnitude of their difference.

$$ROC = \left| \text{Slope}(|A|) - \text{Slope}(1/\beta) \right|_{f=f_x}$$

The ROC is used to estimate the phase margin and therefore the stability (How?)
The Rate of Closure (ROC)

- Observing a single-root transfer function

\[ H(jf) = \frac{1}{1 + jf / f_0} \]

- For \( f \leq f_0 / 10 \), \( \text{Slope}(|H|) \to 0 \text{dB/dec} \) \( \angle H \to 0^\circ \)
- For \( f > 10f_0 \), \( \text{Slope}(|H|) \to -20 \text{dB/dec} \) \( \angle H \to -90^\circ \)
- For \( f = f_0 \), \( \text{Slope}(|H|) \to -10 \text{dB/dec} \) \( \angle H \to -45^\circ \)

- **Empirical Equation**

\[ \angle H \approx 4.5 \times \text{Slope}(|H|) \]

This correlation holds also if \( H(s) \) has more than one root, provided the roots are real negative, and well separated, say, at least a decade apart.
The Rate of Closure (ROC)

- In a feedback system, suppose both $|A|$ and $|1/\beta|$ have been graphed.

$$\angle T(jfx) = \angle A(jfx) - \angle \beta^{-1}(jfx)$$

$$\cong -4.5 \times ROC$$

Thus, the ROC can be used to estimate the phase margin (Page 4)

$$\phi_m = 180^\circ + \angle T(jfx)$$

- Cases

  - $ROC \cong 20\,dB/\,dec \implies \phi_m \cong 90^\circ$
  - $ROC \cong 30\,dB/\,dec \implies \phi_m \cong 45^\circ$
  - $ROC \cong 40\,dB/\,dec \implies \phi_m \cong 0^\circ$
  - $ROC > 40\,dB/\,dec \implies \phi_m < 0^\circ$
The Rate of Closure (ROC)

\[ A(s) = \frac{A_{ol}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)} \]

- * 20dB/dec ROC → “Stability”
- ** 40dB/dec ROC → “Marginal Stability”
Stability in Constant-GBP OpAmp

- Constant-GBP OpAmp (i.e. \( A(s)|_{s=j\omega} = \frac{\omega}{j\omega} \))
  - Unconditionally stable with frequency-independent feedback, or \( \angle \beta = 0 \). (e.g. in a non-inverting or inverting amplifier, the feedback network contains only resistors)
  - Stable for any \( \beta \leq 1 \).
  - In feedback systems, since now we have
    \[ \angle T = \angle (A\beta) = \angle A \text{ , } \angle A \approx -90^\circ \]
    these circuits enjoy
    \[ \phi_m = 180^\circ + \angle A(jf_x) \approx 180^\circ - 90^\circ \approx 90^\circ \]
  - Typically, due to additional high-order poles in OpAmps,
    \[ 60^\circ \leq \phi_m \leq 90^\circ \]
Feedback Pole

- Feedback Pole
  Feedback network includes reactive elements $\rightarrow$ Stability may no longer be unconditional

  $$\beta(jf) = \frac{\beta_0}{1 + jf / f_p}$$

- A pole $f_p$ (or a zero) of $\beta$ becomes a zero $f_z$ (or a pole) for $1/\beta$.

- For the case $f_z << \beta_0f_t$

The effect of a pole within the feedback loop
Feedback Pole

Examine the error function

\[ E(s) = \frac{H_{CL}(s)}{A_{ideal}} = \frac{1}{1+1/T}, \quad T = A\beta, \quad A_{ideal} = \frac{1}{\beta} \]

Using OpAmp high-frequency approximation: \( A(j\omega) = \frac{GB}{j\omega} = \frac{f_t}{jf} \)

\[ E(s) = \frac{1}{1+\frac{1}{A\beta}} = \frac{1}{1+j\frac{f}{f_i\beta_0} - \frac{f^2}{f_i\beta_0f_z}} \]

Refer to page 14, and we have \( s=j2\pi f \). The peak value of \( E(s) \) can be obtained. For \( Q >> 1 \), the approximate result is

\[ f_x = \sqrt{f_z\beta_0f_t}, Q = \sqrt{\beta_0f_t / f_z} \]
Feedback Pole

- The lower $f_z$ compared to $\beta_0 f_t$, the higher the Q and, hence, the more pronounced the peaking and ringing.

