Consider a nonlinear system described by the following equation:

\[ y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \]  

Where \( y(t) \) and \( x(t) \) is the output and input of the system respectively.

Assume \( x(t) = A \cos(\omega t) \), then from equation (A.1) we get:

\[ y(t) = \alpha_0 + \alpha_1 A \cos(\omega t) + \alpha_2 A^2 \cos^2(\omega t) + \alpha_3 A^3 \cos^3(\omega t) \]  

Note that the DC (fundamental) magnitude is affected by the even (odd) harmonic components.
In equation (A.2b), the term with the input frequency is called the fundamental and the higher order terms the harmonics. LNAs are typical examples of eq. (a.2). Harmonic distortion factors (HD) provide a measure for the distortion introduced by each harmonic for a given input signal level (using a single tone at a given frequency). HD is defined as the ratio of the output signal level of the \( i^{th} \) harmonic to that of the fundamental. The THD is the geometric mean of the distortion factors. Assuming \( \alpha_1 A >> 3\alpha_3 A^3/4 \) the second harmonic distortion \( HD_2 \), the third harmonic distortion \( HD_3 \) and the total harmonic distortion \( THD \) are defined as:

\[
HD_2 = \frac{\alpha_2 A}{2\alpha_1} \quad \text{(A.3a)}
\]

\[
HD_3 = \frac{\alpha_3 A^2}{4\alpha_1} \quad \text{(A.3b)}
\]

\[
THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \ldots} \quad \text{(A.3c)}
\]
For fully differential systems, ideally even harmonics will vanish and only odd harmonics remain. In reality, however, mismatches corrupt the symmetry, yielding finite even order harmonics. In a fully differential system with $\varepsilon\%$ mismatch and from equation (A.3a), $HD_2$ is given by:

$$HD_2 = \varepsilon \frac{\alpha_2 A}{2\alpha_1}$$ \hspace{2cm} (A.4)

**1-dB compression point**

The 1-dB compression point is defined as the point where the fundamental gain deviates from the ideal small signal gain by 1 dB, as shown in Fig. A.1. From the previous definition and from equation (A.2b), we have:

$$20\log\left(\alpha_1 A_{1-dB} + \frac{3\alpha_3 A_{1-dB}^3}{4}\right) = 20\log(\alpha_1 A_{1-dB}) - 1 = 20\log(0.89125\alpha_1 A_{1-dB})$$ \hspace{2cm} (A.5)

Note that $20 \log 0.89125 = -1$ dB, $|1-0.89125| = 0.10875$
\[ A_{1-dB}^2 = 0.10875 \frac{4 |\alpha_1|}{3 |\alpha_3|} = k \frac{|\alpha_1|}{|\alpha_3|} \]  

(A.6)

Note that \( A_{1-dB} \) often occurs for a certain \( V_{pp} \) value in a typical resistor \( R_s=50\Omega \).

Thus at the 1-dB compression point, the value of \( HD_3 \) can be calculated as:

\[ HD_3 = \frac{1}{4} \frac{k}{A_{1-dB}^2} = \frac{0.145}{4} \frac{|\alpha_3| |\alpha_1|}{|\alpha_1| |\alpha_3|} = \frac{0.145}{4} \times 100\% = 3.6\% \]  

(A.7)

Fig. A.1 Definition of the 1-dB compression point
Intermodulation Distortion

Consider \( x(t) = A \cos(\omega_1 t) + A \cos(\omega_2 t) \), then from equation (A.1) and trigonometric identities:

