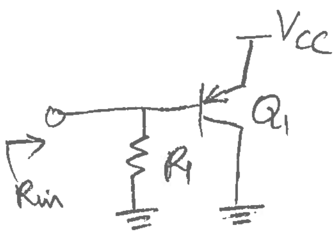
ECEN 326.

Homework 1

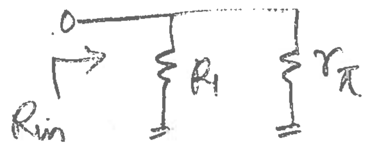
Solutions

5.5) Find R_{in}

(a)

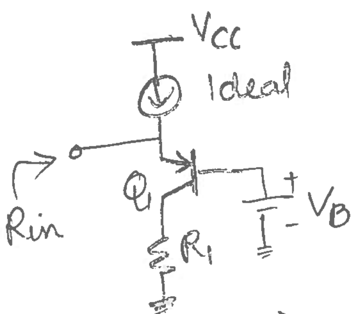


$\Rightarrow$



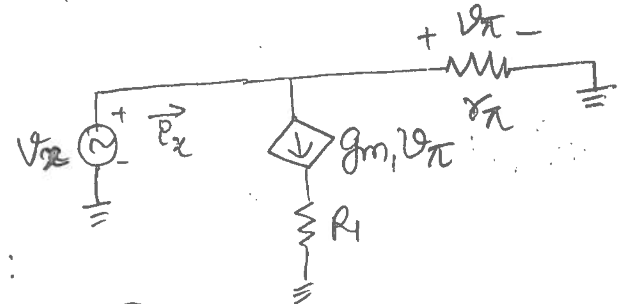
$$\boxed{R_{in} = R_1 \,||\, r_\pi}$$

(b)



$\Rightarrow$

Ideal current source has Infinite Impeda



Applying KCL in node (A):

$$i_x = \frac{v_\pi}{r_\pi} + g_{m_1} v_{\pi_1} \quad \text{--- (1)}$$

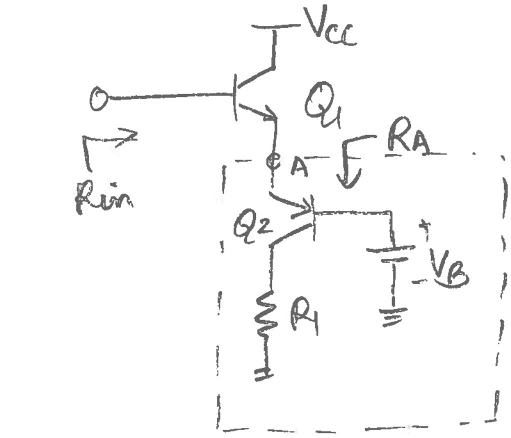
Also $\quad v_x = v_\pi \quad$ --- (2)

Using (1) & (2),

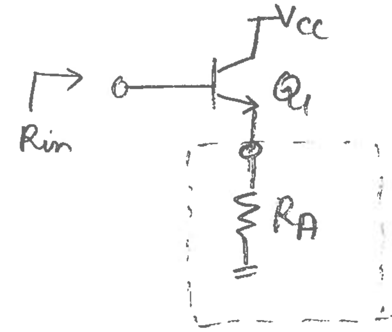
$$i_x = \frac{v_x}{r_\pi} + g_{m_1} v_x.$$

$$\therefore \quad R_{in} = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi} + g_{m_1}} \quad \text{or} \quad \boxed{R_{in} = r_\pi \,||\, \frac{1}{g_{m_1}}}$$

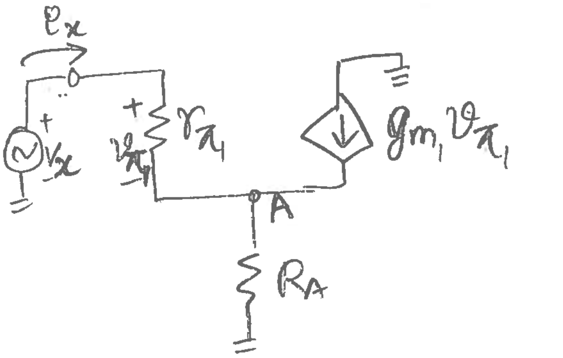
c)