- Derive the phase margin

\[ \angle T(jf_x) = \angle A(jf_x) - \angle|1/\beta(jf_x)| \approx -90^\circ - \tan^{-1}\left(f_x/f_z\right) \]

\[ \frac{f_x}{f_z} = \sqrt[\beta_0 f_t]{f_z} \]

- As $f_z << \beta_0 f_t$, $\angle T(jf_x) \approx -180^\circ$ and ROC = 40dB/dec

  The circuit is on the verge of oscillation!
**Differentiator**

- Feedback pole example: differentiator

- Assume constant-GBP OpAmp
  
  \[ A(jf) \approx GB \frac{1}{j\omega} f \]

  \[ \beta = \frac{Z_C}{Z_C + R} = \frac{1}{1 + jf/f_z} \]

  \[ Z_C = \frac{1}{j\omega C} \]

- To stabilize the differentiator, add a series resistance Rs.
Differentiator

- At low frequency, $R_S$ has little effect because $R_S << |1/j\omega C|$
- At high frequency, $C$ acts as a short compared to $R_S$, the feedback network becomes $|1/\beta| = 1 + R/R_S$. 

![Differentiator Diagram](image-url)
**Differentiator**

- Assume $R_S << R_C$, the series resistor $R_S$ introduces an extra pole frequency $f_e$

\[
\frac{1}{\beta} = \frac{Z_C + R}{Z_C} \approx \frac{1}{\beta_0} \frac{1 + jf / f_z}{1 + jf / f_e}
\]

\[
Z_C = \frac{1}{j\omega C} + R_S, \beta_0 = 1, f_x = \sqrt{f_t f_z}
\]

- Choose $R_S \approx R / \sqrt{f_t / f_z}$, we have $f_e = \sqrt{f_t f_z}$

\[
\angle T(jf_x) = \angle A(jf_x) - \angle \frac{1}{\beta(jf_x)}
\]

\[
\approx -90^\circ + \tan^{-1}(f_x/f_e) - \tan^{-1}(f_x/f_z)
\]

\[
= -135^\circ
\]

Therefore, ROC = 30 dB/dec, $\phi_m \approx 45^\circ$
**Stray Input Capacitance Compensation**

- All practical OpAmps exhibit stray input capacitance. The net capacitance $C_n$ of the inverting input toward ground is

$$C_n = C_d + C_c / 2 + C_{ext}$$

$C_d$ is the differential Cap between input pins, $C_c/2$ is the common-mode cap of each input to ground, and $C_{ext}$ is the external parasitic cap.

- In the absence of $C_f$, there’s a pole in feedback

$$\frac{1}{\beta} = (1 + R_2 / R_1)(1 + j\beta [2\pi \left(\frac{R_1}{R_2}\right)C_n])$$

ROC ≈ 40 dB/dec

(See page 29)
Stray Input Capacitance Compensation

- **Solution:** Introduce a feedback capacitance $C_f$ to create feedback phase lead.

- In the presence of $C_f$ we have

$$\frac{1}{\beta} = \left(1 + \frac{R_2}{R_1}\right) \frac{1 + jf/f_z}{1 + jf/f_p}$$

$$f_z = \frac{1}{2\pi(c_n+c_f)(R_1||R_2)}, \quad f_p = \frac{1}{2\pi c_f R_2}$$

- To have $\phi_m = 45^\circ$ (i.e. ROC=30 dB/dec):

Make the cross-over frequency exactly at $f_p$

$$|A(jf_p)| = \left|\frac{1}{\beta(jf_p)}\right| \approx \frac{1}{\beta_\infty} \rightarrow |A(jf_p)| \approx 1 + \frac{c_n}{c_f}$$

Since

$$|A(jf_p)| = \frac{f_t}{f_p} \rightarrow \frac{1}{f_p} = 2\pi C_f R_2 = \frac{1}{f_t} \left(1 + \frac{c_n}{c_f}\right)$$

Solve the equation to get $C_f$

$$C_f = \frac{1 + \sqrt{1 + 8\pi R_2 C_n f_t}}{4\pi R_2 f_t}$$
Stray Input Capacitance Compensation

To have $\phi_m = 90^\circ$ (i.e. ROC=20 dB/dec):

Place $f_p$ exactly on the top of $f_z$ to cause a pole-zero cancellation

$$f_z = f_p$$

$$\left( C_n + C_f \right) \left( R_1 \parallel R_2 \right) = C_f R_2$$

Thus using simple algebra

$$C_f = \frac{R_1}{R_2} C_n \quad (Neutral\ Compensation)$$
Capacitive-Load Isolation

- There’re applications in which the external load is heavily capacitive.

