

$$\textcircled{3} \quad \text{closed-loop gain} = \left(1 + \frac{R_1}{R_2}\right)$$

$$= 8$$

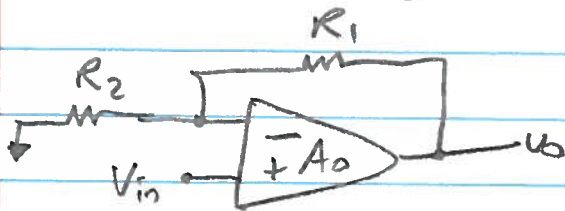
$$\text{Gain error} = \left(1 + \frac{R_1}{R_2}\right) (A_0)^{-1}$$

$$= \frac{8}{2000}$$

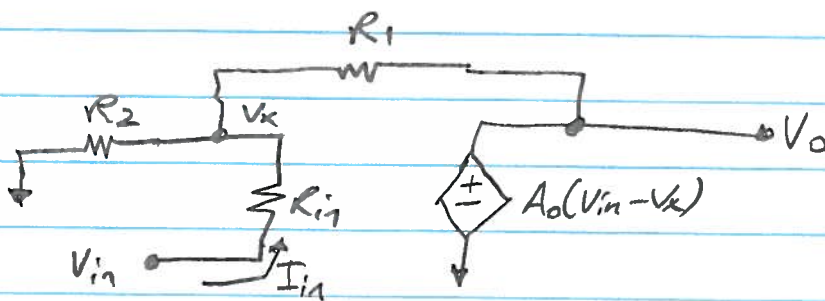
$$= \underline{\underline{0.4\%}}$$

8.7

Non-Inverting Amp



w/ Opamp Model



① KCL @ V_x

$$\frac{V_x}{R_2} + \frac{V_x - V_{in}}{R_{in}} + \frac{V_x - V_o}{R_1} = 0$$

② Use $V_o = A_o (V_{in} - V_x)$

$$V_x = V_{in} - \frac{V_o}{A_o}$$

Plug V_x Eq into ①

$$\frac{V_{in} - \frac{V_o}{A_o}}{R_2} + \frac{V_{in} - \frac{V_o}{A_o} - V_{in}}{R_{in}} + \frac{V_{in} - \frac{V_o}{A_o} - V_o}{R_1} = 0$$

$$V_{in} \left[\frac{1}{R_2} + \frac{1}{R_1} \right] = V_o \left[\frac{1}{A_o R_2} + \frac{1}{A_o R_{in}} + \frac{1}{R_1} + 1 \right]$$

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{R_1} + \frac{1}{R_2}}{\frac{1}{A_o R_2} + \frac{1}{A_o R_{in}} + \frac{1}{A_o} + 1}$$

$$\frac{V_o}{V_{in}} = \frac{1 + \frac{R_1}{R_2}}{\frac{R_1}{A_o R_2} + \frac{R_1}{A_o R_{in}} + \frac{1}{A_o} + 1}$$

$$A_s A_o \rightarrow \infty$$

$$\frac{V_o}{V_{in}} = 1 + \frac{R_1}{R_2}$$

$$\text{Input Resistance} = \frac{V_{in}}{I_{in}}$$

$$I_{in} = \frac{V_{in} - V_x}{R_{in}} = \frac{V_{in} - (V_{in} - \frac{V_o}{A_o})}{R_{in}} = \frac{V_o}{A_o R_{in}}$$

$$\text{Input Resistance} = \frac{V_{in}}{I_{in}} = A_o R_{in} \left(\frac{V_o}{V_{in}} \right)^{-1}$$

where $\frac{V_o}{V_{in}}$ is given above.

$$\text{as } A_o \rightarrow \infty$$

$$\text{Input Resistance} = \infty$$

(11) if $A_o = \infty$,

$$V_+ = V_- = V_{in}$$

$$V_- = \left(\frac{R_2}{R_2 + R_3} \right) \left[\frac{R_4 \parallel (R_2 + R_3)}{R_1 + R_4 \parallel (R_2 + R_3)} \right] V_{out}$$

