Chapter IV
Diodes and their Applications

An important mindset of practical engineers is to build systems from simple and reliable components while maximizing functionality and performance with a minimal number of basic devices. Diodes are among those building blocks that are frequently part of electrically circuits. They are simple two-terminal devices which conditionally pass current in one direction. Despite their simplistic nature, you can find them in diverse electrical systems where they play key roles in power conversion, temperature measurements, protection of circuits from excessive voltages, logic operations, and radiation detection. There are many other applications for diodes, especially for those with special properties such as laser diodes used in optical communication systems. In this chapter, you will learn about the basic properties of conventional diodes and how to exploit those properties to construct various circuits with different functions.

IV.1. The PN junction (diode).

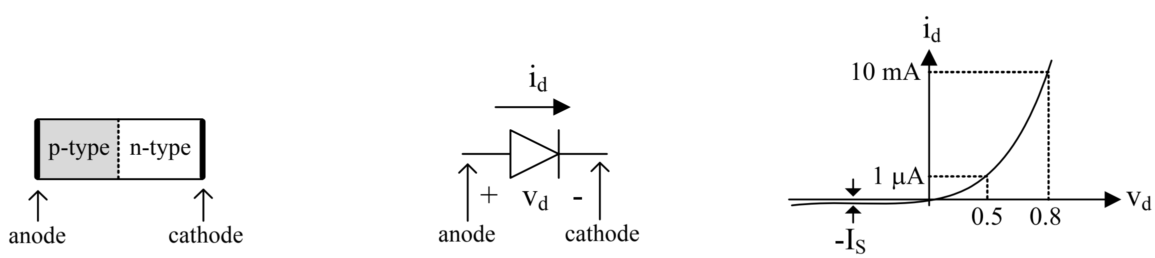
A PN junction diode is manufactured as semiconductor device, in which p-type and n-type semiconductor materials (e.g. silicon) are joined together as visualized in Figure 4.1a. Current flow across the boundary between the two types of semiconductor materials depends on the voltage applied at the two terminals called anode and cathode by convention. This labeling convention also applies to the diode symbol shown in Figure 4.1b. The PN diode is a unidirectional device with two modes of operation: The first one is referred to as "forward biased", which is when current can flow through the device and its intrinsic resistance is small. The second mode of operation is called "reverse biased". Under reverse bias conditions, the current flowing through the device is extremely small (less than 0.1 nA for most of the diodes in microelectronics applications), and the diode can be regarded as an open circuit.
In the forward bias region, the current flowing through the device depends on the voltage applied across the diode terminals. This voltage-dependent current can be determined from the following approximation:

$$i_d = I_S\left(e^{\frac{v_d}{nV_{th}}} - 1\right) \approx I_S\left(e^{\frac{v_d}{V_{th}}} - 1\right), \tag{4.1}$$

where I_S is the saturation current, V_{th} is the thermal voltage, and n is the diode constant. Here and throughout this chapter it is assumed that $n \approx 1$, as is the case for many silicon diodes. Since n depends on the diode's material and the fabrication procedure used by the manufacturer, it is one of numerous reasons for you to consult the datasheet before using a specific diode in the lab. The saturation current I_S is a function of diode dimensions and other physical parameters. Its value is strongly affected by temperature variations, typically ranging in the order of 10^{-10} - 10^{-15} A. The value of I_S usually increases by factor of two when the temperature increases by 10 degrees. The thermal voltage V_{th} is also temperature-dependent as can be observed from its definition:

$$V_{th} = \frac{kT}{q}, \tag{4.2}$$

where k is the Boltzmann constant ($=1.38\times10^{-23}$ J/K); T is the temperature in Kelvin degrees (300° K = 27° centigrade), and q is the fundamental charge of the electron ($=1.6\times10^{-19}$ C). At room temperature (300° K), the thermal voltage V_{th} is roughly 26 mV. A diode's typical i-v curve is plotted in Figure 4.1c.



(a) (b) (c)

Fig. 4.1. (a) Physical representation of a semiconductor diode, (b) the symbol for a diode, (c) typical i_d-v_d curve.

Since the i-v curve is obtained by sweeping the dc voltage across the diode at DC or very low frequencies, none of the diode's high-frequency limitations are captured in this plot. Hence, the i-v plot is known as the diode's DC characteristics. For negative voltages, the diode behaves like a very large resistor by allowing a current flow around $-I_S$ (less than -1 nA in most cases). Such a small current is negligible in most practical cases. For instance, if -1 V is applied to the diode's terminals, then the current flowing through it is around $-I_S$. If $I_S = 0.1$ nA, the diode's equivalent resistance is in the order of $r_{diode} = 1V/0.1\ nA = 10^{10}\Omega$, which can be treated like an open circuit.
A diode can be used for the processing of AC signals as well. The example circuit shown in Figure 4.2 operates as an AC rectifier if the amplitude of the input signal is larger than 0V in the simplified diode model discussed here. It can be described by the following equations:

$$\begin{cases} v_0 = i_d R_L = I_S R_L \left(e^{\frac{v_i - v_0}{V_{TH}}} - 1 \right) & if \quad v_i > 0 \\ v_0 = -I_S R_L \cong 0 & if \quad v_i < 0 \end{cases} \tag{4.3}$$

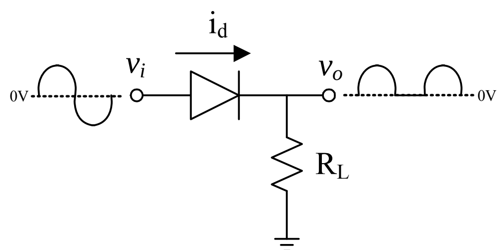


Fig. 4.2. A signal rectifier circuit with a diode.

Notice in equation 4.3 that the resulting output voltage is non-linear and there is no closed form solution for the circuit's operation, even if we try solve it for the two separate cases $v_i > 0$ and $v_i < 0$. Computer-based methods are often used for plotting the output voltage as function of the input voltage. Even if the solution can be found for this simple case, practical circuits might contain more diodes, making the solutions for the sets of non-linear equations quite complex. Thus, many designers use a simpler model to quickly analyze and construct more complex systems with diodes. This basic switch model is explained next.

IV.2. Simple switch model.

In a first approximation, the diode can be considered as a switch: if current is flowing from anode to cathode, then it can be regarded as a short circuit ($v_d = 0$); otherwise, it is treated like an open circuit ($i_d = 0$) and current cannot flow through it. A simple model of this idealization is depicted in Figure 4.3.

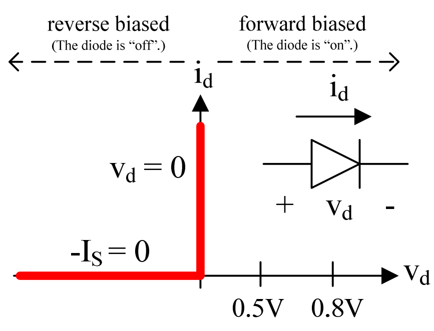


Fig. 4.3. Switch model for a diode.

