ECEN 326
Electronic Circuits

Frequency Response

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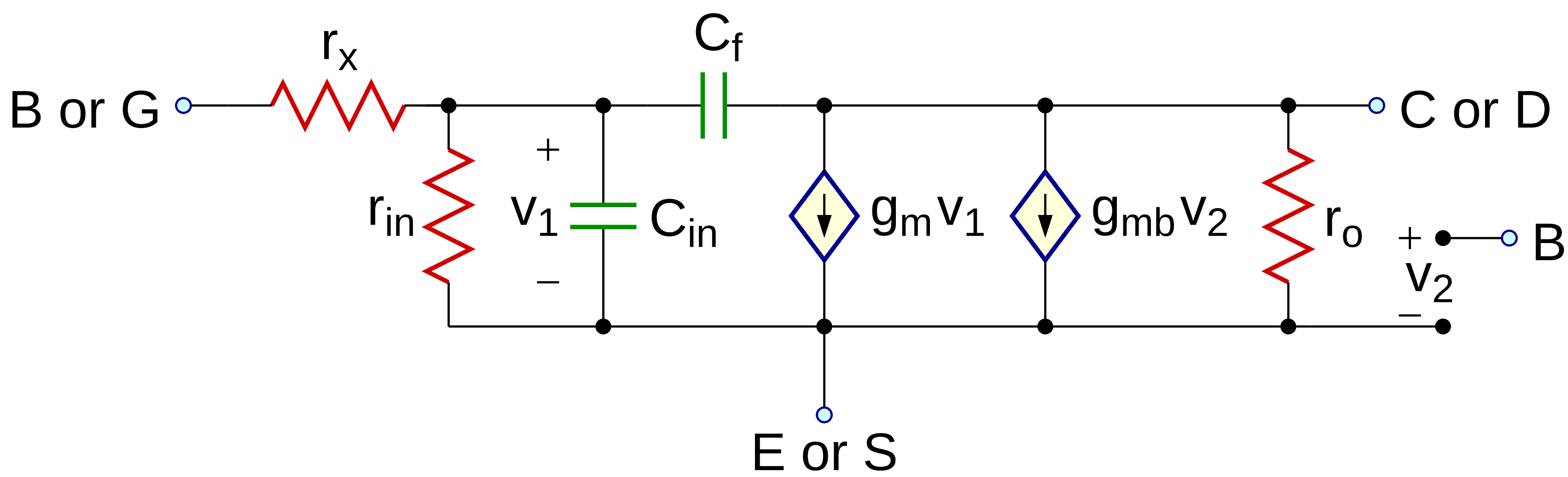
High-Frequency Model



General	BJT	MOS
r_x	r_b	0
r_{in}	r_π	∞
C_{in}	C_π	C_{gs}
C_f	C_μ	C_{gd}
r_o	r_o	r_o
g_m	g_m	g_m
g_{mb}	0	g_{mb}

BJT & MOS Capacitances

BJT:
$$C_\pi = C_b + C_{je} = \tau_F g_m + C_{je}$$

$$C_\mu = \frac{C_{\mu o}}{\left(1 - \frac{V}{\psi_o}\right)^n}$$

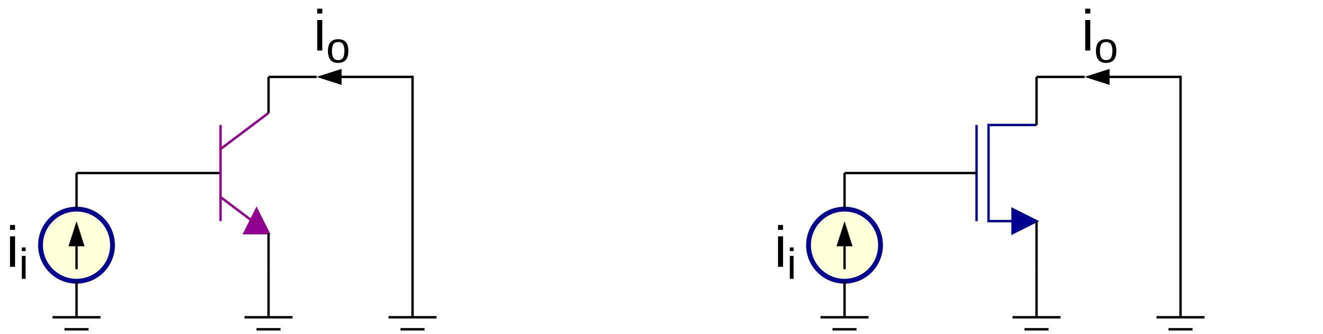
$$C_\pi \gg C_\mu$$

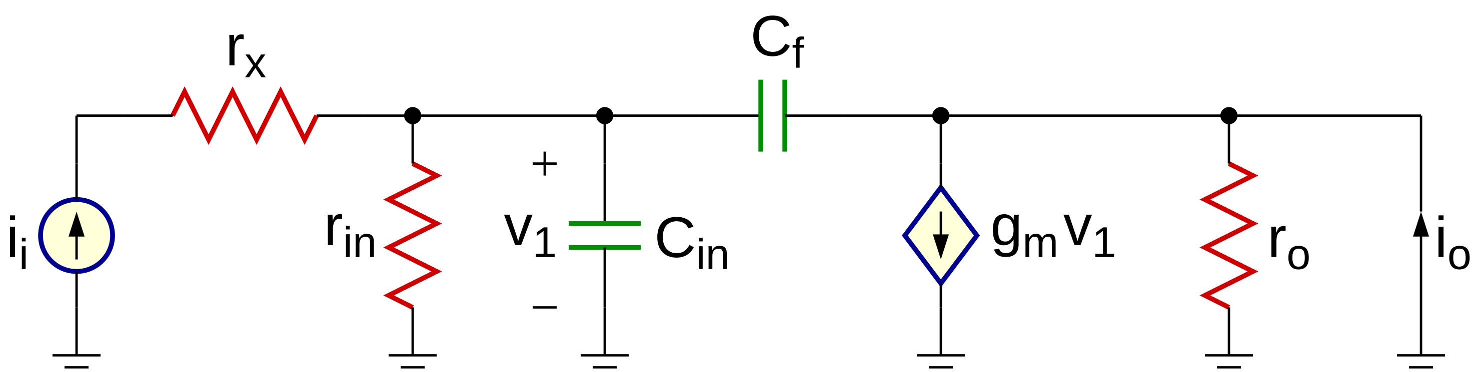
MOS:
$$C_{gs} = \frac{2}{3} W L C_{ox}$$

C_{gd} : Overlap capacitance

$$C_{gs} \gg C_{gd}$$

Transition Frequency, f_T





$$\frac{i_o}{i_i}(s) \approx \frac{g_m r_{in}}{1 + r_{in}(C_{in} + C_f)s}$$

Transition Frequency, f_T

$$\frac{i_o}{i_i}(j\omega) \approx \frac{g_m r_{in}}{1 + r_{in}(C_{in} + C_f)j\omega}$$

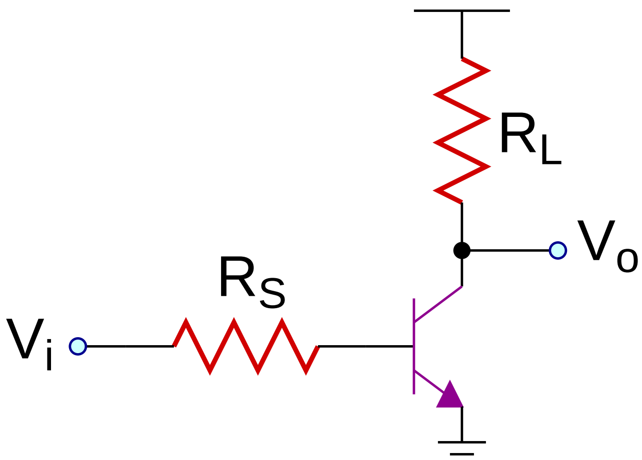
At high frequencies:

$$\frac{i_o}{i_i}(j\omega) \approx \frac{g_m}{(C_{in} + C_f)j\omega}$$

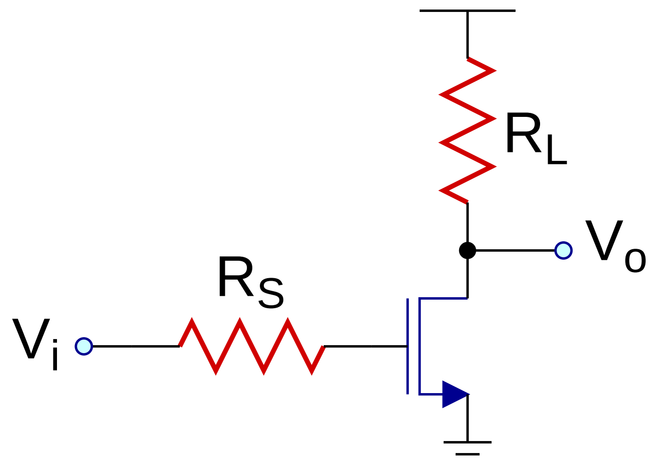
$$\left|\frac{i_o}{i_i}(j\omega)\right|_{\omega=\omega_T} = 1 \;\Rightarrow\; \frac{g_m}{(C_{in} + C_f)\omega_T} = 1$$

$$\boxed{\omega_T = \frac{g_m}{C_{in} + C_f} \quad , \quad f_T = \frac{\omega_T}{2\pi}}$$

