ELEN 326

Differential Pairs

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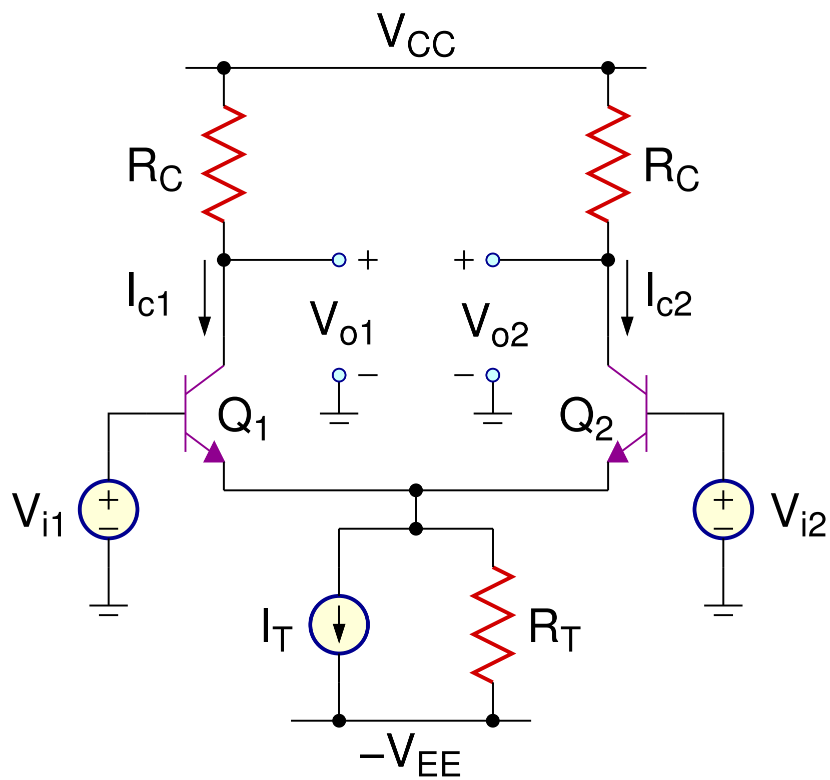
Emitter-Coupled Pair



$$V_{i1} - V_{be1} + V_{be2} - V_{i2} = 0$$

$$V_{be1} = V_T \ln \frac{I_{C1}}{I_{S1}} \quad , \quad V_{be2} = V_T \ln \frac{I_{C2}}{I_{S2}}$$

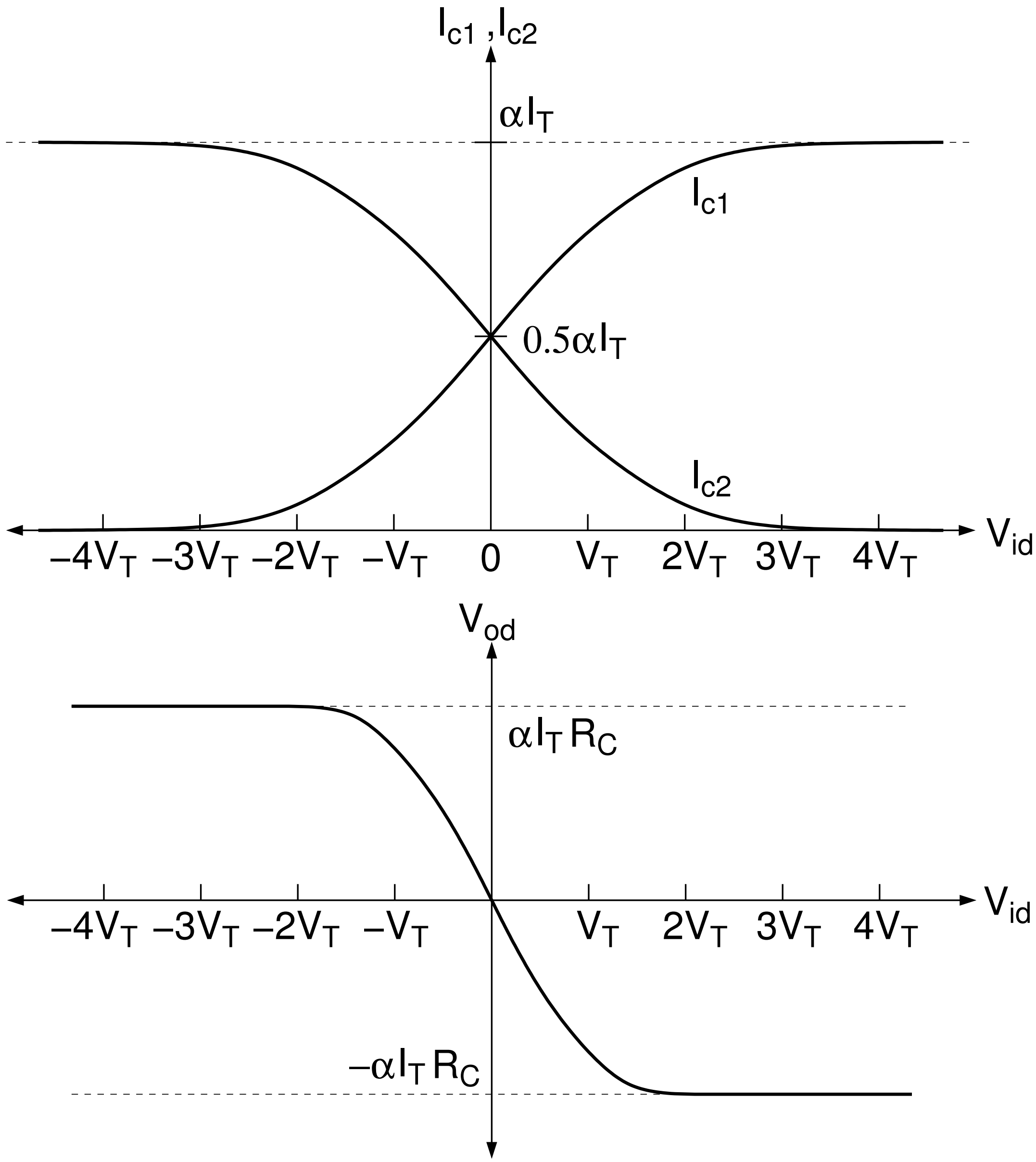
$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{be1} - V_{be2}}{V_T}\right) = \exp\left(\frac{V_{i1} - V_{i2}}{V_T}\right) = \exp\left(\frac{V_{id}}{V_T}\right)$$