Although the idealized switch model is very simple and inaccurate, especially when dealing with small signals, it is very useful in understanding a circuit's operation. The results can be easily extrapolated to designs with more elaborate diode models for higher accuracy. To get some insights into the use of the ideal model, let us reconsider the circuit shown in Figure 4.2. For positive input signals, the current flows from anode to cathode. Hence, the diode operates in "on" condition, leading to the equivalent circuit shown in Figure 4.4a where the output voltage is $v_o = v_i$. Therefore, the current flowing through the diode and resistor is $i_d = v_i/R_L$.

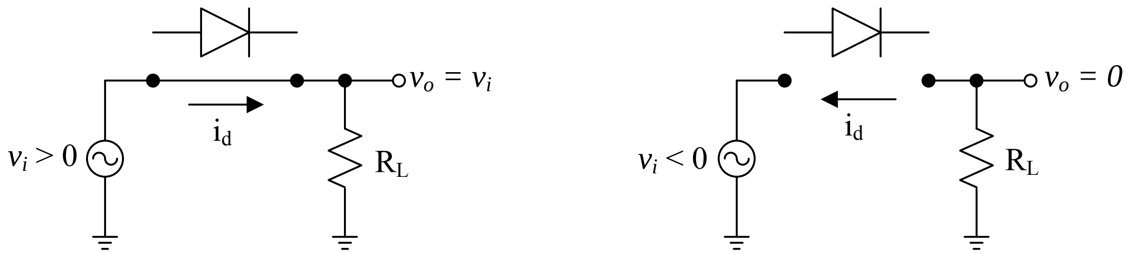


Fig. 4.4. Equivalent circuit for the two cases: (a) $v_i > 0$, (b) $v_i < 0$.

For the case $v_i < 0$, the current, if any, should flow from v_o to v_i to ground; that is in the diode's reverse direction. Due to the diode's characteristic, the reverse current is very small and assumed to be zero in the ideal model. The diode is represented as an open circuit as shown in Figure 4.4b. Since $i_d = 0$, the voltage drop across the resistor is zero and consequently the output voltage is zero. Since only the positive output voltage appears at the output of the circuit and the output is zero during the negative half of the sinusoidal input signal in Figure 4.2, this topology is known as a half-wave rectifier circuit.

The negative half rectifier circuit can be obtained by flipping the diode, as shown in Figure 4.5. When the input signal is negative, the current flows from v_o to v_i to ground. Hence the diode operates in forward bias when $v_i < 0$. The diode behaves as a short circuit, leading to v_i and to v_o, where $v_o = v_i$. During the positive input cycle, the current should flow from input to output, which is in the diode's reverse direction. This very small current is neglected in the ideal model, leading to zero output voltage since $i_d = 0$.

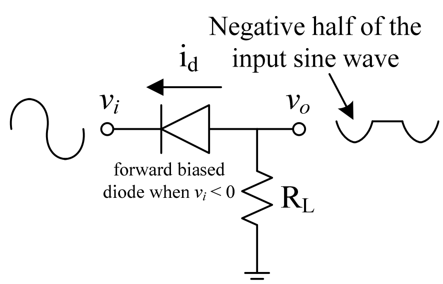


Fig. 4.5. Half-wave rectifier for the negative half-cycle.

Let us consider some other examples to get more intuition for the use of diodes in other circuits such as the topologies shown in Figure 4.6. The circuit in Figure 4.6a is fed with a DC current source of 10 mA. Two resistors and a diode are connected at the output. Since the current flows through the combination of the resistors and the diode, the current is expected to flow in the diode's forward direction, assuming a short circuit for the diode. Thus, the load consists of two 1kΩ resistors connected in parallel, yielding an equivalent load resistance of 500Ω. The output voltage is then approximately equal to v_{out} = 0.01 A × 500 Ω = 5 V. When it is forward biased, a real diode has a small voltage drop across its terminals, with the anode being at a higher voltage. This fact can help you to check the intuitive assumption that was made in the beginning of this idealized analysis: In the solved circuit, imagine a very small virtual resistance between the anode and cathode. The current flowing through the diode in the direction from anode to cathode now causes a very small voltage drop ($v_{d1} > 0$) when a small finite resistance is assumed between the diode terminals. Hence, the "on" condition $v_{d1} > 0$ for the ideal switch model characteristics visualized in Figure 4.3 holds, and you can approximate the diode with a short circuit ($v_{d1} = 0$) under the idealization that this voltage drop is very small. On the contrary, without analysis experience, one might also attempt to analyze the circuit in Figure 4.6a under the assumption that the diode is turned "off" by replacing it with an open circuit. This would mean that the entire 10 mA current would flow through resistor R_1 at the output, creating v_{out} = 10 V. Checking your assumption as before, the anode of the diode is connected at a node with 10 V while the cathode is connected to a node with 0 V because no current flows through R_2 due to the open circuit and consequently the voltage drop across R_2 is zero. Thus, the resulting $v_{d1} > 0$ from the analysis does not match the assumed condition for reverse bias operation, which requires $v_{d1} < 0$ according to Figure 4.3. This check tells you that the second analysis approach with the diode assumed to be in "off" condition is not valid. Since it is not always possible to predict the bias conditions of the diodes when beginning to analyze a circuit, the chances are high to make false assumption after which you have to analyze the circuit again while changing one or more diodes from an open circuit to a short circuit or vice versa. Fortunately the probability of making the correct assumption on the first attempt will increase as you analyze more circuits.

If the diode is flipped in the example circuit as shown in Figure 4.6b, a good initial guess is to consider the diode to be turned "off" and replacing it with an open circuit. As a result, the load consists of a simple 1kΩ resistor, leading to an output voltage of 10 V. Applying the same check, the cathode is connected to a 10 V node and the anode is connected to a 0V node since the open circuit prevents current flow through R_2. With the new diode orientation and analysis in "off" state, its terminal voltages give $v_{d2} = 0\text{ V} - 10\text{ V} = -10\text{ V} < 0$. This verification result agrees with the reverse bias condition, indicating that the flipped diode in this circuit is in fact in "off" state according to the ideal switch model. By analyzing the circuit again, you can prove to yourself that the diode in Figure 4.6b cannot be "on" because the terminal voltages would violate the condition required for forward bias.

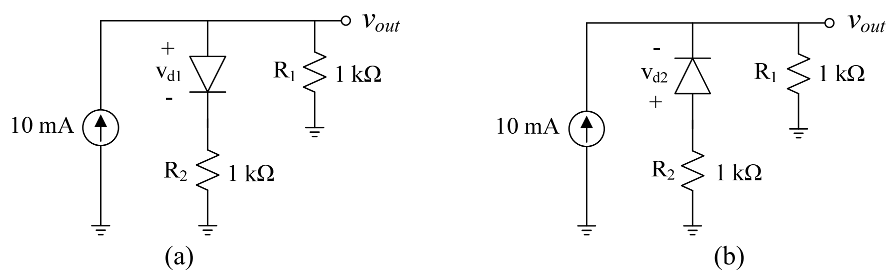


Figure 4.6. Current driven circuit with (a) forward biased diode and (b) with reverse biased diode.