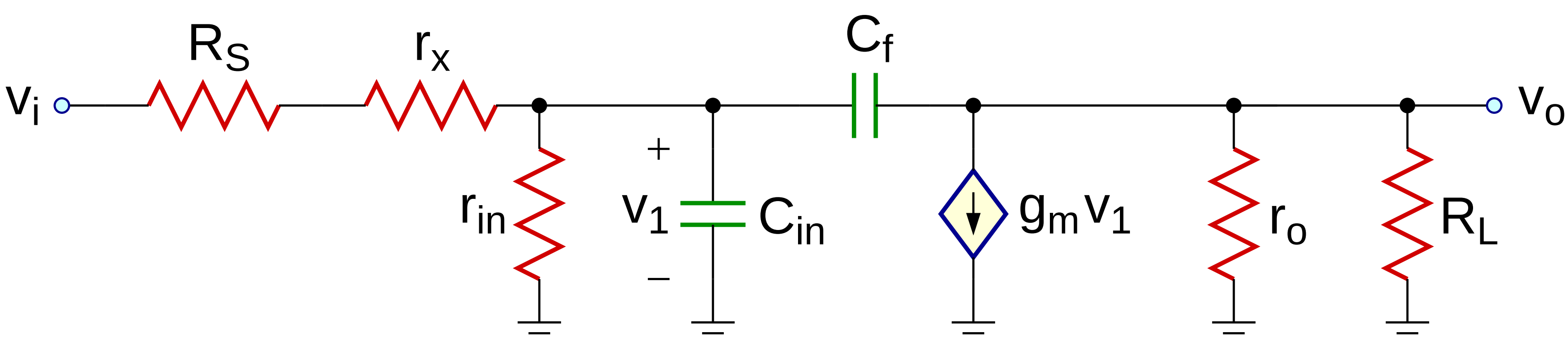
BJT: $\omega_T = \dfrac{g_m}{C_\pi + C_\mu}$ MOS: $\omega_T = \dfrac{g_m}{C_{gs} + C_{gd}}$



RL
Vo
RS
Vi



RL
Vo
RS
Vi



RS
rx
Cf
vi
vo
+
rin
v1
Cin
gmv1
ro
RL
−

$$\frac{v_o}{v_i}(s) = K \; \frac{1 - s\dfrac{C_f}{g_m}}{1 + a_1 s + a_2 s^2}$$

$$K = -\frac{g_m R_{OL} R}{R_S + r_x} = -g_m R_{OL} \frac{r_{in}}{R_S + r_x + r_{in}}$$

$$a_1 = C_f R_{OL} + C_f R + C_{in} R + g_m R_{OL} R C_f$$

$$a_2 = R_{OL} R C_f C_{in}$$

$$R = (R_S + r_x) \parallel r_{in}$$

$$R_{OL} = r_o \parallel R_L$$

$$D(s) = 1 + a_1 s + a_2 s^2 = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)$$

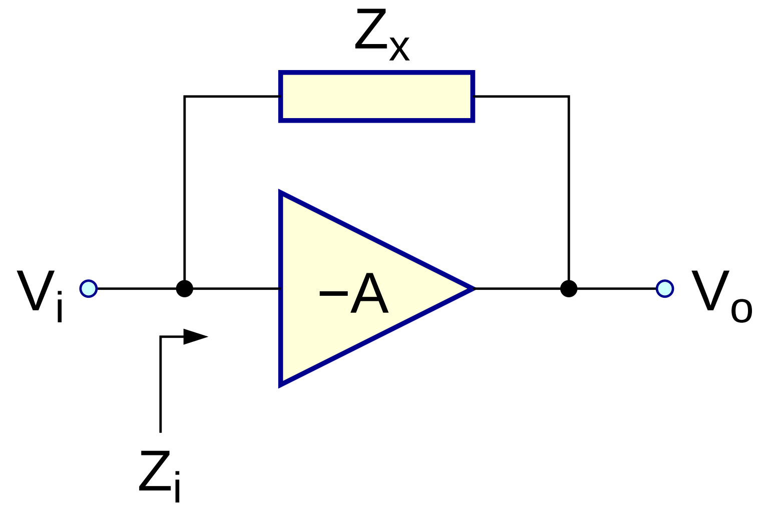
$$|p_2| \gg |p_1| \;\Rightarrow\; p_1 \approx -\frac{1}{a_1} \quad , \quad p_2 \approx -\frac{a_1}{a_2}$$

$$p_1 = -\frac{1}{R\left(C_{in} + C_f\left(1 + g_m R_{OL} + \frac{R_{OL}}{R}\right)\right)}$$

$$p_2 = -\left(\frac{1}{R_{OL}C_f} + \frac{1}{RC_{in}} + \frac{1}{R_{OL}C_{in}} + \frac{g_m}{C_{in}}\right)$$

