ACTIVE - RC FILTERS

The basic building block is illustrated below

\[ H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-Z_2}{Z_1} \left( 1 + \frac{Z_2}{Z_1} \right) \]

Let us assume that \( A \rightarrow \infty \), then

\[ H(s) = -\frac{Z_2}{Z_1} \]

Next we consider particular cases

**Integrator**

\[ H(s) = -\frac{1}{sR_1C_2} \]

**Differentiator**

\[ H(s) = -sR_2C \]
\( Z_1 \) or 
\( Z_F = Z_2 \)
Can be:

\[
\begin{align*}
\text{EXAMPLE: } & Z_1 = \frac{R_1}{1 + sR_1C_1}, \quad Z_F = \frac{R_F}{1 + sR_FC_F} \\
& \text{Assuming ideal op amp A } \rightarrow \infty. \text{ Then using} \\
& H_1 = \frac{V_{01}}{V_1} = -\frac{R_F/R_1(1+sR_1C_1)}{(1+sR_FC_F)} = -\frac{K_p(1+s/\omega_z)}{(1+s/\omega_p)}
\end{align*}
\]
Particular cases are easily derived from (3) and (4)

— Integrator: $C_1 \to 0$, $R_F \to \infty$

$$H_1 \approx -\frac{R_F}{R_1} \frac{1}{sR_FC_F} = -\frac{1}{sC_FR_1}$$

— Differentiator; $R_1 \to \infty$, $C_F \to 0$

$$H_1 \approx -\frac{R_F}{R_1} sR_1C_1 = -sR_FC_1$$

— Low-Pass: $C_1 = 0$

$$H_1 = \frac{R_F}{R_1} \frac{1}{1 + sR_FC_F}$$

— High-Pass: $R_1 \to \infty$

$$H_1 \approx -\frac{R_F}{R_1} \frac{sR_1C_1}{1 + sR_FC_F}$$
— One pole and one zero

\[
\frac{V_o}{V_i} = \frac{-1 + sR_F C_F}{sC_F} \frac{sC_1}{1 + sR_1 C_1} = \frac{-C_1}{C_F} \frac{1 + sR_F C_F}{1 + sR_1 C_1}
\]  

(5)

What are the key differences between Eqs. (4) and (5)?

Exercise 1. Obtain the transfer function of the following circuit.
Second-Order Filters Based on a Two-Integrator Loop.

- We can design a second-order filter by cascading two inverters. i.e.

\[
\frac{V_o}{V_i} = \frac{-R_{F1}}{R_1} \left( \frac{-R_{F2}}{R_2} \right) = \frac{R_{F1} R_{F2}}{R_1 R_2} \left( 1 + sC_{F1}R_{F1} \right) \left( 1 + sC_{F2}R_{F2} \right) = \frac{R_{F1} R_{F2}}{R_1 R_2} \left( 1 + sC_{F1}R_{F1} \right) \left( 1 + sC_{F2}R_{F2} \right) + 1
\]

(6)

What are the locations of the poles?

\[
s_{p1,2} = \frac{- \left( C_{F1}R_{F1} + C_{F2}R_{F2} \right) \pm \sqrt{\left( C_{F1}R_{F1} + C_{F2}R_{F2} \right)^2 - 4C_{F1}R_{F1}C_{F2}R_{F2}}}{2C_{F1}R_{F1}C_{F2}R_{F2}}
\]
To have complex poles it requires that

$$(C_{F1}R_{F1})^2 + (C_{F2}R_{F2})^2 - 2C_{F1}R_{F1}C_{F2}R_{F2} < 0?$$

Which it is impossible to satisfy. Therefore, cascading two first-order filter yield a second-order filter with only real poles.

The general form of the second order two-integrator loop has the following topology.

$$H = \frac{-Z_{F1}Z_{F2}}{Z_3Z_2} \frac{Z_3Z_2}{1+Z_{F1}Z_{F2}} \frac{Z_{F1}Z_{F2}}{Z_1Z_2}$$

(7a)
Note the similarity of Eq. (7a) with (2). Also observe that A “-1” needs to be inserted before or after the second inverter to yield a negative feedback loop.

Let us consider the following filter where

\[ Z_3 = R_3, \quad Z_1 = R_1, \quad Z_2 = R_2, \quad Z_{F1} = \frac{1}{sC_{F1}}, \quad Z_{F2} = \frac{R_{F2}}{1 + sC_{F2}R_{F2}} \]

Thus Eq. (7a) yields:

\[
H = \frac{-\frac{1}{sC_{F1}R_3 (1 + sC_{F2}R_{F2})} \frac{R_{F2}/R_2}{1 + \frac{R_{F2}/R_2}{sC_{F1}R_1 (1 + sC_{F2}R_{F2})}}}{}
\]

\[
H = \frac{-\frac{1}{C_{F1}R_3C_{F2}R_2}}{s^2 + \frac{s}{C_{F2}R_{F2}} + \frac{1}{C_{F1}R_1C_{F2}R_2}} = \frac{-\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}
\]

