ECEN325: Electronics
Spring 2024

Lecture 2: Linear Circuit Analysis Review

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Announcements

- Reading
  - Chapter 1 (Razavi)
  - Fundamentals of Circuit Analysis (Dr. Silva)
- Homework 1 due Feb 1
- Prelab 1 due in lab the week of 1/29 – 2/2

Agenda

- Laplace Transform
- Passive Circuit s-Domain Models
- Transfer Functions
- Sinusoidal Steady-State Response
- Poles & Zeros
- Bode Plots
- Second-Order Systems

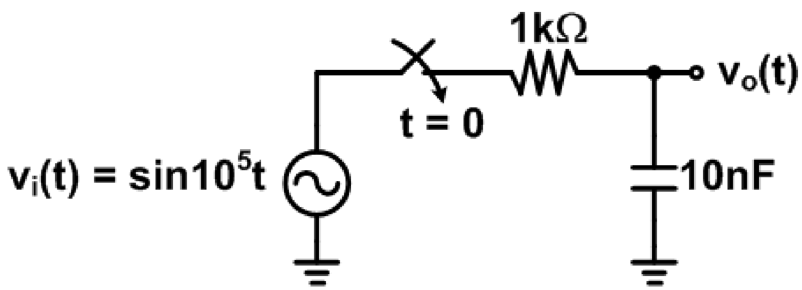
References

- *Continuous & Discrete Signal & System Analysis, 3rd Ed.,* C. McGillem and G. Cooper, Saunders College Publishing, 1991.

- *Feedback Control of Dynamic Systems, 3rd Ed.,* G. Franklin, J. Powell, and A. Emami-Naeini, Addison-Wesley, 1994.

- *Design of Analog Filters,* R. Schaumann and M. Van Valkenburg, Oxford University Press, 2001.

Motivation Example



Given $v_o(0) = 0$

Write a KCL at v_o

$$\frac{v_o(t) - \sin 10^5 t}{1k\Omega} + 10nF \frac{dv_o(t)}{dt} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_o(t)}{(1k\Omega)(10nF)} = \frac{\sin 10^5 t}{(1k\Omega)(10nF)}$$

Now, if we remember anything from out Diff. Eq. class, we can solve this equation.

Note, this is not trivial.

$$v_o(t) = \frac{1}{2}e^{-10^5 t} - \frac{1}{2}\cos 10^5 t + \frac{1}{2}\sin 10^5 t = \frac{1}{2}e^{-10^5 t} + \frac{1}{\sqrt{2}}\sin\left(10^5 t - 45^\circ\right)$$

transient response
(can go to zero quickly)

sinusoidal steady-state response

- Now, let's look at Laplace Transforms to make this easier

Laplace Transform

- Laplace transforms are useful for solving differential equations
- One-Sided Laplace Transform

$$\mathcal{L}\{x(t)\} = X(s) \equiv \int_0^{\infty} x(t) e^{-st} dt$$

where *s* is a complex variable

$$s = \sigma + j\omega$$

Note, $j = \sqrt{-1}$ and ω is the angular frequency (rad/s)

- *s* has units of inverse seconds (s^{-1})

Laplace Transform of Signals

Laplace Transforms of Signals

$X(s)$	$x(t)$	$X(s)$	$x(t)$
s^n	$\delta^{(n)}(t)$	$\frac{\beta}{s^2 + \beta^2}$	$\sin \beta t u(t)$
s	$\delta'(t)$	$\frac{s}{s_2 + \beta^2}$	$\cos \beta t u(t)$
1	$\delta(t)$	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$	$e^{-\alpha t} \sin \beta t u(t)$
$\frac{1}{s}$	$u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$	$e^{-\alpha t} \cos \beta t u(t)$
$\frac{1}{s^2}$	$tu(t)$	$\frac{1}{(s + a)(s + b)}$	$\frac{e^{-at} - e^{-bt}}{b - a} u(t)$
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n - 1)!} u(t)$	$\frac{s + c}{(s + a)(s + b)}$	$\frac{(c - a)e^{-at} - (c - b)e^{-bt}}{b - a} u(t)$
$\frac{1}{s + \alpha}$	$e^{-\alpha t}u(t)$		
$\frac{1}{(s + \alpha)^2}$	$te^{-\alpha t}u(t)$		

[McGillem]

Laplace Transform of Operations

Laplace Transforms of Operations

$x(t)$	$X(s)$
$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
$x'(t)$	$sX(s) - x(0^-)$
$\int_0^t x(\xi)\,d\xi$	$\frac{1}{s}X(s)$
$tx(t)$	$-\frac{dX(s)}{ds}$
$\frac{1}{t}x(t)$	$\int_s^\infty X(\xi)\,d\xi$
$x(t - t_0)u(t - t_0)$	$e^{-st_0}X(s)$
$e^{-at}x(t)$	$X(s + a)$
$x(at),\ a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
$x_1 * x_2 = \int_0^t x_1(\lambda)x_2(t - \lambda)\,d\lambda$	$X_1(s)X_2(s)$
$x(0^+)$	$\lim_{s\to\infty} sX(s)$
$x(\infty)$	$\lim_{s\to 0} sX(s)$ [$X(s)$ left-half-plane poles only]
$x''(t)$	$s^2X(s) - sx(0^-) - x'(0^-)$
$x_1(t)x_2(t)$	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s - \lambda)X_2(\lambda)\,d\lambda$

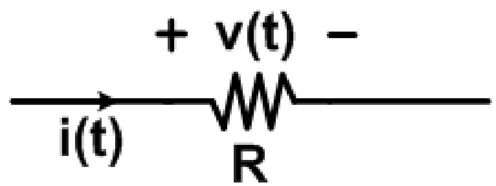
[McGillem]

Resistor s-Domain Equivalent Circuit

Time-domain Representation:

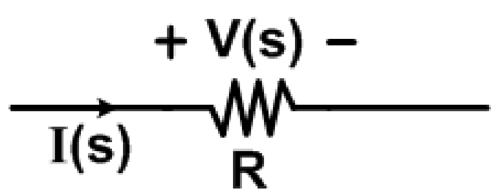
$$v(t) = Ri(t)$$

$$i(t) = \frac{1}{R} v(t)$$

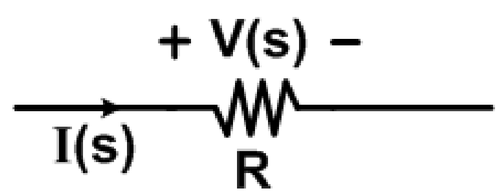


Complex Frequency Representation:

$$V(s) = RI(s)$$



$$I(s) = \frac{1}{R} V(s)$$

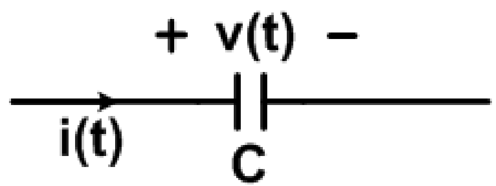


Capacitor s-Domain Equivalent Circuit

Time-domain Representation:

$$i(t) = C\frac{dv(t)}{dt}$$

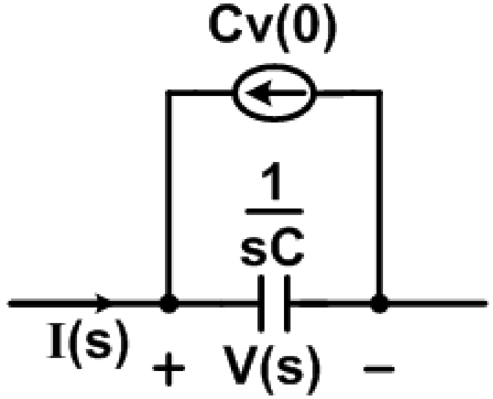
$$v(t) = \frac{1}{C}\int_0^t i(\lambda)d\lambda + v(0)$$

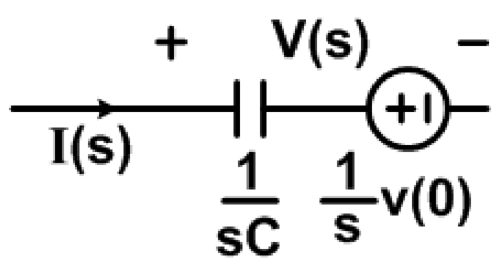


Complex Frequency Representation:

$$I(s) = CsV(s) - Cv(0)$$

$$V(s) = \frac{1}{sC}I(s) + \frac{1}{s}v(0)$$



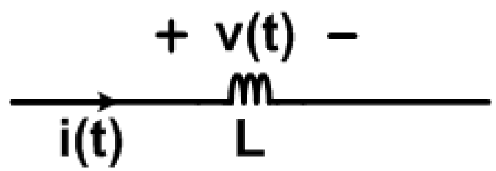


Inductor s-Domain Equivalent Circuit

Time-domain Representation:

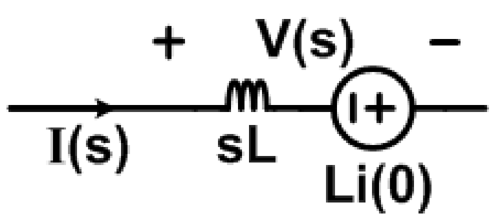
$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t v(\lambda)d\lambda + i(0)$$