\[
y(t) = \left(\alpha_0 + \alpha_2 A^2\right) + \left(\alpha_4 A + \frac{9\alpha_3 A^3}{4}\right) \cos(\omega_1 t) + \left(\alpha_4 A + \frac{9\alpha_3 A^3}{4}\right) \cos(\omega_2 t) + \\
\left(\frac{\alpha_2 A^2}{2}\right) \cos(2\omega_1 t) + \left(\frac{\alpha_2 A^2}{2}\right) \cos(2\omega_2 t) + \left(\alpha_2 A^2\right) \cos((\omega_1 + \omega_2) t) + \\
\left(\alpha_2 A^2\right) \cos((\omega_1 - \omega_2) t) + \left(\frac{3\alpha_3 A^3}{4}\right) \cos((2\omega_1 - \omega_2) t) + \left(\frac{3\alpha_3 A^3}{4}\right) \cos((2\omega_2 - \omega_1) t) + \\
\left(\frac{3\alpha_3 A^3}{4}\right) \cos((2\omega_1 + \omega_2) t) + \left(\frac{3\alpha_3 A^3}{4}\right) \cos((2\omega_2 + \omega_1) t) + \\
\left(\frac{\alpha_3 A^3}{4}\right) \cos(3\omega_1 t) + \left(\frac{\alpha_3 A^3}{4}\right) \cos(3\omega_2 t)
\]

(A.8)

The third order input intercept point \( \text{IIP}_{3i} \) is defined as the intercept point of the fundamental component with the third order intermodulation component, as shown in Fig. A.2. From the previous definition and from equation (A.8), we have:
(\alpha_1 A^3_{IIP3i}) = \left(\frac{3\alpha_3 A^3_{IIP3i}}{4}\right) \Rightarrow A^2_{IIP3i} = \frac{4}{3} \left|\frac{\alpha_1}{\alpha_3}\right| \quad \text{(A.9)}

The input \( I_P \) is \( A_{IIP3} = \left[\frac{4}{3} \left|\frac{\alpha_1}{\alpha_3}\right|\right]^{\frac{1}{2}} \) and the output \( I_P \) is \( \alpha_1 A_{IIP3} \)

The third order intermodulation distortion \( IM_3 \) is defined as:

\[
IM_3 = \frac{3}{4} \left|\frac{\alpha_3}{\alpha_1}\right| A^2 = 3HD_3 \quad \text{(A.10)}
\]

Note from equations (A.6) and (A.9) that:

\[
\frac{A^2_{IIP3i}}{A_{1-dB}^2} = 9.195 \Rightarrow A_{IIP3i} \text{(dB)} = A_{1-dB} \text{(dB)} + 10 \quad \text{(A.11)}
\]

Fig. A.2 Definition of the third order intercept point
How to Measure IIP3

Let us assume that the input of a nonlinear device characterized by (A.2a) which consists of three components, the desired input signal with amplitude $A_s$ at $f_o$, and two interferers at $f_1$ and $f_2$, where $f_1 = f_o + \Delta f$ and $f_2 = f_o + 2\Delta f$. The amplitude of the IM3 products are denoted by $A_{IM3}$. Then one can write from (A.10).

$$\frac{A_{1,2}}{A_{IM3}} = \frac{\alpha_1 |A_s|}{3|\alpha_3|A_s^3 / 4} = \frac{4|\alpha_1|}{3|\alpha_3|} \frac{1}{A_s^2}$$  \hspace{1cm} (B.1)

Since

$$A_{IP3}^2 = \frac{4|\alpha_1|}{3|\alpha_3|}$$

Then (B.1) can be written as

$$\frac{A_{1,2}}{A_{IM3}} = \frac{A_{IP3}^2}{A_s^2}$$  \hspace{1cm} (B.2)

Thus by taking the log of (B.1) and (B.2) and equating them, yields

$$20 \log A_{1,2} - 20 \log A_{IM3} = 20 \log A_{IP3}^2 - 20 \log A_s^2$$

or equivalent
\[ 20 \log A_{IP3} = \frac{1}{2} (20 \log A_{1,2} - 20 \log A_{IM3}) + 20 \log A_s \] (B.3)

- Observe that \( A_{IP3} \) can be determined with only one input level, thus there is no need for extrapolation.