By injecting in different current summing nodes a general biquad filter can be obtained.

\[ V_o = \frac{V_3}{sC_{F1}R_3} \left( 1 + sC_{F2}R_{F2} \right) + \frac{-K_2 R_{F2}}{R_2} - \frac{K_3 R_{F2} V_1}{R_{F2}} \]

\[ V_o = \frac{1}{sC_{F1}R_3C_{F2}R_2} + V_2 sC_{F1}R_1K_2 - V_3 sC_1 R_1 K_3 \]

\[ V_o = \frac{1}{s^2 + \frac{s}{C_{F2}R_{F2}} + \frac{1}{C_{F1}R_1C_{F2}R_2}} \]
Exercise 2. Obtain the expressions of $V_{o1}$ and $V_{o2}$.

More general biquad expressions and topologies can be obtained by adding a summer.

$$V_{oT} = -K_{10} V_{in} - K_{20} V_{o1} - K_{30} V_{o}$$

Exercise 3. Draw an active-RC topology of the block diagram shown above.

Exercise 4 a) For only $V_1 \neq 0$ obtain $V_o$ and $V_{o1}$ when instead of the resistor $R_{F2}/K_3$ a capacitor $K_4 C_{F2}$ is used. b) For only $V_3 \neq 0$ obtain $V_{o1}$ when the resistor $R_3$ is replaced by a capacitor $K_{HP} C_{F1}$. 
By using also the positive input of the op amp other useful filters can be obtained.

\[
V_o = \frac{-\frac{Z_2}{Z_1} V_1 + \frac{Z_R}{Z_R + Z_C} V_2 \left(1 + \frac{Z_2}{Z_1}\right)}{1 + \frac{1}{A \left(1 + \frac{Z_2}{Z_1}\right)}}
\]

Example. Phase shifter \(Z_2=R_2=R_1\), \(Z_1=R_1\), \(Z_R=R\) \(Z_C = \frac{1}{sC}\) and \(A \to \infty\) with \(V_1 = V_2\)

\[
\frac{V_o}{V_1} = -1 + \frac{sRC}{1 + sRC} \cdot 2 = -\frac{1 - sRC + 2sRC}{1 + sRC} = -\frac{1 - sRC}{1 + sRC}
\]
Sallen and Key Bandpass Filter

\[ K \text{ is a non-inverting amplifier} \]

Using Nodal Analysis

\[
V_1 \left( s(C_1 + C_2) + \frac{1}{R_1} + \frac{1}{R_3} \right) - sC_2 V_2 - \frac{V_o}{R_3} = \frac{V_i}{R_1} \quad (1)
\]

\[
- V_1(sC_2) + V_2 \left( sC_2 + \frac{1}{R_2} \right) = 0 \quad (2)
\]

\[
V_o = KV_2 \quad (3)
\]

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{K \frac{s}{R_1 C_1}}{s^2 + \left[ \frac{1}{R_2 C_2} + \left( 1 - K \frac{R_3}{R_1} + \frac{R_3}{R_2} \right) \frac{1}{R_3 C_1} + \frac{R_1 R_3}{R_1 R_3 R_2 C_1 C_2} \right]}
\]
A particular case is for $R_1=R_2=R_3=R$, $C_1=C_2=C$

Then

$$\omega_o^2 = \frac{2}{(RC)^2} ; \quad Q = \frac{\sqrt{2}}{4-K}$$

or for a given $\omega_o$ and $Q$

$$RC = \frac{\sqrt{2}}{\omega_o} \quad \text{and} \quad K = 4 - \frac{\sqrt{2}}{Q}$$

Exercise 5. Prove the transfer function is a BP filter of the following circuit

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-K}{K+1} \frac{s}{R_1C_1} \left( \frac{1}{s^2 + \frac{1}{K+1} \left[ R_1 + R_2 \frac{1}{R_1R_2C_1} \right]} + \frac{1}{s + (K+1)R_1R_2C_1C_2} \right)$$
In the past before IC fabrication, active filters implementation preferred one op amp structure. One very popular type is the Sallen and Key unity gain implementations.

\[ H_{LP}(s) = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2} \]

\[ R_1 = R_2 = R \]
\[ C_1 = C \]
\[ C_2 = 4Q^2C \]
\[ RC = \frac{1}{2} \omega_oQ \]

One also popular topology is the Rauch Filter

\[ H(s) = \frac{1}{s^2 + \frac{s}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \frac{1}{R_1C_1R_2C_2}} \]

LP Sallen - Key

LP Rauch Filer
Another technique for analysis and design based on state-variable uses building blocks.
Let us apply to a two-integrator loop plus Mason’s Rule.

For Second-topology

\[
V_{o1} = \frac{-K_1 V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1} K_{o2}}{s^2}} = \frac{-K_1 s^2 V_{in}}{s^2 + K_Q s + K_{o1} K_{o2}} \quad \text{HP}
\]

\[
V_{o2} = \frac{-K_1 V_{in}}{1 + \frac{K_Q}{s} + \frac{K_{o1} K_{o2}}{s^2}} = \frac{-K_1 s V_{in}}{s^2 + K_Q s + K_{o1} K_{o2}} \quad \text{BP}
\]
Next we show that we can go from an Active-RC representation into a block diagram or vice versa.