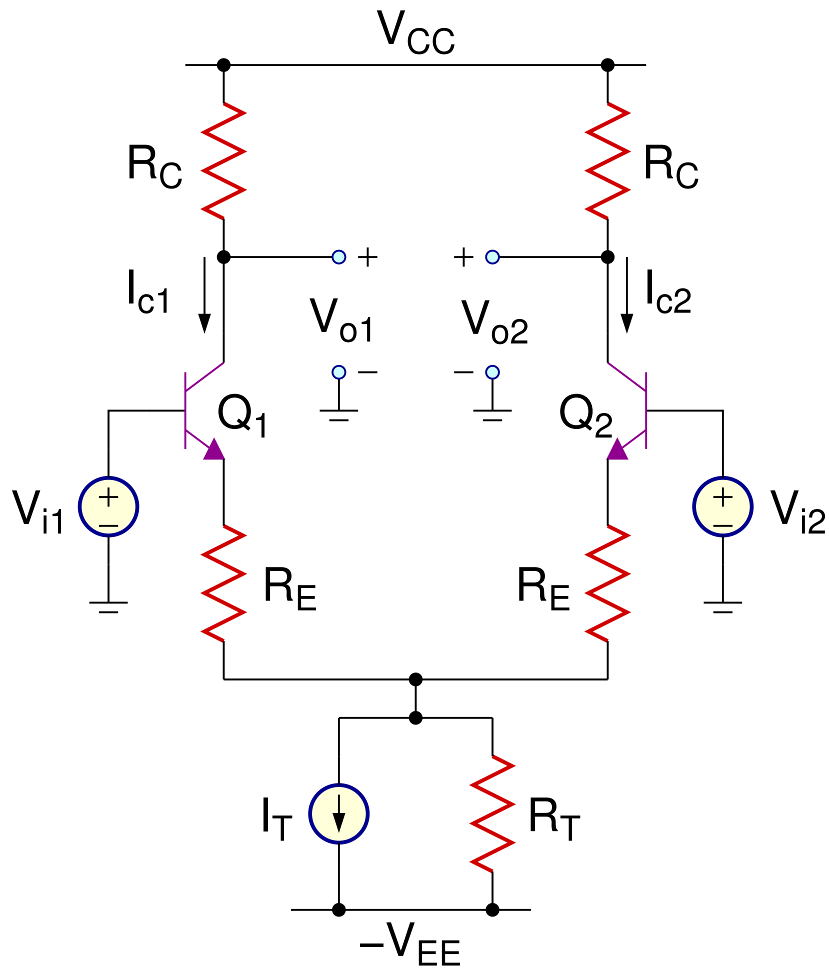
$$I_{c1} + I_{c2} = \alpha I_T$$

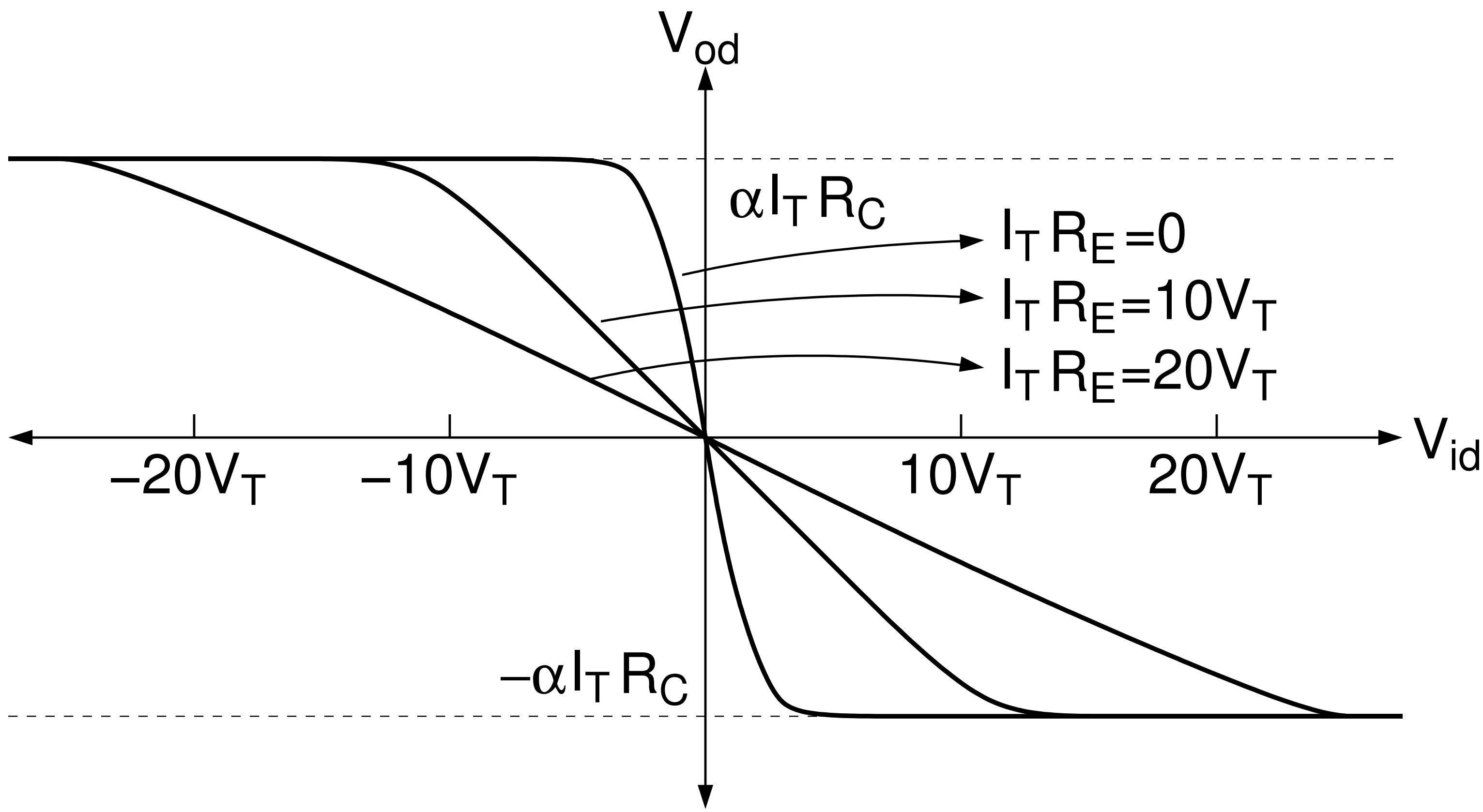
$$\boxed{I_{c1} = \frac{\alpha I_T}{1 + \exp\left(-\frac{V_{id}}{V_T}\right)} \quad , \quad I_{c2} = \frac{\alpha I_T}{1 + \exp\left(\frac{V_{id}}{V_T}\right)}}$$

$$V_{o1} = V_{CC} - I_{c1}R_C \qquad V_{o2} = V_{CC} - I_{c2}R_C$$

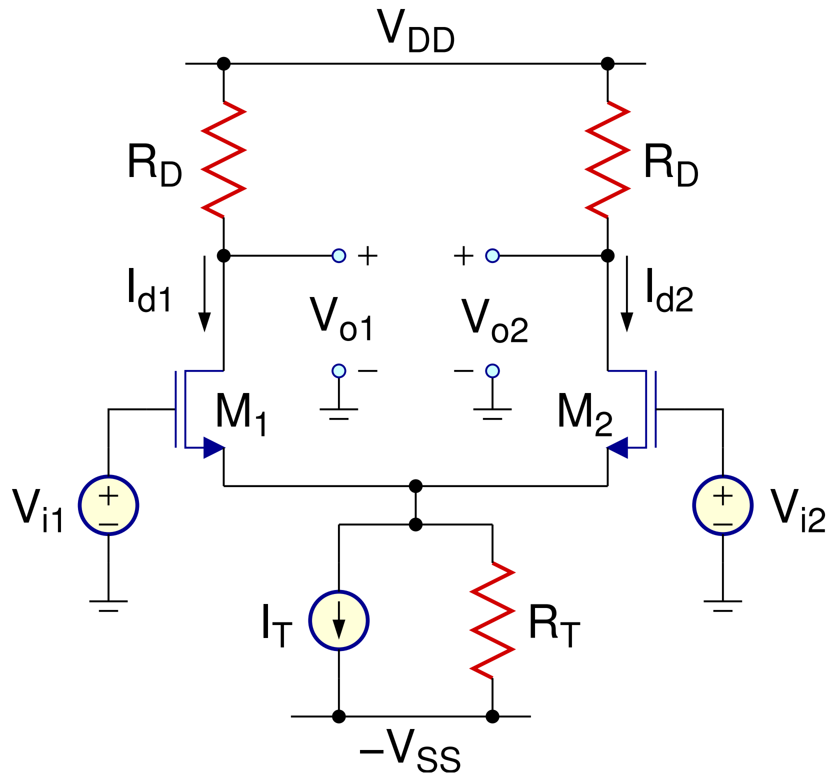
$$\boxed{V_{od} = V_{o1} - V_{o2} = \alpha I_T R_C \tanh\left(-\frac{V_{id}}{2V_T}\right)}$$