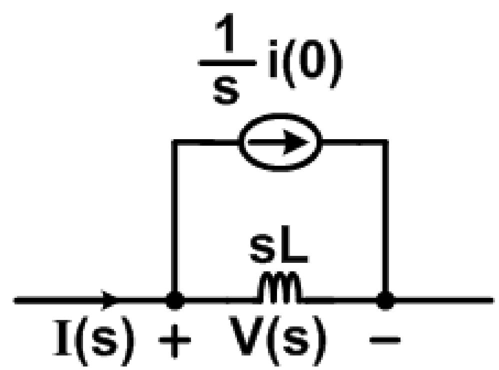


Complex Frequency Representation:

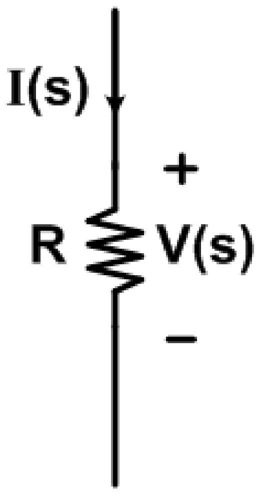
$$V(s) = LsI(s) - Li(0)$$

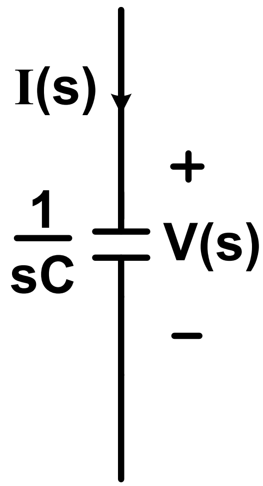


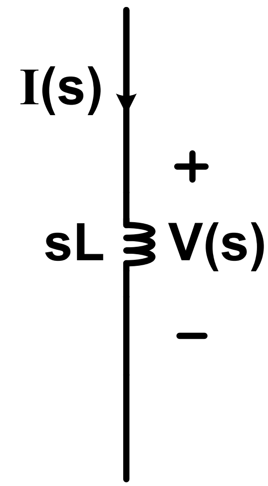
$$I(s) = \frac{1}{sL}V(s) + \frac{1}{s}i(0)$$



s-Domain Impedance w/o I.C.







$$V(s) = I(s)R$$

$$Z(s) = R$$

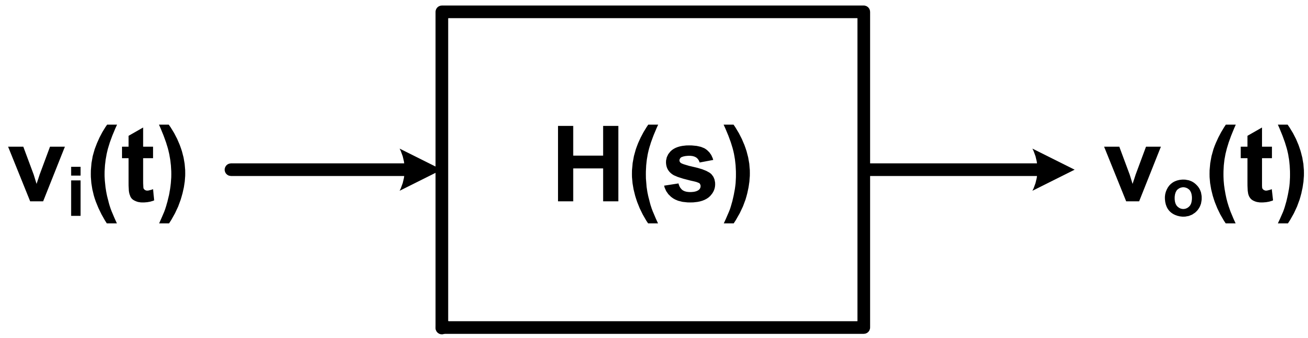
$$V(s) = I(s)\frac{1}{sC}$$

$$Z(s) = \frac{1}{sC}$$

$$V(s) = I(s)sL$$

$$Z(s) = sL$$

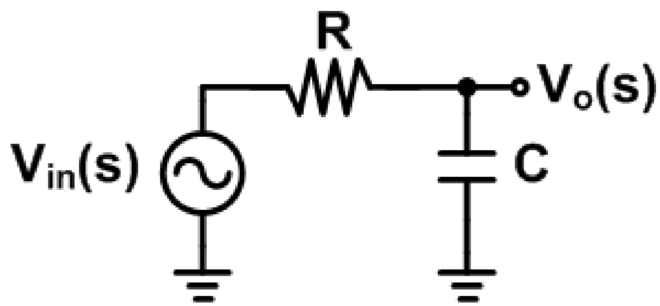
Transfer Function



$$H(s) = \frac{\mathcal{L}\{v_o(t)\}}{\mathcal{L}\{v_i(t)\}} = \frac{V_o(s)}{V_i(s)}$$

- The transfer function H(s) of a network is the ratio of the Laplace transform of the output and input signals when the initial conditions are zero
- This is also the Laplace transform of the network's impulse response

RC Transfer Function



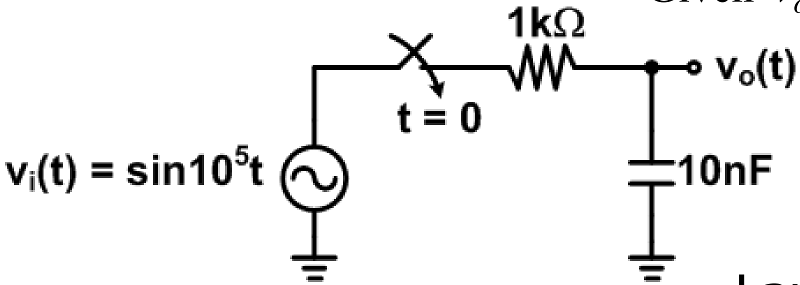
$$V_o(s) = \frac{Z_C}{Z_R + Z_C} V_{in}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}(s) = \frac{1}{1 + sRC} V_{in}(s)$$

<u>AC Transfer Function, H(s)</u>

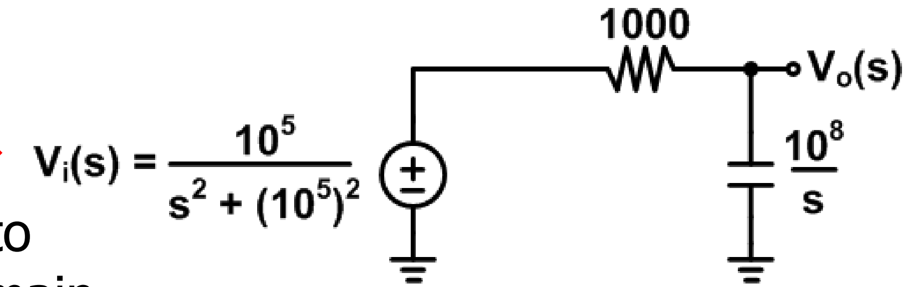
$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}$$

Laplace Transform Circuit Example

Given $v_o(0) = 0$



Convert to Laplace Domain



$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1+sRC} = \frac{1}{1+\frac{s}{10^5}} = \frac{10^5}{s+10^5}$$

$$V_o(s) = H(s)V_i(s) = \left(\frac{10^5}{s+10^5}\right)\left(\frac{10^5}{s^2+\left(10^5\right)^2}\right)$$

with partial fraction expansion

$$V_o(s) = \frac{\frac{1}{2}}{s+10^5} - \frac{\frac{1}{2}s}{s^2+\left(10^5\right)^2} + \frac{\frac{1}{2}\left(10^5\right)}{s^2+\left(10^5\right)^2}$$

with inverse Laplace Transform

$$\mathrm{v_o}(t) = \frac{1}{2}e^{-10^5 t} - \frac{1}{2}\cos 10^5 t + \frac{1}{2}\sin 10^5 t = \frac{1}{2}e^{-10^5 t} + \frac{1}{\sqrt{2}}\sin\left(10^5 t - 45^\circ\right)$$