KHN Biquad Filter
\[
\frac{V_{HP}}{V_i} = \frac{-\frac{R_5}{R_3}}{1 + \frac{K_Q}{R_6C_1} s + \frac{R_5/R_4}{R_6C_1R_7C_2s^2}} = \frac{-\frac{R_5}{R_3}}{s^2 + \frac{K_Q}{R_6C_1} s + \frac{R_5/R_4}{R_6C_1R_7C_2}}
\]
ELEN 622 (ESS)  **Non-Ideal Active-RC Integrators**

Op Amp Non-Idealities

![Op Amp Non-Idealities Diagram](image)

\[ H(s) = \frac{-Z_2}{Z_1} \left( 1 + \frac{1}{A} \left( 1 + \frac{Z_2}{Z_1} \right) \right) = -A\beta \frac{Z_1}{1 + A\beta} \]

where

\[ \beta = \frac{Z_1}{Z_1 + Z_2} \]

Integrator

Case 1

\[ Z_1 = R_1 ; Z_2 = \frac{1}{sC_2} ; \frac{GB}{s} = A(s) \]

\[ H(s) \approx \frac{-1}{sRC \left( s + \frac{1}{RC} + GB \right)} \approx -\frac{1}{sRC(1 + s/GB)} ; GBRC >> 1 \]

\[ H(j\omega) = -\frac{1}{-\omega^2 RC + j\omega RC} ; \quad \phi = -\frac{\pi}{2} - tan^{-1}(\omega/GB) \]

\[ \phi = -90^\circ + \Delta\phi \]
\[ \Delta \phi \approx -\tan^{-1} \frac{\omega}{GB} \approx -\tan^{-1} \frac{1}{|A(j\omega)|} ; \text{ i.e. } \frac{\omega_o}{GB} = \frac{1}{10} \]

\[ \Delta \phi \approx -5.7^\circ \]

\[ |H(j\omega_o)| = \frac{1}{|\frac{-\omega_o}{GB} + j\omega_o|} = \frac{1}{\sqrt{1 + \frac{\omega_o^2}{GB^2}}} = 1 + \Delta_M \]

\[ \Delta_M = \frac{1 - \sqrt{1 + \frac{\omega_o^2}{GB^2}}}{\sqrt{1 + \frac{\omega_o^2}{GB^2}}} \]

\[ \text{ i.e. } \frac{\omega_o}{GB} = 0.1 \]

\[ \Delta_M \sim 5\% \text{ error} \]
It follows that the ideal -6 dB/octave roll-off expected from an ideal integrator changes to -12 dB/octave at the frequency of the parasitic pole given by

\[ s_p = -\left(\omega_t + \frac{1}{CR}\right) \]

which may be approximated by,

\[ s = -\omega_t \quad \text{for} \quad \omega_t \gg \frac{1}{CR} \]
In general

\[ Q_L = -\left( \frac{\omega_t}{\omega} \right) = -\left( \frac{GB}{\omega} \right) \]

then we define the integrator Q-factor by

\[ T(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \]

\[ Q_I = \frac{X(\omega)}{R(\omega)} \]

\[ Q_I = -\left( \frac{\omega_t}{\omega} \right) = -|A(j\omega)| \]
Making an analogy of $Q_L$ of an inductor

$$Q_L = \frac{\omega_o L}{R_L}$$

Lossy Part

For an integrator one can obtain

$$Q_I = \left( \frac{\omega R C}{-\omega^2 R C} \right) GB = -\frac{1}{\omega} GB = -A(j\omega)$$

Miller Integrator
How can we compensate this degradation of performance?

If we make

\[ R_C = \frac{1}{GB \cdot C} \]

Ideally we obtain

\[ \frac{V_o(s)}{V_i(s)} = -\frac{1}{RC} \]

This integrator yields a positive

\[ Q_I = +|A(j\omega)| \]
ACTIVE – RC INTEGRATOR: Pole Shift and Predistortion

\[
\frac{V_o}{V_i} = \left. \frac{-1}{sRC} \right|_{A(s) \to \infty} = -\frac{1}{sRC} \quad (1a)
\]

\[
\frac{V_o}{V_i} = \frac{-1}{sRC + \frac{1}{A(s)}(sRC + 1)} \quad (1b)
\]

Let \( A(s) = \frac{A_o \omega_{3dB}}{s + \omega_{3dB}} \); where \( A_o \) is the DC gain and \( \omega_{3dB} \) the dominant pole in open loop.