This can be redrawn as:



We already know from 5.5 (b) that $R_A = r_{\pi_2} \| \frac{1}{g_{m_2}}$



We have:

$$v_A = v_x - v_{\pi_1}$$

or $$v_A = v_x - i_x r_{\pi_1} \quad (1)$$

Also,

$$v_A = R_A (i_x + g_{m_1} v_{\pi_1})$$

$$= R_A (i_x + g_{m_1} i_x r_{\pi_1})$$

$$\Rightarrow v_A = R_A i_x (1 + g_{m_1} r_{\pi_1}) \quad (2)$$

Using (1) & (2): $R_A i_x (1 + g_{m_1} r_{\pi_1}) = v_x - i_x r_{\pi_1}$

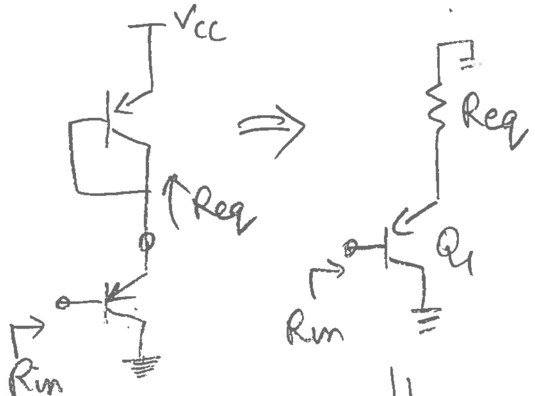
or $i_x \left[R_A (1 + g_{m_1} r_{\pi_1}) + r_{\pi_1} \right] = v_x$

or $R_{in} = \frac{v_x}{i_x} = r_{\pi_1} + R_A (1 + g_{m_1} r_{\pi_1})$

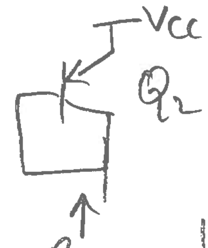
or $$\boxed{R_{in} = r_{\pi_1} + (1 + g_{m_1} r_{\pi_1}) \left[r_{\pi_2} \| \frac{1}{g_{m_2}} \right]}$$

or $$\boxed{R_{in} = r_{\pi_1} + (1 + \beta_1) \left[r_{\pi_2} \| \frac{1}{g_{m_2}} \right]} = r_{\pi_1} + (\beta + 1) r_{e2}$$

(d)



Where

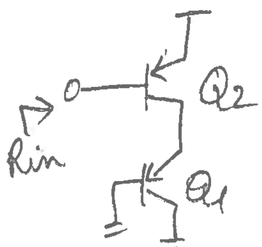


$R_{eq} = \frac{1}{g_{m_2}} \parallel r_{\pi_2}$

$R_{in} = r_{\pi_1} + (1+\beta) R_{eq} \Rightarrow \boxed{R_{in} = r_{\pi_1} + (1+\beta_1)\left[\frac{1}{g_{m_2}} \parallel r_{\pi_2}\right]}$

$= r_{\pi_1} + (\beta+1) r_{e2}$

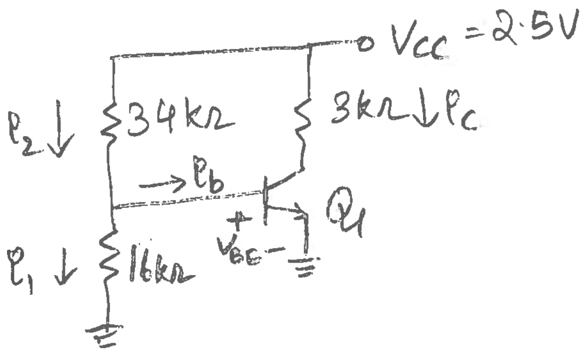
(e)



$R_{in} = r_{\pi_2}$

Since Q_1 is connected to the collector of Q_2, the input impedance of Q_1 plays no role in the input impedance of the circuit.