\therefore closed-loop gain $\frac{V_{out}}{V_{in}}$

$$= \frac{(R_2 + R_3) [R_1 + R_4 \parallel (R_2 + R_3)]}{R_2 [R_4 \parallel (R_2 + R_3)]}$$

if $R_1 = 0$,

$$G|_{R_1=0} = 1 + \frac{R_3}{R_2} //$$

if $R_3 = 0$,

$$G|_{R_3=0} = \frac{R_2 [R_1 + R_4 \parallel R_2]}{R_2 [R_4 \parallel R_2]}$$

$$= 1 + \frac{R_1}{R_4 \parallel R_2} //$$

$$\textcircled{12} \quad \text{Gain Error} = \frac{1}{A_0} \left(1 + \frac{R_1}{R_2} \right)$$

$$= \frac{1}{A_0} (1 + 8)$$

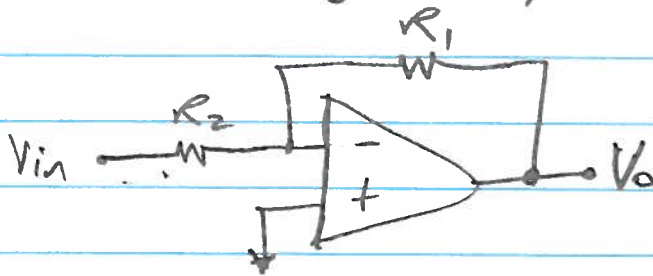
$$= 0.2 \%$$

$$\frac{1}{A_0} (9) = 0.2 \%$$

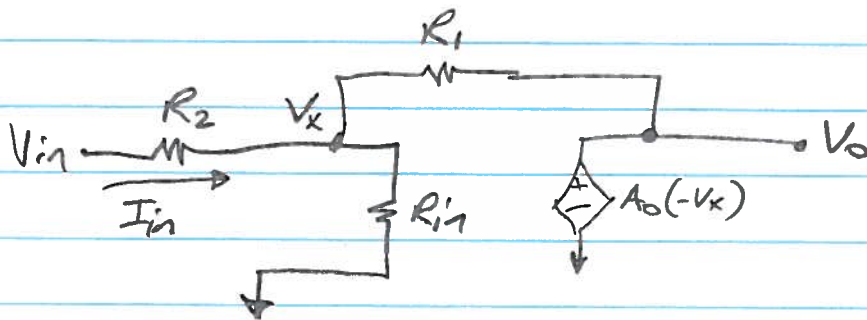
$$A_0 = 4500$$

8.13

Inverting Amp



w/ OpAmp Model



① KCL @ V_x

$$\frac{V_x - V_{in}}{R_2} + \frac{V_x}{R_{i1}} + \frac{V_x - V_o}{R_1} = 0$$

② Use $V_o = -A_o V_x$

$$V_x = -\frac{V_o}{A_o}$$

Plug V_x Eq into ①

$$-\frac{\frac{V_o}{A_o} - V_{in}}{R_2} - \frac{\frac{V_o}{A_o}}{R_{i1}} + \frac{-\frac{V_o}{A_o} - V_o}{R_1} = 0$$

$$V_{in} \left[\frac{1}{R_2} \right] = -V_o \left[\frac{1}{A_o R_2} + \frac{1}{A_o R_{in}} + \frac{1}{R_1} + 1 \right]$$

$$\frac{V_o}{V_{in}} = - \frac{\frac{1}{R_2}}{\frac{1}{A_o R_2} + \frac{1}{A_o R_{in}} + \frac{1}{R_1} + 1}$$

$$\frac{V_o}{V_{in}} = \frac{\left(-\frac{R_1}{R_2} \right)}{\frac{R_1}{A_o R_2} + \frac{R_1}{A_o R_{in}} + \frac{1}{A_o} + 1}$$

$$\text{Input Resistance} = \frac{V_{in}}{I_{in}}$$

$$I_{in} = \frac{V_{in} - V_x}{R_2} = \frac{V_{in} - \left(-\frac{V_o}{A_o} \right)}{R_2} = \frac{V_{in} + \frac{V_o}{A_o}}{R_2}$$

$$\text{Input Resistance} = \frac{V_{in} R_2}{V_{in} + \frac{V_o}{A_o}} = \frac{R_2}{1 + \left(\frac{V_o}{V_{in}} \right) \left(\frac{1}{A_o} \right)}$$

$$\text{Input Resistance} = \frac{R_2}{1 + \left(\frac{V_o}{V_{in}} \right) \left(\frac{1}{A_o} \right)}$$

