Appendix

Three-State Device Networks

All derivations in the appendix are based on the binomial expansion \((P+q)^n\). In the case of three-state device structures, the expansion is modified to \((P+q_o+q_s)^n\), where \(P\) is the component probability of success, \(q_o\) open failure probability, \(q_s\) short failure probability, and \(n\) is the number of independent elements.

A.1 SERIES STRUCTURE

A.1.1 Reliability Expression

Let \(n=2\). Therefore

\[
(P + q_o + q_s)^2 = 1
\]

Thus

\[
P^2 + 2Pq_o + 2Pq_s + q_o^2 + 2q_oq_s + q_s^2 = 1
\]

The number of state combinations for \((n=2) = 3^2 = 9\). The state combination truth table may be represented as follows:

<table>
<thead>
<tr>
<th>Table A1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
</tr>
<tr>
<td>ON</td>
</tr>
<tr>
<td>SN</td>
</tr>
</tbody>
</table>

where \(N\) = the normal mode of the device (success)
\(O\) = the open mode failure state
\(S\) = the short mode failure state

The reliability terms, by inspection of the truth table, Table A1, are

\[
PP + Pq_s = Pq_s = P^2 + 2Pq_s
\]
Since
\[
P = 1 - q_o - q_s, \quad (A.1)
\]
\[
R = (1 - q_o^2) - q_s^2 \quad (A.2)
\]

Now let \( n = 3 \):

Therefore
\[
(P + q_o + q_s)^3 = 1
\]

Thus
\[
P^3 + 3P^2q_o + 3Pq_o^2 + 3Pq_s^2 + 6Pq_oq_s + q_o^3 + 3q_o^2q_s + 3q_oq_s^2 + q_s^3 = 1
\]

Number of state combinations for \((n = 3) = 3^3 = 27.\) The new state combinations truth table is as shown in Table A2.

### Table A2.

<table>
<thead>
<tr>
<th>NNN</th>
<th>SSS</th>
<th>OOO</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOO</td>
<td>SSO</td>
<td>OSS</td>
</tr>
<tr>
<td>OON</td>
<td>OOS</td>
<td>SSO</td>
</tr>
<tr>
<td>NON</td>
<td>OSO</td>
<td>ONS</td>
</tr>
<tr>
<td>ONO</td>
<td>SOS</td>
<td>NNO</td>
</tr>
<tr>
<td>NSS</td>
<td>SNN</td>
<td>NSO</td>
</tr>
<tr>
<td>SSN</td>
<td>NSN</td>
<td>SNO</td>
</tr>
<tr>
<td>SNS</td>
<td>NSN</td>
<td>SNO</td>
</tr>
<tr>
<td>SON</td>
<td>OSN</td>
<td>ONS</td>
</tr>
</tbody>
</table>

The reliability terms by inspection of Table A2

\[
PPP + P_2q_o + P_2q_s + P_4q_o + P_4q_s + PPq_o + PPq_s
\]

therefore

\[
R = P^3 + 3P^2q_s + 3P^2q_o \quad (A.3)
\]

By using (A.1) and (A.3):

\[
R = (1 - q_o)^3 - q_s^3 \quad (A.4)
\]

Obviously, from (A.2) and (A.4), the general equation for the reliability of the series structure can be written as follows. For identical components:

\[
R = (1 - q_o)^n - q_s^n \quad (A.5)
\]

and in the case of nonidentical components:

\[
R = \prod_{i=1}^{n} (1 - q_{oi}) - \prod_{i=1}^{n} q_{si} \quad (A.6)
\]

### A.1.2 Probabilities of Failure

At \( n = 2 \). From Table A1 open mode failure terms

\[
q_oP + Pq_o + q_oq_o + q_qo + q_oq_s
\]

Since

\[
P = 1 - q_o - q_s \quad (A.7)
\]

Therefore the series system open mode failure probability

\[
Q_o = 1 - (1 - q_o)^2 \quad (A.8)
\]

and similarly for short-mode failure probability

\[
Q_s = q_s^2 \quad (A.9)
\]

At \( n = 3 \). According to Table A2, open mode failure terms

\[
3Pq_o^2 + 3P^2q_o + 6Pq_oq_s + 3q_o^2q_s + 3q_oq_s^2 + q_s^3
\]