Ic1 ,Ic2
αIT
Ic1
0.5αIT
Ic2
Vid
–4VT –3VT –2VT –VT 0 VT 2VT 3VT 4VT
Vod
αIT RC
Vid
–4VT –3VT –2VT –VT VT 2VT 3VT 4VT
–αIT RC


V_{CC}
R_C
R_C
I_{c1}
I_{c2}
V_{o1}
V_{o2}
Q_1
Q_2
V_{i1}
V_{i2}
R_E
R_E
I_T
R_T
$-V_{EE}$



V_od
αI_T R_C
I_T R_E =0
I_T R_E=10V_T
I_T R_E=20V_T
–20V_T
–10V_T
10V_T
20V_T
V_id
–αI_T R_C



$$V_{i1} - V_{gs1} + V_{gs2} - V_{i2} = 0$$

$$V_{gs1} = V_t + \sqrt{\frac{I_{d1}}{\frac{k'}{2}\frac{W}{L}}}, \quad V_{gs2} = V_t + \sqrt{\frac{I_{d2}}{\frac{k'}{2}\frac{W}{L}}}$$

$$I_{d1} + I_{d2} = I_T$$

$$V_{id} = V_{i1} - V_{i2} = V_{gs1} - V_{gs2} = \frac{\sqrt{I_{d1}} - \sqrt{I_{d2}}}{\sqrt{\frac{k'}{2}\frac{W}{L}}}$$

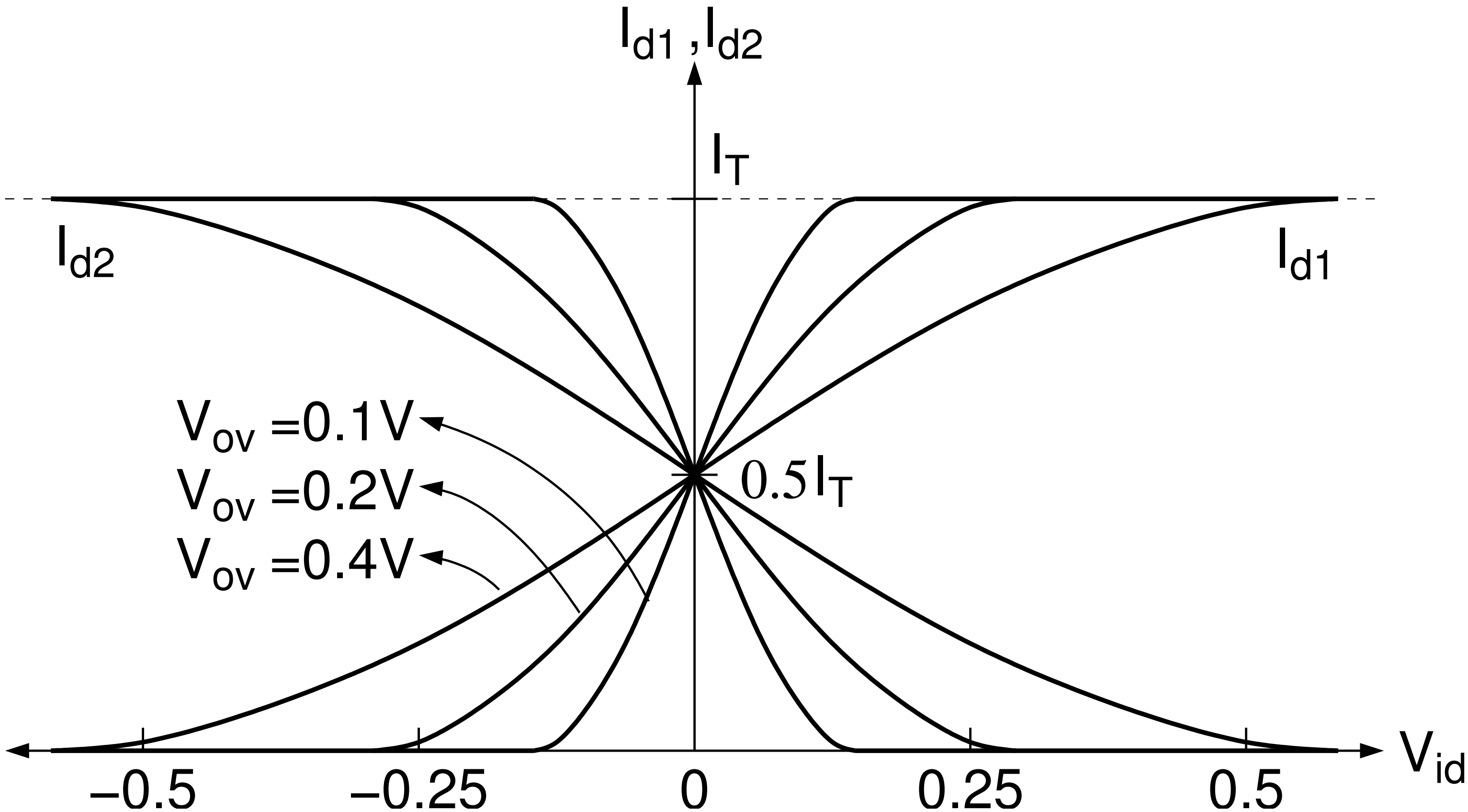
$$\boxed{I_{d1} = \frac{I_T}{2} + \frac{k'}{4}\frac{W}{L}V_{id}\sqrt{\frac{4I_T}{k'(W/L)} - V_{id}^2}}$$

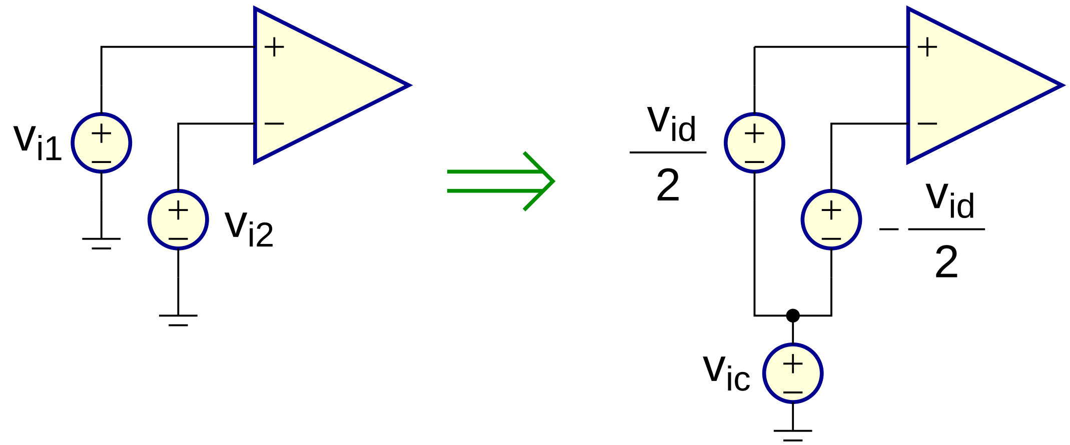
$$\boxed{I_{d2} = \frac{I_T}{2} - \frac{k'}{4}\frac{W}{L}V_{id}\sqrt{\frac{4I_T}{k'(W/L)} - V_{id}^2}}$$