5.9)
(a)



$V_{BE} = 0.7V$; $\beta = 50$

$i_c = \beta i_b = 100 i_b$

$i_1 = \frac{V_{BE}}{16k} = \frac{0.7V}{16k} = \underline{43.75\mu A}$

$i_2 = \frac{V_{cc} - V_{BE}}{34k} = \frac{2.5 - 0.7}{34k} = \underline{52.94\mu A}$

$\therefore i_b = i_2 - i_1 = 52.94\mu - 43.75\mu$

$\Rightarrow \underline{i_b = 9.19\mu A}$

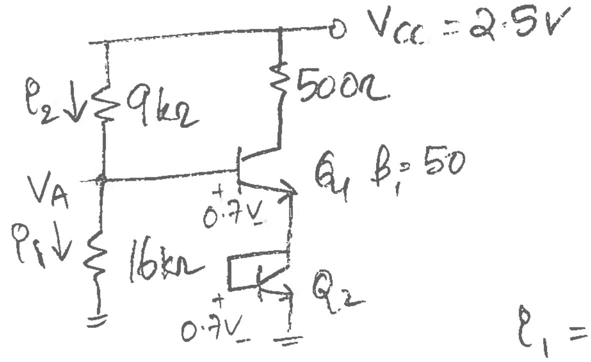
$\therefore i_c = 9.19\mu A \times 50 \Rightarrow \boxed{i_c = 459.5\mu A}$

$V_{CE} = V_{cc} - i_c \times 3k\Omega \Rightarrow V_{CE} = 2.5V - 1.3785V$

or $\boxed{V_{CE} = 1.112V}$

Q_1 is in forward active

(b)



$V_A = V_{BE1} + V_{BE2}$

$= 0.7 + 0.7 \Rightarrow \underline{\underline{V_A = 1.4V}}$

$i_1 = \frac{V_A}{16k} = \frac{1.4V}{16k} = \underline{\underline{87.5\mu A}}$

$i_2 = \frac{V_{cc} - V_A}{9k} = \frac{2.5 - 1.4}{9k} = \frac{1.1}{9k} = \underline{\underline{122.22\mu A}}$

$\therefore I_{B_1} = i_2 - i_1 = \underline{\underline{34.72\mu A}}$

$I_{C_1} = \beta I_{B_1} = 50 \times 34.72\mu A \quad \therefore \boxed{I_{C_1} = 1.736mA}$

Since $I_{E_1} = I_{E_2} \Rightarrow I_{C_1} = I_{C_2} \quad \therefore \boxed{I_{C_2} : 1.736mA}$

$\boxed{V_{CE_2} = V_{BE_2} = 0.7V}$

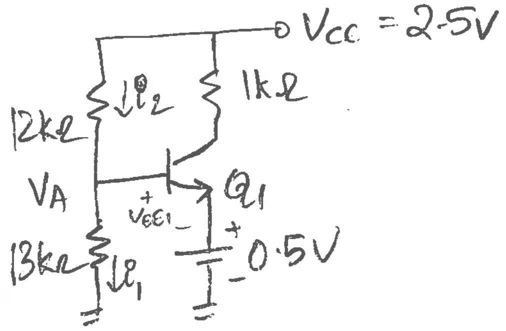
$V_{CE_1} = V_{cc} - I_{C_1}(500\Omega) - V_{CE_2}$

$= 2.5V - 1.736m \times 0.5k - 0.7V$

$\Rightarrow \boxed{V_{CE_1} = 0.932V}$

$\therefore$ Both Q_1 & Q_2 are in forward active region of transistor operation.

(c)



$V_{BE1} = 0.7V$, $\beta = 50$ (given)

$V_A = V_{BE1} + 0.5V = 0.7 + 0.5 = 1.2\,V$

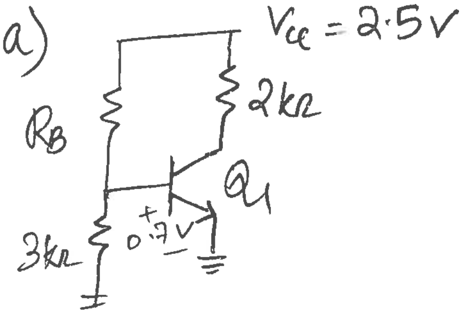
$i_B = i_2 - i_1 = \frac{2.5V - 1.2V}{12k\Omega} - \frac{1.2V}{13k\Omega}$

$\underline{i_B = 16\mu A} \Rightarrow \boxed{i_C = 0.8mA}$

$V_{CE_1} = 2.5V - (0.8m \times 1k) - 0.5V \Rightarrow \boxed{V_{CE_1} = 1.2V}$

$\therefore Q_1$ is in forward active.