- Load capacitance $C_L$
  - A new pole is formed with output resistance $r_o$ and $C_L$
  - Ignore loading by the feedback network
  - The loaded gain is $A_{\text{loaded}} \approx A\left(1 + j\frac{f}{f_p}\right)^{-1}, f_p = \left(2\pi r_o C_L\right)^{-1}$
  - ROC is increased and thus invite instability

- Solution: Add a small series resistance $R_S$ to decouple the output from $C_L$
Capacitive-Load Isolation

\[ R_S = \frac{R_1}{R_2} r_o \]

\[ C_f = \left(1 + \frac{R_1}{R_2}\right)^2 \left(\frac{r_o}{R_2}\right) C_L \]
Uncompensated OpAmp

- The poles of the uncompensated OpAmp are located close together thus accumulating about 180° of phase shift before the 0-dB crossover frequency $f_x$.
- Unstable device, thus efforts must be done to stabilize it.
- Example of uncompensated OpAmps is 748, which is the uncompensated version of 741.
- They can be approximated as a three-pole system

\[
a(jf) = \frac{a_0}{(1 + jf/f_1)(1 + jf/f_2)(1 + jf/f_3)}
\]

Three-pole OpAmp model
Stability of Uncompensated OpAmp

- With a frequency-independent feedback (i.e. $1/\beta$ curve is flat) around an uncompensated OpAmp, we have
  $$|T| = |a|\beta$$
  $|T|$ curve can be visualized as the $|a|$ curve with the $1/\beta$ line is the new 0-dB axis
  - For $1/\beta \geq |a(jf_{-135^\circ})|$
    ROC $\leq 30$ dB/dec
    $\phi_m \geq 45^\circ$
  - For $|a(jf_{-180^\circ})| \leq 1/\beta < |a(jf_{-135^\circ})|$
    $30$ dB/dec $\leq$ ROC $\leq 40$ dB/dec
    $0^\circ \leq \phi_m \leq 45^\circ$
  - For $1/\beta \leq |a(jf_{-180^\circ})|$
    ROC $< 40$ dB/dec
    $\phi_m < 0^\circ$

![Three-pole open-loop response diagram](image-url)
The uncompensated OpAmp provides only adequate phase margin only in high-gain applications (i.e. high $1/\beta$)

To provide adequate phase margin in low-gain application, frequency compensation is needed.

- Internal compensation $\rightarrow$ Achieved by changing $a(jf)$
- External compensation $\rightarrow$ Achieved by changing $\beta(jf)$
How to stabilize the circuit by modifying the open loop response $a(jf)$?
- Dominant-Pole Compensation
- Shunt-Capacitance Compensation
- Miller Compensation
- Pole-Zero Compensation
- Feedforward Compensation
Dominant-Pole Compensation

- An additional pole at sufficiently low frequency is created to insure a roll-off rate of -20 dB/dec all the way up to the crossover frequency.

- Cases:
  - $f_{x(new)} = f_1$: 
    ROC = 30 dB/dec $\Rightarrow \phi_m = 45^\circ$
  - $f_{x(new)} < f_1$: 
    ROC = 20 dB/dec $\Rightarrow \phi_m \approx 90^\circ$

- This technique causes a drastic gain reduction above $f_d$
Dominant-Pole Compensation

- Numerical example:
  - \( r_d = \infty, r_0 = 0 \)
  - \( g_1 = 2 \) mA/V, \( R_1 = 100 \) k\( \Omega \), \( g_2 = 10 \) mA/V, \( R_2 = 50 \) k\( \Omega \)
  - \( f_1 = 100 \) kHz, \( f_2 = 1 \) MHz, \( f_3 = 10 \) MHz
  - Find the required value of \( f_d \) for \( \phi_m = 45^\circ \) with \( \beta = 1 \)

