Brief Note on 2nd-Order System w/ Complex Poles

$$F(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow f(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\, t)$$

ω_n = Natural Frequency

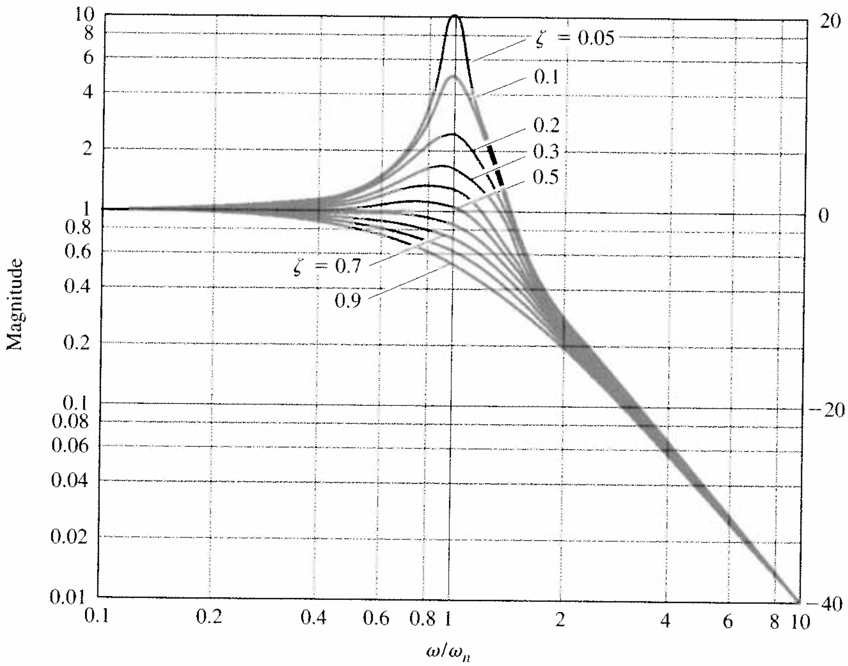
ζ = Damping Ratio

For Bode Plots

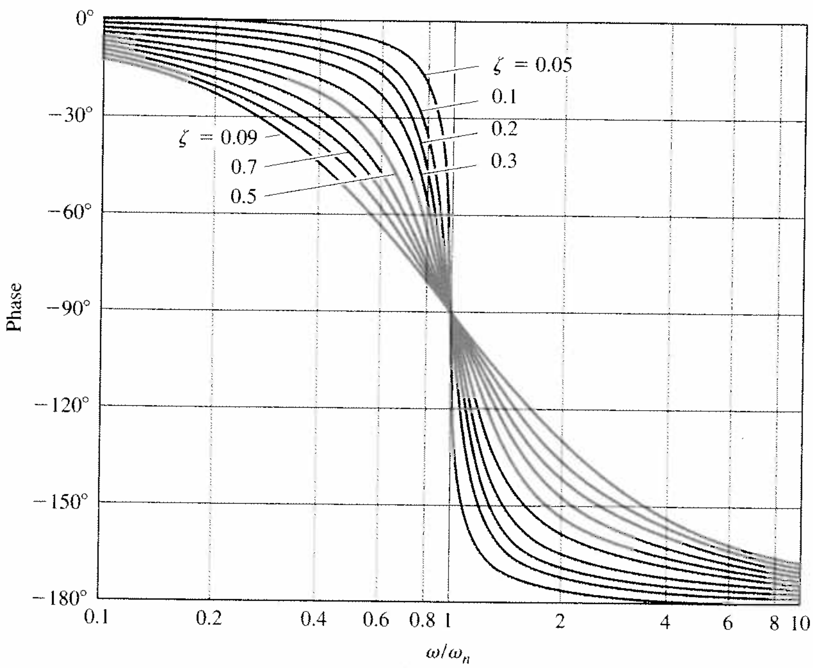
* If the denominator has 2 real poles, plot w/ normal Bode tech.
* If the denominator has 2 complex poles:
  - Approximate break point at ω_n
  - Past ω_n the magnitude roll-off will be −40 dB/dec
  - From $0.1\,\omega_n$ to $10\,\omega_n$ the phase slope will be −90 dB/dec
* More exact plot w/ peaking

  At $\omega = \omega_n \Rightarrow |F(j\omega)| = \frac{1}{2\zeta}$

FIGURE 6.2
(a) Magnitude and (b) phase of Eq. (6.7)



(a)



(b)