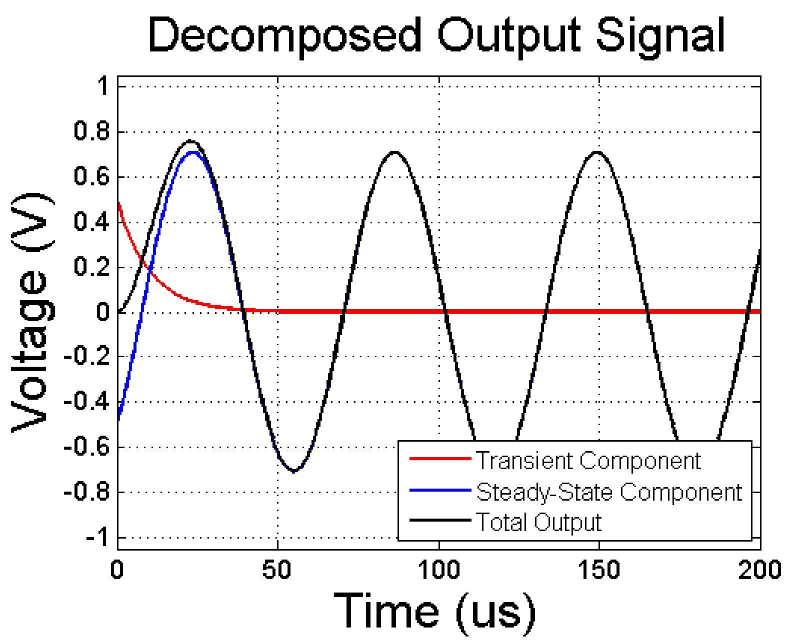
Laplace Transform Circuit Example

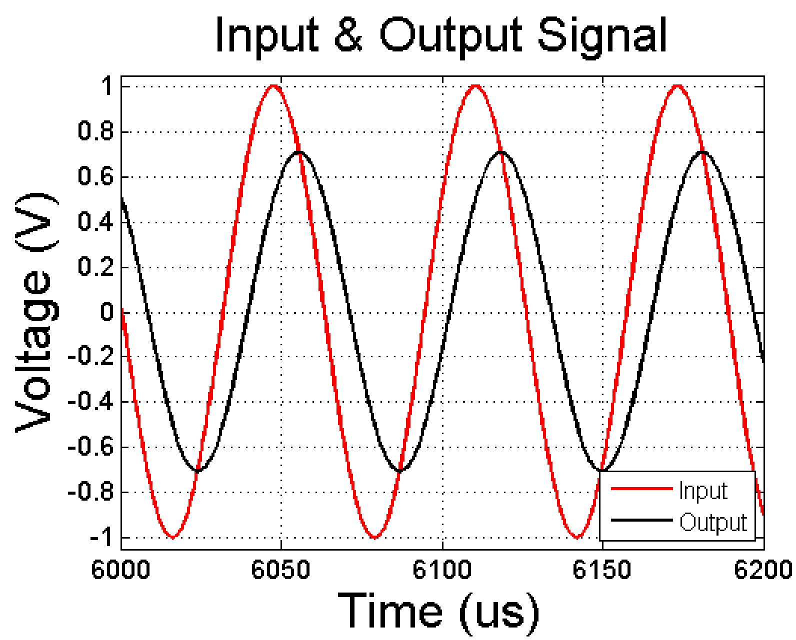
We can decompose the output into it's transient and steady - state response

$$v_o(t) = \frac{1}{2}e^{-10^5 t} + \frac{1}{\sqrt{2}}\sin\left(10^5 t - 45^\circ\right) = v_{tr}(t) + v_{ss}(t)$$

$$v_{tr}(t) = \frac{1}{2}e^{-10^5 t}$$

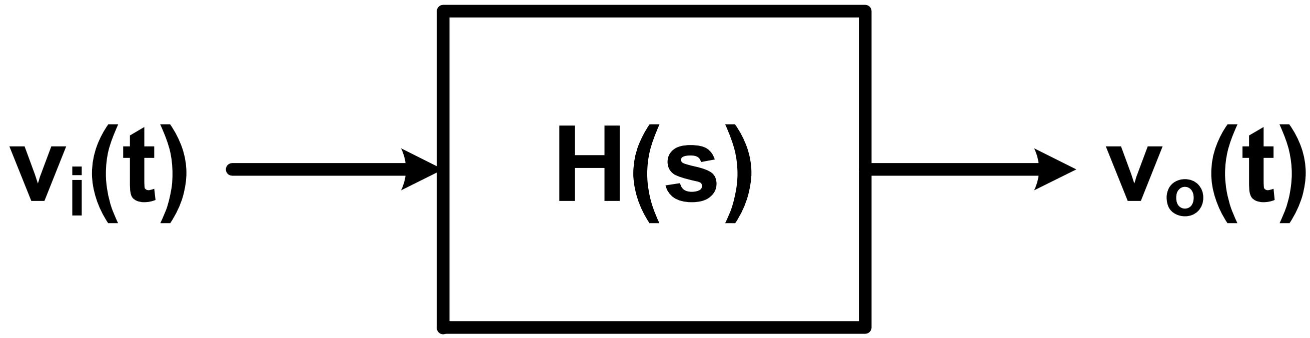
$$v_{ss}(t) = \frac{1}{\sqrt{2}}\sin\left(10^5 t - 45^\circ\right)$$





- Note that the transient response decays very quickly!

Sinusoidal Steady-State Response



If input $v_i(t)$ is sinusoidal

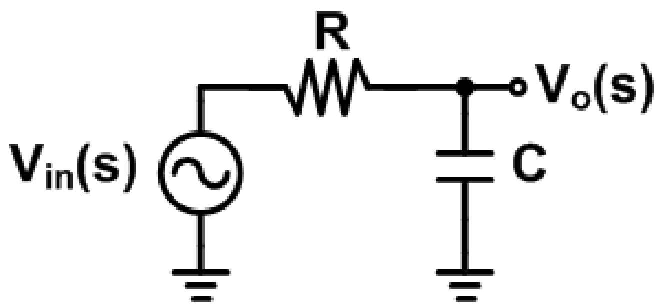
$$v_i(t) = A\cos(\omega t + \phi)$$

The steady-state output will be

$$v_{ss}(t) = |H(j\omega)| A\cos(\omega t + \phi + \angle H(j\omega))$$

- Here we have evaluated the transfer function H(s) with s=jω
- The magnitude of H(jω) scales the input amplitude
- The phase of H(jω) shifts the input phase

RC Circuit Sinusoidal Steady-State Response



$$H(s)=\frac{V_o(s)}{V_{in}(s)}=\frac{1}{1+sRC}\overset{s=j\omega}{\Rightarrow}H(j\omega)=\frac{1}{1+j\omega RC}$$

Output Magnitude

$$|H(j\omega)|=\sqrt{H(j\omega)H^*(j\omega)}=\sqrt{\left(\frac{1}{1+j\omega RC}\right)\left(\frac{1}{1-j\omega RC}\right)}$$

$$|H(j\omega)|=\sqrt{\frac{1}{1+(\omega RC)^2}}$$

Output Phase

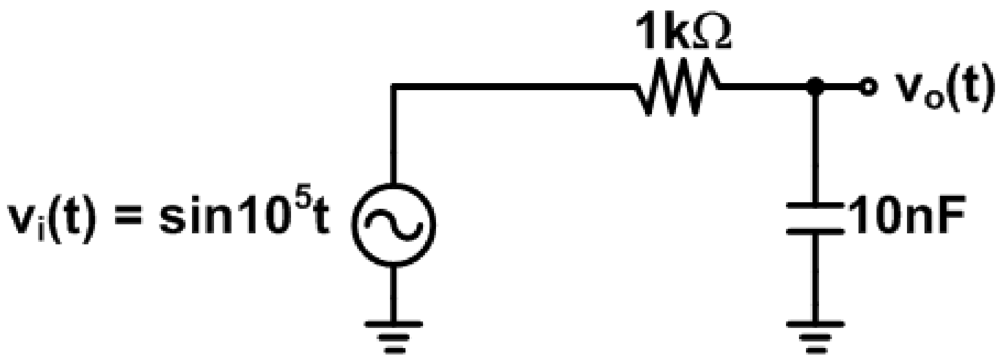
$$\angle H(j\omega)=\tan^{-1}\left(\frac{\mathrm{Im}(H(j\omega))}{\mathrm{Re}(H(j\omega))}\right)=\tan^{-1}\left(\frac{\mathrm{Im}(Num)}{\mathrm{Re}(Num)}\right)-\tan^{-1}\left(\frac{\mathrm{Im}(Den)}{\mathrm{Re}(Den)}\right)$$

where $\mathrm{Num}=\mathrm{Numerator}$ and $\mathrm{Den}=\mathrm{Denominator}$ of $H(j\omega)$

$$\angle H(j\omega)=\tan^{-1}\left(\frac{0}{1}\right)-\tan^{-1}\left(\frac{\omega RC}{1}\right)=-\tan^{-1}(\omega RC)$$

$$\angle H(j\omega)=-\tan^{-1}(\omega RC)$$

RC Circuit Sinusoidal Steady-State Response Example



$$H(s) = \frac{1}{1 + \frac{s}{10^5}}$$

with $s = j\omega = j10^5$

$$H\left(j10^5\right) = \frac{1}{1 + j}$$

$$\left|H\left(j10^5\right)\right| = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\angle H\left(j10^5\right) = -\tan^{-1}(1) = -45^\circ$$

$$v_{ss}(t) = \frac{1}{\sqrt{2}} \sin\left(10^5 t - 45^\circ\right)$$

Complex Numbers Properties

[Silva]

