Let us consider a voltage follower and determine its step response

\[ \frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{a|a = \frac{GB}{s}}} = \frac{1}{1 + \frac{s}{GB}} \]

Thus for \( v_i(t) = V_m u(t) \) or \( V_i(s) = \frac{V_m}{s} \)

Then \( V_o = \frac{V_m}{s} \frac{1}{1 + \frac{s}{GB}} = \frac{V_m}{s} \frac{GB}{s + GB} \)

Taking the inverse Laplace it yields

\[ v_o(t) = V_m \left( 1 - e^{-t/\tau} \right) \quad \text{where} \quad \tau = \frac{1}{\omega_t} = \frac{1}{GB (rad)} = \frac{1}{2\pi \times f_t} \]

The rising time \( t_R \) is defined as the time for the output \( v_o(t) \) to reach 90% of \( V_m \) and starting at 10% \( V_m \).

\[ t_R = \tau (\ln(0.9) - \ln(0.1)) = \frac{2.2}{(2\pi \times f_t)} \]
Let us consider next a Voltage Gain $\neq 1$ for the non-inverting amplifier

![Amplifier Diagram]

$$V_x = \frac{1}{R_1} + \frac{1}{R_2} - \frac{V_o}{R_2} = 0$$

and

$$v_a = v_x - v_{in}, \quad v_o(s) = -a(s)V_a(s)$$

$$V_o(s) = -\frac{a(s)V_i(s)}{1 - \frac{a(s)}{R_2}} \frac{R_2}{1 + \frac{R_2}{R_1}}$$

$$H(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{a(s)}\left(1 + \frac{R_2}{R_1}\right)}$$

Let $a(s) = \frac{GB}{s}$; $-v_a(t) = -v_x(t) + v_{in}$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{s}{GB}\left(1 + \frac{R_2}{R_1}\right)}$$

---

Now for \( a(s) = \frac{-GB}{s} \), \( V_o(s) \) and \( V_a(s) \) become

\[
V_a(s) = \frac{-V_i(s)}{1 + \frac{GB}{G/B}} = \left(1 + \frac{R_2}{R_1}\right) \frac{-V_i(s)}{s\left(1 + \frac{R_2}{R_1}\right)} + \frac{GB}{s} = \frac{-s V_i(s)}{s + GB/(1 + R_2/R_1)}
\]

\[
V_o(s) = a(s)V_a(s) = \frac{GBV_i(s)}{s + GB/(1 + \frac{R_2}{R_1})}
\]

then if \( V_i(s) = \frac{V_m}{s} \) (a step of magnitude \( V_m \))

\[
V_a(s) = \frac{-V_m}{s + GB/(1 + R_2/R_1)} \quad ; \quad V_o(s) = \frac{V_m}{s(s + GB/(1 + R_2/R_1))}
\]

The corresponding time-domain expressions become

\[
v_a(t) = -V_m e^{-tGB/(1+R_2/R_1)}
\]

\[
v_o(t) = (1 + \frac{R_2}{R_1})V_m \left[1 - e^{-tGB/(1+R_2/R_1)}\right]
\]
**Transient Response Expression Derivations**

- **Finding delay time** $\tau_D$:
  \[
  0.5K_m = K_m (1 - e^{-\tau_D \omega_{3dB}}) \]
  \[
  \frac{1}{2} = e^{-\tau_D \omega_{3dB}}; \quad \tau_D = \frac{0.693}{\omega_{3dB}}
  \]

- **Finding rising time** $\tau_R = t_2 - t_1$
  \[
  0.1K_m = K_m (1 - e^{-t_1 \omega_{3dB}}); \quad t_1 = \frac{0.1}{\omega_{3dB}}
  \]
  \[
  0.9K_m = K_m (1 - e^{-t_2 \omega_{3dB}}); \quad t_2 = \frac{2.3}{\omega_{3dB}}
  \]
  Thus
  \[
  \tau_R = t_2 - t_1 = \frac{2.2}{\omega_{3dB}} = \frac{2.2}{2\pi f_{3dB}} = \frac{0.35}{f_{3dB}}
  \]
\[
\max \frac{dv_o(t)}{dt} = V_m GB = \text{amplitude step} \times \text{gain - bandwidth product}
\]

Note that the Gain Effect has been cancelled for the non-inverting case. GB must be in r/s units.

In order to keep the circuit operating in the linear region the following inequality must be satisfied.

\[ V_m GB < \text{Slew Rate (SR)} \]

where GB is expressed in r/s. If this condition is not satisfied the step response is slewing-rate-limited.