For \( \phi_m = 45^\circ \), we have

\[
f_x = f_1
\]

Draw a straight line of slope -20 dB/dec until it intercepts with the DC gain asymptote at point D and get \( f_d \).

\[
\frac{|a(j f_x(new))|}{|a(j f_d)|} = \frac{f_d}{f_x(new)}
\]

Thus:

\[
f_d = \frac{f_x(new)}{\beta a_0} = \frac{f_1}{\beta \times (g_1 R_1 g_2 R_2)} = 1 \text{ Hz}
\]

Requires EXTREMELY LARGE passive components
Shunt-Capacitance Compensation

- The dominant-pole technique adds a fourth pole → Extra cost and less bandwidth.
- This technique rearranges the existing rather than creating a new pole.
- It decreases the first (dominant) pole to sufficiently low frequency to insure a roll-off rate of -20 dB/dec all the way up to the crossover frequency.

\[
f_1 = \frac{1}{2\pi R_1(C_1 + C_c)}
\]

\[
f_2 = \frac{1}{2\pi R_2C_2}
\]

\[
f_3 = \frac{1}{2\pi R_3C_3}
\]
Shunt-Capacitance Compensation

Cases:

- \( f_x(\text{new}) = f_2 \):
  - ROC = 30 dB/dec \( \rightarrow \phi_m = 45^\circ \)

- \( f_x(\text{new}) < f_2 \):
  - ROC = 20 dB/dec \( \rightarrow \phi_m \approx 90^\circ \)

Since \( f_{1(\text{new})} \) is chosen to insure roll-off rate of -20 dB/dec all the way up to the crossover frequency.

Thus:

\[
\frac{|a(jf_x(\text{new}))|}{|a(jf_{1(\text{new})})|} = \frac{f_{1(\text{new})}}{f_x(\text{new})} \quad \rightarrow \quad f_{1(\text{new})} = \frac{f_x(\text{new})}{\beta a_0}
\]
Shunt-Capacitance Compensation

- The first pole is decreased by adding an extra capacitance to the internal node causing it.

- Given the value of $f_{1(new)}$ from the desired value of $\phi_m$, we can find $C_c$

\[
f_{1(new)} = \frac{f_x}{\beta a_0} = \frac{1}{2\pi R_1 (C_1 + C_c)} \rightarrow C_c \approx \frac{\beta a_0}{2\pi R_1 f_x}
\]
Shunt-Capacitance Compensation

**Numerical example:**

- \( r_d = \infty, \ r_o = 0 \)
- \( g_1 = 2 \text{ mA/V}, \ R_1 = 100 \text{ k}\Omega \)
- \( g_2 = 10 \text{ mA/V}, \ R_2 = 50 \text{ k}\Omega \)
- \( f_1 = 100 \text{ kHz}, \ f_2 = 1 \text{ MHz}, \ f_3 = 10 \text{ MHz} \)
- Find the required value of \( C_c \) for \( \phi_m = 45^\circ \) with \( \beta = 1 \)

![Circuit Diagram]

For \( \phi_m = 45^\circ \), \( f_x = f_2 = 1 \text{ MHz} \)

Then, \( f_{1(new)} = \frac{f_2}{a_0\beta} = \frac{f_2}{g_1R_1g_2R_2} = 10 \text{ Hz} \) → \( C_c = \frac{1}{2\pi R_1 f_{1(new)}} = 159 \text{ nF} \)

**EXEMPLARY LARGE**
Unsuitable for monolithic fabrication
Miller’s Theorem

If $A_v$ is the voltage gain from node 1 to 2, then a floating impedance $Z_F$ can be converted to two grounded impedances $Z_1$ and $Z_2$:

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} \Rightarrow Z_1 = Z_F \frac{V_1}{V_1 - V_2} = Z_F \frac{1}{1 - A_v}$$

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2} \Rightarrow Z_2 = -Z_F \frac{V_2}{V_1 - V_2} = Z_F \frac{1}{1 - \frac{1}{A_v}}$$
Applying Miller’s theorem to a floating capacitance connected between the input and output nodes of an amplifier.

\[
Z_1 = \frac{Z_F}{1 - A_v} = \frac{1}{j \omega C_F} = \frac{1}{j \omega (1 - A_v)C_F}
\]

\[
Z_2 = \frac{Z_F}{1 - \frac{1}{A_v}} = \frac{1}{j \omega \frac{1}{A_v}} = \frac{1}{j \omega \left(1 - \frac{1}{A_v}\right)C_F}
\]

The floating capacitance is converted to two grounded capacitances at the input and output of the amplifier.