Function	Evaluation
$f(x) = R + j\,Im$	$f(x) = \lvert f(x)\rvert e^{j\phi_f}$ $\lvert f(x)\rvert = \sqrt{R^2 + Im^2}$ $\phi_f = tan^{-1}(Im/R)$
$f(x)\cdot g(x)$	$\lvert f(x)\rvert \cdot \lvert g(x)\rvert e^{j(\phi_f + \phi_g)}$
$\dfrac{f(x)}{g(x)}$	$\dfrac{\lvert f(x)\rvert}{\lvert g(x)\rvert} e^{j(\phi_f - \phi_g)}$
$\dfrac{f_1(x)\cdot f_2(x)\ldots f_n(x)}{g_1(x)\cdot g_2(x)\ldots g_m(x)}$	$\dfrac{\lvert f_1(x)\rvert \cdot \lvert f_2(x)\rvert \ldots \lvert f_n(x)\rvert}{\lvert g_1(x)\rvert \cdot \lvert g_2(x)\rvert \ldots \lvert g_m(x)\rvert} e^{j\left(\sum_{i=1}^{n}\phi_{fi} - \sum_{k=1}^{m}\phi_{gk}\right)}$

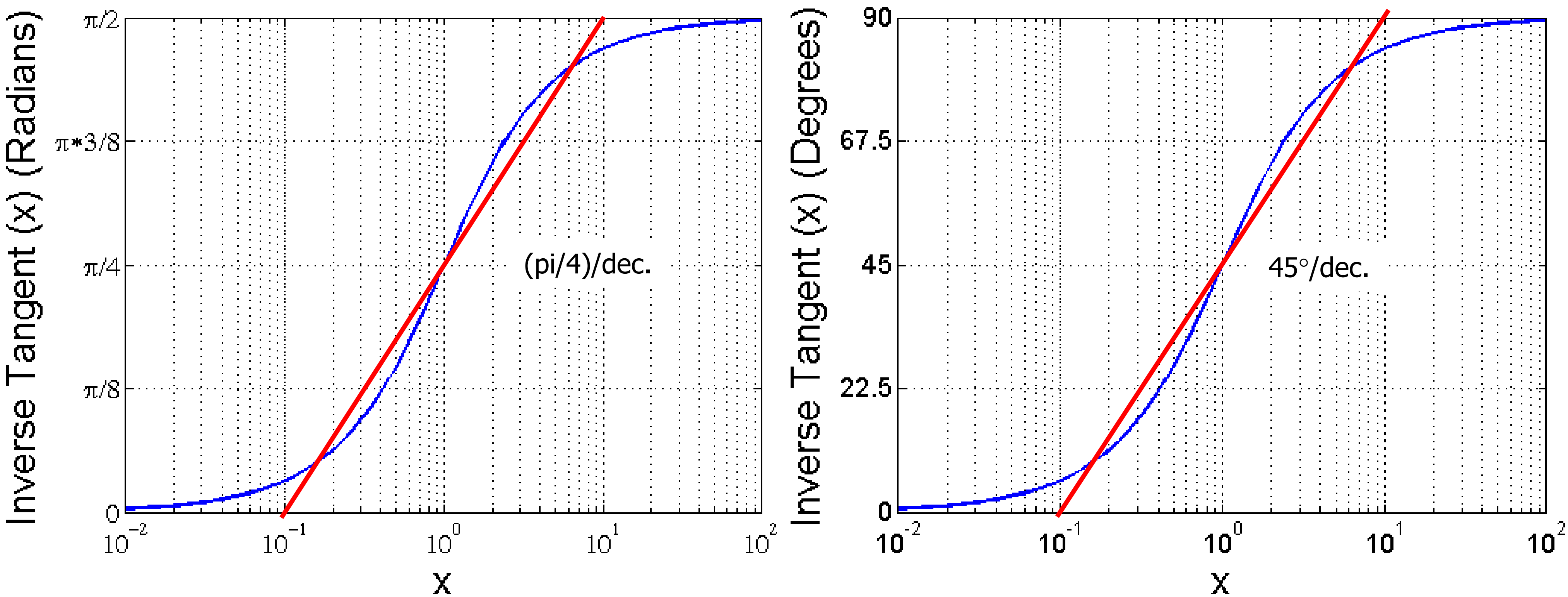
<u>Numerical Example</u>

$$\frac{(1+j10)(10+j10)}{(100+j10)(1000+j10)}$$

$$\left|\frac{(1+j10)(10+j10)}{(100+j10)(1000+j10)}\right| = \frac{\sqrt{1^2+10^2}\sqrt{10^2+10^2}}{\sqrt{100^2+10^2}\sqrt{1000^2+10^2}} = 1.41\times10^{-3}$$

$$\angle\frac{(1+j10)(10+j10)}{(100+j10)(1000+j10)} = \tan^{-1}\left(\frac{10}{1}\right) + \tan^{-1}\left(\frac{10}{10}\right) - \tan^{-1}\left(\frac{10}{100}\right) - \tan^{-1}\left(\frac{10}{1000}\right) = 123^\circ$$

Inverse Tangent Function



- For small values approximately 0
- For large values saturates at π/2 or 90°
- Between 0.1 and 10 can be approximated as changing with a slope of 45° per decade

Poles & Zeros

$$H(s) = A\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

- Poles are the roots of the denominator (p_1, p_2, … p_n) where H(s)→∞
- Zeros are the roots of the numerator (z_1, z_2, … z_m) where H(s)→0

$$\text{Example 1: } H(s) = \frac{10^5}{s+10^5}$$

$$s + 10^5 = 0$$

$$p_1 = s = -10^5 \, rad/s$$

$$\text{Example 2: } H(s) = \frac{s}{s+10^5}$$

$$z_1 = s = 0 \, rad/s$$

$$s + 10^5 = 0$$

$$p_1 = s = -10^5 \, rad/s$$

$$\text{Example 3: } H(s) = \frac{100(s+15)}{s^2+50s+1500}$$

$$s + 15 = 0$$

$$z_1 = s = -15 \, rad/s$$

$$s^2 + 50s + 1500 = 0$$

$$p_{1,2} = s_{1,2} = \frac{-50 \pm \sqrt{2500-6000}}{2} = -25 \pm j29.6 \, rad/s$$

Bode Plots

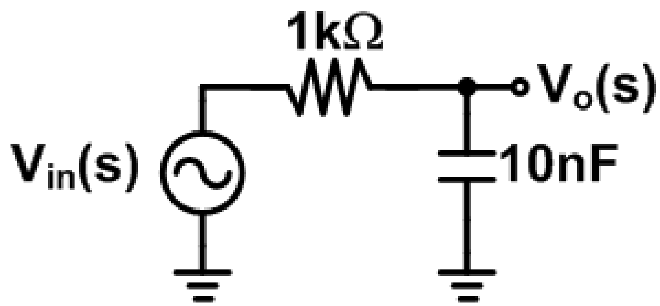
- Technique to plot the **Magnitude** (squared) and **Phase** response of a transfer function
  - Magnitude is plotted in Decibels (dB), which is a power ratio unit

$$|H(j\omega)|^2 \overset{dB}{\Rightarrow} 10\log_{10}\left(|H(j\omega)|^2\right)(\text{dB}) = 20\log_{10}(|H(j\omega)|)(\text{dB})$$

  - Phase is typically plotted in degrees

$$\angle(H(j\omega)) = \tan^{-1}\left(\frac{\text{Im}(H(j\omega))}{\text{Re}(H(j\omega))}\right)$$

RC Bode Plot Example



$$H(s)=\frac{V_o(s)}{V_{in}(s)}=\frac{1}{1+sRC}=\frac{1}{1+s10^{-5}}\overset{s=j\omega}{\Rightarrow}\frac{1}{1+j\omega 10^{-5}}$$

$$H(j\omega)=\frac{1}{1+j\omega 10^{-5}}=\frac{1}{1-\frac{j\omega}{p_1}},\ \text{where } p_1=-10^5 \text{ rad/s}$$

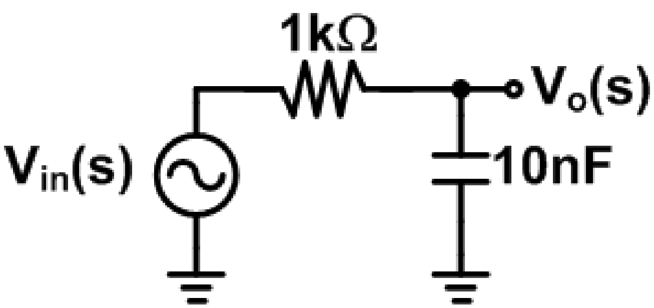
Magnitude Squared (dB):

$$20\log_{10}\left|H(j\omega)\right|=20\log_{10}\left|\frac{1}{\sqrt{1+\left(\omega 10^{-5}\right)^2}}\right|=20\log_{10}(1)-20\log_{10}\left(\sqrt{1+\left(\omega 10^{-5}\right)^2}\right)$$

Phase: $\text{Phase}(H(j\omega))=-\tan^{-1}\left(\omega 10^{-5}\right)$

RC Bode Plot Example

$$H(j\omega)=\frac{1}{1+j\omega 10^{-5}}$$



Magnitude:

$$20\log_{10}|H(j\omega)| = 20\log_{10}\left|\frac{\sqrt{1^2}}{\sqrt{1^2+(\omega 10^{-5})^2}}\right| = 20\log_{10}(1) - 20\log_{10}\left(\sqrt{1+(\omega 10^{-5})^2}\right)$$