By substituting for \( P \) in (A.10), open mode failure probability

\[
Q_o = 1 - (1 - q_o)^3 \quad (A.11)
\]

and in the same way for short-mode failure

\[
Q_s = q_s^3 \quad (A.12)
\]

In the case of identical components, according to (A.8) and (A.11), the general form of open mode system failure

\[
Q_o = 1 - (1 - q_o)^n \quad (A.13)
\]

Similarly, for nonidentical components

\[
Q_s = 1 - \prod_{i=1}^{n} (1 - q_{oi}) \quad (A.14)
\]
In the case of short mode system failure for identical components

\[ Q_s = q_s^n \]  \hspace{1cm} (A.15)

and for the nonidentical elements case

\[ Q_s = \prod_{i=1}^{n} q_{si} \]  \hspace{1cm} (A.16)

### A.2 PARALLEL STRUCTURE

#### A.2.1 Reliability Expression

Let the number of parallel elements \( m = 2 \)

\[ \therefore (P + q_s + q_s)^2 = 1 \]

Thus

\[ P^2 + 2Pq_s + 2Pq_s + q_s^2 + 2q_s q_s = 1 \]

The reliability terms with inspection from Table A1

\[ PP + q_s P + P q_s = P^2 + 2Pq_s \]  \hspace{1cm} (A.17)

Since \( P = 1 - q_s - q_o \), therefore

\[ R = (1 - q_s)^2 - q_s^2 \]  \hspace{1cm} (A.18)

At \( m = 3 \)

\[ \therefore (P + q_s + q_s)^3 = 1 \]

Thus

\[ P^3 + 3P^2q_s + 3Pq_s^2 + 3q_s^3 + 6Pq_s q_s + q_s^3 + 3q_s q_s^2 + q_s^3 = 1 \]

By inspecting Table A2, system reliability

\[ = P^3 + 3q_s^2 P + 3P^2 q_s \]  \hspace{1cm} (A.19)

Replace \( P \) with \( (A.7) \); therefore

\[ R = (1 - q_s)^3 - q_s^3 \]  \hspace{1cm} (A.20)

With the aid of \( (A.18) \) and \( (A.20) \), the general system reliability formula for identical elements connected in the parallel configuration becomes as follows:

\[ R = (1 - q_s)^n - q_s^n \]  \hspace{1cm} (A.21)

The above equations for nonidentical elements may be rewritten as

\[ R = \prod_{i=1}^{m} (1 - q_{si}) - \prod_{i=1}^{m} q_{oi} \]  \hspace{1cm} (A.22)

### A.2.2 Failure Probabilities

For \( m = 2 \). Collected short-mode failure terms from Table A1

\[ = q_s P + q_s q_s + q_s q_s + q_s q_s + P q_s = q_s^2 + 2Pq_s + 2q_s q_s \]

Thus:

\[ \therefore Q_s = 1 - (1 - q_s)^2 \]  \hspace{1cm} (A.23)

Similarly, for open-mode failure

\[ Q_o = q_o^2 \]  \hspace{1cm} (A.24)

At \( m = 3 \). Short-mode system failure terms from Table A2

\[ = q_s^3 + 3Pq_s^2 + 6Pq_s q_s + 3q_s^2 q_s + 2q_s q_s^2 + 3q_s^2 q_s + 3q_s^2 q_o \]

Thus

\[ Q_s = 1 - (1 - q_s)^3 \]  \hspace{1cm} (A.25)

Similarly, in the case of open mode failure

\[ Q_o = q_o^3 \]  \hspace{1cm} (A.26)

As seen from \( (A.23) \) and \( (A.25) \), the short-mode failure equation for identical elements can be generalized as follows:

\[ Q_s = 1 - (1 - q_s)^n \]  \hspace{1cm} (A.27)

Similarly, for the nonidentical elements case

\[ Q_s = 1 - \prod_{i=1}^{m} (1 - q_{si}) \]  \hspace{1cm} (A.28)
The corresponding generalized open mode system failure probability

\[ Q_o = q_o^m \quad \text{identical components} \quad (A.29) \]

and

\[ Q_o = \prod_{i=1}^{m} q_{oi} \quad \text{nonidentical components} \quad (A.30) \]