The circuits shown in Figure 4.7 are two examples of voltage driven circuits. Notice that the current flows from the positive 10V voltage source down to the ground terminal. In Figure 4.7a, the current flows through the diode in the forward direction, hence the ideal diode's voltage drop is zero. Furthermore, the output current is limited to $i_{out} = 0$ because the voltage-drop across the resistor in parallel with the diode is zero. The voltage drop and current flowing through the upper resistor of 1kΩ are 10 V and 10 mA, respectively. The diode's current is also 10 mA.

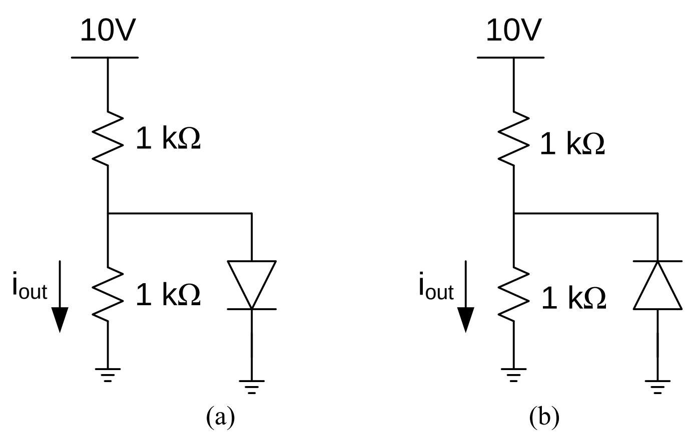


Fig. 4.7. Voltage driven circuit with (a) forward biased diode and (b) with reverse biased diode.

The diode in the circuit of Fig. 4.7b is connected in the reverse biased direction. Since current cannot flow through the diode, it can be considered as an open circuit. Therefore, the current flowing through both resistors is the same, and it can be computed as $i_{out} = 10\text{V} / 2\text{k}\Omega = 5\text{mA}$. With the diode's anode being at 0V (grounded) and its cathode at 5 V, the voltage measured across its terminals is +5 V.

IV.3. A more accurate diode model.

A rudimentary but more accurate model takes the voltage drop across the diode terminals into account. As can be observed in Figure 4.8, the diode's voltage drop is current-dependent and has to be computed using the diode equation. However, the forward voltage drop across the diode is usually in the range of 0.5-0.8 V. Although it is still an approximate approach, the results are typically closer to the actual values when a voltage drop of 0.7 V is assumed for forward-biased diodes in hand calculations. This more accurate model is depicted in Figure 4.8, in which the red line indicates the approximate current-voltage relationship. If the diode's current is positive, then the diode is modeled by a fixed DC voltage source of 0.7 V. Be aware that other voltage drops such as 0.65 V are often assumed or specified in different textbooks and datasheets because this property varies among component manufacturers. As a rule of thumb, the diode voltage drop can be approximated as 0.65 V when small DC currents (< 1mA) flow through it, as 0.7 V when 1 mA < i_d < 10 mA, and as 0.8 V when i_d > 10 mA.

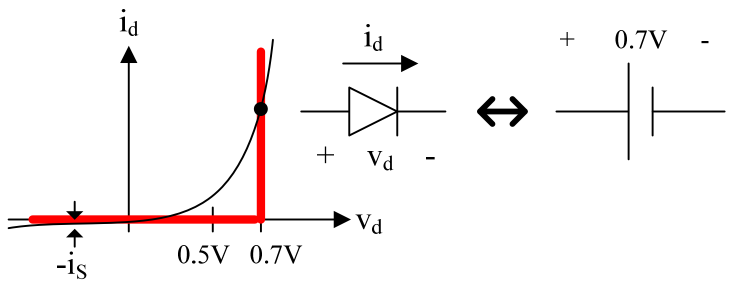


Figure 4.8. Model of a forward-biased diode model as a constant DC voltage source.

The numerical results obtained from the constant voltage drop model are certainly more accurate than those obtained with the ideal switch model. For example, let us reexamine the circuit shown in Figure 4.7a in which the diode's voltage drop is approximated by a 0.7 V battery. The equivalent circuit using this model is shown in Figure 4.9. In this case, i_{out} = 0.7V / 1kΩ = 0.7 mA while i_x = 9.3V / 1kΩ = 9.3 mA. Notice that these values are quite different than the ones obtained from the switch model: i_{out} = 0 mA and i_x = 10 mA.

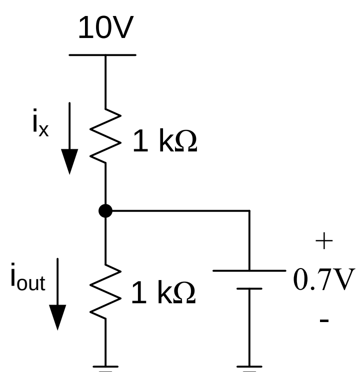


Fig. 4.9. Circuit with the 0.7 V constant voltage source model for the forward biased diode.

A diode characteristic that has been ignored up to this point is the breakdown that occurs under large reverse bias voltage, which is labeled as "breakdown voltage" in Figure 4.10. Due to the large current in the breakdown region, conventional semiconductor diodes typically experience permanent damage under this condition. Even though it often takes 50-150 V for the diodes to break down, you should always refer to the manufacturer's datasheet if you anticipate reverse voltage drops in your design. A special type of diode that is applied in some voltage regulators and reference circuits is called a Zener diode, for which the symbol in Figure 4.11 is widely used. Zener diodes have almost identical forward bias characteristics as the previously described conventional diodes, but they are fabricated to ensure a controlled breakdown voltage as well as safe operation in the breakdown region. For silicon Zener diodes, the voltage drop under reverse breakdown is usually below 10V, making it possible to create stable reference voltages for a wide range of currents in the reverse direction.

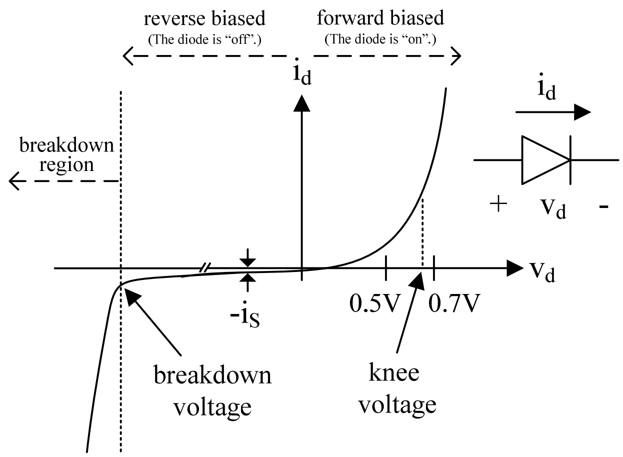


Figure 4.10. Diode i_d-v_d characteristics with breakdown region.

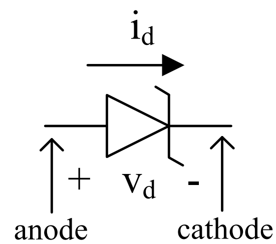


Figure 4.11. Symbol for a Zener diode that can be operated in breakdown region.

Although useful, the previous models do not consider the fact that the diode's voltage is a function of its current. A more accurate model should consider this fact. The simplest model that is used for this purpose is based on the combination of the DC battery and a series resistor, as depicted in Figure 4.12. The question that arises now is how to compute resistor value r_d.