- This approach is not accurate, but it provides a good estimate of \( IP_3 \).
**Dynamic Range**

The are many definitions for the dynamic range. We define here the 1-dB compression dynamic range $\text{DR}_{1-dB}$ and the spurious free dynamic range (SFDR). The SFDR is the difference, in dB, between the fundamental tone and the highest spur, which could be an intermodulation harmonic, in the bandwidth of interest.

\[
\text{DR}_{1-dB} = P_{i,1dB} - P_{i,mds}
\]  

(A.12)

\[
\text{SFDR} = \frac{2}{3}(IIP_{3i} - P_{i,mds})
\]

(A.13)

where, $P_{i,mds} = -174 dBm + 10 \log B + \text{NF}$  

$P_{o,mds} = P_{i,mds} + \alpha_1|_{dB}$
Remarks on Dynamic Range.

The upper end of the dynamic range is defined as the maximum input level in a two tone test for which the IM\textsubscript{3} products do not exceed the noise floor.

Recall (B.3):

$$20 \log A_{\text{IP3}} = \frac{1}{2}(20 \log A_{1,2} - 20 \log A_{\text{IM3}}) + 20 \log A_s$$
This can be written, assuming all the parameters in dBm, as follows:

\[ P_{IIIP3} = P_s + \frac{P_{out} - P_{IM, out}}{2} \]

Where \( P_{IM, out} \) is the power of \( IM_3 \) components at the output. \( G \) is the circuit power gain in dB, and \( P_{IM, in} \) is the input referred level of the \( Im_s \) products. Thus

\[ P_{out} = P_s + G \]

\[ P_{IM, out} = P_{IM, in} + G \]

\[ P_{IIIP3} = P_s + \frac{P_s - P_{IM, in}}{2} = \frac{3P_s - P_{IM, in}}{2} \]

Whereby

\[ P_s = P_{in} = \frac{2P_{IIIP3} + P_{IM, in}}{3} \]
Observe that the input level for which the IM products become equal to the noise floor yields:

\[ P_{in,max} = \frac{2P_{IP3} + NF_{total}}{3} \]

Where \( NF_{total} = -174 dBm + NF + 10 \log B \)

Therefore

\[ SFDR = \frac{2P_{IP3} + NF_{total}}{3} - (NF_{total} + SNR_{min}) \]

\[ SFDR = \frac{2(P_{IP3} - NF_{total})}{3} - SNR_{min} \]

**Example**

For a simple differential pair as shown in Fig. A.4.

\[ I_0 = I_1 - I_2, \quad I_{DC} = I_1 + I_2 \quad \text{(A.14)} \]
where, \( I_1 = \frac{I_{DC}}{2} + \frac{I_0}{2}, \) \( I_2 = \frac{I_{DC}}{2} - \frac{I_0}{2} \)

\[
I_1 = \frac{\beta}{2} (V_{GS1} - V_T)^2, \quad I_2 = \frac{\beta}{2} (V_{GS2} - V_T)^2, \quad \text{where} \quad \beta = K_n \frac{W}{L}
\]  \hspace{1cm} (A.15)

\[
V_{GS1} = V_T + \sqrt{\frac{2I_1}{\beta}}, \quad V_{GS2} = V_T + \sqrt{\frac{2I_2}{\beta}}
\] \hspace{1cm} (A.16)

\[
V_{GS1} - V_{GS2} = V_{in}^+ - V_{in}^- = v_d
\] \hspace{1cm} (A.17)

Fig. A.4 Simple differential pair
Substituting equation (A.16) in (A.17), we have:

\[ v_d = \sqrt{\frac{2}{\beta}} \left( \sqrt{I_1} - \sqrt{I_2} \right) \]  
(A.18)

Substituting equation (A.14) in (A.18), we have:

\[ v_d = \sqrt{\frac{2}{\beta}} \left( \sqrt{\frac{I_{DC}}{2} + \frac{I_0}{2}} - \sqrt{\frac{I_{DC}}{2} - \frac{I_0}{2}} \right) \]  
(A.19)

