High-Order Filters

- There are two main approaches for implementing high-order filters:
  - **Cascade**
  - Multiple Feedback Architecture: Leap Frog, Follow the Leader, etc

- Cascade approach

\[ T(s) = \prod_{i=1}^{N} T_i(s) \]

where

\[ T_i(s) = K_i \frac{s^2 + C_i s + d_i}{s^2 + \left( \frac{\omega_o}{Q} \right)_i s + \omega^2_{oi}} \]
• T(s) can be realized by a cascade of circuits blocks, each of these blocks realizes the biquadratic function \( T_i(s) \).

• Each biquad is independent of any biquad if \( Z_{oi} << Z_{in}(i+1) \)

\[
T_i \quad T_{i+1}
\]

\( Z_{oi} \quad Z_{in, i+1} \)

Condition \( Z_{oi} << Z_{in, i+1} \)

• **Degrees of freedom**
  - Physical position of each biquad in the cascade
  - Distribution of the overall gain in the different biquads
  - Pole-Zero pairing
• Optimal goals for cascade filters
  - Maximization of dynamic range
  - Maximization of the signal-to-noise ration

• Additional desirable features
  - Simplification of the tuning procedure
  - Minimization of the pass band attenuation

• Pole-zero pairing
  - Pair each complex pole with its nearest complex zero. This will maximize the dynamic range of each biquad.
  - Starting with the pole of highest Q factor, i.e.,
• How about the cases for zeros at zero and infinity?

\[ T(s) = \frac{ks^3}{(s^2 + b_1s + C_1)(s^2 + b_2s + C_2)(s^2 + b_3s + C_3)} \]

- Options

\[ T(s) = \frac{k_1}{s^2 + b_1s + C_1} \frac{k_2s^2}{s^2 + b_2s + C_2} \frac{k_3s}{s^2 + b_3s + C_3} \]

\[ T(s) = \frac{k_1s}{s^2 + b_1s + C_1} \frac{k_2s}{s^2 + b_1s + C_1} \frac{k_3s}{s^2 + b_1s + C_1} \]

- Advantages and disadvantages

• Cascade sequence

  - Several options N!
  - Cascade in increasing Q factors.
    
    \[ \text{LP with } Q_1 \]
    \[ \text{BP with } Q_1 \]
    \[ \text{HP with } Q_3 \]
    \[ Q_1 < Q_2 < Q_3 \]