Then (1b) becomes

\[
\frac{V_o}{V_i} = \frac{-A_o \omega_{3dB}}{s^2 + s\left(A_o \omega_{3dB} + \frac{\omega_{3dB}}{RC}\right) + \frac{\omega_{3dB}}{RC}} \approx \frac{-A_o \omega_{3dB}}{s^2 + sA_o \omega_{3dB} + \frac{\omega_{3dB}}{RC}} \quad (2)
\]

Let \( GB = A_o \omega_{3dB} \)

The roots of the denominator are

\[ P_{1,2} = -\frac{GB}{2} \pm \frac{GB}{2} \left[ 1 - \frac{4\omega_{3dB}}{(GB)^2 RC} \right]^{1/2} \]  

(3a)

Using the approximation \((1-X)^{1/2} \approx 1 - X/2\) for \(X<<1\), then

\[ P_{1,2} = -\frac{GB}{2} \pm \frac{GB}{2} \left[ 1 - \frac{2\omega_{3dB}}{(GB)^2 RC} \right] \]  

(3b)

Thus the roots yield

\[ P_1 = -\frac{1}{A_0 RC} \]

\[ P_2 = -GB + \frac{1}{A_0 RC} \approx -GB \]
The Bode Plot Looks Like
In order to relax the bandwidth op amp requirement one can use a $R_C$ or $C_C$ on the Miller Integrator. That is

$$V_i \rightarrow A(s) \rightarrow V_o$$

Use $R_C \frac{1}{C} = \frac{1}{GB}$ or $C_C R \approx \frac{1}{GB}$

Using VCVS vs. VCIS in Active-RC Filters

- The motivation is to use OTA (VCIS) instead of more power hungry Op Amp (VCVS)

For $R_o = Z_o = 0$

$$\frac{V_o}{V_i} = \frac{-Z_F}{Z_1}$$

If $A \to \infty$, then

$$\frac{V_o}{V_i} = -\frac{Z_F}{Z_1}$$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \frac{1-\frac{1}{g_mZ_2}}{1 + \frac{Z_1 + Z_2}{Z_1Z_L} + \frac{g_m}{Z_1}}$$

$$g_m \gg \frac{1}{Z_2} \quad \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

$$g_m \gg \frac{1}{Z_1 + \frac{Z_1 + Z_2}{Z_1Z_L}} \quad \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$
It can be shown that for equal \( A(s) = \frac{GB}{s} \), the Tow-Thomas filter has the following deviations

\[
Q_a = Q_o \frac{1}{1 - 4Q_o \omega_o/GB}
\]

or

\[
\frac{4Q_o \omega_o}{GB} < 1; \quad 4Q_o \omega_o < GB
\]

and

\[
\frac{\Delta \omega_o}{\omega_o} \approx \frac{2 + k}{2} \frac{\omega_o}{GB}
\]

Improved version by replacing noninverting integrator:

\[
Q_a = Q_o \frac{1 - \frac{1}{2}(2 + k)\omega_o/GB}{1 + \frac{\omega_o}{GB} + \frac{kQ_o \omega_o^2}{GB^2}}
\]
How to generate Fully-Differential Filters based on Single-Ended Version?

\[
\frac{V_o}{V_i} = -\frac{1}{CRs}
\]

Single Ended

Fully-Differential Version
Particular Case. Assume no $V_i^-$ is Available.

Symmetric conditions

\[
\frac{1}{R_{03}} + \frac{1}{R_3} + \frac{1}{R_2} = \frac{1}{R_Q} + \frac{1}{R_X}
\]

Read fully balanced - fully symmetric circuits from 607.
KHN State Variable Two-Integrator Filter

Use Mason’s Rule:

\[
\frac{V_{LP}}{V_i} = \frac{-K_{32}K_{01}K_{02}/s^2}{1 + \frac{K_{01}K_Q}{s} + \frac{K_{01}K_{02}K_{03}}{s^2}} = \frac{-K_{32}K_{01}K_{02}}{s^2 + K_{01}K_Qs + K_{01}K_{02}K_{03}}
\]

Next we consider the fully-differential version of the KHN filter.

\[
K_{02} = \frac{1}{R_0C_2}, \quad K_{01} = \frac{1}{R_0C_1}, \quad K_{03} = \frac{R_3}{R_0}, \quad K_{32} = \frac{R_3}{R_2}
\]

\[
K_Q = \frac{R_3}{R_Q}
\]
KHN Fully-Differential Version

\[ V_i^+ \text{ to } V_{LP}^+ \text{ and } V_i^- \text{ to } V_{LP}^- \]

\[ R_X, R_{03}, R_3, R_Q, C_1, C_2 \]
How can we take advantage of improved combination of $\pm Q_i$ in fully differential versions?
Effects of Non-Ideal Op Amps on the Tow-Thomas Biquad

When \( A_i (i = 1,2,3) \) are finite, the denominator becomes of the transfer function yields:

\[
D(s) = s^2 + s \frac{\omega_0}{Q} + \frac{1}{1 + \frac{2Q+1}{A_1} + \frac{1+Q}{A_2} + \frac{3Q+1}{A_1A_2}} + \omega_0^2 \frac{1}{1 + \frac{2}{A_1A_2} + \frac{1}{QA_2} + \frac{1}{QA_1A_2}}
\]
Let \( A_i = \frac{GB_i}{s} \), \( i = 1, 2, 3 \)