5.12) a)



To operate in active mode,

$V_{CE} \geq 0.3V$

$\therefore V_{CC} - i_C \times 2k\Omega = V_{CE} \geq 0.3V$

or $i_C \leq \frac{V_{CC} - 0.3V}{2k\Omega} = 1.1mA$

$\therefore$ To operate in active mode

$i_C \leq 1.1mA \Rightarrow \beta i_B \leq 1.1mA$

or $i_B \leq 11\mu A$ (with $\beta = 100$)

We know that $i_B = \frac{2.5V - 0.7V}{R_B} - \frac{0.7V}{3k} = \frac{1.8V}{R_B} - 233.33$

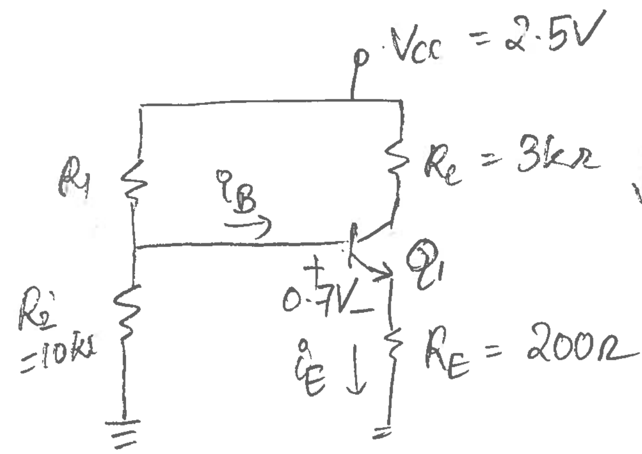
$\therefore \frac{1.8}{R_B} - 233.33\mu \leq 11\mu A$ or $\frac{1.8}{R_B} \leq 244.33\mu$

$\Rightarrow R_B \geq \frac{1.8}{244.33\mu} \Rightarrow \boxed{R_B \geq 7.36\,k\Omega}$

To guarantee active mode in Q_1

(5) $\boxed{R_{B\,min} = 7.36\,k\Omega}$

5.16)
a)



Given
$i_C = 0.25mA$
or
$250\mu A$

$\beta = 100$

$\therefore \; i_B = 2.5\mu A$

$$V_A = V_{BE} + i_E R_E = V_{BE} + 101\, i_B R_E$$

$$\therefore \; V_A = 0.7 + 252.5\mu \times 200 = 0.7505\,V$$

KCL at node A:

$$\frac{V_{CC} - 0.7505}{R_1} = \frac{0.7505}{R_2} + i_B$$

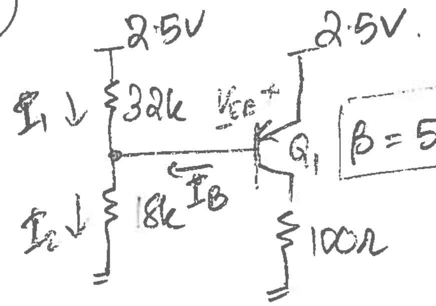
$$\frac{2.5 - 0.7505}{R_1} = \frac{0.7505}{10k} + 2.5\mu =$$

$$\Rightarrow \; \frac{1.7495}{R_1} = 77.55\mu$$

or $\boxed{R_1 = 22.56\,k\Omega}$

5.28)

(a)



$\boxed{\beta = 50}$

$\boxed{V_{EB} = 0.7}$ Given/assum

$I_B = I_2 - I_1$

$= \dfrac{V_{CC} - V_{EB}}{18k} - \dfrac{V_{EB}}{32k}$

$\Rightarrow I_B = \dfrac{2.5 - 0.7}{18k} - \dfrac{0.7}{32k} = \dfrac{1.8}{18k} - \dfrac{0.7}{32k}$