Phase: $\mathrm{Phase}(H(j\omega)) = -\tan^{-1}(\omega 10^{-5})$

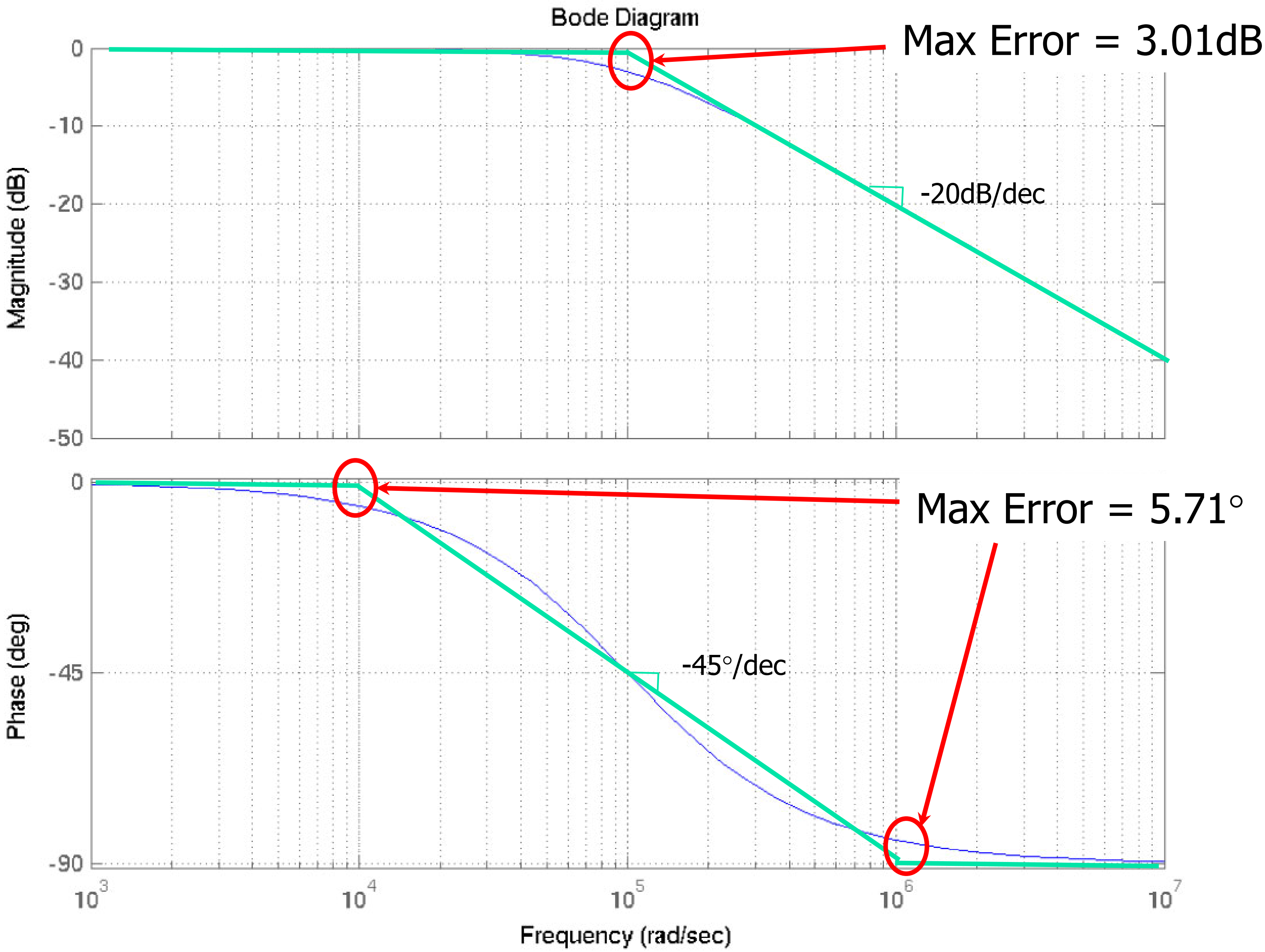
ω (rad/s)	|H(jω)|	|H(jω)|^2	$20\log_{10}$|H(jω)| (dB)	Phase (H(jω)) (°)
10^3	0.9999	0.9999	~0	~0
10^4	0.995	0.990	-0.043	-5.71
$5\text{x}10^4$	0.894	0.800	-0.969	-26.6
10^5	0.707	0.500	-3.01	-45.0
$5\text{x}10^5$	0.196	0.039	-14.2	-78.7
10^6	0.100	0.010	-20.0	-84.3
10^7	10^{-2}	10^{-4}	-40.0	-89.4
10^8	10^{-3}	10^{-6}	-60.0	-89.9

$\sim 20\log_{10}(1)$ = 0dB

$\sim -20\log_{10}(\omega 10^{-5})$ = -20dB/dec

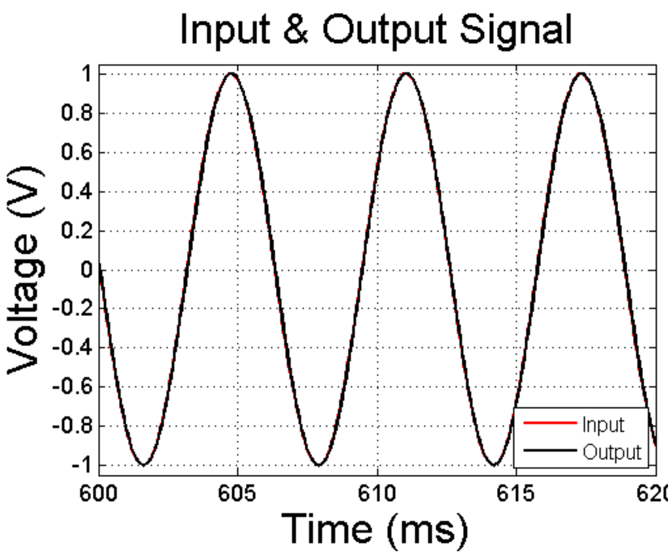
-45°/dec

RC Bode Plot Example



Transient Response

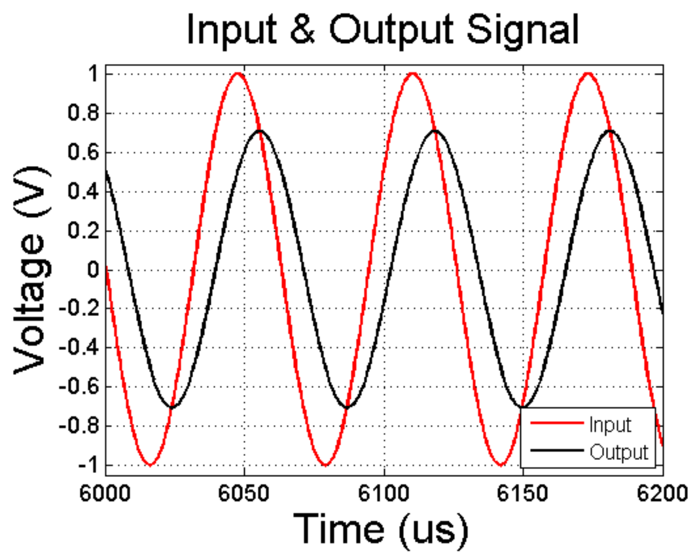
$\omega = 10^3$ rad/s = -p1/100



$$|v_o(t)| \approx 1$$

$$\text{Phase Shift} \approx 0^\circ$$

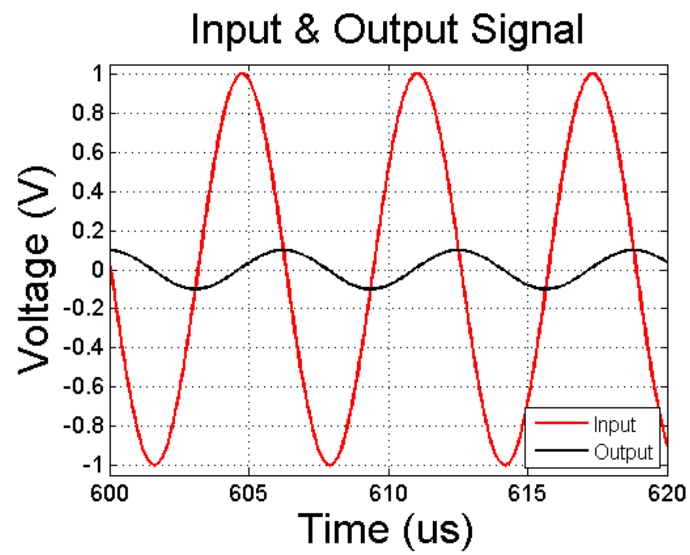
$\omega = 10^5$ rad/s = -p1



$$|v_o(t)| = \frac{1}{\sqrt{2}}$$

$$\text{Phase Shift} = -45^\circ$$

$\omega = 10^6$ rad/s = 10*p1



$$|v_o(t)| \approx 0.1$$

$$\text{Phase Shift} = -84.3^\circ$$

Bode Plot Algorithm - Magnitude

1. Where is a good starting point?
   a. Calculate DC value of $|H(j\omega)|$
   b. If not a reasonable value, I like to calculate $|H(j\omega)|$ at ω equal to the lowest non-zero value of p1/10 or z1/10
2. Where to end?
   a. Calculate $|H(j\omega)|$ as $\omega \rightarrow \infty$
3. Where are the poles and zeros?
   a. Beginning at each pole frequency, the magnitude will decrease with a slope of -20dB/dec
   b. Beginning at each zero frequency, the magnitude will increase with a slope of +20dB/dec
4. Note, the above algorithm is only valid for real poles and zeros. We will discuss complex poles later.