Furthermore assume the range of interest \( \omega >> \frac{GB_i}{A_o} \) and \( \frac{\omega_o}{GB_i} << 1, Q >> 1 \). Then \( D(s) \) becomes:

\[
D(s) = \frac{1}{\omega_o} \left( \frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} + 2 \frac{\omega_o}{GB_3} \right) s^3 + \left( 1 + 2 \frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} \right) s^2 + \omega_o \left( 1 + \frac{\omega_o}{GB_2} \right) s + \omega_o^2
\]

Thus

\[
\omega_{oa} \Delta \omega_o \left( 1 + \Delta_\omega \right)
\]

or \( \Delta_\omega = \frac{\omega_{oa} - \omega_o}{\omega_o} \)

for \( \Delta_\omega << 1 \) and \( Q_a >> 1 \), then

\[
\Delta_\omega = -\frac{\omega_o}{GB_1} - \frac{1}{2} \frac{\omega_o}{GB_2} \bigg|_{GB_1=GB_2=GB} = -\frac{3}{2} \left( \frac{\omega_o}{GB} \right)
\]
\[
\frac{Q_a}{Q} \approx \frac{1}{1 - Q \left( \frac{\omega_o}{GB_1} + \frac{\omega_o}{GB_2} + 2 \frac{\omega_o}{GB_3} \right)}
\]

For equal \( GB_1 = GB_2 = GB_3 \)

\[
Q_a = \frac{Q}{1 - 4Q \left( \frac{\omega_o}{GB} \right)}
\]

\[
Q_a \approx Q \left( 1 + 4Q \left( \frac{\omega_o}{GB} \right) \right), \quad \text{for } 4Q \left( \frac{\omega_o}{GB} \right) \ll 1
\]

Note that for a stable filter

\( 4Q \frac{\omega_o}{GB} < 1 \)

or

\[ Q < \frac{GB}{4\omega_o} \]
KEY FILTER PARAMETERS IN ACTIVE-RC FILTERS

• Dynamic Range
• Signal-To-Noise Ratio
• Total Output Noise
• Noise Power Spectral Density
• Total Area

Resistor and Capacitors can be expressed as:

\[
R_\ell = r_\ell R, \quad C_\ell = c_\ell C
\]

where \(r_\ell\) and \(c_\ell\) are the normalized filter values.

The resistor power dissipation for a sinusoidal input yields

\[
P_R(f) = \sum_\ell \frac{|V_i H_{i\ell}|^2}{2R_\ell} = \frac{|V_i|^2}{2R} \sum_\ell \frac{|H_{i\ell}(f)|^2}{r_\ell}
\]

(1)

Where \(H_{i\ell}(f)\) is the transfer function from the input to the terminals of resistor \(R_\ell\)

Focusing on the noise resistor, the power spectral density is given by

\[ S_R(f) = \sum_{\ell} 4kT R_\ell |H_{\ell o}(f)|^2 = 4kT R \sum_{\ell} r_\ell |H_{\ell o}(f)|^2 \]  

(2)

The definition of \( H_{\ell o}(f) \) is pictorially shown below:

Thus, the total output noise (mean squared value) due to the resistors become

\[ N_R = \int_0^\infty S_R(f) df \]

In practice the upper limit of the integration is limited to a useful practical value.
The signal-to-ratio for a given $V_i$ and frequency $f$ is given by

$$\text{SNR} = \frac{|V_i H(f)|^2}{2N_R}$$

(3)

and

$$\max_f P_R(f) \leq P_{R,\text{max}}$$

where $P_{R,\text{max}}$ is the maximum specified power dissipation in the resistors. Then

$$\text{DR} = \frac{|V_i|_{\text{max}}^2 \max_f |H(f)|^2}{2N_R}$$

(4)

Let us consider a second-order BP filer example

$$H_{\text{BP}}(s) = \frac{\omega_c s}{Q} \frac{s^2 + \frac{\omega_c}{Q} s + \frac{\omega_c^2}{Q}}$$

Using the following notation the above $H_{\text{BP}}(s)$ yields

$$H_{\text{BP}}(f) = \frac{Q^{-1}f_c(jf)}{(jf)^2 + Q^{-1}f_c(jf) + f_c^2}$$
\[ P_R(f_c) = \frac{|V_i|^2}{R} a (a + Q^{-1}) \]

For the biquad shown below

![Biquad Circuit Diagram](image)

and

\[ N_R = \frac{2kT (1 + Q/a)}{C} a \]

\( a \uparrow \Rightarrow N_R \downarrow \Rightarrow a V_{OBP} \) limited by linearity and by resistor power dissipation which is proportional to \((a)^2\).
Fully Differential Fully Balanced Circuits

What is the problem with single-input / single-output?

\[ V_n = \frac{V_o Z_1}{Z_1 + Z_F} \]

\[ V_1 - V_o = \frac{V_o}{A} \Big|_{A \to \infty} = 0 \]

No elimination of common-mode signal.