$= 100\mu A - 21.875\mu$

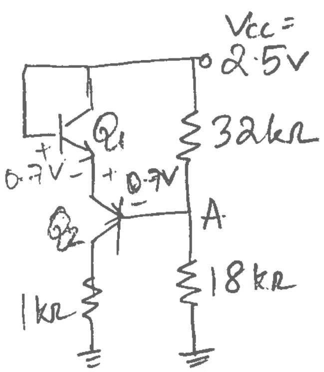
$\therefore \boxed{I_B = 78.125\mu A}$

$I_C = 50 \times 78.125\mu \Rightarrow \boxed{I_C = 3.9mA}$

$V_{EC} = V_{CC} - I_C \times 100. \Rightarrow V_{EC} = 2.5 - (3.9m \times 0.1k)$

or $\boxed{V_{EC} = 2.11V}$

(b)



$V_A = V_{CC} - V_{BE1} - V_{EB2}$

$= 2.5V - 0.7V - 0.7V$

$\Rightarrow V_A = 1.1V$

Given:

$\beta_1 = 100$

$\beta_2 = 50$

$\boxed{V_{BE_1} = V_{EB_2} = 0.7V}$

$I_{B_2} = \dfrac{V_A}{18k} - \dfrac{V_{CC} - V_A}{32k} = \dfrac{1.1}{18k} - \dfrac{1.4}{32k} \Rightarrow \boxed{I_{B_2} = 17.36\mu A}$

$I_{C_2} = \beta_2 \cdot I_{B_2} = 50 \times 17.36\mu A$

$\Rightarrow \boxed{I_{C_2} = 868\mu A}$

$\boxed{V_{CE_1} = V_{BE_1} = 0.7V}$

$V_{CE_2} = V_{CC} - V_{BE_1} - (I_{C_2} * 1k\Omega) = 2.5V - 0.7V - (868\mu \times 1k)$

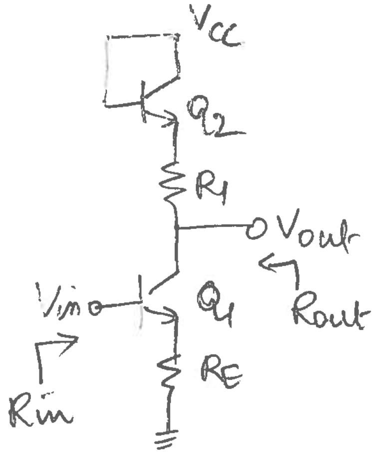
$\Rightarrow \boxed{V_{CE2} = 0.932V}$

$I_{E_1} = I_{E_2} = (1+50)\, 17.36\mu \Rightarrow I_{E_1} = 885.36\mu A \Rightarrow I_{C_1} = \dfrac{I_{E_1} \times 100}{101}$

or $\boxed{I_{C_1} = 876.59\mu A}$

5.46)

(a)

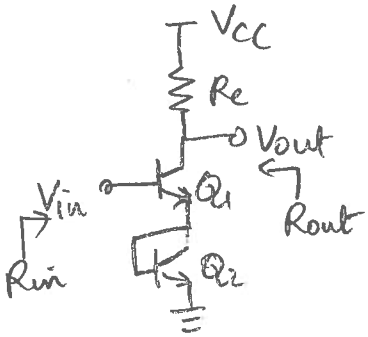


$$R_{in} = r_{\pi_1} + (1+\beta_1) R_E$$

$$R_{out} = R_1 + \left(\frac{1}{g_{m_2}} \| r_{\pi_2}\right)$$

$$A_V = -\frac{R_1 + \left(\frac{1}{g_{m_2}} \| r_{\pi_2}\right)}{R_E + \frac{1}{g_{m_1}}} = -\frac{\alpha(R_1 + r_{e2})}{R_E + r_{e1}}$$

(b)