Squaring both sides and after some algebraic manipulation, we can write:

\[ I_0 = v_d \sqrt{\beta I_{DC}} \left( 1 - \frac{\beta v_d^2}{4I_{DC}} \right)^{1/2} \]  
(A.20)

Expanding \( I_0 \) in a power series in terms of \( v_d \), we have:

\[ I_0 = \alpha_1 v_d + \alpha_3 v_d^3 + O(v_d^5) \]  
(A.21)
where \( \alpha_1 = \sqrt{\beta I_{DC}} = G_{in} \), and

\[
\alpha_3 = \frac{1}{8} \sqrt{\frac{\beta}{I_{DC}}} = \frac{1}{8} \left( \frac{G_{in}}{V_{GS} - V_T} \right)^2 = \frac{1}{8} \frac{G_{in}}{V_{DSAT}^2}
\]

According to (A.3b), HD_3 is given by:

\[
HD_3 = \frac{1}{32} \frac{A^2}{V_{DSAT}^2}
\]  \hspace{1cm} (A.22)

According to (A.6), the 1-dB compression point \( A_{1-dB}^2 \) is given by:

\[
A_{1-dB}^2 = 8k \frac{\alpha_1}{\alpha_3} = 8k V_{DSAT}^2 \Rightarrow A_{1-dB} = 1.077 \times V_{DSAT}
\]  \hspace{1cm} (A.22)

According to (A.9), the third order intercept point \( A_{IP3i}^2 \) is given by:

\[
A_{IP3i}^2 = \frac{4}{3} \frac{\alpha_1}{\alpha_3} = 10.66 \times V_{DSAT}^2 \Rightarrow A_{IP3i} = 3.266 \times V_{DSAT}
\]  \hspace{1cm} (A.23)

According to (A.10), the third order intermodulation distortion IM_3 is given by:

\[
IM_3 = \frac{3}{4} \frac{a_3}{a_1} A^2 = \frac{3}{32} \left( \frac{A}{V_{DSAT}} \right)^2
\]  \hspace{1cm} (A.24)
According to the derivation in section 2.3.3, an expression for IM₃ of a pseudo-differential pair (removing the tail current source of Fig. A.4) can be obtained as:

\[
IM₃ = \frac{3}{16} \frac{A^2 \theta}{V_{DSAT} (1 + \theta V_{DSAT})^2 (2 + \theta V_{DSAT})} 
\]  \hspace{1cm} (A.25)

**Cascaded Nonlinear Stages:**

Consider two nonlinear stages in cascade as shown in Fig. A.5. It can be shown that the overall third order intercept point Aᵢᵢᵢ₃ is given by:

\[
\frac{1}{Aᵢᵢᵢ₃^2} \equiv \frac{1}{Aᵢᵢᵢ₃,1^2} + \frac{G_i^2}{Aᵢᵢᵢ₃,2^2} 
\]  \hspace{1cm} (A.26)

where Aᵢᵢᵢ₃,ᵢ is the input IIP3 point of the iᵗʰ stage, and Gᵢ is the gain of the iᵗʰ stage.
Equation (A.26) can be generalized for more stages as:

\[
\frac{1}{A_{III}^2} \approx \frac{1}{A_{III,1}^2} + \frac{G_1^2}{A_{III,2}^2} + \frac{G_1^2 G_2^2}{A_{III,3}^2}
\]  

(A.27)

Let us consider the case of three blocks.
Where

\[ A_{IM3,1} = \frac{\alpha_1 A_I^3}{A_{IP3,1}^2} \quad ; \quad B_{IM3} = b_1 A_{IM3,1} + A_{IM3,2} \]

\[ D_{IM3} = d_1 b_1 A_{IM3,1} + d_1 A_{IM3,2} + A_{IM3,3} \]

\[ A_{IM3,2} = \frac{(\alpha_1 A_I)^3 b_1}{A_{IP3,2}^2} \quad ; \quad A_{IM3,2} = \frac{(\alpha_1 b_1 A_I)^3 d_1}{A_{IP3,3}} \]