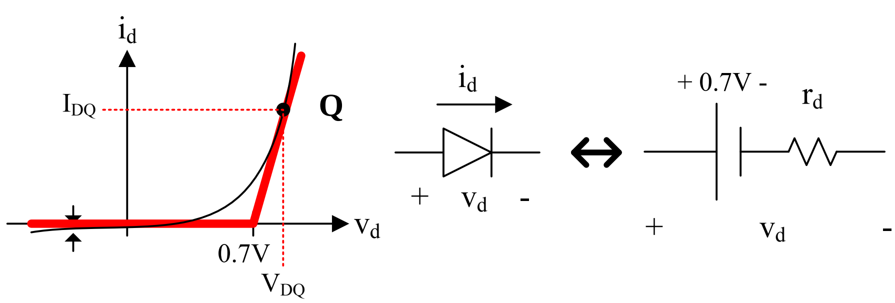


Figure 4.12. More accurate diode model with a battery and a series resistor.

To analyze the issue of the r_d calculation, let us consider the diode equation (4.1) repeated below for convenience.

$$i_d = I_S\left(e^{\frac{v_d}{V_{th}}} - 1\right) \tag{4.4}$$

In Figure 4.12, the non-liner diode equation is approximated by a piece-wise linear function composed of two straight lines such that

i) $i_d = 0$ if $v_d \leq 0.7V$,

ii) $i_d = (v_d - 0.7V) / r_d$ if $v_d > 0.7V$.

Notice in Figure 4.12 that the actual value of the current (equation 4.4) and the one resulting from the piecewise linear model are the same at the point Q, which is usually termed as the operating point. The Q point is defined by I_{DQ} and V_{DQ} (the diode's current and voltage evaluated at the operating point), and it is selected based on the application and the characterization conditions of your device. Since the current-voltage diode relationship is non-linear, it can be expanded in a Taylor series around the proper operation point (Q) as follows:

$$i_D = I_S\left(e^{\frac{v_d}{V_{th}}} - 1\right) = i_d\big|_Q + \left.\frac{\partial i_d}{\partial v_d}\right|_Q i_d + \frac{1}{2}\left.\frac{\partial^2 i_d}{\partial v_d^{\,2}}\right|_Q i_d^{\,2} + \ldots = I_{DQ} + \left.\frac{\partial i_d}{\partial v_d}\right|_Q i_d + \frac{1}{2}\left.\frac{\partial^2 i_d}{\partial v_d^{\,2}}\right|_Q i_d^{\,2} + \ldots \qquad (4.5a)$$

The first component in this series is the DC (constant) current component that defines the operating point Q. The second component measures the variation of the current when small voltage variations are applied, since $\partial i_d/\partial v_d|_Q$ represents the slope of the curve at the operating point Q. In the previous equation, it is assumed that the diode's current I_{DQ} is much larger than I_S. This approximation is reasonable since $I_S < 1nA$ for most of the diodes. Due to the higher order derivatives, expression 4.5a is very accurate but difficult to manipulate in complex circuits. In many applications the *signal applied to the diode is small* ($v_d < V_{th}$). Hence, the higher order terms can be ignored, leading to the approximated but linear model of the diode:

$$i_D \cong I_{DQ} + \left.\frac{\partial i_d}{\partial v_d}\right|_Q i_d = I_{DQ} + g_m i_d\,, \qquad (4.5b)$$

where g_m is known as the small-signal transconductance of the diode. This is the most popular diode model for small-signal modeling. Notice that small-signal diode applications imply voltage variations around a DC value of no more than 26mV. According to equation 4.5, you should understand very well how to compute the two fundamental parameters: given a time-invariant DC voltage V_{DQ}, you must find I_{DQ} (also time-invariant) using equation 4.4. If the diode is operated around the Q-point determined by V_{DQ} and I_{DQ}, the slope of the curve at Q determines g_m, which you can calculate by differentiating equation 4.4, rearranging the result, and making the appropriate substitutions:

$$g_m = \left.\frac{\partial i_d}{\partial v_d}\right|_Q = \left.\frac{\partial I_S\left(e^{\frac{v_d}{V_{th}}} - 1\right)}{\partial v_d}\right|_Q = \left.\frac{I_S e^{\frac{v_d}{V_{th}}}}{V_{th}}\right|_Q = \frac{\left.I_S\left(e^{\frac{v_d}{V_{th}}} - 1\right)\right|_Q + I_S}{V_{th}} = \frac{I_{DQ} + I_S}{V_{th}} \cong \frac{I_{DQ}}{V_{th}} = \frac{1}{r_d}\;. \qquad (4.6)$$

Manipulating equation 4.6 leads to the following formula for the computation of diode's AC resistance:

$$r_d = \frac{1}{g_m} \cong \frac{V_{th}}{I_{DQ}}\,. \qquad (4.7)$$

Solving circuits with diodes usually involves two or three iterations before we arrive at a reasonable result. For instance, let us reconsider the half-wave rectifier described in the previous section. Assuming that a sinusoidal input signal with a peak value of 10 V and a load resistor R_L = 10 kΩ are used in the circuit of Figure 4.2, the output signal for the three discussed diode models can be computed as follows. These three cases are plotted in Figure 4.13.

i) Ideal switch model: The diode is ON during the positive half-cycle, and the output voltage follows the input; where the output's peak value is 10 V as depicted in the trace labeled "ideal rectifier" that is shown in Figure 4.11. The peak diode's current is then equal to 1 mA. During the negative half-cycle, the diode is OFF and does not conduct current, resulting in a 0 V output voltage.
ii) Constant voltage drop model: When the input signal is greater than 0.7V, the diode is ON and the voltage drop across it is approximately 0.7 Volts. The output peak value is therefore equal to 9.3 Volts. Under these

conditions, the diode's peak current is computed as 0.93 mA. The conditions and output voltage during the negative half-cycle are identical as with the constant voltage drop model in i).

iii) Model with diode resistance and constant voltage drop: In this case, the diode's current must be computed before the output voltage can be computed. For intuitive insights, let us first make an *incorrect* assumption by ignoring that the diode's peak current is affected by the series resistor. Then, from the peak current in case ii) and equation 4.7, the diode's resistance would be r_d = 26 mV / 0.93 mA = 28Ω. With this first order approximation, a more accurate peak current value can be computed as I_{pk} = 9.3V / 10.028kΩ = 0.927 mA, which is relatively close to the previous value. Next, the voltage drop across the resistor can be calculated as 28Ω × 0.927mA = 26mV. Thus, the output peak's value becomes $V_{out\text{-}pk}$ = 10V - 0.7V - 0.026V = 9.274 V. In comparison to the 9.3 V obtained in ii), this result is more accurate since it contains the resistor's effect. Depending on the accuracy requirements of the application, this value could still have a significant error because the diode voltage drop is not considered a small-signal voltage ($v_d > V_{th} \approx$ 26 mV). Also notice that we substituted the peak current into equation 4.7 instead of the quiescent current I_{DQ}. With large voltage and current variations, the resistance r_d actually varies dynamically as a function of the diode current during the positive half-cycle, which is not taken into account here. Nevertheless, this approximation is more accurate than the other two cases and it is typically sufficient for hand calculations to select initial design parameters for circuit components prior to simulations.