Where $\frac{V_o}{V_{in}}$ is given above,

(31) By KCL,

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = - \frac{V_{out} - V_x}{R_F}$$

$$\therefore V_{out} = -A_0 V_x$$

$$V_x = - \frac{V_{out}}{A_0}$$

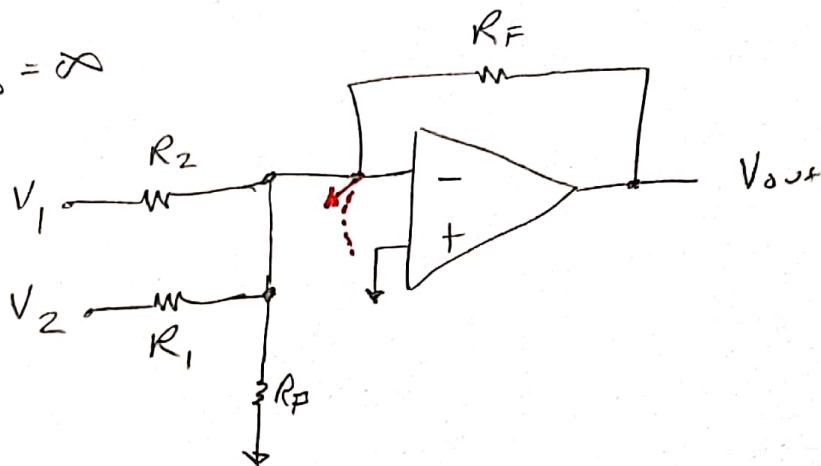
$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) + \frac{V_{out}}{A_0} \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} \right) = - \frac{V_{out}}{R_F}$$

$$- \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) = V_{out} \left[\frac{1}{R_F} + \frac{1}{A_0} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F} \right) \right]$$

$$\therefore V_{out} = - \left(\frac{1}{R_F} + \frac{1}{A_0} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F} \right) \right)^{-1} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