The capacitance at the input node is larger than the original floating capacitance (Miller multiplication effect).
Miller Compensation

- This technique places a capacitor $C_c$ in the feedback path of one of the internal stages to take advantage of Miller multiplication of capacitors.

The reflected capacitances due to $C_c$ and the DC voltage gain between $V_2$ and $V_1$ ($a_2 = -g_2R_2$) yields

$$C_{1,c} = C_c(1 + g_2R_2) \quad \text{and} \quad C_{2,c} = C_c\left(1 + \frac{1}{g_2R_2}\right)$$

$$\approx |a_2|C_c \quad \text{and} \quad \approx C_c$$

- A low-frequency dominant pole can be created with a moderate capacitor value.
### Miller Compensation

#### Accurate transfer function

\[ \frac{v_2}{v_d} \approx g_1 R_1 g_2 R_2 \frac{1-jf/f_z}{(1-jf/f_1(new))(1-jf/f_2(new))} \]

![Miller Compensation Circuit Diagram](image)

#### Pole/zero locations

- **RHP zero**
  \[ \omega_z = \frac{g_2}{C_c} \]

- **Dominant Pole**
  \[ \omega_1(new) = \frac{1}{R_1 C_1 + g_2 R_2 R_1 C_c + R_2 C_2} \approx \frac{1}{R_1 g_2 R_2 C_c} = \frac{1}{|a_2| C_c R_1} \]

- **Second Pole**
  \[ \omega_2(new) = \frac{R_1 C_1 + R_1 g_2 R_2 C_c + R_2 C_2}{R_1 R_2 (C_1 C_c + C_1 C_2 + C_c C_2)} \approx \frac{g_2 C_c}{C_1 C_c + C_1 C_2 + C_c C_2} \]
**Miller Compensation**

**Right-half plane zero:**

- The RHP zero is a result of the feedforward path through $C_c$

\[ C_c \]

- The circuit is no longer a minimum-phase system.
- It introduces excessive phase shift, thus reduces the phase margin.
- In bipolar OpAmps, it is usually at much higher frequency than the poles $\rightarrow 1 - f/f_z \cong 1$
Pole Splitting

- Increasing $C_c$ lowers $f_{1(new)}$ and raises $f_{2(new)}$
- The shift in $f_2$ eases the amount of shift required by $f_1 \rightarrow$ Higher bandwidth

![Diagram showing pole splitting and Miller Compensation](image)

- Increasing $C_c$ above a certain limit makes $f_2$ stops to increase.

$$\omega_{2(new)} = 2\pi f_{2(new)} = \frac{g_2}{C_1 + \frac{C_1 C_2}{C_c} + C_2} \approx \frac{g_2}{C_1 + C_2} \mid C_c \gg C_1, C_2$$
Miller Compensation

Numerical example:

- \( r_d = \infty, r_o = 0 \), \( g_1 = 2 \text{ mA/V}, R_1 = 100 \text{ k}\Omega, g_2 = 10 \text{ mA/V}, R_2 = 50 \text{ k}\Omega \)
- \( f_1 = 100 \text{ kHz}, f_2 = 1 \text{ MHz}, f_3 = 10 \text{ MHz} \)
- Find the required value of \( C_c \) for \( \phi_m = 45^\circ \) with \( \beta = 1 \)

From \( f_1 \) and \( f_2 \), we can calculate \( C_1 = 15.9 \text{ pF} \) and \( C_2 = 3.18 \text{ pF} \)

Assume \( C_c \) is large \( \rightarrow f_2(\text{new}) = \frac{g_2}{2\pi(C_1+C_2)} = 83.3 \text{ MHz} > f_3 \)

Since \( f_2(\text{new}) > f_3 \) \( \rightarrow f_3 \) is the first non-dominant pole

For \( \phi_m = 45^\circ, f_x = f_3 = 10 \text{ MHz} \)