$$N(s) = 1 - \frac{s}{z_1} \;\Rightarrow\; z_1 = \frac{g_m}{C_f}$$

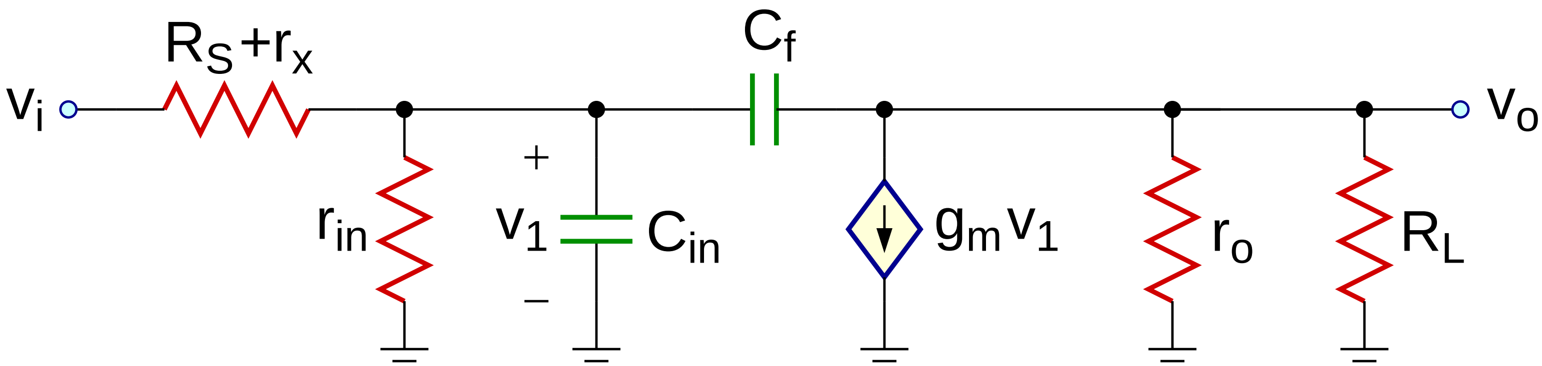
Miller Approximation



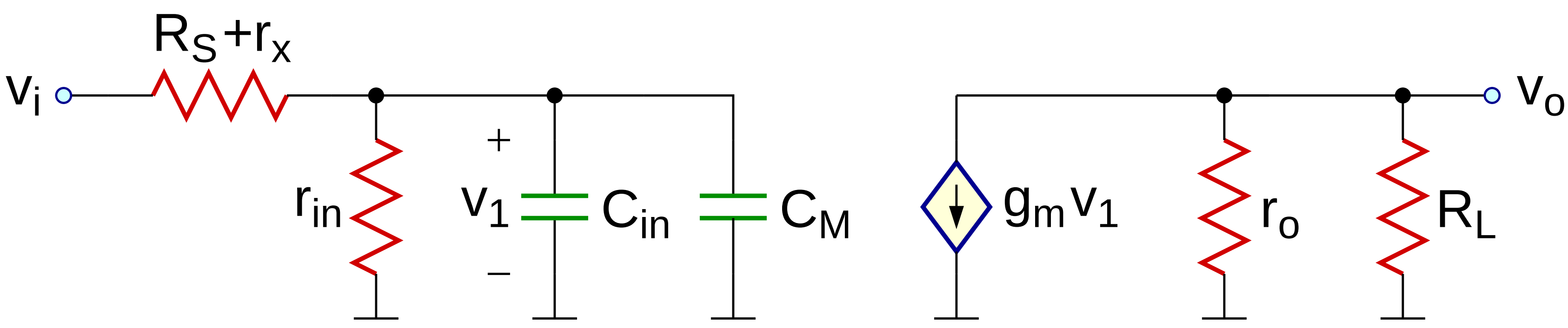
$$Z_i = \frac{V_i}{I_i} = \frac{V_i}{\frac{V_i - V_o}{Z_x}} = \frac{V_i}{\frac{V_i - (-AV_i)}{Z_x}} = \frac{Z_x V_i}{(A+1)V_i} = \frac{Z_x}{A+1}$$

$$Z_x = \frac{1}{sC} \quad \Rightarrow \quad Z_i = \frac{1}{s(A+1)C} = \frac{1}{sC'} \quad , \quad C' = (A+1)C$$

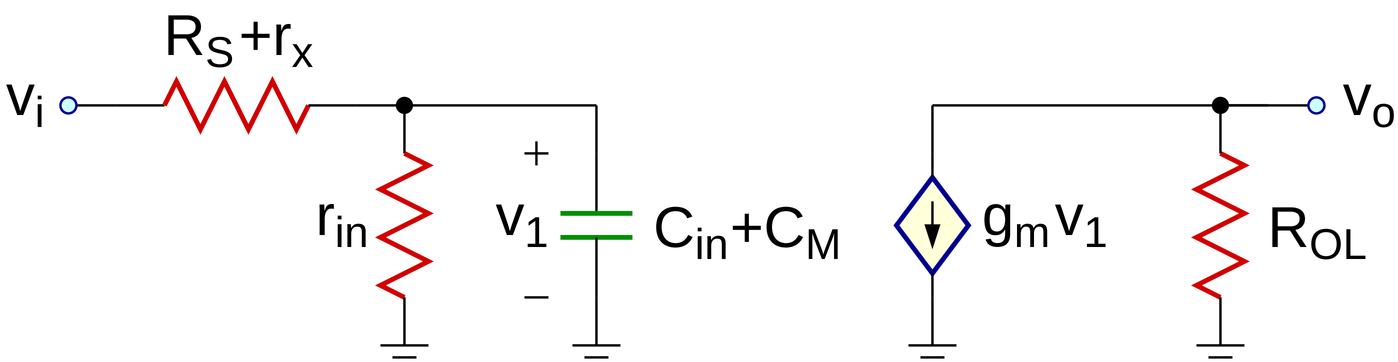
Miller Capacitance



⇓ Miller Approximation ⇓



$$\mathbf{C_M = (A+1)C_f \quad , \quad A = -\frac{v_o}{v_1} = g_m(r_o \parallel R_L)}$$



$$\frac{v_o}{v_i} = -g_m R_{OL} \frac{r_{in} \parallel \dfrac{1}{s(C_{in}+C_M)}}{R_S + r_x + \left(r_{in} \parallel \dfrac{1}{s(C_{in}+C_M)}\right)}$$

$$= -g_m R_{OL} \frac{r_{in}}{R_S + r_x + r_{in}} \frac{1}{1 + s(C_{in}+C_M)R}$$

$$R = (R_S + r_x) \parallel r_{in} \quad , \quad R_{OL} = r_o \parallel R_L$$

Approximate vs. Exact Solution

Miller approximation:

$$p_1 = -\frac{1}{R(C_{in} + C_f(1 + g_m R_{OL}))}$$

Exact derivation:

$$p_1 = -\frac{1}{R\left(C_{in} + C_f\left(1 + g_m R_{OL} + \frac{R_{OL}}{R}\right)\right)}$$

$$p_2 = -\left(\frac{1}{R_{OL}C_f} + \frac{1}{RC_{in}} + \frac{1}{R_{OL}C_{in}} + \frac{g_m}{C_{in}}\right)$$

$$z_1 = \frac{g_m}{C_f}$$

$$p_1 = -\frac{1}{((R_S + r_b) \parallel r_\pi)\left(C_\pi + C_\mu\left(1 + g_m R_{OL} + \frac{R_{OL}}{R}\right)\right)}$$

$$p_2 = -\left(\frac{1}{R_{OL} C_\mu} + \frac{1}{R C_\pi} + \frac{1}{R_{OL} C_\pi} + \frac{g_m}{C_\pi}\right)$$

$$z_1 = \frac{g_m}{C_\mu}$$

$$\omega_T = \frac{g_m}{C_\pi + C_\mu}$$

$$|p_2| \gg |p_1| \quad , \quad |p_2| > \omega_T \quad , \quad |z_1| > \omega_T$$

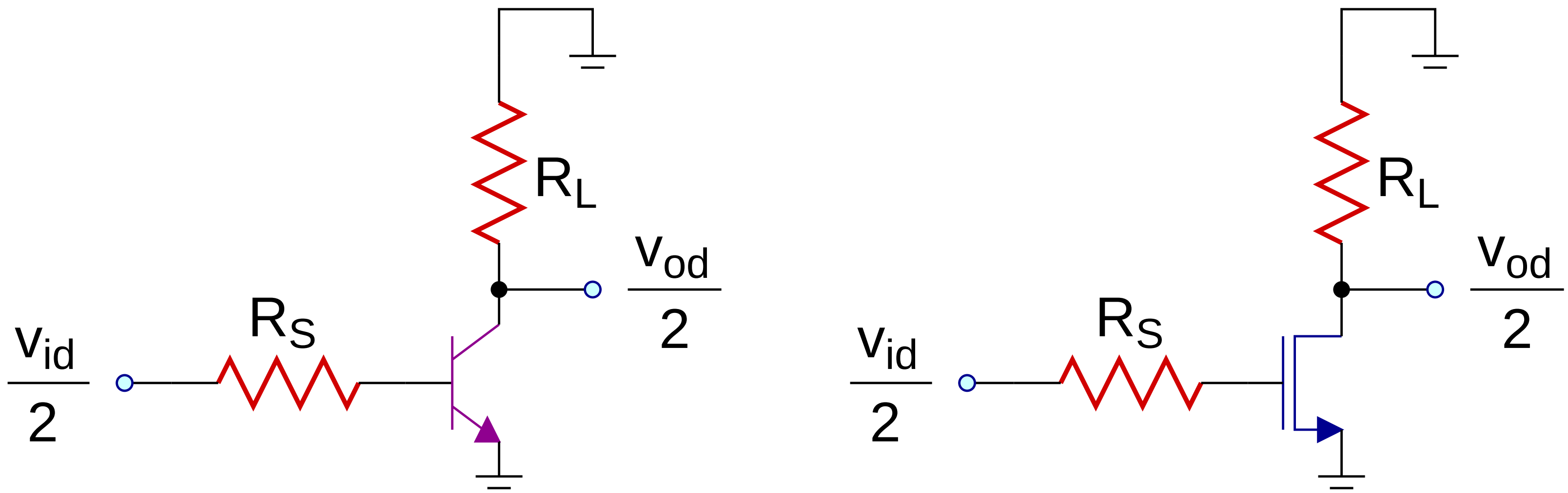
$$p_1 = -\frac{1}{R_S\left(C_{gs} + C_{gd}\left(1 + g_m R_{OL} + \frac{R_{OL}}{R_S}\right)\right)}$$

$$p_2 = -\left(\frac{1}{R_{OL}C_{gd}} + \frac{1}{R_S C_{gs}} + \frac{1}{R_{OL}C_{gs}} + \frac{g_m}{C_{gs}}\right)$$

$$z_1 = \frac{g_m}{C_{gd}}$$

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}}$$

$$|p_2| \gg |p_1| \quad , \quad |p_2| > \omega_T \quad , \quad |z_1| > \omega_T$$