Both transistors operate in the active region if

$$|V_{id}| \leq \sqrt{\frac{2I_T}{k'(W/L)}} = \sqrt{2}V_{ov}|_{V_{id}=0}$$

Source-Coupled Pair





$$v_{id} = v_{i1} - v_{i2}$$

$$v_{ic} = \frac{v_{i1} + v_{i2}}{2}$$

$$v_{od} = v_{o1} - v_{o2}$$

$$v_{oc} = \frac{v_{o1} + v_{o2}}{2}$$

$$v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

$$v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

$$v_{o1} = v_{oc} + \frac{v_{od}}{2}$$

$$v_{o2} = v_{oc} - \frac{v_{od}}{2}$$

Differential-mode gain:

$$A_{dm} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0}$$

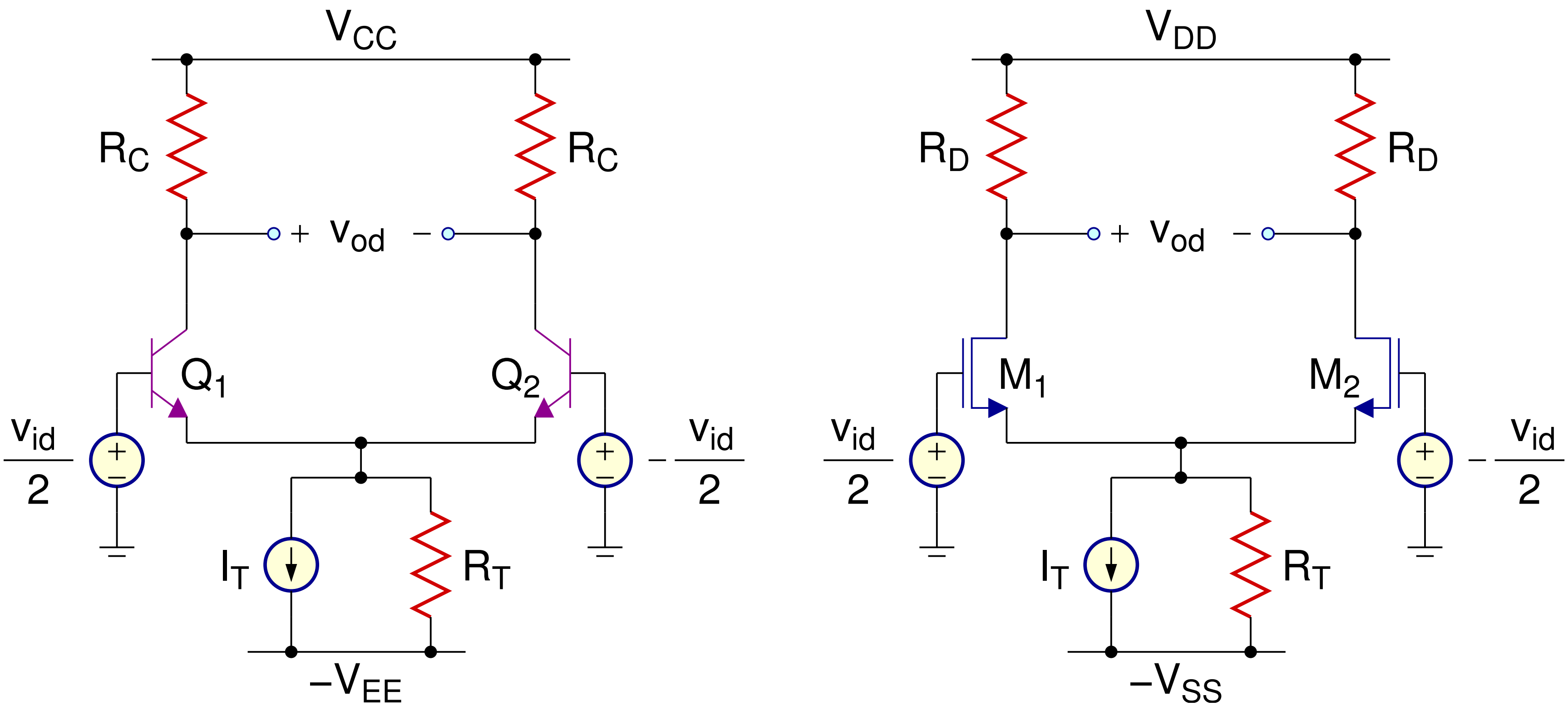
Common-mode gain:

$$A_{cm} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0}$$

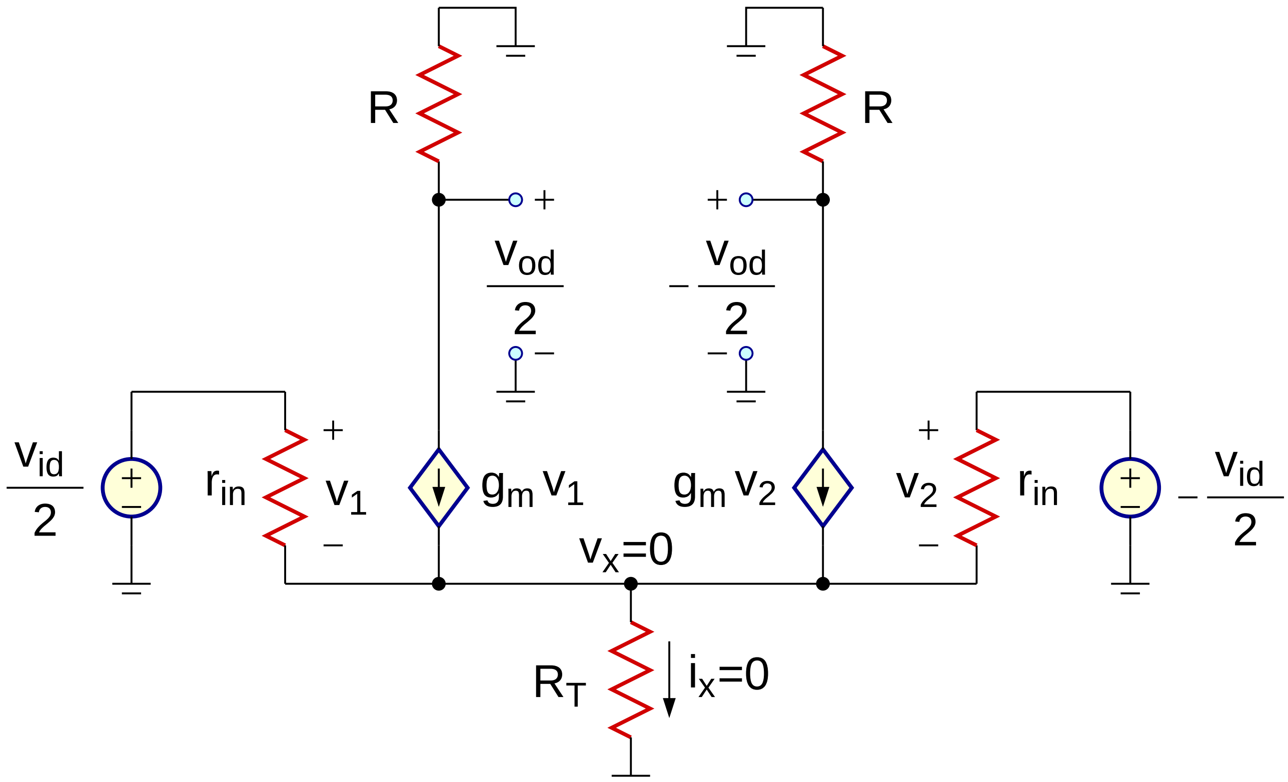
Common-mode-rejection ratio:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$$