How to solve this problem?

For \( V_i = V_{id} + V_{icm} \)

\[ V_o = \left(1 + \frac{Z_F}{Z_1}\right)(V_{id} + V_{icm}) \]

For \( V_i = V_{id} + V_{icm} = (V_1 - V_2) + \frac{(V_1 + V_2)}{2} \)

\[ V_o = \frac{Z_F}{Z_1}(V_1 - V_2) \]

No common-mode output.
How to obtain a fully differential circuit?
We will discuss two potential approaches

**Approach 1**

![Circuit Diagram](image)

\[
V_{o1} = -\frac{R_F}{R_1} (V_2 - V_1)
\]

\[
= \frac{R_F}{R_1} (V_1 - V_2)
\]

\[
V_{o2} = +\frac{R_F}{R_1} (V_2 - V_1)
\]

\[
V_{o1} - V_{o2} = \frac{R_F}{R_1} (V_1 - V_2 - V_2 + V_1) = 0
\]

\[
V_{oD} = \frac{2R_F}{R_1} (V_1 - V_2)
\]

**Remark:** sensitive to CM signals

**Approach 2**

![Circuit Diagram](image)

\[
V_{o1} = \frac{R_F}{R_1} (V_2 - V_1)
\]

\[
V_{o2} = \frac{R_F}{R_1} (V_1 - V_2)
\]

\[
V_{p1} = V_{p2}
\]

**Remark:** More robust to reject common-mode signals
First-Order FB Low Pass with Op Amp

* 
.subckt opamp non inv out
rin non inv 100K
egain 1 0 (non, inv) 200K
ropen 1 2 2K
copen 2 0 15.9155u
eout 3 0 (2, 0) 1
rout 3 out 50
.ends
*vin 3 31 ac 1.0
vin 31 0 ac 1.0
x1 4 1 2 opamp
x2 4 11 22 opamp
R1 3 1 1K
R11 3 4 1K
R1B 31 4 1K
R1BB 31 11 1K
RF1 2 1 1K
RF1B 22 11 1K
RF11 4 0 1K
RF11B 4 0 1K
C1 2 1 0.159155u
C1B 22 11 0.159155u
C1A 4 0 0.159155u
C11B 4 0 0.159155u
rdummy 3 31 1
.ac dec 10 10Hz 10KHz
.probe
.ends
Fully Balanced T-T Active-RC Implementation
Introduction to Matlab and Simulink For Filter Design
Example 1: Ideal Integrator

\[ R = 1K\Omega \quad C = 0.159\text{mF} \]
s=tf('s');
R=1e3;
C=0.159e-3;
hs=1/(R*C*s);
figure(1)
bode(hs) %Create Bode Plot
grid minor %Add grid to plot
H= gcr;
units
h.AxesGrid.Xunits = 'Hz'; %Set units to Hz
pole(hs); %calculates hs poles
zero(hs); %calculates hs zeros
Bode Plot: Ideal Integrator (Simulink)

1) Create Model using Gain, Integrator, and In/Out blocks

2) Go to: Tools => Control Design => Linear Analysis

3) Then press: Linearize model
Tow-Thomas Biquad (Simulink)
Output Waveform (Scope)
Integrator Non-ideal amplifier

clear
clc
s=tf('s');
R=1;                           %Resistor Value
C=0.159e-3;              %Capacitor Value
hs1=-1/(R*C*s);       %hs= Vo(s)/Vi(s)
figure(1)
bodemag(hs1)
hold on
f=1e3;
for i=1:5;
GBW=2*pi*f;
A=GBW/s;
Beta=R/(R+1/(s*C));
hs2=-1/(R*C*s)*1/(1+1/(A*Beta));
hold on
bodemag(hs2,{2*pi*1,2*pi*1e5})
f=10*f;
end
grid minor       %Add grid to plot
h= gcr;          %change X-axis units
h.AxesGrid.Xunits = 'Hz'; %Set units to Hz
legend('ideal', 'GBW=1kHz','GBW=10kHz', 'GBW=100kHz','GBW=1MHz', 'GBW=10MHz',1)
Filter Approximation: Low-Pass Butterworth

- E.g.: Use Matlab to find the numerator b and denominator a coefficients for a third-order Butterworth low-pass filter prototype with normalized cutoff frequency.

\[ [z,p,k]=\text{buttap}(3); \quad \% \text{To get gain and poles} \]
\[ [b,a]=\text{zp2tf}(z,p,k); \quad \% \text{To get b and a coefficients} \]
\[ H=\text{tf([b],[a])}; \quad \% \text{to generate transfer function} \]
\[ \text{figure}(1) \]
\[ \text{bode}(H); \quad \% \text{Bode plot} \]
\[ \text{grid minor;} \]
\[ \text{figure}(2) \]
\[ \text{pzmap}(H); \quad \% \text{plot poles and zeros} \]
\[ \text{grid minor;} \]