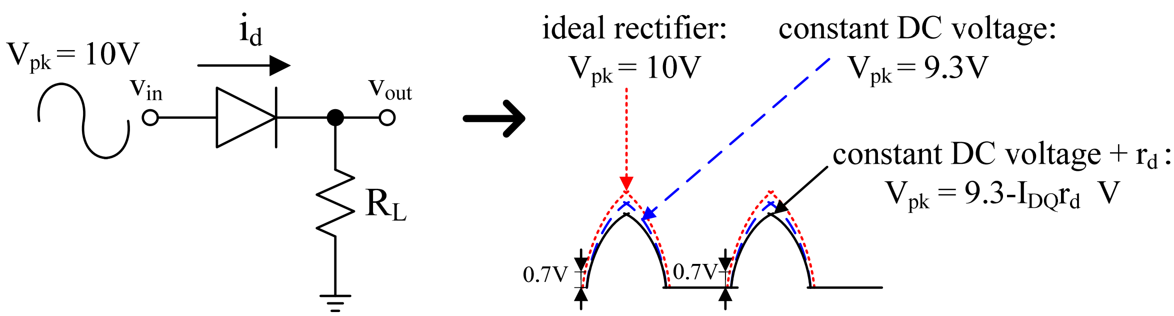


Fig. 4.13. Half-wave rectifier and its output voltage illustrating the differences between the three diode models.

IV.4. Peak detectors and AC-to-DC converters.

The peak value of the input signal can be detected if the load resistor of the half-wave rectifier is replaced by a grounded capacitor, resulting in the circuit shown in Figure 4.14. To understand the operation of this peak detector, let us assume that the diode is ideal. If the initial condition of the capacitor is zero, the diode is on as long as the input signal is greater than zero, and the output voltage follows the input signal provided that the input signal increases. As soon as the input signal begins to decrease, the polarity of the voltage across the diode changes, causing the diode to be off. In that state, the output voltage is held by the capacitor. The only way the output voltage can decrease is when current is extracted from the capacitor, but the diode is unable to do so because it does not allow current flow in the reverse bias condition. Therefore, the diode is off until the input voltage becomes greater than the voltage held by the capacitor. Theoretically, if the input signal does not exceed the voltage value stored in the capacitor C_L, then the capacitor should hold this value indefinitely. In practice, the diode has a negative leakage current I_S that extracts a small amount of current ($i_d = -I_S$), which will eventually discharge the capacitor.

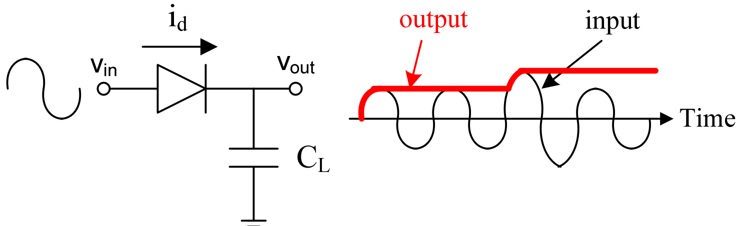


Fig. 4.14. Simple peak detector circuit.

Notice that the previous circuit is able to extract the peak value of the signal. However, if glitches are present in the system, the peak value might not be the real information of interest. Hence, some losses can be included on purpose to partially discharge the capacitor, providing a more robust solution at the expense of reduced accuracy. The circuit in Figure 4.15 includes a resistor to introduce such a loss. As before, the diode is on when the value of the input signal increases beyond the previously stored output voltage such that the output follows the input. Assuming an ideal diode, the equation for the output voltage in Figure 4.15 is

$$v_{out} = v_{in} \quad \text{for} \quad t_1 - T \le t \le t_0 . \tag{4.8}$$

At the time $t = t_0$, $v_{in} = V_{peak}$. After this time, the input signal decreases, but the output voltage is partially sustained by the capacitor. Hence, the diode is reverse biased and operating as an open circuit. Since the diode is off when $t_0 < t < t_1$, the capacitor is discharged through the load resistor R_L. During this time period, the output voltage is determined by the first order differential equation $i_{CL} + i_{RL} = C_L \cdot (dV_{out}/dt) + V_{out}/R_L = 0$, whose solution is given by the following expression

$$v_{out} = V_{peak} e^{-\frac{t-t_0}{R_L C_L}} \quad if \quad t_0 \le t \le t_1 . \tag{4.9}$$

Important parameters to be determined are $t_1 - t_0$ and the voltage ripple $V_{ripple} = V_{peak} - v_{out}(t_1)$, where the output voltage ripple (V_{ripple}) is a quality indicator for AC-to-DC converters. The smaller the ripple, the better is the converter's efficiency. Likewise, a small ripple voltage improves the accuracy when the circuit serves as a peak detector. To facilitate the calculations, let us assume that our time scale is such that $t_0 = 0$. The output signal reaches the peak value at $t = t_0 = 0$ and it decreases until the input signal increases to the point that the following condition is met for the sinusoidal input signal and the output voltage according to equation 4.9:

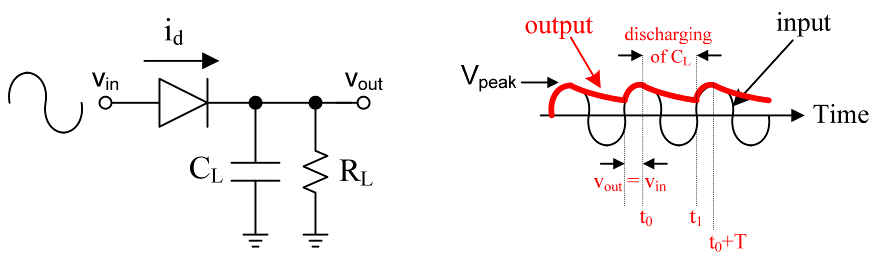


Fig. 4.15. AC-to-DC converter using a half-wave rectifier. This circuit is also known as peak/envelope detector with resistive losses, which was frequently used in old FM radio demodulators.

$$V_{peak} \cos(\omega_0 t_1) = V_{peak} \cos\left(2\pi\left(\frac{t_1}{T}\right)\right) = V_{peak}\, e^{-\frac{t_1}{R_L C_L}} . \tag{4.10}$$

Finding the solution of the previous equation at $t = t_1$ is not trivial since it is a non-linear equation. One option would be to solve it using numerical methods. A simple and useful alternative solution is obtained if we assume that the ripple is small, hence $t_1 \approx T$. An easy way to find an approximate solution is by expanding the exponential function in a Taylor series

$$e^{-\left(\frac{t_1}{R_L C_L}\right)} \cong 1 - \left(\frac{t_1}{R_L C_L}\right) - \frac{1}{2}\left(\frac{t_1}{R_L C_L}\right)^2 - .... \tag{4.11}$$