Bode Plot Algorithm - Magnitude

$$H(s) = -\frac{10^4(s+1)}{(s+10)(s+100)} = -\frac{10(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$

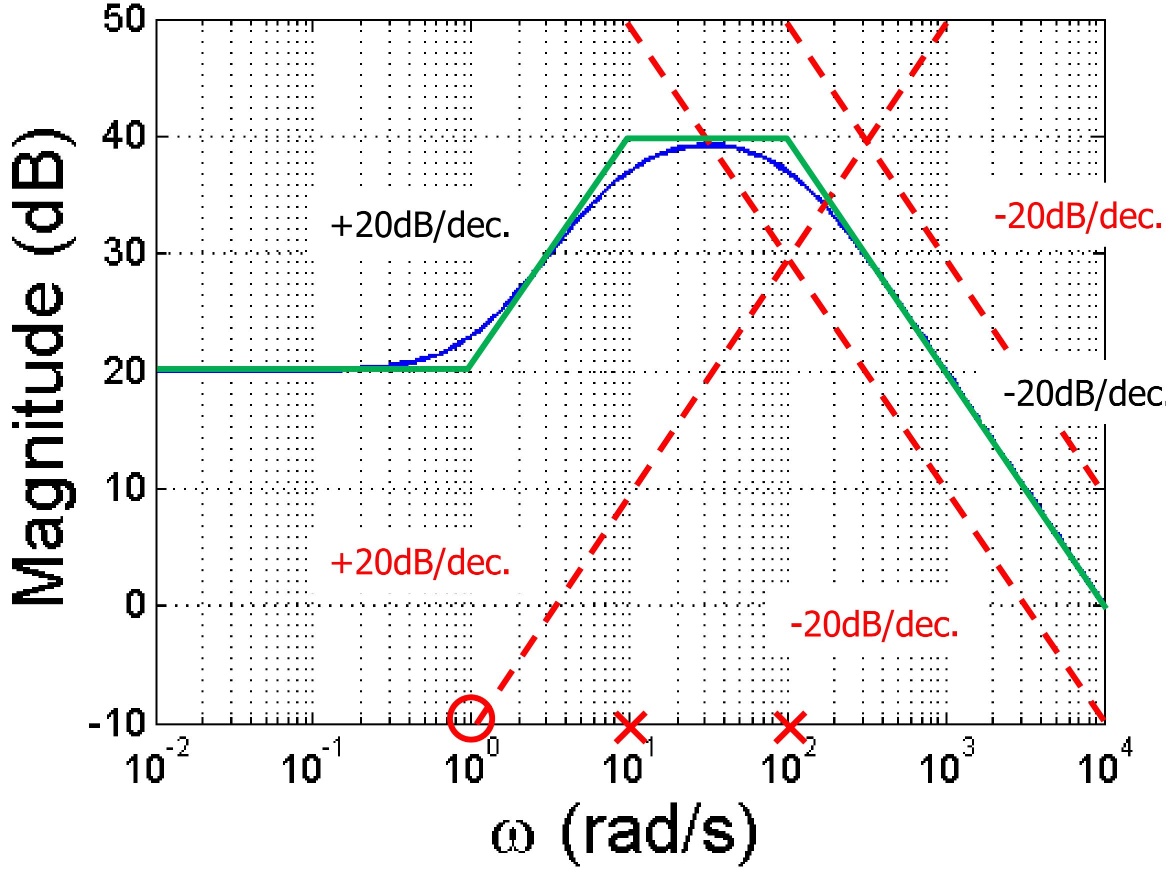
$$20\log_{10}|H(j\omega)| = 20\log_{10}\left|\frac{10\sqrt{1+\omega^2}}{\sqrt{1+(\omega 10^{-1})^2}\sqrt{1+(\omega 10^{-2})^2}}\right| =$$

$$20\log_{10}(10) + 20\log_{10}\left(\sqrt{1+\omega^2}\right) - 20\log_{10}\left(\sqrt{1+(\omega 10^{-1})^2}\right) - 20\log_{10}\left(\sqrt{1+(\omega 10^{-2})^2}\right)$$

$$\text{DC Magnitude} = 10 = 20dB$$

$$\text{HF Magnitude} = 0 = -\infty dB$$

$$z_1 = -1,\ p_1 = -10,\ p_2 = -100$$



Bode Plot Algorithm - Phase

1. Calculate low frequency value of Phase(H(jω))
   a. An negative sign introduces -180° phase shift
   b. A DC pole introduces -90° phase shift
   c. A DC zero introduces +90° phase shift
2. Where are the poles and zeros?
   a. For negative poles: 1 dec. before the pole freq., the phase will decrease with a slope of -45°/dec. until 1 dec. after the pole freq., for a total phase shift of -90°
   b. For negative zeros: 1 dec. before the zero freq., the phase will increase with a slope of +45°/dec. until 1 dec. after the zero freq., for a total phase shift of +90°
   c. Note, if you have positive poles or zeros, the phase change polarity is inverted
3. Note, the above algorithm is only valid for real poles and zeros. We will discuss complex poles later.

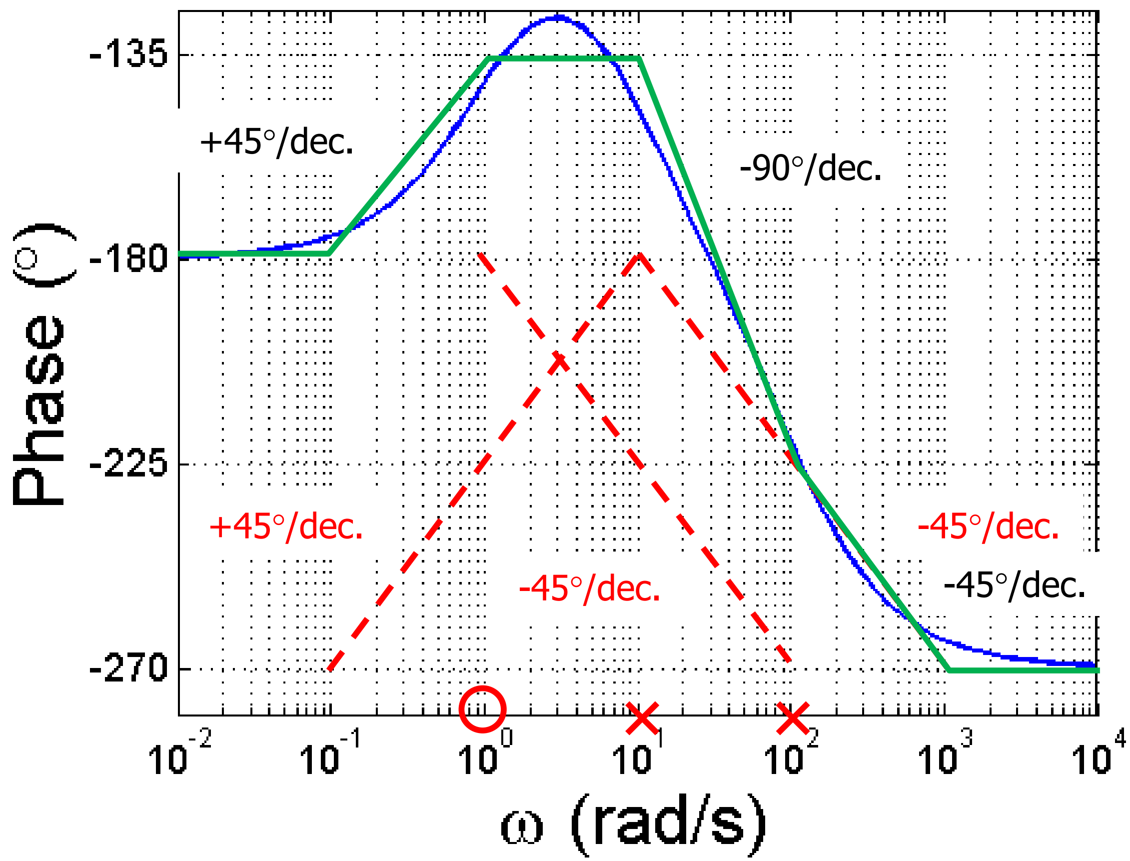
Bode Plot Algorithm - Phase

$$H(s) = -\frac{10^4(s+1)}{(s+10)(s+100)} = -\frac{10(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$