Results:

\[ b = 0 \ 0 \ 0 \ 1 \]
\[ a = 1 \ 2 \ 2 \ 1 \]

Thus,

\[ H(s) = \frac{1}{s^3+2s^2+2s+1} \]

The squared magnitude of a low-pass butterworth filter is given by:

\[ |H(\omega)|^2 = \frac{1}{1 + (\omega/\omega_0)^{2n}} \]
Low-pass Chebyshev Filter

- Use the Matlab `cheb1ap` function to design a second order Type I Chebyshev low-pass filter with 3dB ripple in the pass band

```matlab
w=0:0.05:400;  % Define range to plot
[z,p,k]=cheb1ap(2,3);
[b,a]=zp2tf(z,p,k);  % Convert zeros and poles of G(s) to polynomial form
bode(b,a)
grid minor;
```
Low-pass Chebyshev Filter
Low-pass Chebyshev Filter

w=0:0.01:10;
[z,p,k]=cheb1ap(2,3);
[b,a]=zp2tf(z,p,k);
Gs=freqs(b,a,w);
xlabel('Frequency in rad/s');
ylabel('Magnitude of G(s)');
semilogx(w,abs(Gs));
title('Type 1 Chebyshev Low-Pass Filter');
Grid;

% Another way to write the code!
Low-pass Chebyshev Filter
Inverse Chebyshev

• Using the Matlab cheb2ap function, design a third order Type II Chebyshev analog filter with 3dB ripple in the stop band.

```matlab
w=0:0.01:1000;
[z,p,k]=cheb2ap(3,3);
[b,a]=zp2tf(z,p,k);Gs=freqs(b,a,w);
semilogx(w,abs(Gs));
xlabel('Frequency in rad/sec');
ylabel('Magnitude of G(s)');
title('Type 2 Chebyshev Low-Pass Filter, k=3, 3 dB ripple in stop band');
grid
```
Inverse Chebyshev

Type 2 Chebyshev Low-Pass Filter, $k=3$, 3 dB ripple in stop band
Elliptic Low-Pass Filter

- Use Matlab to design a four pole elliptic analog low-pass filter with 0.5dB maximum ripple in the pass-band and 20dB minimum attenuation in the stop-band with cutoff frequency at 200 rad/s.

```matlab
w=0: 0.05: 500;
[z,p,k]=ellip(4, 0.5, 20, 200, 's');
[b,a]=zp2tf(z,p,k);
Gs=freqs(b,a,w);
plot(w,abs(Gs))
title('4-pole Elliptic Low Pass Filter');
grid
```
Elliptic Low-Pass Filter
Transformation Methods

- Transformation methods have been developed where a low pass filter can be converted to another type of filter by simply transforming the complex variable s.

- Matlab lp2lp, lp2hp, lp2bp, and lp2bs functions can be used to transform a low pass filter with normalized cutoff frequency, to another low-pass filter with any other specified frequency, or to a high pass filter, or to a band-pass filter, or to a band elimination filter, respectively.
LPF with normalized cutoff frequency, to another LPF with any other specified frequency

• Use the MATLAB **buttap** and **lp2lp** functions to find the transfer function of a third-order Butterworth low-pass filter with cutoff frequency \( fc = 2 \text{kHz} \).

```matlab
% Design 3 pole Butterworth low-pass filter (wcn=1 rad/s)
[z,p,k]=buttap(3);
[b,a]=zp2tf(z,p,k); % Compute num, den coefficients of this filter
(wcn=1rad/s)
f=1000:1500/50:10000; % Define frequency range to plot
w=2*pi*f;
fc=2000; % Define actual cutoff frequency at 2 KHz
wc=2*pi*fc; % Convert desired cutoff frequency to rads/sec
[bn,an]=lp2lp(b,a,wc); % Compute num, den of filter with fc = 2 kHz
Gsn=freqs(bn,an,w); % Compute transfer function of filter with fc = 2 kHz
semilogx(w,abs(Gsn));
grid;
xlabel('Radian Frequency w (rad/sec)')
ylabel('Magnitude of Transfer Function')
title('3-pole Butterworth low-pass filter with fc=2 kHz or wc = 12.57 \text{kr/s}')
```
LPF with normalized cutoff frequency, to another LPF with any other specified frequency
High-Pass Filter