If the resistor and capacitor are selected such that t_1, $T << 1/R_L C_L$, the higher order terms can be neglected and the output voltage can be approximated as

$$v_{out}(t) = V_{peak}\, e^{-\frac{t}{R_L C_L}} \cong \left(1 - \left(\frac{t}{R_L C_L}\right)\right) V_{peak} \,. \tag{4.12}$$

If $t_1 \approx T$, then the normalized ripple voltage $(V_{peak} - v_{out})/ V_{peak}$ can be determined by evaluating equation 4.12 at $V_{peak} = v_{out}(t = t_0 = 0)$ and $v_{out}(t = t_1 \approx T)$:

$$V_{ripple}(\%) = \frac{V_{peak} - v_{out}(t = t_1 \approx T)}{V_{peak}} \approx \frac{V_{peak} - V_{peak}\left(1 - T/(R_L C_L)\right)}{V_{peak}} = \frac{T}{R_L C_L} = \frac{T}{\tau} \,, \tag{4.13}$$

where T is the period of the input signal and $R_L C_L = \tau$ is the circuit's time constant. To decrease the ripple, we should increase τ as much as possible. Potential drawbacks are the associated increases of component size and cost. Equation 4.13 is very simple and can give you a clear indication on how to design an AC-to-DC converter with small ripple. Just select the resistor and capacitor component values to ensure that the T/τ ratio is less than the ripple specification. For instance, a 10% ripple limit requires that $T/\tau < 0.1$.

IV.5. Full-wave rectifiers.

To reduce the ripple of the rectified output voltage, a transformer is typically combined with diodes to achieve full-wave rectification. The classic full-wave rectifier without output filter is shown in Figure 4.16. A center-tapped transformer is employed to generate fully-differential output signals that are rectified by the diodes. To understand the operation of the circuit, let us assume that the voltage on the secondary side of the transformer is such that the voltage at the upper terminal is positive while the voltage at the bottom terminal is negative. Under these conditions, the upper diode is ON and the lower diode is OFF, as labeled in Figure 4.16. The positive signal is then transferred to the output, leading to a peak output voltage equal to the peak value of the secondary terminal of the transformer minus the 0.7V voltage drop across the forward-biased diode. The operation repeats during the following half cycle, but now the bottom diode is ON while the top diode is OFF. If the circuit is not capacitively loaded, the output will be a rectified version of the input signal. In AC-to-DC conversion applications, the advantage of a full-wave rectifier is its better conversion efficiency compared to a half-wave rectifier. This is explained by the fact that both the positive and negative half-cycles of the input are rectified with a full-wave rectifier, resulting in a higher RMS output voltage.

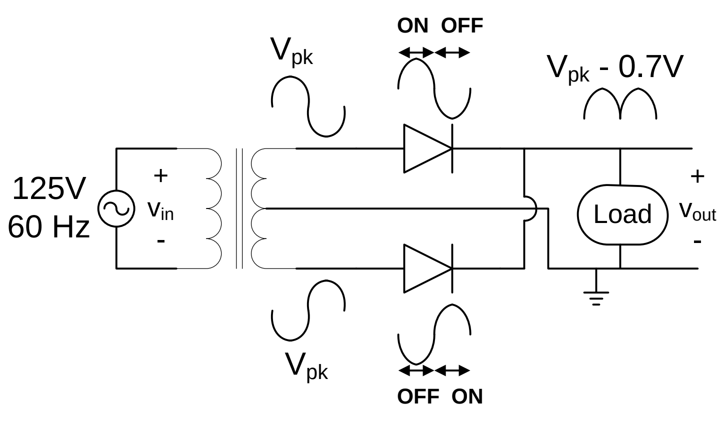
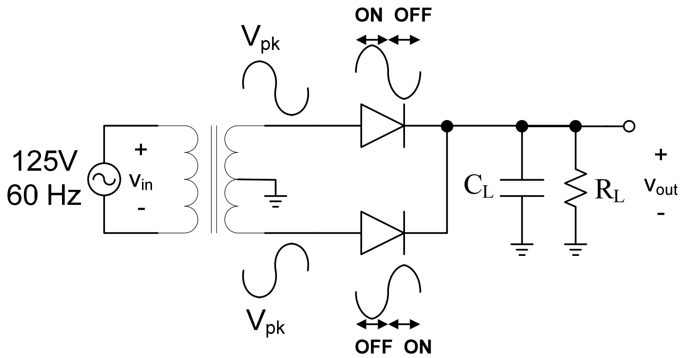


Fig. 4.16. Typical full-wave rectifier with a center-tapped transformer.

The ripple can be further minimized by connecting a capacitor C_L in parallel with the load, as depicted in Figure 4.17. This circuit can be analyzed in the same manner as the one in Figure 4.15, where the main difference is that the full-wave rectified output voltage has a period of T/2. As a result, the expression to calculate the output ripple becomes:



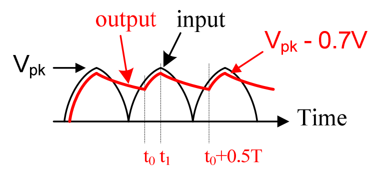


Fig. 4.17. Full-wave rectifier with a center-tapped transformer and filter capacitor C_L.

$$V_{ripple}(\%) = \frac{V_{peak} - v_{out}(t = t_1 \approx T/2)}{V_{peak}} \approx \frac{T}{2R_L C_L} = \frac{T}{2\tau}. \quad (4.14)$$

Evidently, the ripple of the output from the full-wave rectifier is half of that from the half-wave rectifier. This is a major benefit of this circuit, and it is often a good reason to invest in a center-tapped transformer and two diodes. However, if the transformer is not fully balanced such that the peak voltage of both outputs is the same, then the output ripple will increase. Since it is not easy to maintain a good output balance for center-tapped transformers, the circuit displayed in Figure 4.18 can be utilized to overcome this drawback because it includes a transformer without center tap. This structure is a full-wave rectifier with an array of four diodes such that two diodes are ON and two are OFF depending on the polarity of the input signal. The capacitor C_L minimizes the output ripple as discussed previously. Since this is a full-wave rectifier, the ripple should be calculated with equation 4.14.

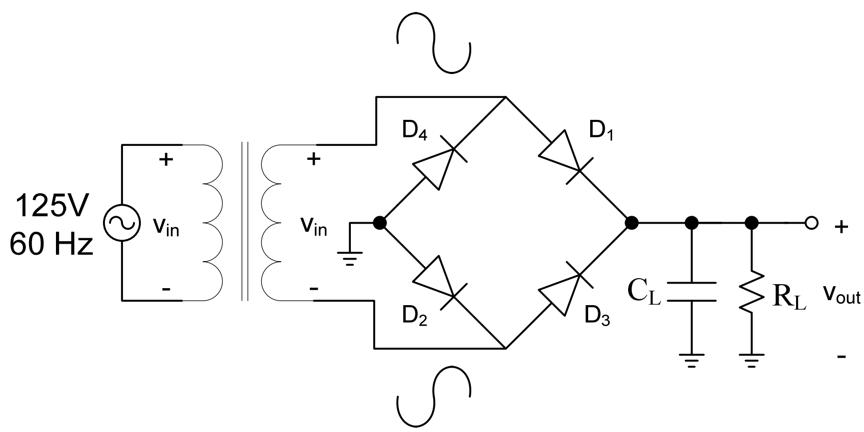


Fig. 4.18. AC-to-DC converter based on a full-wave rectifier that is comprised of a transformer without center tap.