Differential-Mode



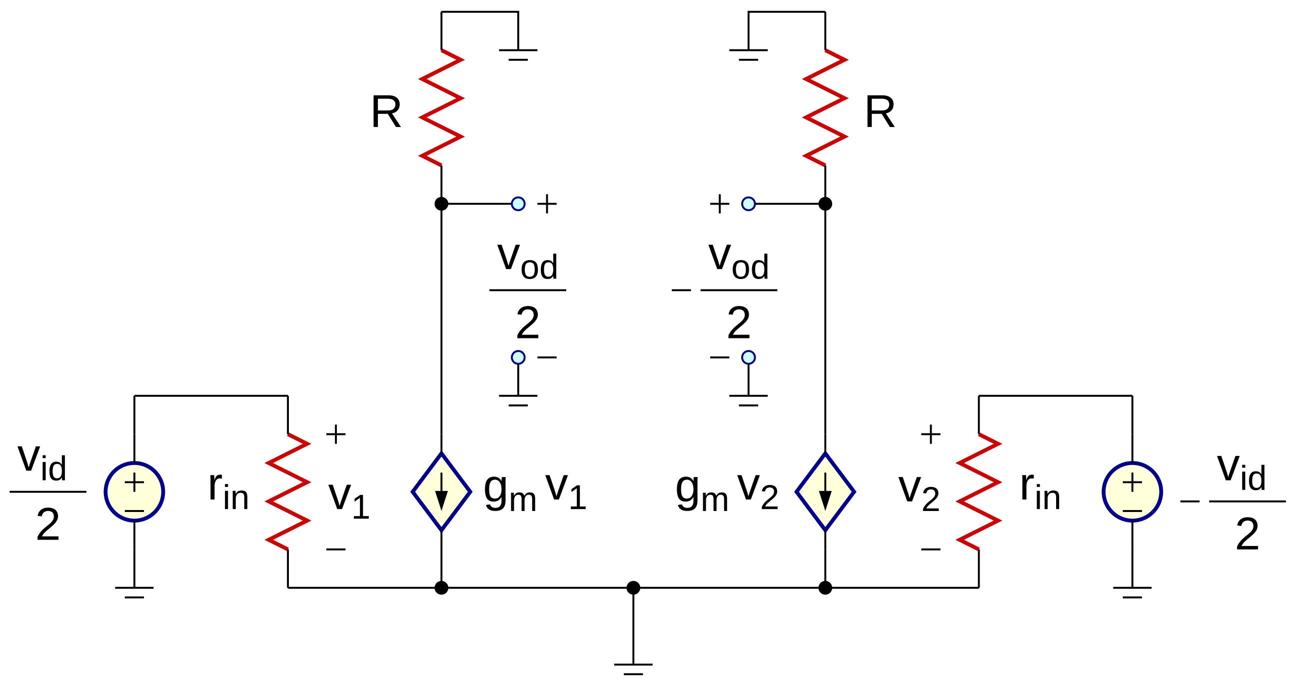
Differential-Mode



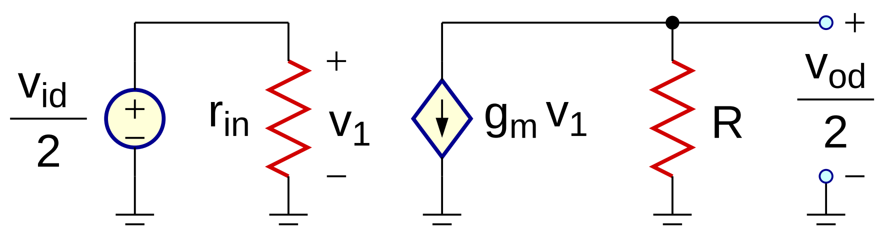
BJT: $\mathbf{r_{in} = r_\pi}$ MOSFET: $\mathbf{r_{in} = \infty}$

$\mathbf{R = R_C}$ $\mathbf{R = R_D}$

Differential-Mode



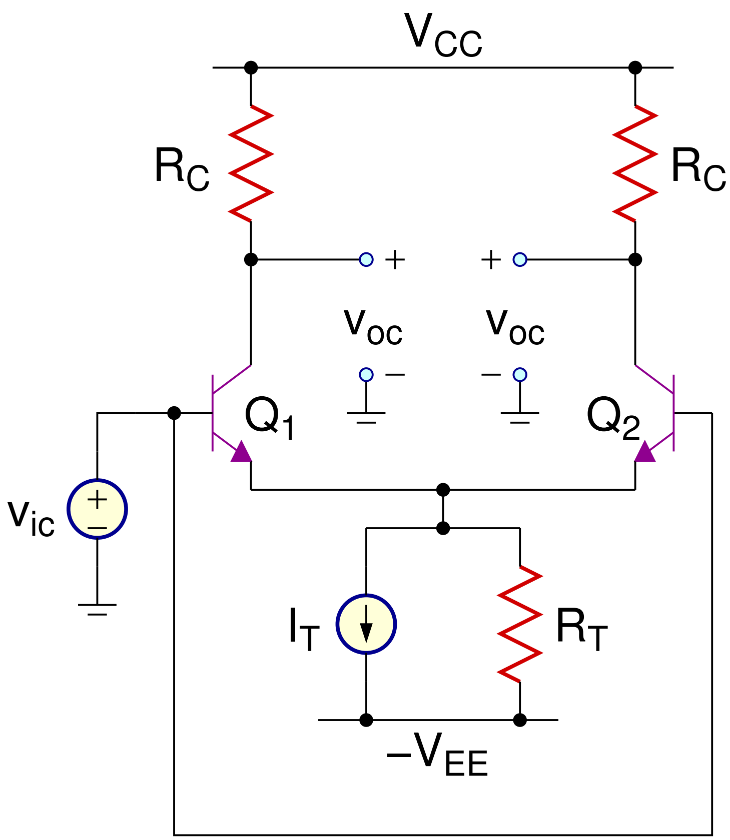
Differential-Mode



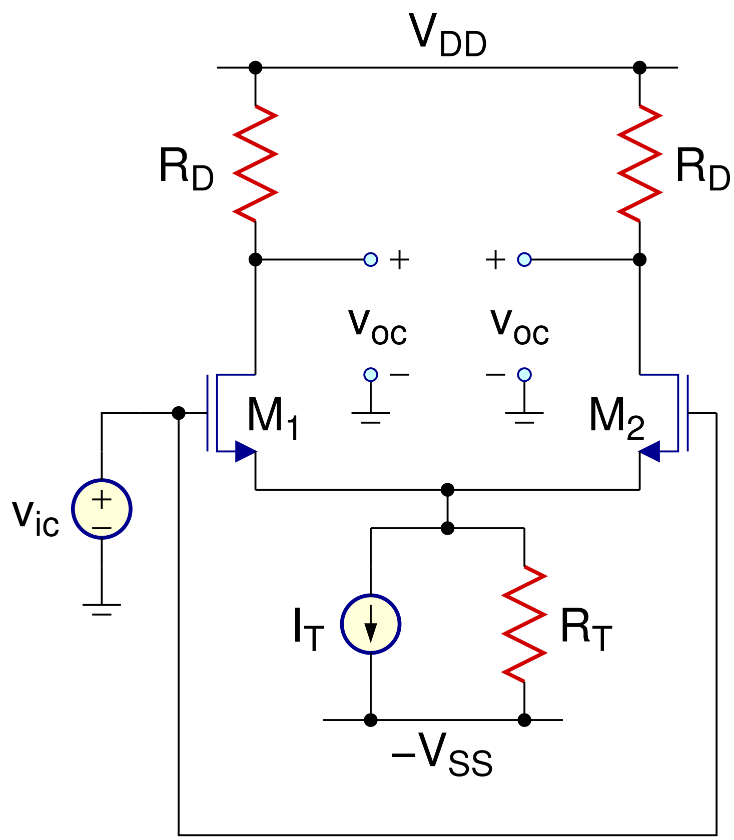
$$\frac{v_{od}}{2} = -g_m R \, \frac{v_{id}}{2}$$