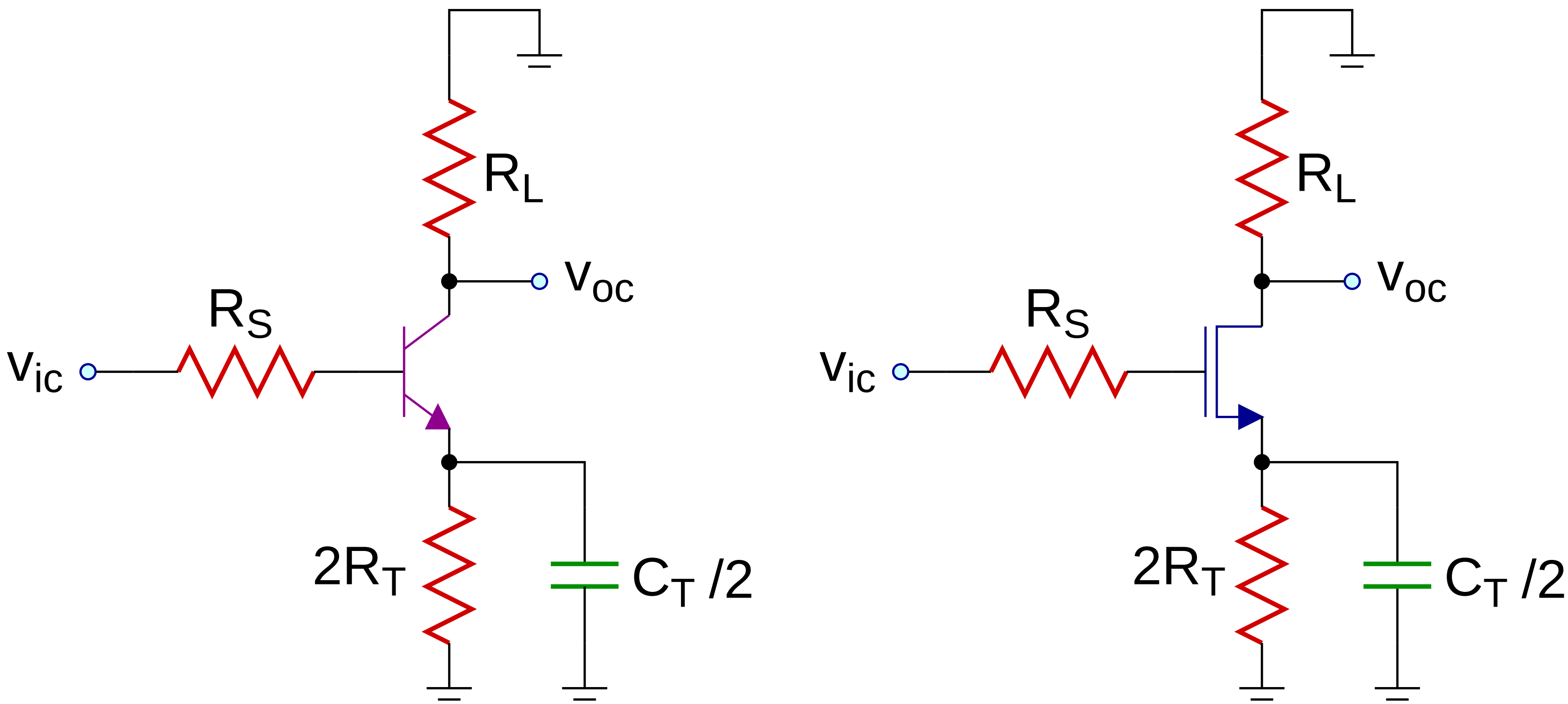


Dominant pole of $\dfrac{v_{od}}{v_{id}}(s)$:

$$p_1 = -\frac{1}{R\left(C_{in} + C_f\left(1 + g_m R_{OL} + \dfrac{R_{OL}}{R}\right)\right)}$$

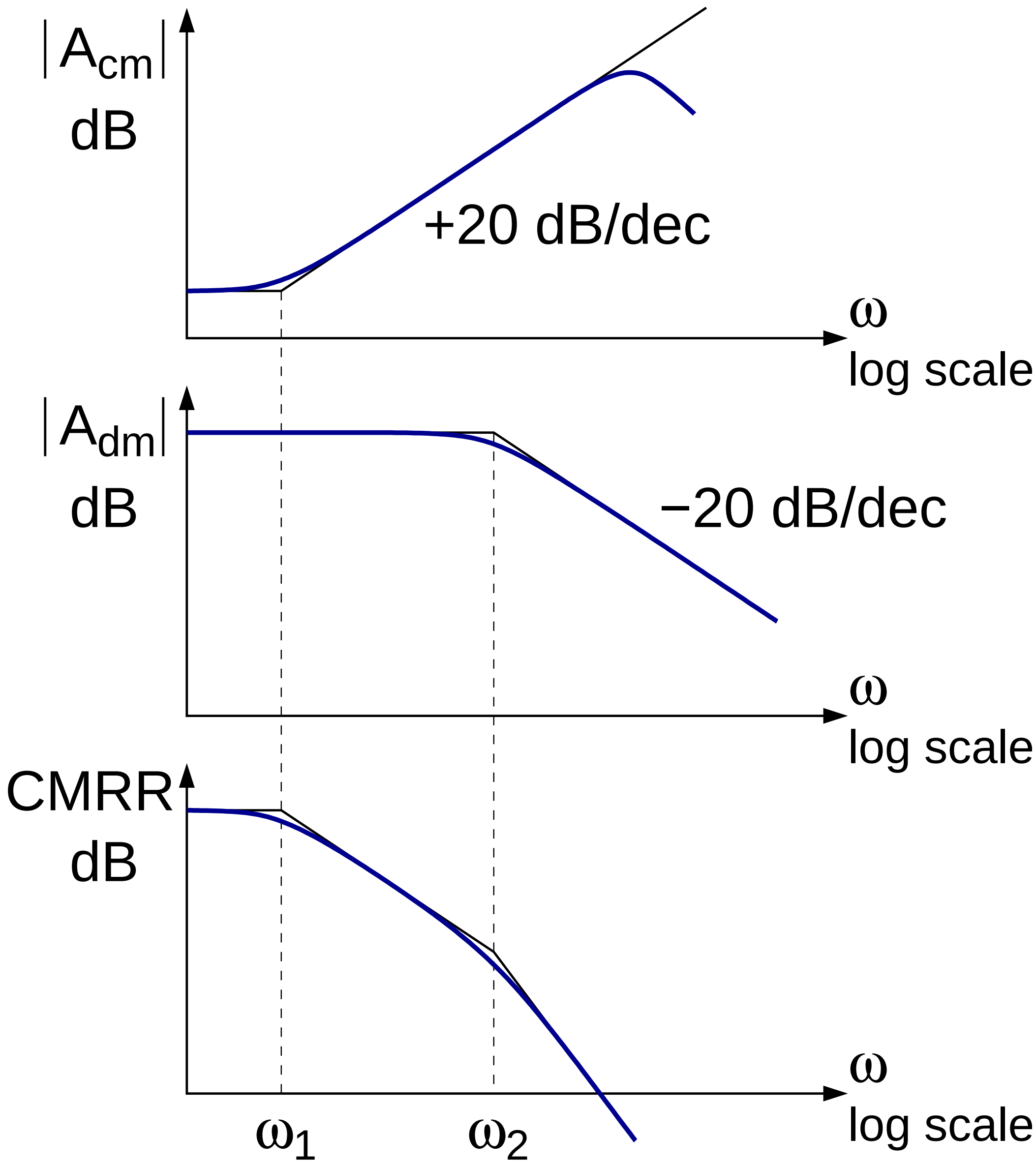
$$R = (R_S + r_x) \parallel r_{in} \quad , \quad R_{OL} = r_o \parallel R_L$$