To get insights into the operation of the circuit, let us to remove the capacitor and analyze the simplified version of the rectifier depicted in Figure 4.19. During the positive half-cycle, the voltage at the upper transformer terminal is positive. Consequently, diodes D_1 and D_4 are ON and OFF, respectively. Since the voltage at the transformer's bottom terminal is negative, D_2 and D_3 are ON and OFF, respectively, which leads to the equivalent circuit depicted in Figure 4.20.

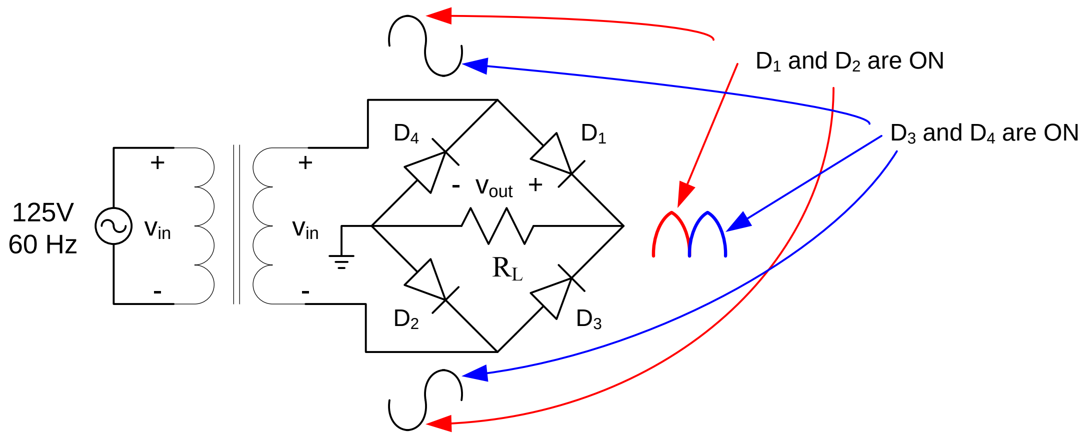


Fig. 4.19. Simplified full-wave rectifier circuit without load capacitor.

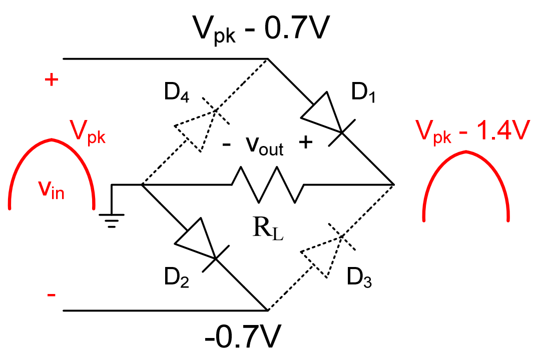


Fig. 4.20. Equivalent circuit for the full-wave rectifier during the positive half-cycle.

Figure 4.21 presents a conceptual diagram for a typical AC-to-DC converter. The actual circuit-level implementation of the rectifier might be more complex than the architectures described in this chapter, but the fundamental principles remain the same. Similarly, the low-pass filter and voltage regulator circuits could be very complex for demanding applications. The design of these circuits is outside of the scope in this introductory book, but you will encounter these types of circuits if you are going to work in the power generation or power management fields.

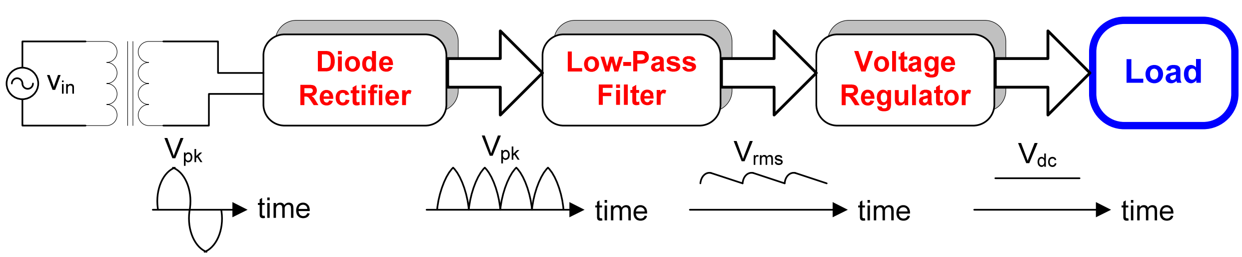


Fig. 4.21. Typical block diagram of an AC-to-DC converter.

IV.6. Other diode application examples.

i) Protection diodes. Diodes are commonly used to protect integrated circuits against electrostatic discharge (ESD). To illustrate the role of the diodes in such applications, Figure 4.22 shows a chip with four pins, on-chip diodes at the inputs/outputs, and internal integrated circuits that process a signal. An ESD event is a rapid discharge of charge between two bodies at different potentials. You have probably experienced such an event many times while touching a door knob after walking on carpet. ESD incidents are very likely to occur during the manufacturing and lifetime of a chip whose pins come into contact with equipment and sometimes humans. To protect the chip from large damaging voltages, a pair of diodes can be inserted at input and output pins. Here, the additional protection circuitry at the positive supply (V_{DD}) and negative supply (V_{SS}) pins is not included in Figure 4.22 for simplicity. If undesirable voltage spikes from ESD events appear at the input or output pins, then the discharging current flows through D_1 and D_3 when the spike is larger than V_{DD} + 0.7V, given that the diodes have a forward voltage drop of 0.7V. Likewise, diodes D_2 and D_4 prevent that the discharging current flows into the internal circuits when the spike's voltage is below V_{SS} – 0.7V. The V_{SS} pin is grounded in this example, thus D_2 and D_4 will turn on if the voltage at the respective pins falls below -0.7V. An additional consideration with protection diodes is that the signal must now remain within V_{DD} + 0.7V and V_{SS} – 0.7V as visualized in Figure 4.22. Otherwise, one of the diodes at the input/output will turn on and clamp the signal, which leads to signal loss and distortion (clipped signals).

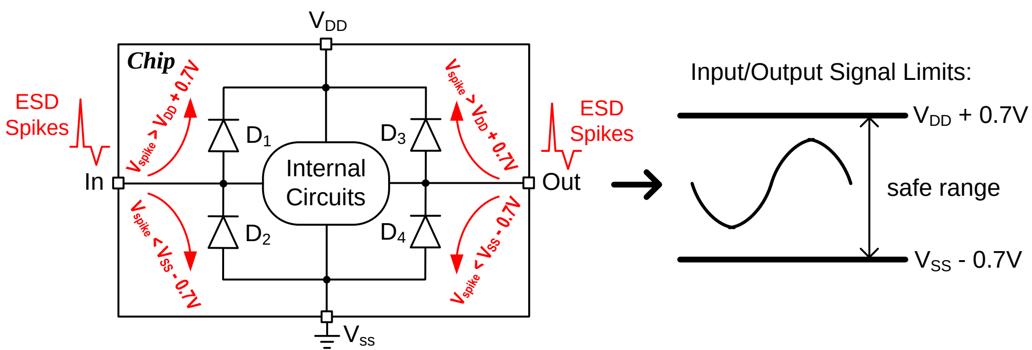


Fig. 4.22. Diode protecting a circuit from electrostatic discharge (ESD).

ii) Diodes in reference circuits. Another application of diodes is in basic reference voltage generation circuits based on the property that the diode voltage drop only exhibits small variations for different currents in the forward direction. A convenient method is to force a current into a stack of N diodes such that the voltage drop across these diodes is approximately N × 0.7V, where a voltage drop of 0.7V is assumed for each forward-biased diode.