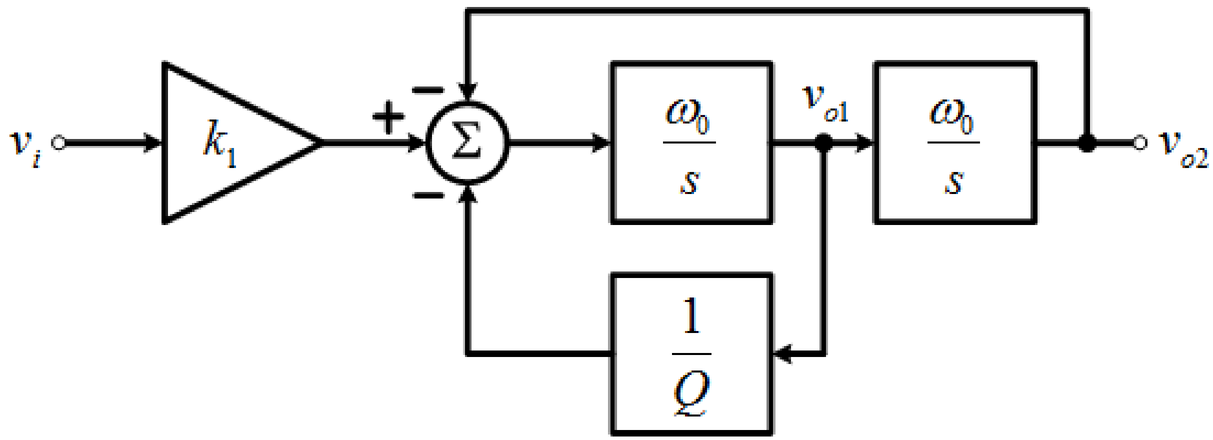
$$\angle H(j\omega) = -180^\circ + \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

$$\text{LF Phase} = -180^\circ$$

$$z_1 = -1,\ p_1 = -10,\ p_2 = -100$$



Second-Order Systems: Real or Complex Poles?



$$H(s) = \frac{k_1 \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$2 \text{ poles} \quad p_1, p_2 = -\frac{\omega_0}{2Q} \pm \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 - \omega_0^2}$$

2 real poles if $Q \leq 0.5$

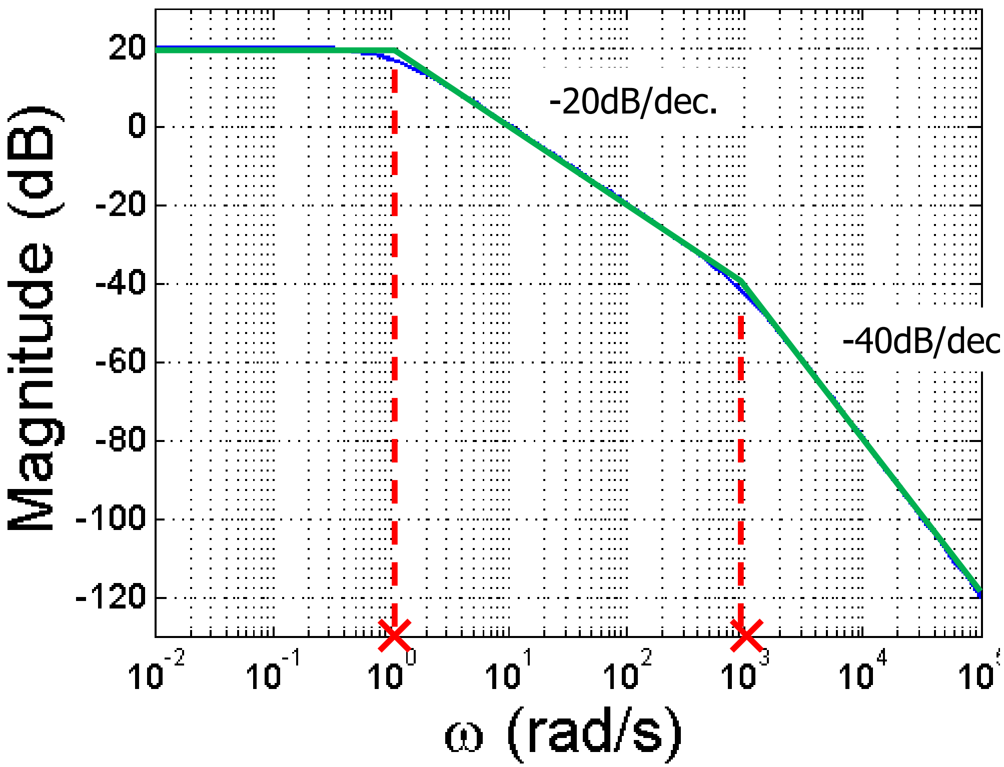
2 complex conjugate poles if $Q > 0.5$

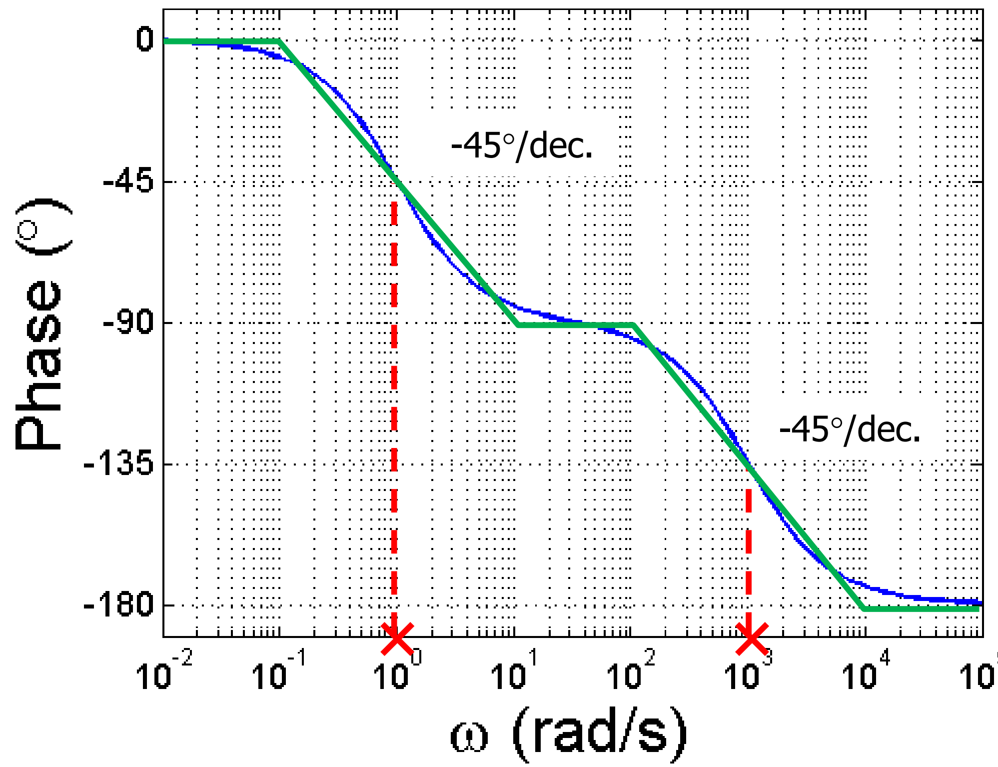
Second-Order Systems – Real Poles (1)

$$H(s)=\frac{10^4}{s^2+1001s+1000}=\frac{10^4}{(s+1)(s+1000)}$$

$$\text{2 poles: } p_1=-1,\ p_2=-1000$$

$$\text{Note, } Q=0.032$$





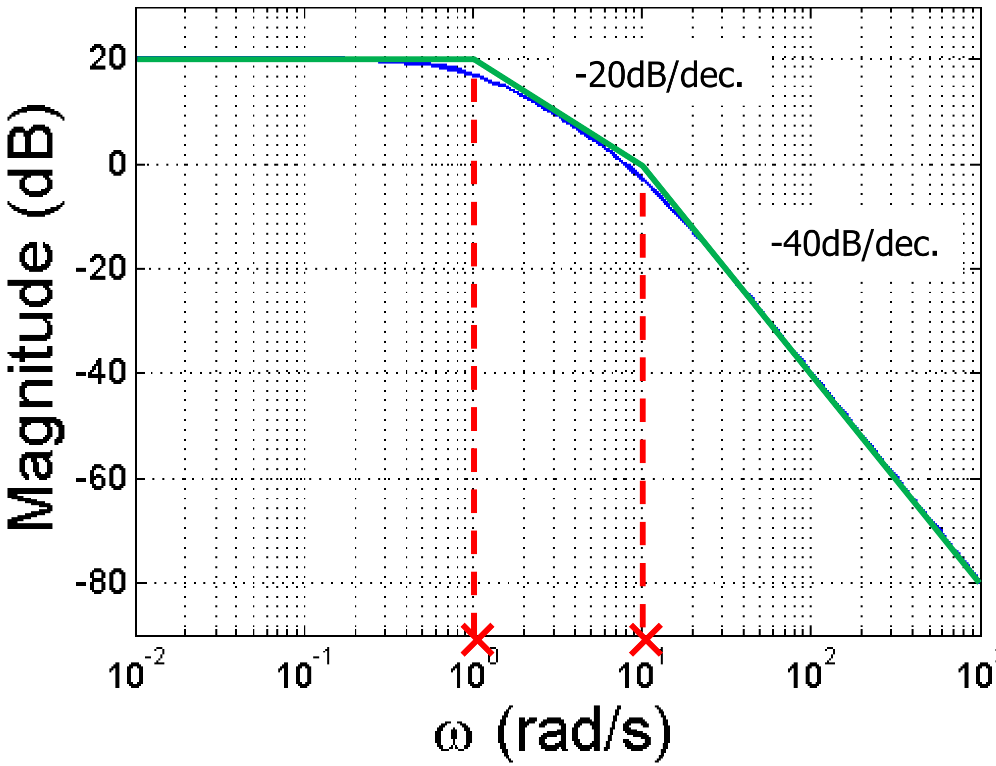
- If poles are spaced by more than 2 decades, there are 2 distinct regions of -45°/dec phase slope