BJT & MOS Differential Pair



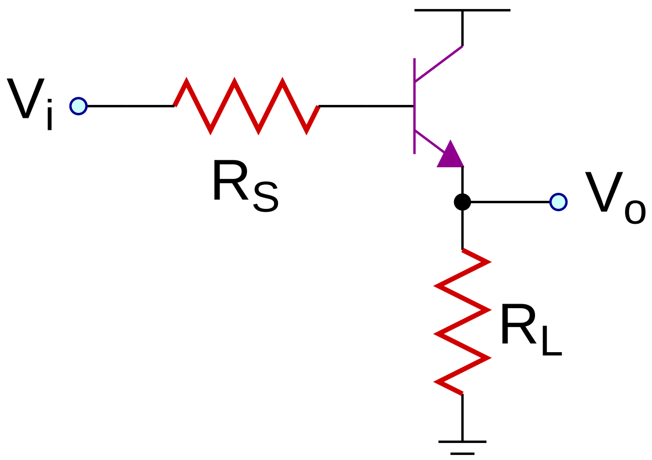
$$A_{cm}(s) \approx -\frac{R_L}{Z_T}\,, \quad Z_T = 2R_T \parallel \frac{1}{sC_T/2} = \frac{2R_T}{1 + sC_T R_T}$$

$$A_{cm}(s) \approx -\frac{R_L}{2R_T}(1 + sC_T R_T)$$

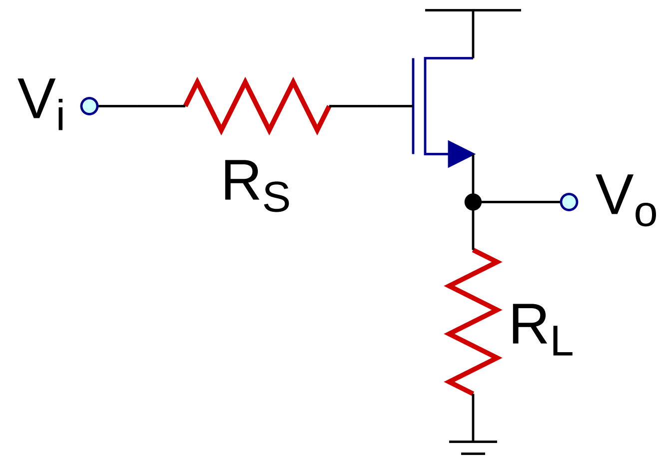


$$\omega_1 \approx \frac{1}{R_T C_T}$$

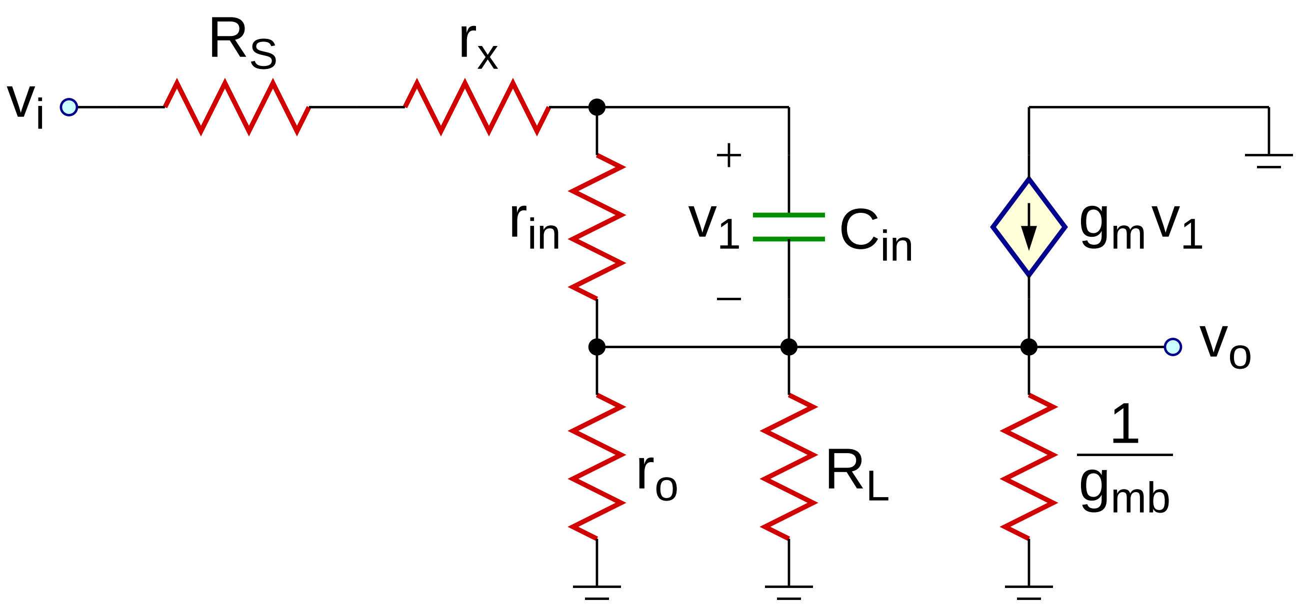
$$\omega_2 \approx \frac{1}{R(C_{in} + C_M)}$$



V_i
R_S
V_o
R_L



V_i
R_S
V_o
R_L



v_i
R_S
r_x
r_{in}
+
v_1
−
C_{in}
$g_m v_1$
v_o
r_o
R_L
$\frac{1}{g_{mb}}$

$$\frac{v_o}{v_i}(s) = \frac{g_m R'_L + \dfrac{R'_L}{r_{in}}}{1 + g_m R'_L + \dfrac{R'_S + R'_L}{r_{in}}} \; \frac{1 - \dfrac{s}{z_1}}{1 - \dfrac{s}{p_1}}$$

$$p_1 = -\frac{1}{R_1 C_{in}} \quad , \quad z_1 = -\frac{g_m + \dfrac{1}{r_{in}}}{C_{in}}$$

$$R'_S = R_S + r_x$$

$$R'_L = r_o \parallel R_L \parallel \frac{1}{g_{mb}}$$

$$R_1 = r_{in} \parallel \frac{R'_S + R'_L}{1 + g_m R'_L}$$

$$z_1 = -\frac{g_m + \dfrac{1}{r_\pi}}{C_\pi} \approx -\frac{g_m}{C_\pi} \approx -\omega_T$$

$$p_1 = -\frac{1}{R_1 C_\pi} \quad , \quad R_1 = r_\pi \parallel \frac{R'_S + R'_L}{1 + g_m R'_L}$$

$$R'_S = R_S + r_b \quad , \quad R'_L = r_o \parallel R_L$$

Typically, $|z_1|$ is slightly larger than $|p_1|$.

If $g_m R'_L \gg 1$ and $R'_S \ll R'_L$, then $R_1 \approx 1/g_m$ and $p_1 \approx -\omega_T$.

If R'_S is large (compared to R'_L), then $|p_1|$ will be significantly less than ω_T.