- Use the MATLAB commands `cheb1ap` and `lp2hp` to find the transfer function of a 3-pole Chebyshev high-pass analog filter with cutoff frequency $fc = 5$KHz.

```matlab
% Design 3 pole Type 1 Chebyshev low-pass filter, wc=1 rad/s
[z,p,k]=cheb1ap(3,3);
[b,a]=zp2tf(z,p,k);  % Compute num, den coef. with wc=1 rad/s
f=1000:100:100000;  % Define frequency range to plot
fc=5000;  % Define actual cutoff frequency at 5 KHz
wc=2*pi*fc;  % Convert desired cutoff frequency to rads/sec
[bn,an]=lp2hp(b,a,wc);  % Compute num, den of high-pass filter with fc =5KHz
Gsn=freqs(bn,an,2*pi*f);  % Compute and plot transfer function of filter with fc = 5 KHz
semilogx(f,abs(Gsn));
grid;
xlabel('Frequency (Hz)');
ylabel('Magnitude of Transfer Function');
title('3-pole Type 1 Chebyshev high-pass filter with fc=5 KHz ')```
High-Pass Filter

3-pole Chebyshev high-pass filter with $f_c=5$ KHz
Band-Pass Filter

- Use the MATLAB functions \texttt{buttap} and \texttt{lp2bp} to find the transfer function of a 3-pole Butterworth analog band-pass filter with the pass band frequency centered at $f_0 = 4$kHz, and bandwidth $BW = 2$KHz.

\[ z,p,k = \texttt{buttap}(3); \]  
\% Design 3 pole Butterworth low-pass filter with $wcn=1$ rad/s
\[ [b,a] = \texttt{zp2tf}(z,p,k); \]  
\% Compute numerator and denominator coefficients for $wcn=1$ rad/s
\[ f=100:100:100000; \]  
\% Define frequency range to plot
\[ f0=4000; \]  
\% Define centered frequency at 4 KHz
\[ W0=2*pi*f0; \]  
\% Convert desired centered frequency to rads/s
\[ fbw=2000; \]  
\% Define bandwidth
\[ Bw=2*pi*fbw; \]  
\% Convert desired bandwidth to rads/s
\[ [bn,an] = \texttt{lp2bp}(b,a,W0,Bw); \]  
\% Compute num, den of band-pass filter
\% Compute and plot the magnitude of the transfer function of the band-pass filter
\[ Gsn = \texttt{freqs}(bn,an,2*pi*f); \]
\% Compute and plot the magnitude of the transfer function
\texttt{semilogx}(f,abs(Gsn));
\% Compute and plot the magnitude of the transfer function
\texttt{xlabel('Frequency f (Hz)' )};
\% Compute and plot the magnitude of the transfer function
\texttt{ylabel('Magnitude of Transfer Function' )};
\% Compute and plot the magnitude of the transfer function
\texttt{title('3-pole Butterworth band-pass filter with $f_0 = 4$ KHz, $BW = 2$KHz')};
Band-Pass Filter

3-pole Butterworth band-pass filter with $f_0 = 4$ KHz, BW = 2KHz
Band-Elimination (band-stop) Filter

- Use the MATLAB functions `buttap` and `lp2bs` to find the transfer function of a 3-pole Butterworth band-elimination (band-stop) filter with the stop band frequency centered at $f_0 = 5 \text{ kHz}$, and bandwidth $BW = 2\text{kHz}$.

```matlab
[z,p,k]=buttap(3); % Design 3-pole Butterworth low-pass filter, wcn = 1 r/s
[b,a]=zp2tf(z,p,k); % Compute num, den coefficients of this filter, wcn=1 r/s
f=100:100:100000; % Define frequency range to plot
f0=5000; % Define centered frequency at 5 kHz
W0=2*pi*f0; % Convert centered frequency to r/s
fbw=2000; % Define bandwidth
Bw=2*pi*fbw; % Convert bandwidth to r/s
% Compute numerator and denominator coefficients of desired band stop filter
[bn,an]=lp2bs(b,a,W0,Bw);
% Compute and plot magnitude of the transfer function of the band stop filter
Gsn=freqs(bn,an,2*pi*f);
semilogx(f,abs(Gsn));
grid;
xlabel('Frequency in Hz'); ylabel('Magnitude of Transfer Function');
title('3-pole Butterworth band-elimination filter with f0=5 \text{ KHz}, BW = 2 \text{ KHz}')
```
Band-Elimination (band-stop) Filter

3-pole Butterworth band-elimination filter with $f_0=5$ KHz, $BW = 2$ KHz
How to find the minimum order to meet the filter specifications?

The following functions in Matlab can help you to find the minimum order required to meet the filter specifications:

- `Buttord` for butterworth
- `Cheb1ord` for chebyshev
- `Ellipord` for elliptic
- `Cheb2ord` for inverse chebyshev
Calculating the order and cutoff frequency of a inverse chebyshev filter

- Design a 4MHz Inverse Chebyshev approximation with Ap gain at passband corner. The stop band is 5.75MHz with -50dB gain at stop band.

```matlab
clear all;
Fp = 4e6; Wp=2*pi*Fp;
Fs=1.4375*Fp; Ws=2*pi*Fs;
Fplot = 20*Fs;
f = 1e6:Fplot/2e3:Fplot ;
w = 2*pi*f;
Ap = 1;
As = 50;
% Cheb2ord helps you find the order and wn (n and Wn) that you can pass to cheby2 command.
[n, Wn] = cheb2ord(Wp, Ws, Ap, As, 's');
[z, p, k] = cheby2(n, As, Wn, 'low', 's');
[num, den] = cheby2(n, As, Wn, 'low', 's');
bode(num, den)
```
Bode Plot
References