Total \( A_{IM3} \) becomes:

\[ A_{IM3,\text{total}} = d_1 b_1 A_{IM3,1} + d_1 A_{IM3,2} + A_{IM3,3} \]

\[ A_{IM3,\text{total}} = \frac{\alpha_1 b_1 d_1 A_I^3}{A_{IP3,\text{casc}}^2} \]
Then

\[
\frac{1}{A_{IP3,casc}^2} = \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{b_1^2 \alpha_1^2}{A_{IP3,3}^2}
\]

\[
A_{IP3,cas}^2 = \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{b_1^2 \alpha_1^2}{A_{IP3,3}^2}
\]

\[
IIP_{3,i}(dBV_{rms}) = 20 \log A_{IP3,i}
\]

i.e., \( \alpha_1^2 = G R_o/R_s \Rightarrow \alpha_1(dB) = G_{dB} + 10 \log(R_o/R_s) \)

**Cadence Simulation**

To simulate the 1-dB compression point or the two-tone intermodulation distortion of the differential amplifier, the setup shown in Fig. A.6 is used. Swept Periodic Steady State (SPSS) analysis simulation of SpectreRF is chosen. The input is applied through **PORT0**. It is a power source and is called “psin” in “analogLib” library. The output resistance of the source is set as 50Ω. A physical resistance of 50Ω (not shown in Fig.
A.5) should be placed in parallel with the source for matching purposes, since the resistance seen from the gate of the MOSFET transistor is infinity. The source type should be “sine” as shown in Fig. A.7. The input consists of two relatively close frequencies ($F_1 = F_{in} = 10\text{MHz}, F_2 = F_{in} + 1\text{MHz} = 11\text{MHz}$) and their power levels are set equal to a design variable called $P_{in}$ (make sure that the Amplitude and Amplitude2 fields are left empty). This is the variable that will be swept in the SPSS simulation.
Fig. A.6 Swept periodic steady state (SPSS) simulation setup
**Fig. A.7 PORTO setup**

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In the simulation window, select SPSS analysis. The **Fundamental (Beat)** frequency is the highest frequency common to all inputs shown in the **Fundamental Tones** section. In the sweep section, select “**variable**” to sweep $P_{in}$ from $-30$dBm to $10$dBm. Only the harmonics of interest (9,10,11,12) are saved to reduce the disk area required for saving, as shown in Fig. A.8.
Fig. A.8 SPSS analysis setup
From “Options” button choose “gear2only” as the integration method. It is important to switch to “flat” netlisting to run SPSS. Select Setup->Environment. Set the netlist type as “flat”. After running the simulation, you can display the IIP$_3$ plot from Analog Artist, select Results->Direct Display->SPSS. Setup the form as shown in Fig. A.9. The 1$^{\text{st}}$ order harmonic is at 10MHz and the 3$^{\text{rd}}$ order harmonic is at 9MHz.
Fig. A.9 IIP3 results setup

Fig. A.10 1-dB Compression results setup
For the 1-dB compression point simulation only one frequency $F_{in}$ is specified and the **Number of harmonics** is set to 2. $P_{in}$ is swept from $-30$dBm to $10$dBm. The form of SPSS results is set as shown in Fig. A.10. The 1$^{\text{st}}$ order harmonic is at 10MHz. Select node $V_{out}$ (of Fig. A.5). The 1-dB compression point plot should look like Fig. A.11. The 1-dB compression point is about $-4.9$dBm, compared to the theoretical value calculated according to (A.22), that is $-3.3$dBm for our case of $V_{DSAT}=200$mV. The IIP$_3$ plot should look like Fig. A.12. IIP$_3$ is about $6.1$dBm, which is very close to the theoretical value calculated according to equation (A.23), that is $6.3$dBm for our case of $V_{DSAT}=200$mV.
Fig. A.11 1-dB compression point plot