$$z_1 = -\frac{g_m}{C_{gs}} \approx -\omega_T$$

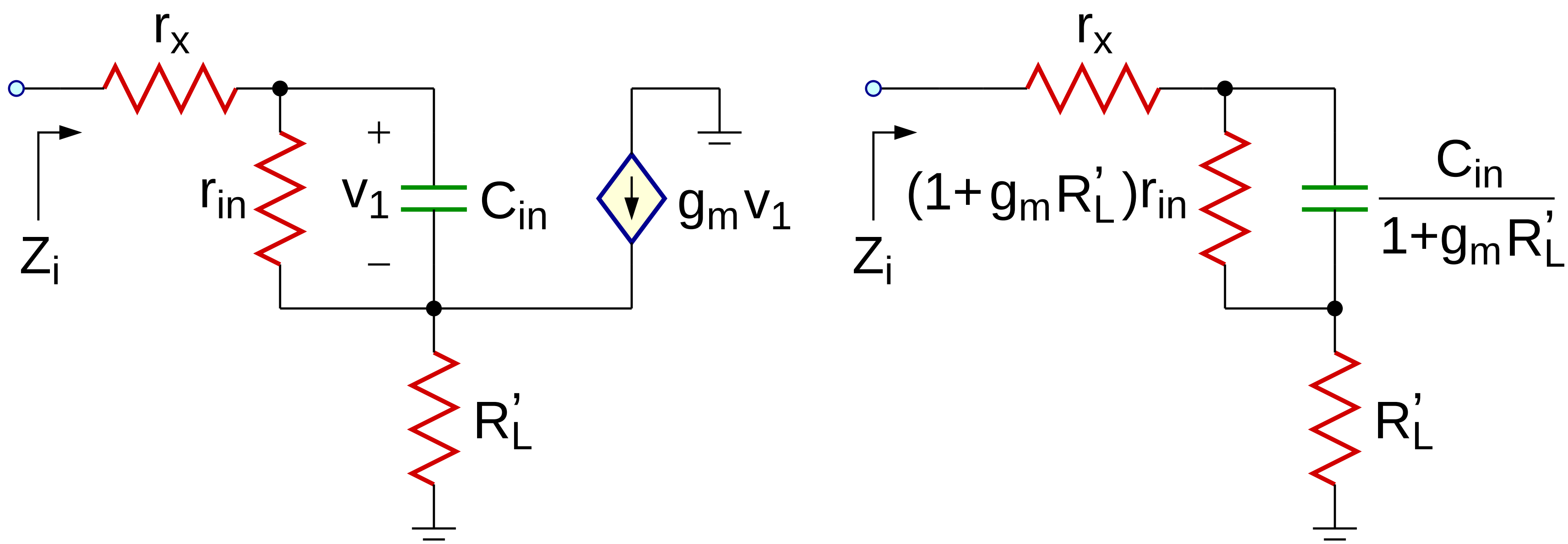
$$p_1 = -\frac{1}{R_1 C_{gs}} \quad , \quad R_1 = \frac{R_S + R_L'}{1 + g_m R_L'}$$

$$R_L' = r_o \parallel R_L \parallel \frac{1}{g_{mb}}$$

Typically, $|z_1|$ is slightly larger than $|p_1|$.

If $g_m R_L' \gg 1$ and $R_S \ll R_L'$, then $R_1 \approx 1/g_m$ and $p_1 \approx -\omega_T$.

If R_S is large (compared to R_L'), then $|p_1|$ will be significantly less than ω_T.



$$Z_i = r_x + \left(R \parallel \frac{1}{sC}\right) + R'_L$$

$$R = (1 + g_m R'_L) r_{in} \quad , \quad C = \frac{C_{in}}{1 + g_m R'_L}$$

$$R'_L = r_o \parallel R_L \parallel \frac{1}{g_{mb}}$$

BJT:

$$Z_i = r_b + \left(R \parallel \frac{1}{sC}\right) + R'_L$$

$$R = (1 + g_m R'_L) r_\pi \quad , \quad C = \frac{C_\pi}{1 + g_m R'_L}$$

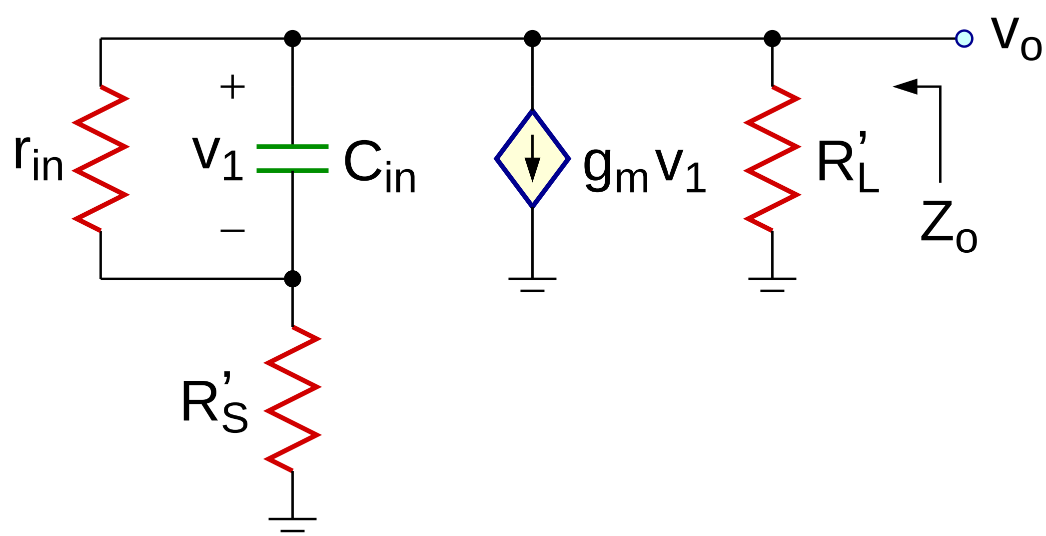
$$R'_L = r_o \parallel R_L$$

MOS:

$$Z_i = \frac{1}{sC} + R'_L$$

$$C = \frac{C_{gs}}{1 + g_m R'_L}$$

$$R'_L = r_o \parallel R_L \parallel \frac{1}{g_{mb}}$$



$$v_1 = \frac{z_{in}}{z_{in} + R_S'} v_o \quad , \quad z_{in} = r_{in} \parallel \frac{1}{sC_{in}}$$

$$Z_o = (z_{in} + R_S') \parallel \left(\frac{z_{in} + R_S'}{g_m z_{in}} \right) \parallel R_L' = \frac{z_{in} + R_S'}{1 + g_m z_{in}} \parallel R_L'$$

$$R_S' = R_S + r_x \quad , \quad R_L' = r_o \parallel R_L \parallel \frac{1}{g_{mb}}$$

$$Z_o = R'_L \parallel \left[\frac{r_{in} + R'_S}{1 + g_m r_{in}} \frac{1 + sC_{in}(R'_S \parallel r_{in})}{1 + sC_{in}\left(\frac{1}{g_m} \parallel r_{in}\right)} \right]$$

$$\frac{1}{g_m} > R'_S \Rightarrow Z_o \text{ is capacitive}$$

$$\frac{1}{g_m} < R'_S \Rightarrow Z_o \text{ is inductive}$$

$$\frac{1}{g_m} = R'_S \Rightarrow Z_o \text{ is resistive}$$

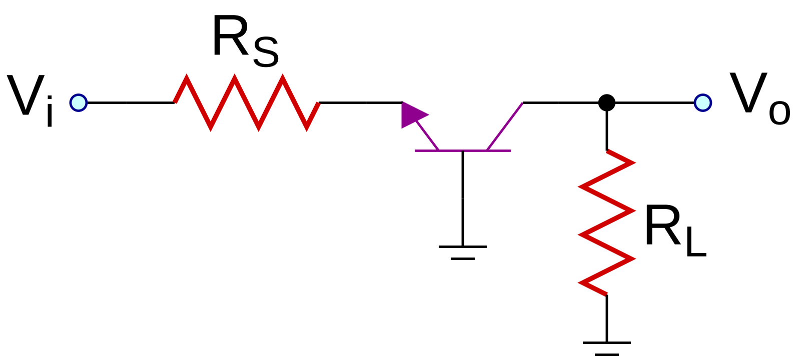
Practical design goals: $r_x \ll R_S$, $\frac{1}{g_m} = R_S$

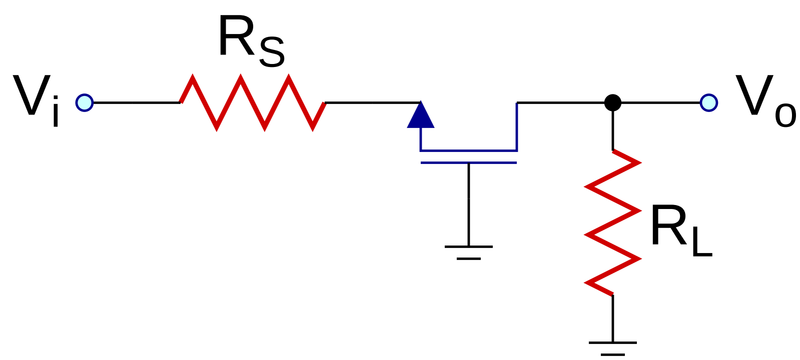
BJT:

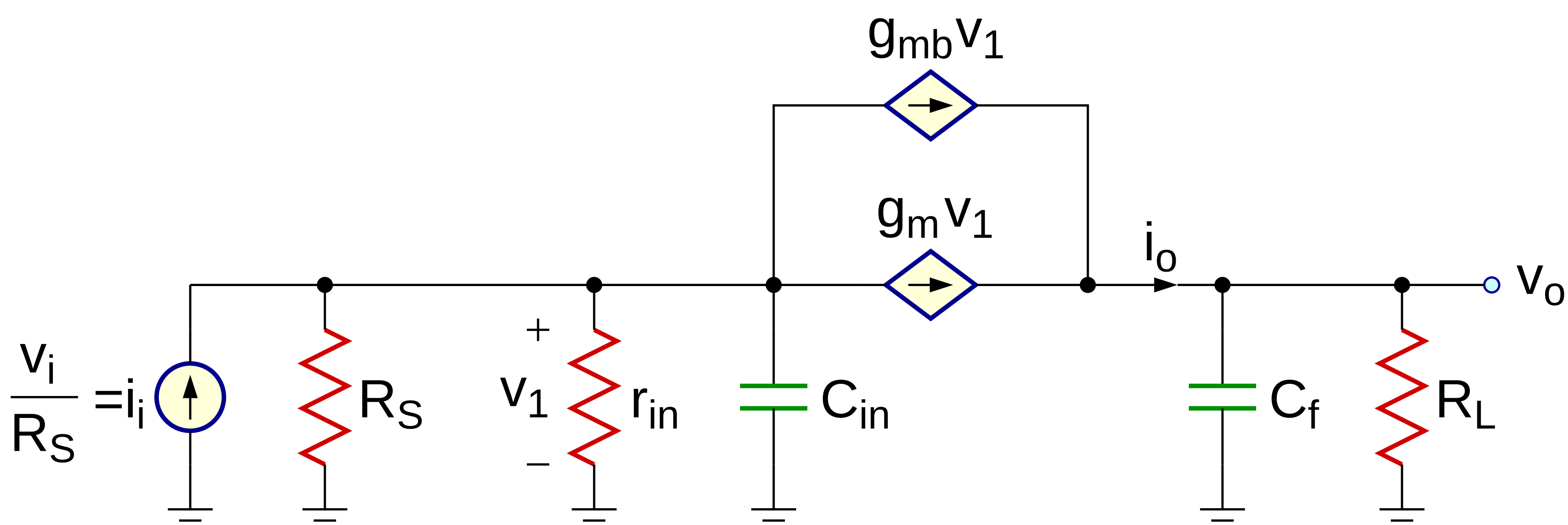
$$Z_o = r_o \parallel R_L \parallel \left[\frac{r_\pi + R_S + r_b}{\beta + 1} \, \frac{1 + sC_\pi((R_S + r_b) \parallel r_\pi)}{1 + sC_\pi\left(\frac{1}{g_m} \parallel r_\pi\right)} \right]$$

MOS:

$$Z_o = r_o \parallel R_L \parallel \frac{1}{g_{mb}} \parallel \left[\frac{1}{g_m} \, \frac{1 + sC_{gs}R_S}{1 + s\frac{C_{gs}}{g_m}} \right]$$


R_S
V_i
V_o
R_L


R_S
V_i
V_o
R_L


$g_{mb}v_1$
$g_m v_1$
i_o
v_o
$\frac{v_i}{R_S} = i_i$
R_S
+
v_1
−
r_{in}
C_{in}
C_f
R_L

$$\frac{i_o}{i_i}(s) = K \frac{1}{1 - \frac{s}{p_1}}$$

$$\frac{v_o}{v_i}(s) = \frac{R_L}{R_S} K \frac{1}{1 - \frac{s}{p_1}} \frac{1}{1 - \frac{s}{p_2}}$$

$$K = \frac{g_m + g_{mb}}{g_m + g_{mb} + \frac{1}{R_S} + \frac{1}{r_{in}}}$$

$$p_1 = -\frac{g_m + g_{mb} + \frac{1}{R_S} + \frac{1}{r_{in}}}{C_{in}} \quad , \quad p_2 = -\frac{1}{C_f R_L}$$

BJT-CB & MOS-CG

BJT:

$$\frac{v_o}{v_i}(s) = \frac{R_L}{R_S}\,\frac{g_m}{\frac{g_m}{\alpha} + \frac{1}{R_S}}\,\frac{1}{1 - \frac{s}{p_1}}\,\frac{1}{1 - \frac{s}{p_2}}$$

$$p_1 = -\frac{\frac{g_m}{\alpha} + \frac{1}{R_S}}{C_\pi}\ , \quad p_2 = -\frac{1}{C_\mu R_L}$$

MOS:

$$\frac{v_o}{v_i}(s) = \frac{R_L}{R_S}\,\frac{g_m + g_{mb}}{g_m + g_{mb} + \frac{1}{R_S}}\,\frac{1}{1 - \frac{s}{p_1}}\,\frac{1}{1 - \frac{s}{p_2}}$$

$$p_1 = -\frac{g_m + g_{mb} + \frac{1}{R_S}}{C_{gs}}\ , \quad p_2 = -\frac{1}{C_{gd} R_L}$$

$$A(s) = \frac{K}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} = \frac{K}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\cdots\left(1 - \frac{s}{p_n}\right)}$$

$$b_1 = \sum_{i=1}^{n}\left(-\frac{1}{p_i}\right)$$

If $|p_1| \ll |p_2|, |p_3|, \ldots$ then

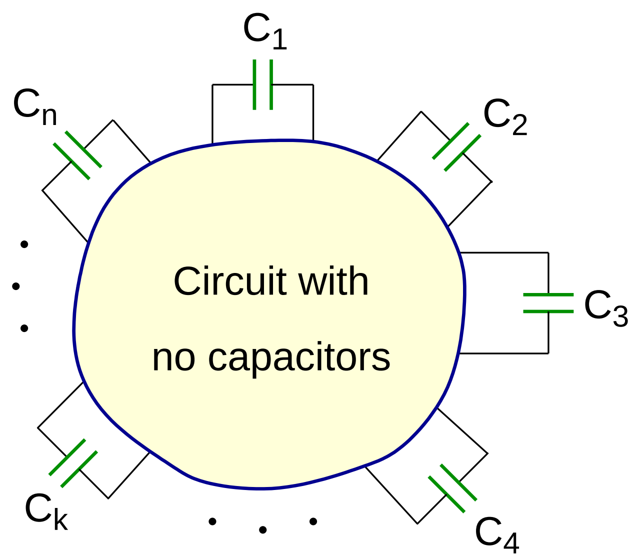
$$b_1 \approx \left|\frac{1}{p_1}\right| \quad , \quad |A(j\omega)| \approx \frac{K}{\sqrt{1 + \left(\frac{\omega}{p_1}\right)^2}}$$

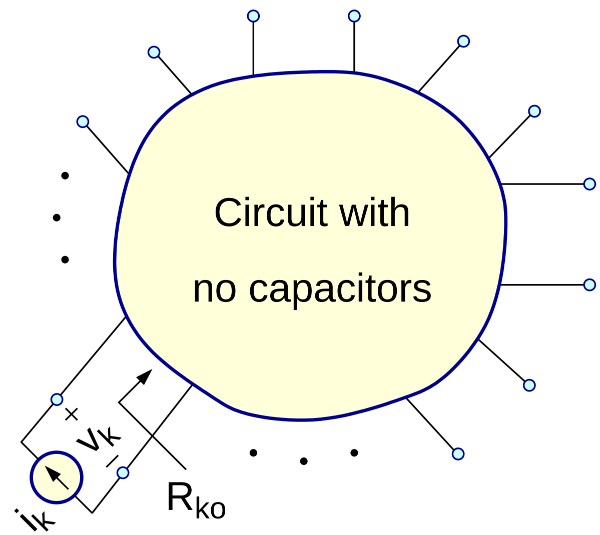
The higher 3-dB frequency is

$$\omega_H \approx |p_1| \approx \frac{1}{b_1}$$

Open-Circuit Time Constant Analysis

Also known as Zero-Value Time Constant Analysis. Can be used to estimate the higher 3-dB frequency (ω_H).