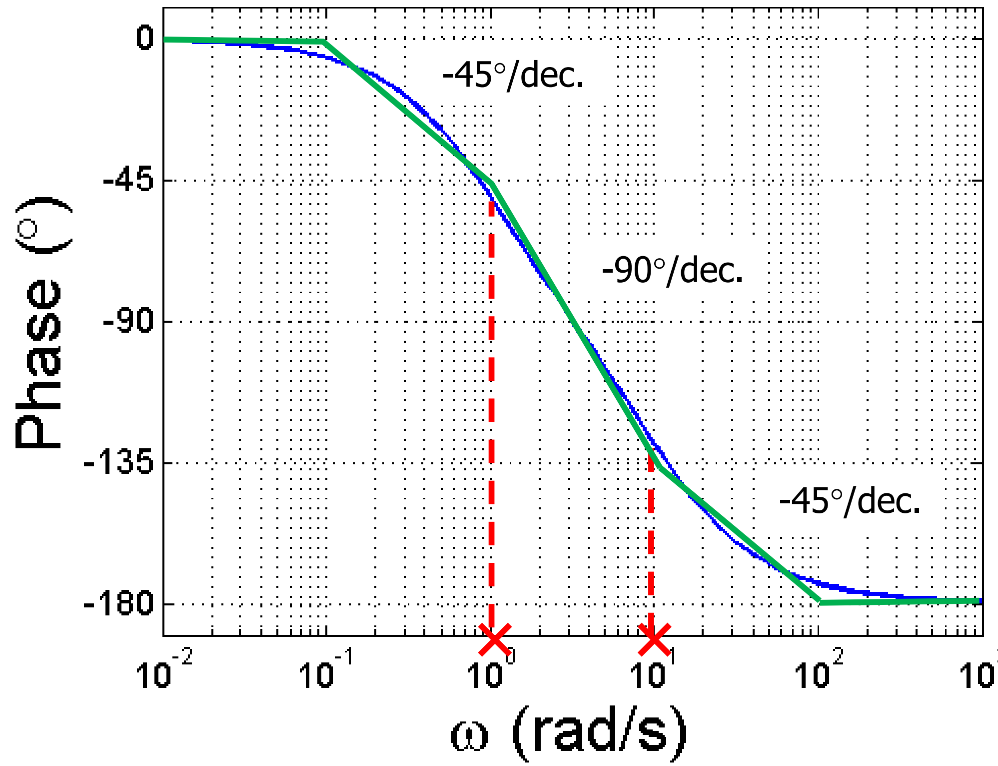
Second-Order Systems – Real Poles (2)

$$H(s)=\frac{100}{s^2+11s+10}=\frac{100}{(s+1)(s+10)}$$

$$2\text{ poles}:\ p_1=-1,\ p_2=-10$$

$$\text{Note, } Q=0.287$$





- If poles are spaced by less than 2 decades, there is a region of -90°/dec phase slope
  - Watch out for system stability!

Second-Order Systems – Complex Poles

$$H(s) = \frac{k_1 \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

What is the low frequency magnitude?

$$|H(j0)| = k_1$$

What is the high frequency magnitude?

$$|H(j\omega)|\Big|_{\omega \Rightarrow \infty} = \frac{k_1 \omega_0^2}{\omega^2} \Rightarrow \text{-40dB/dec. slope at high frequencies}$$

What happens in the middle, particularly near ω_0?

$$|H(j\omega_0)| = \left| \frac{k_1 \omega_0^2}{-\omega_0^2 + j\frac{\omega_0^2}{Q} + \omega_0^2} \right| = k_1 Q$$

Note, if $Q > 1$ then the magnitude exceeds the low frequency value, i.e. frequency peaking occurs!

Frequency Peaking w/ Complex Poles

Where is the peak frequency?

$$\frac{d\left|H(j\omega)\right|^2}{d\omega} = \frac{d}{d\omega}\left(\frac{k_1^2\omega_0^4}{\left(\omega_0^2-\omega^2\right)^2+\left(\frac{\omega_0}{Q}\omega\right)^2}\right)=0$$

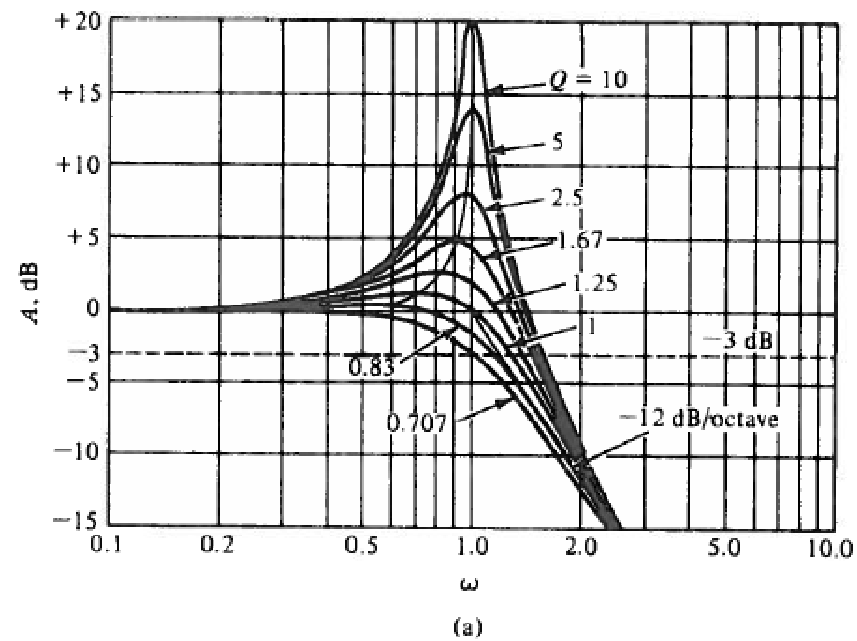
$$\omega_{pk} = \omega_0\sqrt{1-\frac{1}{2Q^2}} \approx \omega_0 \text{ for large } Q$$

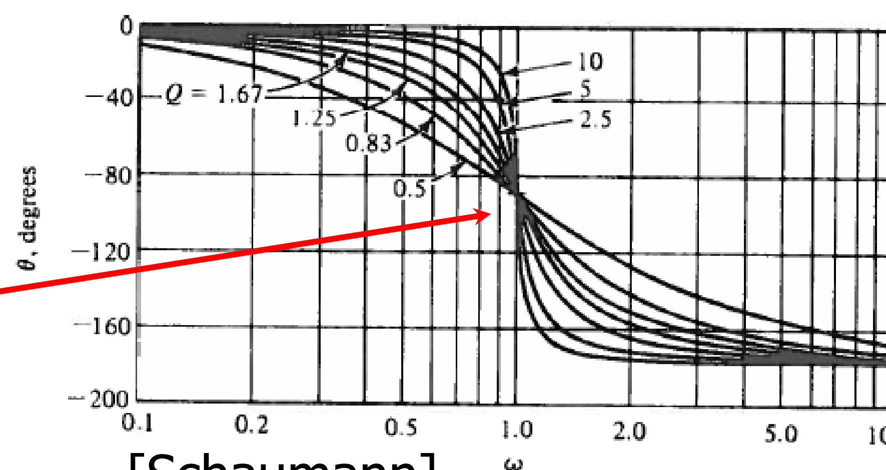
At ω_{pk}, the peak value is

$$T_{pk} = \frac{k_1 Q}{\sqrt{1-\frac{1}{4Q^2}}} \approx k_1 Q \text{ for large } Q$$

- Note, phase always crosses -90° at ω_0

For k_1=1 and ω_0=1





[Schaumann]

Second-Order Systems' Bode Plots Summary

- 2 real poles ⇒ Plot with standard Bode plot techniques
- 2 complex poles ⇒ Approximate as 2 real poles at ω_0
  - Past ω_0 the magnitude decreases at -40dB/dec
  - From $0.1\omega_0$ to $10\omega_0$ the phase slope is -90°/dec
- A more exact plot of second order systems can be obtained by calculating Q and using the reference plots on the previous slide

$$\left|H(j\omega_0)\right| = \left|\frac{k_1\omega_0^2}{-\omega_0^2 + j\frac{\omega_0^2}{Q} + \omega_0^2}\right| = k_1 Q$$

Next Time

- OpAmp Circuits
- Reading
  - Razavi 8.1,2,4,5