Then, \( f_1(\text{new}) = \frac{f_3}{a_0\beta} = \frac{f_3}{g_1R_1g_2R_2} = 100 \text{ Hz} \)

\[ C_c = \frac{1}{2\pi R_1 g_2 R_2 f_1(\text{new})} = 31.8 \text{ pF} \]
Pole-Zero Compensation

- This technique uses a large compensation capacitor \((C_c \gg C_1)\) to lower the first pole \(f_1\).
- It also uses a small resistor \((R_c \ll R_1)\) to create a zero that cancels the second pole \(f_2\).
- The compensated response is then dominated by the lowered first pole up to \(f_3\).
- Transfer function
  \[
  \frac{V_1}{V_d} = -g_1 R_1 \frac{1 + jf/f_z}{(1 + jf/f_{1(new)})(1 + jf/f_4)}
  \]
- Pole/zero locations
  \[
  f_{1(new)} \approx \frac{1}{2\pi R_1 C_c}, \quad f_z = \frac{1}{2\pi R_c C_c}, \quad f_4 \approx \frac{1}{2\pi R_c C_1}
  \]
**Pole-Zero Compensation**

- $C_c$ and $R_c$ lowers the dominant pole $f_{1(new)} \ll f_1$, creates a zero $f_z$, and creates an additional pole $f_4 \gg f_z$
- Choose $R_c$ such that $f_z$ cancels $f_2$
- The open loop gain now becomes

$$a_{new}(j\omega) = \frac{a_0}{(1 + j\omega/f_{1(new)})(1 + j\omega/f_3)(1 + j\omega/f_4)}$$

- To have $\phi_m = 45^\circ$, the cross-over frequency should be $f_3$
- Since the compensated response is dominated by $f_{1(new)}$ pole up to $f_3$

$$\frac{|a(jf_{1(new)})|}{|a(jf_3)|} = \frac{a_0}{1/\beta} = \frac{f_3}{f_{1(new)}}$$

- Thus, $f_{1(new)} = f_3/a_0\beta$
Pole-Zero Compensation

Numerical example:

- $r_d=\infty$, $r_o=0$, $g_1=2$ mA/V, $R_1=100$ kΩ, $g_2=10$ mA/V, $R_2=50$ kΩ
- $f_1 = 100$ kHz, $f_2 = 1$ MHz, $f_3 = 10$ MHz
- Find the required value of $C_c$ for $\phi_m = 45^\circ$ with $\beta = 1$

From $f_1$ and $f_2$, we can calculate $C_1 = 15.9$ pF and $C_2 = 3.18$ pF

For $\phi_m = 45^\circ$, $f_x = f_3 = 10$ MHz

Then, $f_1^{(new)} = \frac{f_3}{a_0\beta} = \frac{f_3}{g_1R_1g_2R_2} = 100$ Hz $\rightarrow$ $C_c = \frac{1}{2\pi R_1 f_1^{(new)}} = 15.9$ nF

$R_c$ is chosen such that $f_z = f_2$ $\rightarrow$ $R_c = \frac{1}{2\pi C_c f_2} = 10$ Ω

Since $f_4 = \frac{1}{2\pi R_c C_1} = 1$ GHz $\gg f_3$ $\rightarrow$ It will not affect the phase margin

Relaxed compared to shunt-capacitance compensation, but still LARGE
Feedforward Compensation

- In multistage amplifiers, usually there is one stage that acts as a bandwidth bottleneck by contributing a substantial amount of phase shift in the vicinity of the cross-over frequency $f_x$.
- This technique creates a high-frequency bypass around the bottleneck stage to suppress its phase at $f_x$, thus improving $\phi_m$.
Feedforward Compensation

- The bypass around the bottleneck stage is a high-pass function
  \[ h(jf) = \frac{jf/f_0}{1+jf/f_0} \]

- The compensated open-loop gain is
  \[ a_{comp}(jf) = [a_1(jf) + h(jf)]a_2(jf) \]

- **At low frequency:** \(|h(jf)| \ll |a_1(jf)|\)
  \[ a_{comp}(jf) \cong a_1(jf)a_2(jf) = a(jf) \]
  The high low-frequency gain advantage of the uncompensated amplifier still hold.