$$A_{dm} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} = -g_m R$$

Common-Mode

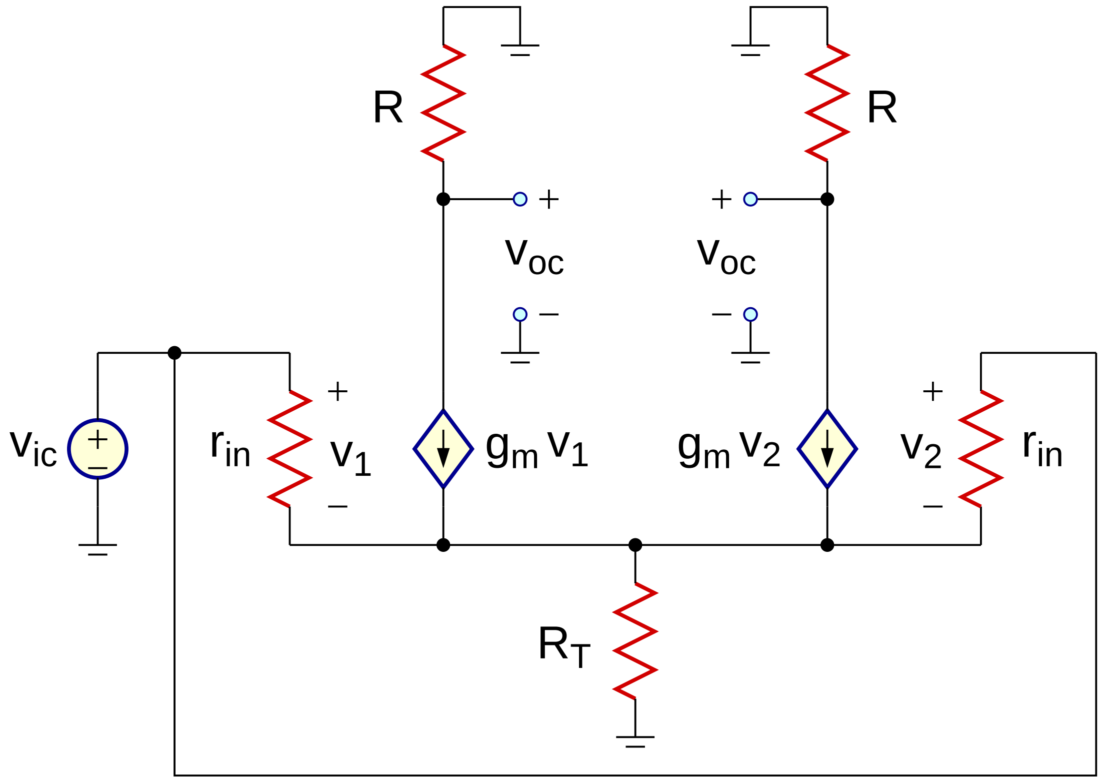


V_{CC}
R_C
R_C
+
+
v_{oc}
v_{oc}
−
−
Q_1
Q_2
v_{ic}
+
−
I_T
R_T
$-V_{EE}$



V_{DD}
R_D
R_D
+
+
v_{oc}
v_{oc}
−
−
M_1
M_2
v_{ic}
+
−
I_T
R_T
$-V_{SS}$

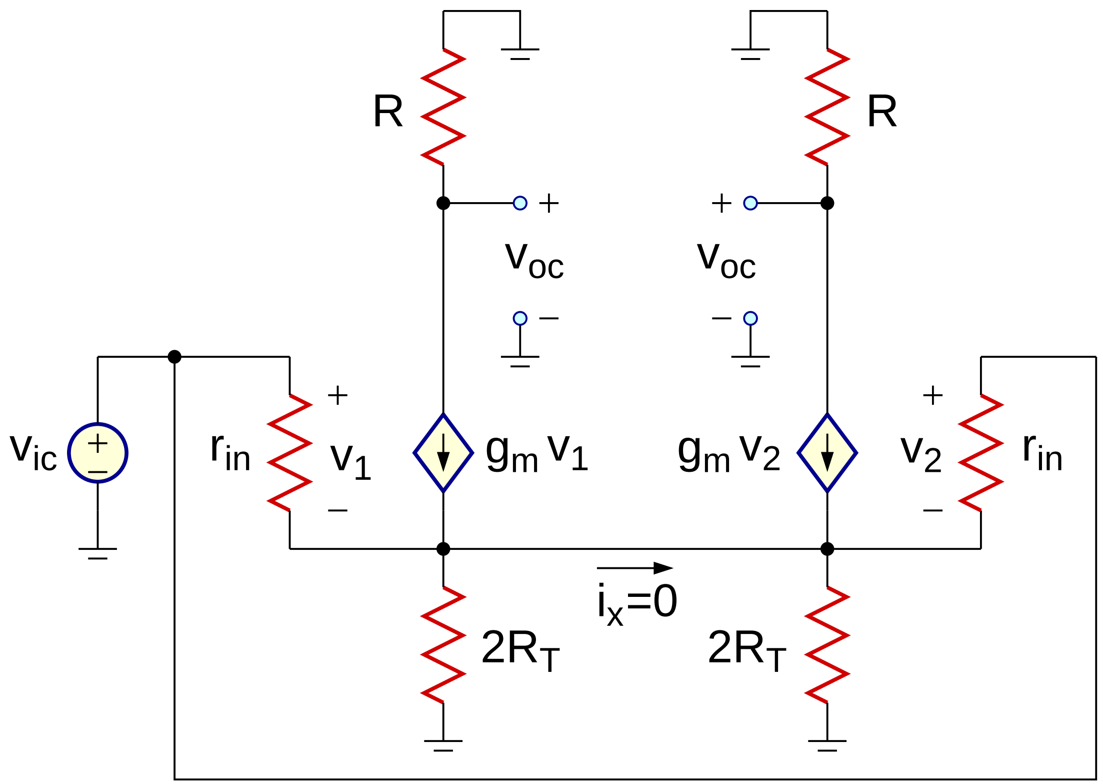
Common-Mode



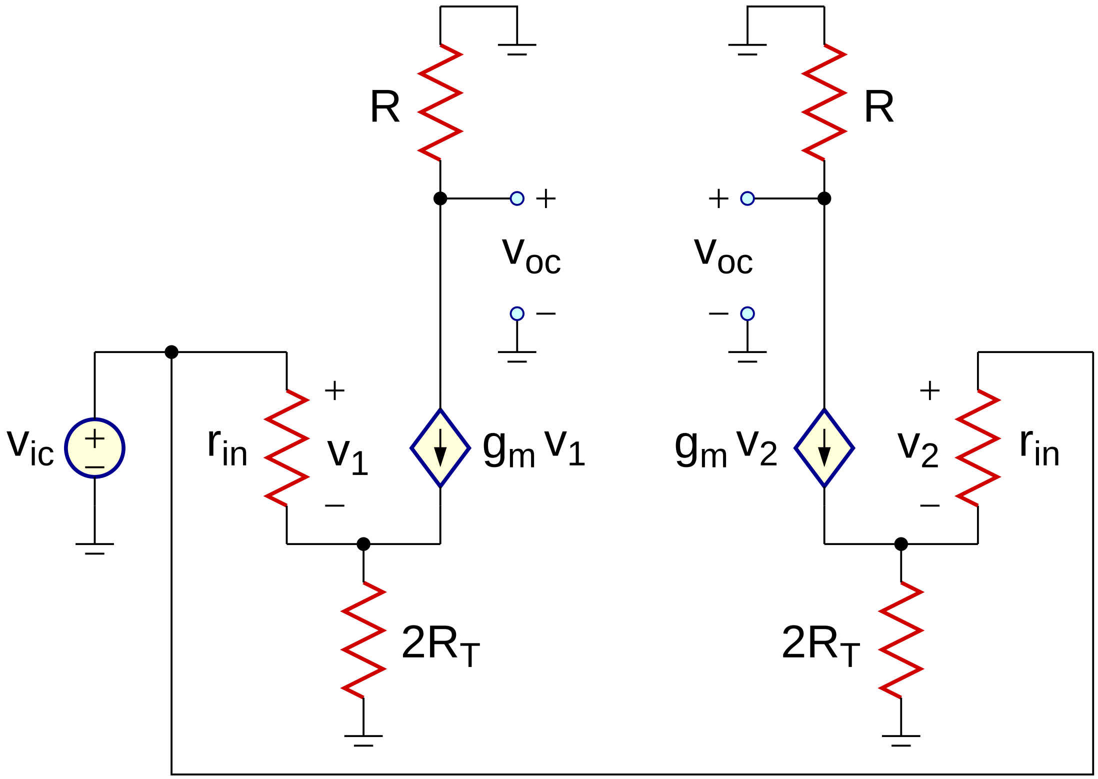
BJT: $\mathbf{r_{in} = r_\pi}$ MOSFET: $\mathbf{r_{in} = \infty}$

