Outline

- Definitions
- Combinatorial probability rules
- Application to power system reliability
Definitions

- Sample space or state space
- Events
  - Union of events
  - Intersection of events
  - Disjoint events
  - Complement of an event
  - Independent events
- Probability of an event
- Conditional probability
Sample Space or State Space

- The set of all possible outcomes of a random phenomenon

Example

- A status of a generator,
- state space = \{Up, Down\}
A Status of Two Transmission Lines

State space = \{ (1U,2U), (1U,2D) , (1D,2U) , (1D,2D) \}
Events

- A set of outcomes or a subset of a sample space.

Example
- Event of a generator fails, $E = \{\text{Down}\}$
An Event of One Transmission Line Fails

$$E = \{ (1\text{U},2\text{D}) , (1\text{D},2\text{U}) \}$$
Union of Events

- Union of event $E_1$ and $E_2$ ($E_1 \cup E_2$) contains outcomes from either $E_1$ or $E_2$ or both.

Examples

- $E_1$ is an event that at least one line is up,
  $$E_1 = \{(1U,2U), (1U,2D) , (1D,2U)\}$$

- $E_2$ is an event that at least one line is down,
  $$E_2 = \{(1U,2D) , (1D,2U) , (1D,2D) \}$$

- Then, union of event $E_1$ and $E_2$ is,
  $$E = E_1 \cup E_2 = \{(1U,2U), (1U,2D) , (1D,2U) , (1D,2D) \}$$
Intersection of Events

- Intersection of event $E_1$ and $E_2$ ($E_1 \cap E_2$) contains outcomes from both $E_1$ and $E_2$.
- Example
  - $E_1$ is an event that at least one line is up, $E_1 = \{(1U,2U), (1U,2D), (1D,2U)\}$
  - $E_2$ is an event that at least one line is down, $E_2 = \{(1U,2D), (1D,2U), (1D,2D)\}$
  - Then, intersection of event $E_1$ and $E_2$ is, $E = E_1 \cap E_2 = \{(1U,2D), (1D,2U)\}$
    Which is that only one line is up
Disjoint Events

- Events that can not happen together.
- Example
  - $E_1$ is an event that two lines are up,
    \[ E_1 = \{ (1U,2U) \} \]
  - $E_2$ is an event that two lines are down,
    \[ E_2 = \{ (1D,2D) \} \]
  - Then, $E_1$ and $E_2$ are disjoint events.
Complement of an Event

- The set of outcomes that are not included in an event.

- Example
  - $E_1$ is an event that two lines are up,
    \[ E_1 = \{ (1U,2U) \} \]
  - $\bar{E}_1$ is a complement of $E_1$,
    \[ \bar{E}_1 = \{ (1U,2D) , (1D,2U) , (1D,2D) \} \]
Independent Events

- Events that happen independently.
- Example
  - Failure of a generator and failure of a line
  - Failure of line 1 and failure of line 2
Probability of an Event

- Number of times that an event occurs divided by total number of occurrences

- Properties,
  - \(0 \leq P(E) \leq 1\)
  - \(P(\text{Impossible event}) = 0\)
  - \(P(\text{Sure event}) = 1\)
Conditional Probability

- The probability of E2 given E1, \( P(E_2 \mid E_1) \), is the probability that event E2 occurs given that E1 has already occurred.

- If \( E_2 \) and \( E_1 \) are independent, then

\[
P(E_2 \mid E_1) = P(E_2).
\]
Probability Rules

- Combinatorial properties
  - Addition rule
  - Multiplication rule
  - Conditional probability rule
  - Complementation rule
Addition Rule

- A method of finding a probability of union of two events: Prob of $E_1$ or $E_2$ or both.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

- If $E_2$ and $E_1$ are mutually exclusive, then

$$P(E_2 \cup E_1) = P(E_1) + P(E_2).$$
A method of finding a probability of intersection of two events: prob of \( E_1 \) and \( E_2 \).

\[
P(E_1 \cap E_2) = P(E_1) \times P(E_2 | E_1)
\]

If \( E_1 \) and \( E_2 \) are independent, then

\[
P(E_1 \cap E_2) = P(E_1) \times P(E_2).
\]
Conditional Probability Rule

- If an event \( E \) depends on a number of mutually exclusive events \( B_j \), then

\[
P(E) = \sum_j [P(E|B_j) \times P(B_j)]
\]
Complementation Rule

- Probability of the set of outcomes that are not included in an event.

\[ P(\bar{E}) = 1 - P(E) \]

- Example

Probability of success = 1 – Probability of failure
Example of Application to Calculation of Loss Of Load Probability - LOLP

- 3 generators
- Each with capacity 50 MW
- Identical probability of failure = 0.01
- Assume that each generator fails and is repaired independently.
- Find probability distribution of generating capacity.
State Space

\[
\begin{align*}
\text{State space} &= \{(1\text{U},2\text{U},3\text{U}), (1\text{D},2\text{U},3\text{U}), (1\text{U},2\text{D},3\text{U}), (1\text{U},2\text{U},3\text{D}), (1\text{D},2\text{U},3\text{D}), (1\text{U},2\text{D},3\text{D}), (1\text{D},2\text{D},3\text{U}), (1\text{D},2\text{U},3\text{D}), (1\text{U},2\text{D},3\text{D}), (1\text{D},2\text{D},3\text{D})\}
\end{align*}
\]
Generating Capacity Probability Distribution