- **At high frequency:** \(|h(jf)| \gg |a_1(jf)|\)
  \[ a_{comp}(jf) \cong a_2(jf) \]
  The dynamics are controlled only by \(a_2 \rightarrow \) Wider bandwidth & Lower phase shift
Dominant-pole compensation:

- It creates an additional pole at sufficiently low frequency.
- It doesn’t take advantage of the existing poles.
- It suffers from extremely low bandwidth.

Shunt-capacitance compensation:

- It rearranges the existing poles rather than creating an additional pole.
- It moves the first pole to sufficiently low frequency.
- The value of the shunt capacitance is extremely large → Extra cost
Miller compensation:

- It takes advantage of Miller multiplicative effect of capacitors, thus requires moderate capacitance to move the first pole to sufficiently low frequency.
- It causes pole splitting, where the dominant pole is reduced and the first non-dominant pole is raised in frequency.
## Summary of Internal Frequency Compensation

### Pole-zero compensation:
- Similar to shunt-capacitance technique, a large capacitor is used to shift the first pole to sufficiently low frequency.
- A small resistance is used to create a zero that cancels the first non-dominant pole.

### Feedforward Compensation
- It places a high frequency bypass around the bottleneck stage that contributes the most phase shift in the vicinity of $f_X$. 

![Circuit Diagram](image-url)
How to stabilize the circuit by modifying its feedback factor $\beta$?

- Reducing the Loop Gain
- Input-Lag Compensation
- Feedback-Lead Compensation
Reducing the Loop Gain

- This method shifts $|1/\beta|$ curve upwards until it intercepts the $|a|$ curve at $f = f_{\phi_m - 180^\circ}$, where $\phi_m$ is the desired phase margin.
- The shift is obtained by connecting resistance $R_c$ across the inputs.

\[
\frac{1}{\beta} = 1 + \frac{R_2}{(R_1 || R_c)} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}
\]

Shifts the curve upwards to improve $\phi_m$.
Reducing the Loop Gain

- $R_c$ is chosen to achieve the desired phase margin $\phi_m$:

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c} = \left| a(jf \phi_m - 180^\circ) \right|$$

Then,

$$R_c = \frac{R_2}{\left| a(jf \phi_m - 180^\circ) \right| - (1 + R_2/R_1)}$$

[Diagram of a circuit with labels and points on a graph indicating phase margin conditions.]
Reducing the Loop Gain

- *Prices that we are paying for stability:*
  - **Gain Error:**
    \[ H_{CL} = \frac{1}{\beta} \frac{T}{1 + T} = \frac{A_{ideal}}{1 + 1/T} \]
    
    The presence of \( R_c \) reduces \( T \), thus resulting in a larger gain error.

  - **DC Noise Gain:**
    \[ H_{CL}(j0) \approx \frac{1}{\beta_0} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c} \]
    
    The presence of \( R_c \) causes an increased DC-noise gain which may result in an intolerable DC output error.

**THERE’S NO FREE LUNCH!**
The high DC-noise gain of the previous method can be overcome by placing a capacitance $C_c$ in series with $R_C$.

- **High frequencies:**
  - $C_c$ is short.
  - $\frac{1}{|\beta|}$ curve is unchanged compared to the previous case.

- **Low frequencies:**
  - $C_c$ is open
  - $\frac{1}{|\beta|} = 1 + \frac{R_2}{R_1}$, we now have much higher DC loop gain & much lower DC output error.
Input-Lag Compensation

- $R_c$ is chosen to achieve the desired phase margin $\phi_m$:

$$\frac{1}{\beta_{\infty}} = 1 + \frac{R_2}{R_1} + \frac{R_2}{R_c} = \left| a(jf \phi_m - 180^\circ) \right|$$

Then,

$$R_c = \frac{R_2}{\left| a(jf \phi_m - 180^\circ) \right| - (1+R_2/R_1)}$$

- To avoid degrading $\phi_m$, it is good practice to position the second breakpoint of $|1/\beta|$ curve a decade below $f \phi_m - 180^\circ$.

$$\frac{1}{2\pi C_c R_c} = \frac{1}{10} f \phi_m - 180^\circ.$$  

Then,

$$C_c = \frac{5}{\pi R_c f \phi_m - 180^\circ}.$$
Input-Lag Compensation

**Advantage(s):**

😊 Lower DC-noise gain due to the presence of $C_c$.
😊 It allows for higher slew rate compared with internal compensation techniques: Op-amp is spared from having to charge/discharge internal compensation capacitance.