8.33

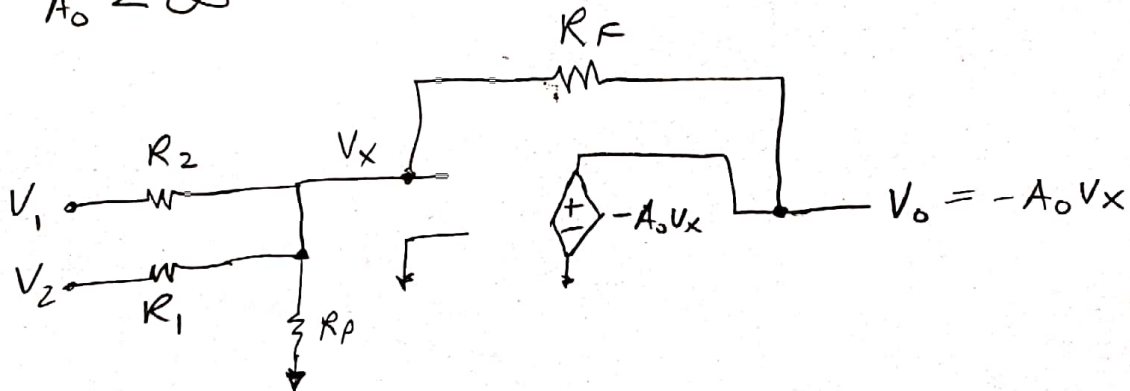
For $A_0 = \infty$



Using Superposition

$$V_{out} = -\frac{R_F}{R_2} V_1 - \frac{R_F}{R_1} V_2$$

If $A_0 < \infty$



KCL at V_x

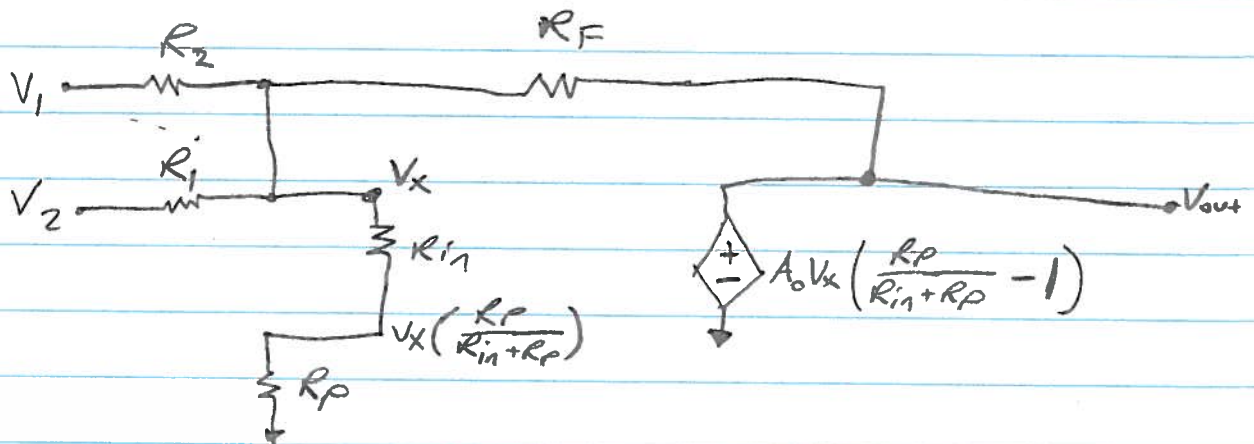
$$\frac{V_x - V_1}{R_2} + \frac{V_x - V_2}{R_1} + \frac{V_x}{R_F} + \frac{V_x(1+A_0)}{R_F} = 0$$

$$V_x \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} + \frac{1+A_0}{R_F} \right] = \frac{V_1}{R_2} + \frac{V_2}{R_1}$$

$$V_x = \left[\frac{V_1}{R_2} + \frac{V_2}{R_1} \right] \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} + \frac{1+A_0}{R_F} \right]^{-1}$$

$$V_0 = -A_0 V_x = -A_0 \left[\frac{V_1}{R_2} + \frac{V_2}{R_1} \right] \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} + \frac{1+A_0}{R_F} \right]^{-1}$$

8.34



Vout Equation

$$V_{out} = A_o V_x \left(\frac{R_p}{R_{in} + R_p} - 1 \right) = A_o V_x \left(\frac{-R_{in}}{R_{in} + R_p} \right)$$

$$V_x = - \frac{V_{out}}{A_o} \left(\frac{R_{in} + R_p}{R_{in}} \right)$$

KCL @ Vx

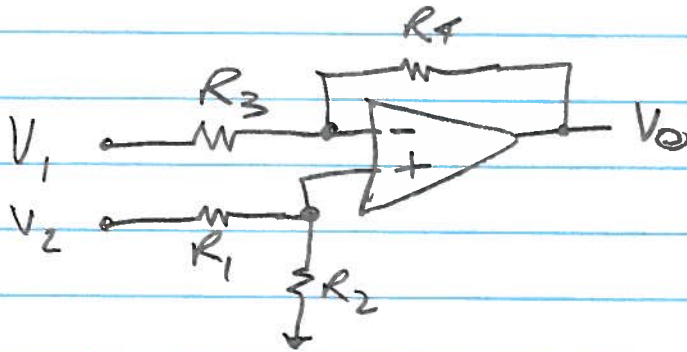
$$\frac{- \frac{V_{out}}{A_o} \left(\frac{R_{in} + R_p}{R_{in}} \right) - V_1}{R_2} + \frac{- \frac{V_{out}}{A_o} \left(\frac{R_{in} + R_p}{R_{in}} \right) - V_2}{R_1}$$