- Find a probability associated with each generating capacity levels
- 4 capacity levels, 0 MW, 50 MW, 100 MW, and 150 MW
- Let
  - $E_0$ be an event that generating capacity is 0 MW
  - $E_1$ be an event that generating capacity is 50 MW
  - $E_2$ be an event that generating capacity is 100 MW
  - $E_3$ be an event that generating capacity is 150 MW
Capacity 50 MW

- \( E_1 = \{(1D,2D,3U),(1D,2U,3D),(1U,2D,3D)\} \)
  
  \[ P(E_1) = P\{(1D,2D,3U)\cup(1D,2U,3D)\cup(1U,2D,3D)\} \]

- Using **addition rule**,  
  
  \[ P(E_1) = P\{(1D,2D,3U)\} + P\{(1D,2U,3D)\} + P\{(1U,2D,3D)\} \]
  
  \[ P(E_1) = P(1D \cap 2D \cap 3U) + P(1D \cap 2U \cap 3D) + P(1U \cap 2D \cap 3D) \]

- Using **multiplication rule**,  
  
  \[ P(E_1) = P(1D) \times P(2D) \times P(3U) + P(1D) \times P(2U) \times P(3D) + P(1U) \times P(2D) \times P(3D) \]

- Using **complementation**,  
  
  \[ P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99 \]

- Then, \( P(E_1) = 0.000297 \)
Capacity 100 MW

- $E_2 = \{(1D, 2U, 3U), (1U, 2D, 3U), (1U, 2U, 3D)\}$
  \[ P(E_2) = P\{(1D, 2U, 3U)\} \cup P\{(1U, 2D, 3U)\} \cup P\{(1U, 2U, 3D)\} \]

- Using **addition rule**,
  \[ P(E_2) = P\{(1D, 2U, 3U)\} + P\{(1U, 2D, 3U)\} + P\{(1U, 2U, 3D)\} \]
  \[ P(E_2) = P(1D \cap 2U \cap 3U) + P(1U \cap 2D \cap 3U) + P(1U \cap 2U \cap 3D) \]

- Using **multiplication rule**,  
  \[ P(E_2) = P(1D) \times P(2U) \times P(3U) + P(1U) \times P(2D) \times P(3U) + P(1U) \times P(2U) \times P(3D) \]

- Using **complementation**,
  \[ P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99 \]

- Then, $P(E_2) = 0.029403$
Capacity 150 MW

- $E_3 = \{(1U, 2U, 3U)\}$

  $$P(E_3) = P(1U \cap 2U \cap 3U)$$

- Using *complementation*,

  $$P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99$$

- Using *multiplication rule*,

  $$P(E_3) = P(1U) \times P(2U) \times P(3U)$$

  $$P(E_3) = 0.99 \times 0.99 \times 0.99 = 0.970299$$
Generating Probability Distribution

<table>
<thead>
<tr>
<th>Capacity (MW)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000001</td>
</tr>
<tr>
<td>50</td>
<td>0.000297</td>
</tr>
<tr>
<td>100</td>
<td>0.029403</td>
</tr>
<tr>
<td>150</td>
<td>0.970299</td>
</tr>
</tbody>
</table>

This is an example of probability density function or probability mass function. Left column: values of discrete random variable ‘capacity’. Right column contains corresponding probabilities.
Loss of Load Probability

- Assume that load has the following distribution, find loss of load probability.
- Let $E$ be the event that system suffers from loss of load, then $E = \{\text{Generation} < \text{Load}\}$

<table>
<thead>
<tr>
<th>Load (MW)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.20</td>
</tr>
<tr>
<td>100</td>
<td>0.75</td>
</tr>
<tr>
<td>150</td>
<td>0.05</td>
</tr>
</tbody>
</table>
LOLP Calculation

- Let
  - \( B_1 \) be an event that load is 50 MW
  - \( B_2 \) be an event that load is 100 MW
  - \( B_3 \) be an event that load is 150 MW

- Then, using conditional probability theory,

\[
P(E) = P(E|B_1) \times P(B_1) + P(E|B_2) \times P(B_2) + P(E|B_3) \times P(B_3)
\]

\[
P(E) = P(E|B_1) \times 0.20 + P(E|B_2) \times 0.75 + P(E|B_3) \times 0.05
\]
Load 50 MW

\[ \text{P}(E|B_1) = P\{(1D,2D,3D)\} = 0.000001 \]
Load 100 MW

\[
P(E|B_2) = \text{P}\{(1U,2D,3U),(1D,2U,3D),(1U,2D,3D),(1D,2D,3D)\}
\]

\[
P(E|B_2) = 0.000297 + 0.000001 = 0.000298
\]
Load 150 MW

\[
P(E|B_3) = 1 - P\{ (1U,2U,3U) \} = 0.029701
\]
LOLP Calculation

- Loss of load probability is,

\[ P(E) = P(E|B_1) \times 0.20 + P(E|B_2) \times 0.75 + P(E|B_3) \times 0.05 \]

\[ P(E) = 0.000001 \times 0.20 + 0.000298 \times 0.75 + 0.029701 \times 0.05 \]

\[ P(E) = 0.00170875 \]