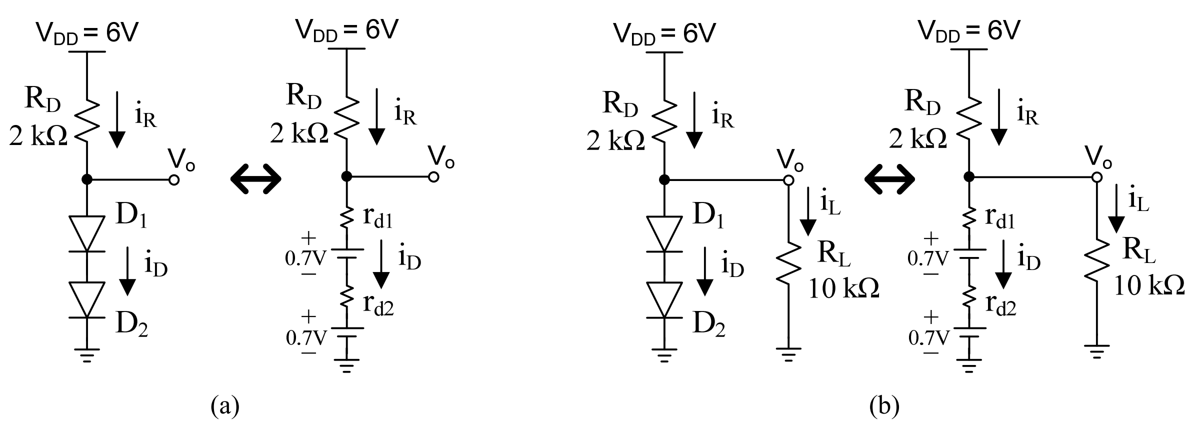


Fig. 4.23. A simple voltage reference circuit: (a) without load, (b) with resistive load.

Figure 4.23a presents a voltage reference circuit consisting of two stacked diodes and one resistor. From the equivalent circuit in the same figure, we can express the output voltage as $V_o = 2 \times 0.7V + i_D \times (r_{d1} + r_{d2})$. Since the output is not loaded by any circuit elements, we know that the current i_D is equal to i_R. Furthermore, i_D flows through both diodes, implying that $r_d = r_{d1} = r_{d2}$ can be substituted in the output voltage equation as well. With these simplifications, we can write $V_o = 2 \times (0.7V + i_D \times r_d)$ for the unloaded case. To approximate the value of r_d, we assume that V_o is around 1.4V. It follows that $i_R = i_D \approx (6V – 1.4V) / R_D = 2.3$ mA, and that $r_d \approx V_{th} / i_D = 11.3\Omega$ at room temperature ($V_{th} \approx 26mV$). Now, the actual output voltage can be calculated as $V_o = 2 \times (0.7V + 2.3mA \times$

11.3Ω) = 1.452 V. For a more accurate result, you could repeat the calculation of V_o with i_R = (6V – 1.452V) / R_D = 2.274 mA and the corresponding new value of r_d, but the first iteration is often sufficient in these hand calculations.

The circuit at the output of a reference generator has a finite input impedance that is high in the ideal case, but it could be in the same order of magnitude as resistor R_D. This raises an important question in practical scenarios: What will happen to the output voltage V_o if a resistor R_L of 10 kΩ for instance is connected to the reference generator under investigation as shown in Figure 4.23b? First of all, you can observe that $i_L \approx V_o / R_L$ = 1.45V / 10kΩ = 0.145 mA in the first order small-signal approximation. From Kirchoff's current law you can also derive that the new current through the diodes is $i_D = i_R - i_L$ = 2.3mA – 0.145 mA = 2.155 mA. Therefore, the diode current change due to the loading effect is Δi_D = 2.155 mA – 2.3mA = -0.145 mA. Since the associated change of the voltage drop across the two series diode resistances directly affects the output voltage, the loading effect of R_L is $\Delta V_o \approx 2 \cdot r_d \times \Delta i_D$ = -3.3mV. The change in DC (average) load voltage as the load conditions vary is one important metric to assess the quality of voltage regulates or reference circuits. This measure of quality is called load regulation, and it is often defined as a percentage:

$$\text{Percent Load Regulation} = \frac{V_{unloaded} - V_{loaded}}{V_{unloaded}} \times 100\% . \qquad (4.15)$$

When providing a load regulation specification based on equation 4.15, it is a proper practice to report the load conditions in terms of current or impedance for the loaded case as well as the permissible load current or impedance range. If the load range is not specified, then you should assume that the given load is the full load, i.e. the maximum allowed current or minimum allowed resistance. Since the supply voltage V_{DD} always has small or large fluctuations due to noise sources in the real world, another important measure of quality for a voltage regulator/reference is its line regulation. As an exercise, you can calculate the output voltage change for the example circuit in Figure 4.23a by changing V_{DD} to 6.2V for example. In general, the line regulation is typically defined as:

$$\text{Percent Line Regulation} = \frac{\Delta V_o}{\Delta V_{DD}} \times 100\% . \qquad (4.16)$$

We have already identified that load and supply voltage variations cause deviations from the desired output voltage. Other sources of variations and inaccuracies are the tolerance of the resistor R_D, the effect of temperature on the diode characteristics, and the modeling error from using a small-signal approximation for the diode resistance r_d as well as assuming a fixed 0.7V diode voltage drop. When designing circuits, you should always consult datasheets and equipment manuals to assess the anticipated variations through hand calculations and simulations. The appropriate design margins and circuit specifications are normally based on thorough evaluations in the design and testing phases during the product development. For example, if you would like to estimate how much the output voltage in the unloaded reference circuit (Figure 4.23a) can vary when you use a resistor with a ±10% tolerance specification, you can quickly do so by calculating the resulting minimum and maximum currents: $i_{D(min)} \approx$ (6V – 1.45V) / (1.1×R_D) = 2.07 mA, $i_{D(max)} \approx$ (6V – 1.45V) / (0.9×R_D) = 2.53 mA. Substituting these currents into the equation for the output voltage without load gives: $V_{o(min)} = 2 \times (0.7V + i_{D(min)} \times r_d)$ = 1.447 V, $V_{o(min)} = 2 \times (0.7V + i_{D(min)} \times r_d)$ = 1.457 V. The roughly 10mV output voltage difference might be significant since voltage reference often have to meet stringent accuracy requirements. You can repeat the calculations to find out how much the output voltage change is reduced when a precision resistor with ±0.5% is available, and to check the impact of resistor variation in the loaded case.