**Disadvantage(s):**

!=( Long settling tail because of the presence of pole-zero doublet of the feedback network ($|\beta|$).
!=( Increased high-frequency noise in the vicinity of the cross-over frequency.
!=( Low closed-loop differential input impedance $Z_d$ which may cause high-frequency input loading

\[
Z_d = z_d || Z_c , \quad Z_c = R_c + 1/sC_c \ll z_d
\]

** $z_d$ is the open loop input impedance of the Op-Amp.**
Feedback-Lead Compensation

- This technique uses a feedback capacitance $C_f$ to create phase lead in the feedback path.

- The phase lead is designed to be in the vicinity of the crossover frequency $f_x$ which is were $\phi_m$ is boosted.
Feedback-Lead Compensation

**Analysis:**

\[
\frac{1}{\beta} = 1 + \left( \frac{R_2 \| Z_C}{R_1} \right) = \left( 1 + \frac{R_2}{R_1} \right) \frac{1 + jf/f_z}{1 + jf/f_p}
\]

where,

\[
f_p = \frac{1}{2\pi C_f R_2}, \quad f_z = \left( 1 + \frac{R_2}{R_1} \right) f_p
\]

- The phase-lag provided by \( \frac{1}{\beta(j\omega)} \) is maximum at \( \sqrt{f_z f_p} \).
- The optimum value of \( C_f \), that maximizes the phase margin, is the one that makes this point at the crossover frequency.

\[
f_x = \sqrt{f_z f_p} = f_p \sqrt{1 + \frac{R_2}{R_1}}
\]

- The cross-over frequency can be obtained from

\[
|a(jf_x)| = \frac{1}{|\beta(jf_x)|} = \sqrt{1 + \frac{R_2}{R_1}}
\]

- Having \( f_x \), the optimum \( C_f \) can be found

\[
C_f = \frac{\sqrt{1 + R_2/R_1}}{2\pi R_2 f_x}
\]
Feedback-Lead Compensation

**How much phase margin can we get?**

- At the geometric mean of $f_p$ and $f_z$, we have
  \[ \angle \left( \frac{1}{\beta} \right) = 90^\circ - 2 \tan^{-1} \left( 1 + \frac{R_2}{R_1} \right) \]
- The larger the value of $1 + \frac{R_2}{R_1}$, the greater the contribution of $1/\beta$ to the phase margin.

- E.g. $1 + \frac{R_2}{R_1} = 10 \rightarrow \angle \left( \frac{1}{\beta} \left( jf_x \right) \right) = -55^\circ$
  
  Thus,
  \[ \angle T(jf_x) = \angle a(jf_x) - \angle \left( \frac{1}{\beta(jf_x)} \right) = \angle a(jf_x) + 55^\circ \]
  - The phase margin is improved by $55^\circ$ due to feedback-lead compensation.
Feedback-Lead Compensation

- **Advantage(s):**
  - 😊 $C_f$ helps to counteract the effect of the input stray capacitance $C_n$ as we discussed beforehand.
  - 😊 It provides better filtering for internally generated noise.

- **Disadvantage(s):**
  - 😞 It doesn’t have the slew-rate advantage of the input-lag compensation.
These OpAmps are compensated for unconditional stability only when used with $1/\beta$ above a specified value

$$\frac{1}{\beta} \geq \left(\frac{1}{\beta}\right)_{\text{min}}$$

They provide a constant GBP only for $|a| \geq (1/\beta)_{\text{min}}$

They offer higher GBP and slew rate.

**Example**

- The fully compensated LF356 OpAmp uses $C_c \approx 10 \, pF$ to provide $GBP = 5 \, MHz$ and $SR = 12 \, V/\mu s$ for any $|a| \geq 1 \, V/V$.

- The decompensated version of the same OpAmp, LF357, uses $C_c \approx 3 \, pF$ and provides $GBP = 20 \, MHz$ and $SR = 50 \, V/\mu s$ but only for any $|a| \geq 5 \, V/V$. 

Decompensated OpAmps