$\mathbf{R = R_C}$ $\mathbf{R = R_D}$

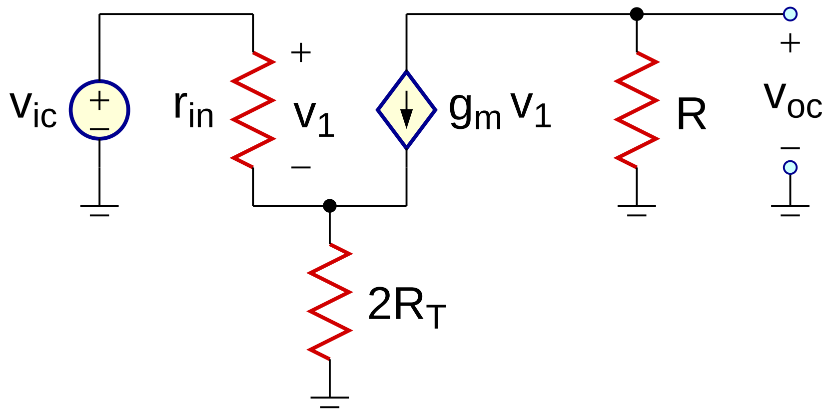
Common-Mode



Common-Mode



Common-Mode



$$v_{oc} = -\frac{R}{\frac{1}{g_m} + 2R_T} v_{ic}$$

$$A_{cm} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0} = -\frac{R}{\frac{1}{g_m} + 2R_T} = -\frac{g_m R}{1 + 2g_m R_T}$$

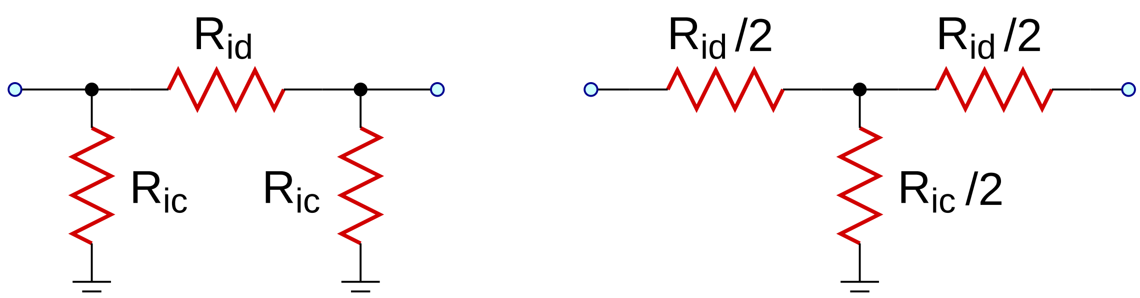
CMRR and R_i

$$\mathrm{CMRR} = \left|\frac{A_{dm}}{A_{cm}}\right| = \boxed{1 + 2 g_m R_T}$$

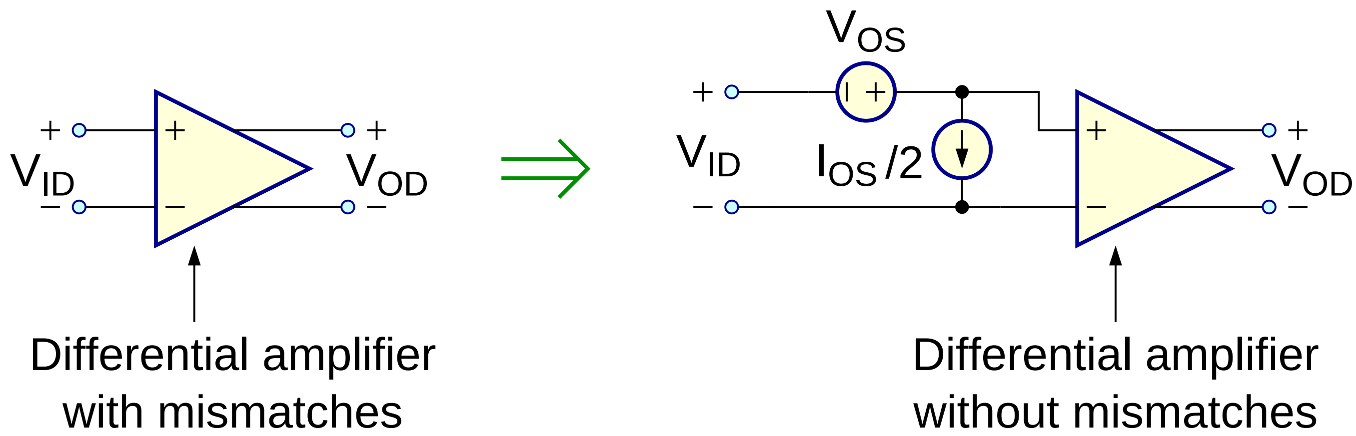
$$\frac{v_{id}}{2} = i_{in} r_{in} \quad \Rightarrow \quad R_{id} = \left.\frac{v_{id}}{i_{in}}\right|_{v_{ic}=0} = \boxed{2 r_{in}}$$

$$R_{ic} = \left.\frac{v_{ic}}{i_{in}}\right|_{v_{id}=0} = \boxed{r_{in} + (\beta + 1) 2 R_T}$$

If $R_{ic} \gg R_{id}$, the input resistance can be represented by:

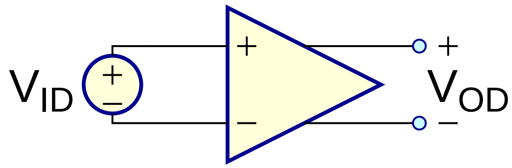


Input Offset Voltage and Current



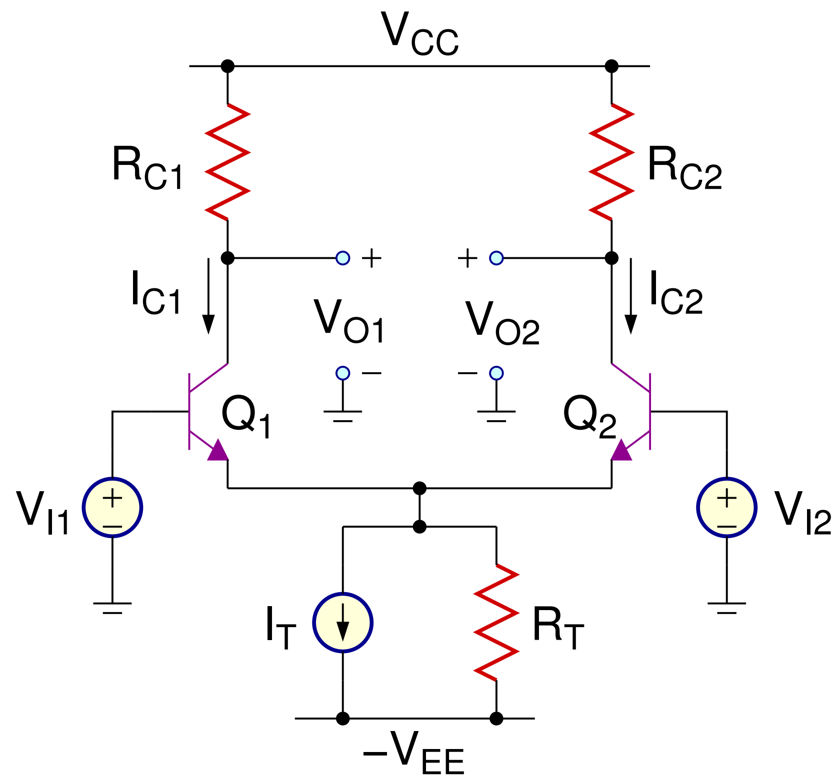
$\mathbf{V_{OS}}$: Input offset voltage

$\mathbf{I_{OS}}$: Input offset current



$$\mathbf{V_{OD} = 0 \quad \Rightarrow \quad V_{OS} = V_{ID}}$$

Input Offset Voltage



$$
\begin{aligned}
V_{I1} - V_{I2} &= V_{BE1} - V_{BE2} \\
&= V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) - V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) \\
&= V_T \ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right)
\end{aligned}
$$

Input Offset Voltage

$$\boxed{V_{ID} = V_{I1} - V_{I2} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right)}$$

$$V_{OD} = V_{O1} - V_{O2} = R_{C2}I_{C2} - R_{C1}I_{C1}$$

$$\boxed{V_{OD} = 0 \quad \Rightarrow \quad V_{OS} = V_{ID}}$$

$$\frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}} \quad \Rightarrow \quad V_{OS} = V_{ID} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right) = V_T \ln\left(\frac{R_{C2}}{R_{C1}}\frac{I_{S2}}{I_{S1}}\right)$$

$$\boxed{V_{OS} = V_T \ln\left[\left(\frac{R_{C2}}{R_{C1}}\right)\left(\frac{I_{S2}}{I_{S1}}\right)\right]}$$

Input Offset Voltage

$$V_{OS} = V_T \ln\left[\left(\frac{R_C - \frac{\Delta R_C}{2}}{R_C + \frac{\Delta R_C}{2}}\right)\left(\frac{I_S - \frac{\Delta I_S}{2}}{I_S + \frac{\Delta I_S}{2}}\right)\right]$$

If $\Delta R_C \ll R_C$ and $\Delta I_S \ll I_S$

$$\begin{aligned} V_{OS} &\approx V_T \ln\left[\left(1 - \frac{\Delta R_C}{R_C}\right)\left(1 - \frac{\Delta I_S}{I_S}\right)\right] \\ &\approx V_T \left[\ln\left(1 - \frac{\Delta R_C}{R_C}\right) + \ln\left(1 - \frac{\Delta I_S}{I_S}\right)\right] \\ &\approx V_T \left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S}\right) \end{aligned}$$

$$\boxed{\sigma_{V_{OS}} = V_T \sqrt{(\sigma_{\Delta R_C/R_C})^2 + (\sigma_{\Delta I_S/I_S})^2}}$$

Input Offset Current

$$I_{OS} = \frac{I_{C1} - I_{C2}}{\beta} \qquad \text{when } \mathbf{V_{OD} = 0}$$

$$I_{C1} = I_C + \frac{\Delta I_C}{2} \quad , \quad I_{C2} = I_C - \frac{\Delta I_C}{2}$$

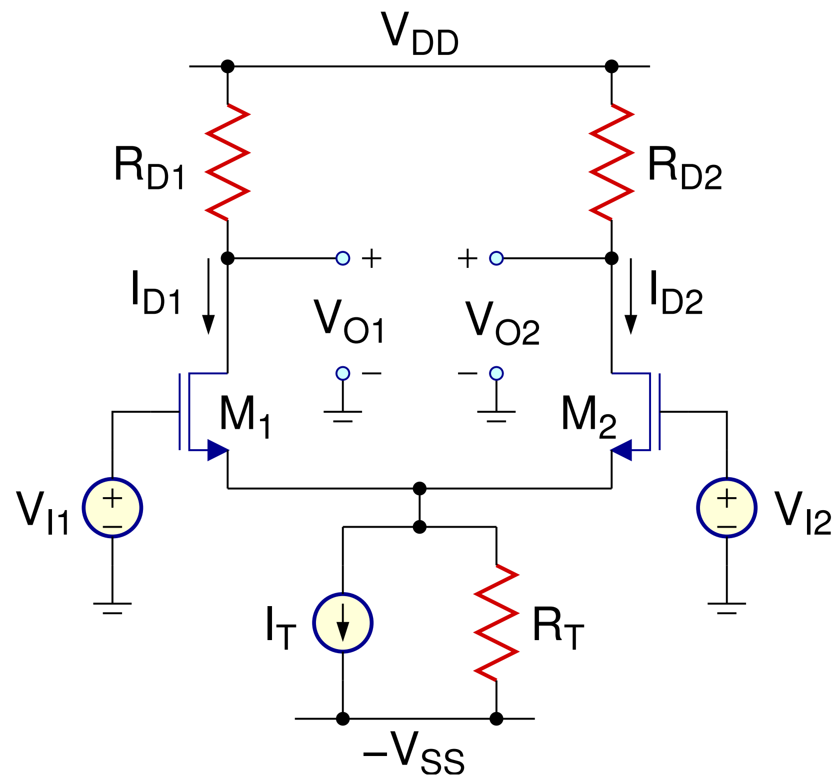
$$\beta_1 = \beta + \frac{\Delta\beta}{2} \quad , \quad \beta_2 = \beta - \frac{\Delta\beta}{2}$$

$$I_{OS} = \left(\frac{I_C + \frac{\Delta I_C}{2}}{\beta + \frac{\Delta\beta}{2}} - \frac{I_C - \frac{\Delta I_C}{2}}{\beta - \frac{\Delta\beta}{2}} \right) \approx \frac{I_C}{\beta}\left(\frac{\Delta I_C}{I_C} - \frac{\Delta\beta}{\beta}\right)$$

For $\mathbf{V_{OD}}$ to be zero, $I_{C1}R_{C1} = I_{C2}R_{C2}$, therefore,

$$\frac{\Delta I_C}{I_C} = -\frac{\Delta R_C}{R_C} \quad \Rightarrow \quad \boxed{I_{OS} \approx \frac{I_C}{\beta}\left(\frac{\Delta R_C}{R_C} - \frac{\Delta\beta}{\beta}\right)}$$

Input Offset Voltage



$$V_{ID} = V_{I1} - V_{I2} = V_{GS1} - V_{GS2}$$

$$= V_{t1} + \sqrt{\frac{2I_{D1}}{k'(W/L)_1}} - V_{t2} - \sqrt{\frac{2I_{D2}}{k'(W/L)_2}}$$

Input Offset Voltage

$$V_{OD} = V_{O1} - V_{O2} = R_{D2} I_{D2} - R_{D1} I_{D1}$$

$$\boxed{V_{OD} = 0 \quad \Rightarrow \quad V_{OS} = V_{ID}}$$

$$V_{OS} = V_{t1} - V_{t2} + \sqrt{\frac{2I_{D1}}{k'(W/L)_1}} - \sqrt{\frac{2I_{D2}}{k'(W/L)_2}}$$

where

$$\frac{I_{D1}}{I_{D2}} = \frac{R_{D2}}{R_{D1}}$$

$$\boxed{V_{OS} \approx \Delta V_t + \frac{(V_{GS} - V_t)}{2} \left(-\frac{\Delta R_D}{R_D} - \frac{\Delta (W/L)}{(W/L)} \right)}$$