$$+ \frac{- \frac{V_{out}}{A_o} \left(\frac{R_{in} + R_p}{R_{in}} \right) - V_{out}}{R_F} + \frac{- \frac{V_{out}}{A_o} \left(\frac{R_{in} + R_p}{R_{in}} \right)}{R_{in} + R_p} = 0$$

$$V_{out} \left[\frac{R_{in} + R_p}{A_o R_{in}} \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} + \frac{1}{R_{in} + R_p} \right] + \frac{1}{R_F} \right] = - \frac{V_1}{R_2} - \frac{V_2}{R_1}$$

$$V_{out} = \frac{- \frac{V_1}{R_2} - \frac{V_2}{R_1}}{\frac{R_{in} + R_p}{A_o R_{in}} \left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F} + \frac{1}{R_{in} + R_p} \right] + \frac{1}{R_F}}$$

$$1. \quad V_0 = -4V_1 + 3V_2$$



$$V_0 = -\frac{R_4}{R_3} V_1 + \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_2$$

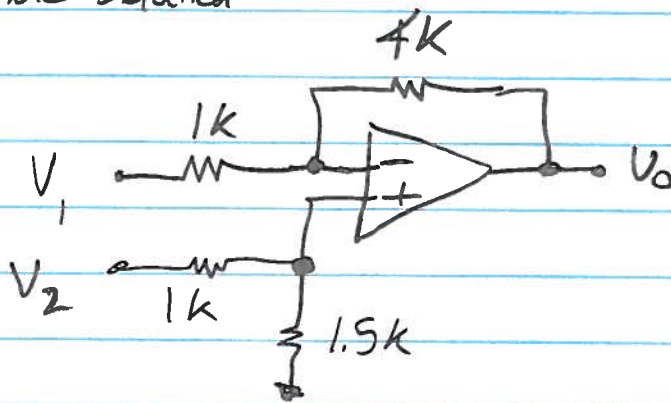
$$\text{For } -4V_1 \Rightarrow R_4 = 4R_3$$

$$\text{For } 3V_2 \Rightarrow \left(1 + \frac{4R_3}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) = 3$$

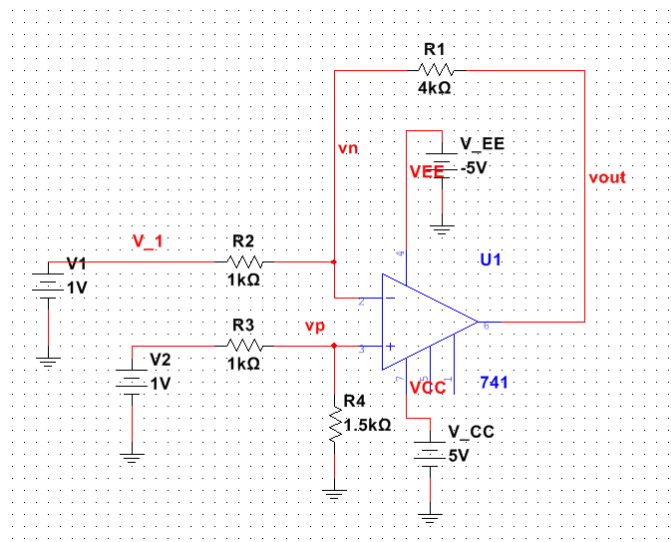
$$\frac{R_2}{R_1 + R_2} = \frac{3}{5}$$

$$R_2 = \frac{3}{2} R_1$$

Possible Solution



Multisim Schematic



Set $V_2=1V$ and Sweep V_1 from $-0.5V$ to $0.5V$. Notice that when $V_1=0$, $V_{out}=3V$ and the curve has a slope of $-4V/V$ over the opamp's linear output range. Also notice that the opamp saturates at $4.1V$, which is below the $5V$ positive power supply.