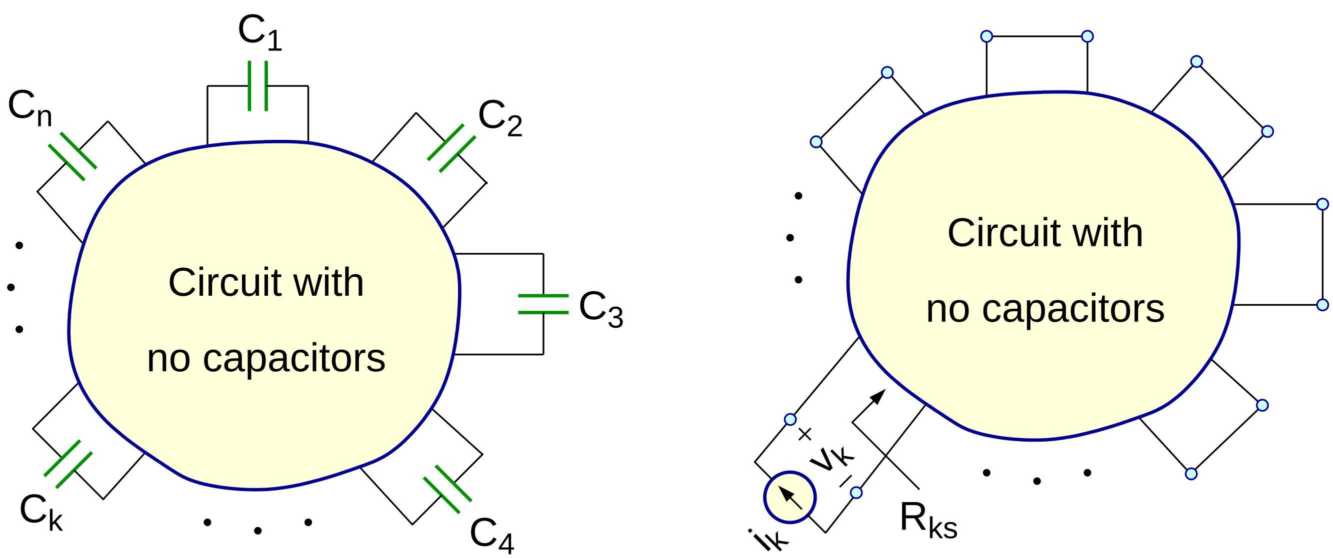
$$R_{ko} = \frac{v_k}{i_k} \quad , \quad T_{ko} = R_{ko}C_k$$

$$b_1 = \sum_{k=1}^{n} R_{ko}C_k = R_{1o}C_1 + R_{2o}C_2 + \cdots + R_{no}C_n$$

$$\omega_H \approx |p_1| \approx \frac{1}{b_1} \approx \frac{1}{\sum_k T_{ko}}$$

Short-Circuit Time Constant Analysis

Can be used to estimate the lower 3-dB frequency (ω_L) of AC-coupled amplifiers.

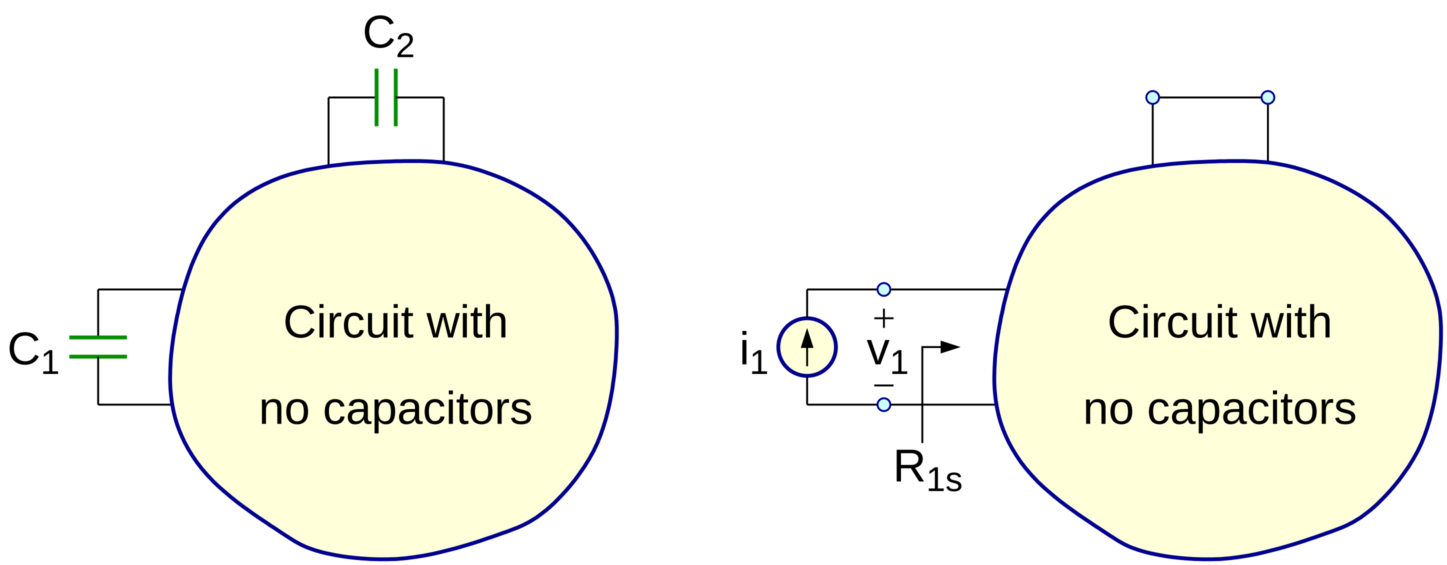


$$R_{ks} = \frac{v_k}{i_k} \quad , \quad \tau_{ks} = R_{ks} C_k$$

$$\omega_L \approx \sum_{k=1}^{n} \frac{1}{\tau_{ks}}$$

Short-Circuit Time Constant Analysis

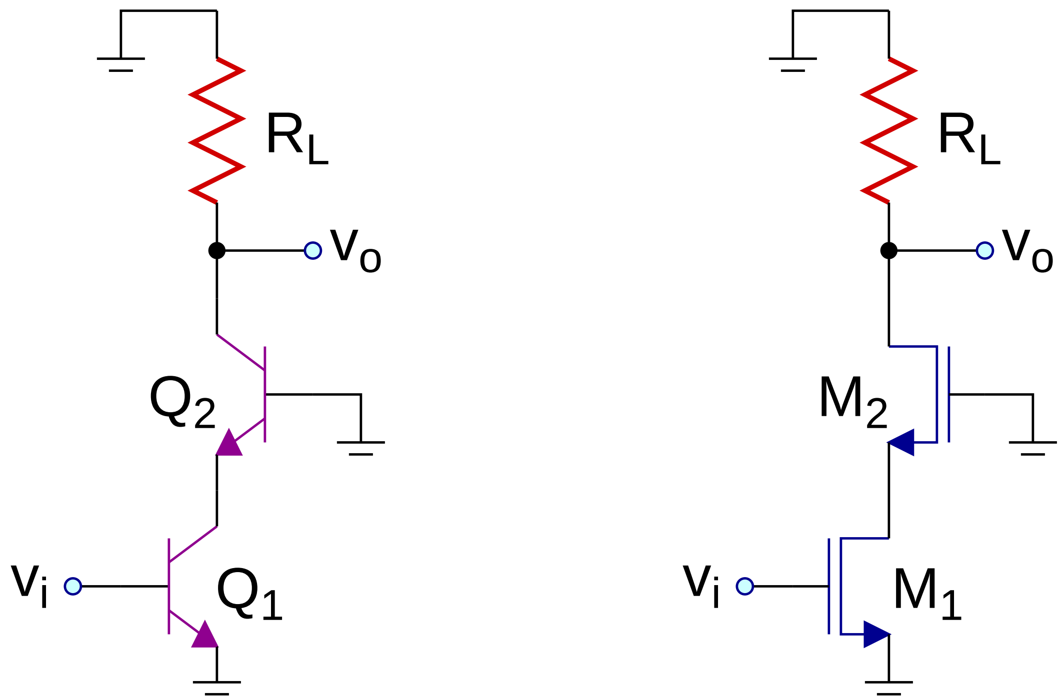
Can be used to estimate the nondominant pole ($\mathbf{p_2}$) in DC-coupled amplifiers that have only two widely-spaced real poles.



$$R_{ks} = \frac{v_k}{i_k} \quad , \quad \tau_{ks} = R_{ks} C_k \quad , \quad k = 1, 2$$

$$p_2 \approx - \sum_{k=1}^{2} \frac{1}{\tau_{ks}}$$

Cascode Frequency Response



Differential Pair