Furthermore, if we consider a sinusoidal input \( V_o \cos \omega t \), an output \( v_o = V_{om} \sin \omega t \)

Then
\[
\max \frac{dv_o}{dt} = \omega V_{om}
\]
\[ 2\pi f V_{om} \leq SR \]

Aside, \( V_{om} \) for op amp becomes the saturation \( V_{SAT} \), then full power bandwidth (FPB) = \[
\frac{SR}{2\pi V_{SAT}}
\]

\[ i.e. \ V_{SAT} = 13V \ , \ SR = 0.5 \times 10^6 \]
then FPB = 6.1 KHz

For the Inverting Amplifier
\[
v_o(t) = -\frac{R_2}{R_1} v_m \left(1 - e^{-\frac{tGB}{R_2+R_1}}\right)
\]
\[
\max \frac{dv_o(t)}{dt} = -\frac{R_2}{R_1+R_2} v_m GB
\]
Then the inequality becomes
\[
\frac{R_2}{R_1+R_2} v_m GB < SR
\]
Unity Gain Voltage Buffer Example

V1
Rser=0

15

U1

LM741
V3

-15
Rser=0

V2

PULSE(0 0.01 0.05m 1p)

.lib LM741.sub
.tran 0 0.1m 0
(a) 10 mV Input Step

\[
\Delta \Delta V_o = 0.01 \approx 0.043 \text{V/\mu s} < SR
\]

\[
\tau_R = \frac{2.2}{GB} = \frac{2.2}{2\pi \times 1.55 \times 10^6} = 0.226 \mu s
\]

\[
\tau_D = \frac{0.693}{9.734 \times 10^6} \approx 0.072 \mu s
\]

NOTE: \( V_{OFFSET} = 1 \text{mV} \)
(b) 5 V Input Square Wave

Rising Slope:
\[ \frac{\Delta v_o}{\Delta t} \bigg|_{v_m=5} \approx 0.325V/\mu s \approx SR \]

Falling Slope:
\[ \frac{\Delta v_o}{\Delta t} \bigg|_{v_m=5} \approx 0.325V/\mu s \approx SR \]
(c) 3dB Cut Off Frequency

\[ f_{3dB} = f_t \approx 1.55\text{MHz} \]

\[ \omega_{3dB} = 2\pi f_t \approx 2\pi \times 1.55\text{MHz} \]
\[ = 9.739\text{Mrad/s} \]
EXAMPLE: NON-INVERTING AMPLIFIER

• An operational amplifier with SR = 0.325 V/µs and $\omega_t = GB = 9.739M$ rad/s, is used to implement a non-inverting amplifier of gain 10, which will limit the rate of rise of the output, the slewing or the bandwidth when
  a) The input $V_i = 10mV$ step
  b) The input $V_i = 0.6$ V step
  c) The input is a sinusoidal $0.5 \sin(2\pi \times 5k \times t)$
  d) The input is a sinusoidal $1.5 \sin(2\pi \times 5k \times t)$
Solution

a)  Determine $V_m GB = 0.01V \times 9.739Mr/s = 0.097V/\mu s$
    
    And $SR = 0.325V/\mu s$
    
    Since $SR$ is the larger of the two, the rate of the output is limited by the BANDWIDTH.

b) In this case, $V_m GB = 0.6V \times 9.739Mr/s = 5.84V/\mu s$
    
    Thus, $SR < V_m GB$. The rate of rise is limited by the slewing rate. Note that the output will change linearly rather than exponentially.
Solution

c) Determine maximum change in closed loop. ($V_m$ is the peak value)

$$V_{om} \omega_{in} = V_m \left(1 + \frac{R_2}{R_1}\right) \omega_{in} = 0.5 \times 10 \times 2\pi \times 5 \times 10^3$$

$$= 0.157 \text{ V/}\mu\text{s} < \text{SR}$$

Since $V_m \omega_{in}$ is smaller than the SR, the rate of rise of the output is limited by the BANDWIDTH.
Non-inverting Amplifier of Gain = 10

\[ SR \approx 0.325V/\mu s \]

\[ GB = 2\pi \times 1.55Mr/s \]

.simulation
.lib LM741.sub
.tran 0 0.1m 0

PULSES(0 0.05 0.05m 0)
**BW LIMIT**

(a) 10 mV Input Step

\[ V_x, V_o, V_{in} \]

\[ \frac{\Delta v_o}{\Delta t} \bigg|_{v_m=0.01} \approx 0.057 V/\mu s < SR \]

\[ \tau = \frac{1 + R_2/R_1}{GB (rad)} = 1.03 \mu s \]

Calculation: \[ 2.2 \times \tau = 2.26 \mu s \approx t_R \sim 3.4 \mu s \]

**NOTE:** \( V_{OFFSET} = 10.9 mV \)
(b) 600 mV Input Step

\[ V_a = V_{in} - V_x \]

\[ \frac{\Delta v_o}{\Delta t}\bigg|_{v_m=0.6} \approx 0.321\text{V/\mu s} \sim SR \]
(c) 5 kHz 0.5V Input Sin Wave

**BW LIMIT**

![Graph 1: Voltage Measurements](image1)

![Graph 2: Spectrum of Output Wave](image2)

No Noticeable Harmonics & Distortion
(d) $V_{SAT}$ @ 5 kHz 1.5V Input Sin

**SR LIMIT**

$V_m \omega_{in} = 1.5 \times 10 \times 2\pi \times 5k = 0.47\ V/\mu s > SR = 0.325\ V/\mu s$

$V_{SAT} = \frac{27.8}{2} = 13.9\ V$

Noticeable Harmonics & Distortion