Fig. A.12 IIP3 plot
Intermodulation Distortion

\[ v = v_1 + v_2 = V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \]

\[ v_o = a_1 \left[ V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \right] + \]

\[ a_2 \left[ V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \right]^2 + \]

\[ a_3 \left[ V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \right]^3 + \]

\[ \cdots \]
Let us first consider only the second-order term

\[
a_2 v^2 = a_2 \left[ V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \right]^2 \\
a_2 V_{1A}^2 \cos^2 \omega_1 t + a_2 V_{2A}^2 \cos^2 \omega_2 t \\
+ 2a_2 V_{1A} V_{2A} \left[ \cos \omega_1 t + \cos \omega_2 t \right] \\
= \frac{1}{2} a_2 \left[ V_{1A}^2 + V_{2A}^2 + \frac{a_2 V_{1A}^2}{2} \cos 2\omega_1 t \right] \\
+ \frac{1}{2} a_2 V_{2A}^2 \cos 2\omega_2 t + a_2 V_{1A} V_{2A} \cos (\omega_1 + \omega_2) t \\
+ a_2 V_{1A} V_{2A} \cos (\omega_1 - \omega_2) t
\]

amp

\[\omega_1 - \omega_2 \quad 2\omega_2 \quad \omega_1 + \omega_2 \quad 2\omega_1\]
The terms with $(\omega_1 + \omega_2)$ and $(\omega_1 - \omega_2)$ are used to determine the second-order intermodulation (IM$_2$).

\[
IM_2 = \frac{a_2 V_{1A} V_{2A}}{a_1 V_{1A}} = \frac{a_2}{a_1} V_{2A} = 2HD_2
\]

For small distortion, assuming that the amplitude of the fundamental component is $V_{oA}$, $V_{oA} \approx a_1 V_{1A}$ Then

\[
IM_2 = \frac{a_2}{a_1^2} V_{oA}
\]
Now let us consider the $a_3 v^3$ term:

$$a_3 v^3 = a_3 \left[ V_{1A} \cos \omega_1 t + V_{2A} \cos \omega_2 t \right]^3$$

$$= a_3 V_{1A}^3 \cos^3 \omega_1 t + 3a_3 V_{1A} V_{2A}^2 \cos \omega_1 t \cos^2 \omega_2 t$$

$$+ 3a_3 V_{1A}^2 V_{2A} \cos^2 \omega_1 t \cos \omega_2 t + a_3 V_{2A}^3 \cos^3 \omega_2 t$$

$$= \cdots + a_{32} \cos(\omega_1 \pm 2\omega_2 t) + a_{33} \cos(2\omega_1 t \pm \omega_2 t)$$

Where

$$a_{32} = \frac{3}{4} a_3 V_{1A} V_{2A}$$

$$a_{33} = 0.75 a_3 V_{1A}^2 V_{2A}$$
Broad-Band amplifiers $V_{1A} = V_{2A}$

$$V_o \left(IM_3\right) = 0.75a_3V_{1A}^3$$

$$IM_3 = \frac{0.75a_3V_{1A}^3}{a_1V_{1A}} = \frac{0.75a_3}{a_1}V_{1A}^2$$

For small distortion

$$IM_3 = \frac{0.75a_3}{a_1^3}V_{oA}^2 = 3HD_3$$

For receivers

$$V_{1A} \ll V_{2A} \Rightarrow Yielding1\%IM_3$$

$$IM_3' = \frac{3a_3V_{1A}V_{2A}^2}{a_1V_{1A}} = \frac{0.75a_3V_{2A}^2}{a_1}$$

$$IM_3 = \frac{3a_3V_{1A}^2V_{2A}}{a_1V_{1A}} = \frac{0.75a_3V_{1A}V_{2A}}{a_1}$$
Other important definitions are:

- Cross modulation i.e.,

\[
v = v_1 + v_2 = V_{1A} \cos \omega_1 t + V_{2A} (1 + m \cos \omega_1 t) \cos \omega_2 t
\]

\[
CM = 4IM_3 = 12HD_3
\]

- Third-order intercept point (IP3)

Let us recall that

\[
v_o(t) = a_1v + a_2v^2 + a_3v^3 + \cdots
\]

In fully differential amplifiers

\[
v_o(t) \approx a_1v(t) + a_3v^3(t)
\]

\[
v_o(t)\big|_{v(t)=V_A \cos \omega t}
\]
Feedback effect on nonlinearity

Feedback system incorporating a nonlinear feedforward amplifier.

The output can be approximated as \( y \approx a \cos \omega t + b \cos 2\omega t \). Our objective is to determine \( a \) and \( b \). The output of the subtractor can be written as

\[
y_s = x(t) - \beta y(t)
\]

\[
= V_m \cos \omega t - \beta (a \cos \omega t + b \cos 2\omega t)
\]

\[
= (V_m - \beta a) \cos \omega t - \beta b \cos 2\omega t.
\]
This signal experiences the nonlinearity of the feedforward amplifier, thereby producing an output given by:

\[
y(t) = a_1 [(V_m - \beta ) \cos \omega t - \beta b \cos 2\omega t] \\
\quad + a_2 [(V_m - \beta ) \cos \omega t - \beta b 2\omega t]^2 \\
= [a_1 (V_m - \beta a) - a_2 (V_m - \beta a) \beta b ] \cos 2\omega t \\
\quad + \left[-a_1 \beta b + \frac{a_2 (V_m - \beta a)^2}{2}\right] \cos 2\omega t + \ldots
\]
The coefficients of $\cos 2\omega t$ in (5) must be equal to $a$ and $b$, respectively:

$$a = (\alpha_1 - \alpha_2 \beta b)(V_m - \beta a)$$  \hspace{1cm} (6)

$$b = -\alpha_1 \beta b + \frac{\alpha_2 (V_m - \beta a)^2}{2}$$  \hspace{1cm} (7)

The assumption of small nonlinearity implies that both $a_2$ and $b$ are small quantities, yielding and hence

$$a = \frac{\alpha_1}{1 + \beta a_1} V_m,$$  \hspace{1cm} (8)

which is to be expected because $\beta \alpha_1$ is the loop gain. To calculate $b$, we write

$$V_m - \beta a \approx \frac{a}{\alpha_1}$$  \hspace{1cm} (9)
Thus expressing (7) as

\[ b = -\alpha_1 \beta b + \frac{1}{2} \alpha_2 \left( \frac{a}{\alpha_1} \right)^2 \]  \hspace{1cm} (10)

That is,

\[ b(1 + \alpha_1 \beta) = \frac{\alpha_2}{2} \left( \frac{a}{\alpha_1} \right)^2 \]  \hspace{1cm} (11)

\[ = \frac{\alpha_2}{2\alpha_1^2} \frac{\alpha_1^2}{(1 + \beta \alpha_1)^2} V_m^2. \]  \hspace{1cm} (12)
It follows that

\[ b = \frac{\alpha_2 V_m^2}{2} \frac{1}{(1 + \beta \alpha_1)^3}. \]  \hspace{1cm} (13)

For a meaningful comparison, we normalize the amplitude of the second harmonic to that of the fundamental:

\[ \frac{b}{a} = \frac{\alpha_2 V_m}{2} \frac{1}{\alpha_1} \frac{1}{(1 + \beta \alpha_1)^2}. \]  \hspace{1cm} (4)
Without feedback, on the other hand, such a ratio would be equal to 
\[(\alpha_2 V_m^2 / 2) / \alpha_1 V_m = \alpha_2 V_m / (2 \alpha_1).\] Thus, the relative magnitude of the second harmonic has dropped by a factor of \((1 + \beta \alpha_1)^2\).