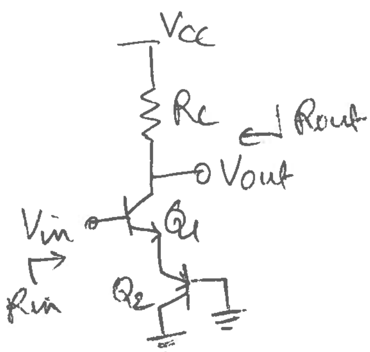


$$R_{in} = r_{\pi_1} + (1+\beta_1)\left[\frac{1}{g_{m_1}} \| r_{\pi_2}\right]$$

$$R_{out} = R_C$$

$$A_V = -\frac{R_C}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}} \| r_{\pi_2}} = -\frac{\alpha R_C}{r_{e1} + r_{e2}}$$

(c)

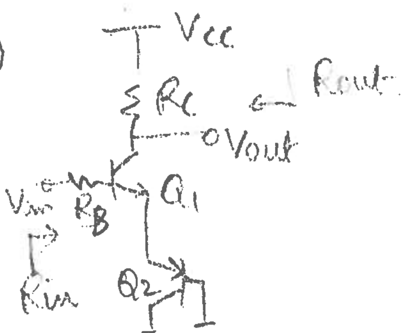


$$R_{in} = r_{\pi_1} + (1+\beta_1)\left[\frac{1}{g_{m_2}} \| r_{\pi_2}\right]$$

$$R_{out} = R_C$$

$$A_V = \frac{-R_C}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}} \| r_{\pi_2}} = -\frac{\alpha R_C}{r_{e1} + r_{e2}}$$

(d)



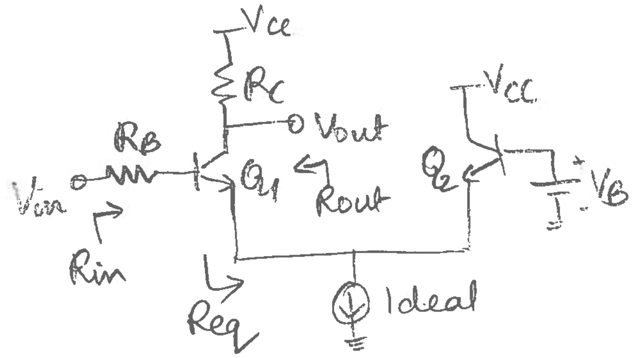
$$R_{in} = R_B + r_{\pi_1} + (1+\beta_1)\left[\frac{1}{g_{m_2}} \| r_{\pi_2}\right]$$

$$R_{out} = R_C$$

$$A_V = \frac{-R_C}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}} \| r_{\pi_2} + \frac{R_B}{1+\beta_1}}$$

$$= -\frac{\alpha R_C}{r_{e1} + r_{e2} + \frac{R_B}{\beta+1}}$$

(e)



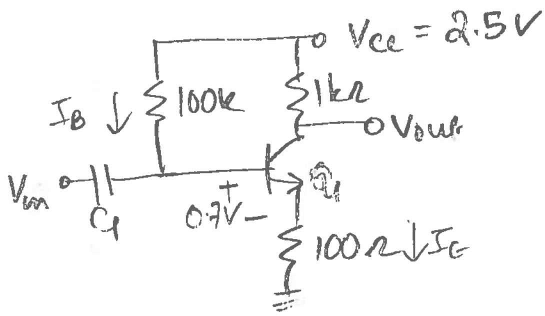
$R_{in} = R_B + r_{\pi_1} + (1+\beta_1) R_{eq}$

where $R_{eq} = \frac{1}{g_{m_2}} \| r_{\pi_2} = r_{e2}$

$R_{out} = R_C$

$$A_V = -\frac{R_C}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}} \| r_{\pi_2} + \frac{R_B}{\beta_1 + 1}} = -\frac{\alpha R_C}{r_{e1} + r_{e2} + \frac{R_B}{\beta + 1}}$$

5.52)
(a)



$$|A_V| = \frac{R_C}{\frac{1}{g_{m_1}} + R_E} = \frac{1k}{100 + \frac{1}{g_{m_1}}}$$

To find g_{m_1} we analyse the DC operating point I_C for Q_1

Assume $V_{BE_1} = 0.7V$ & $\beta = 100$. then,

$V_{CC} - I_B \times 100k - 0.7V - I_E \times 100\Omega = 0$

$2.5V - 0.7V = I_B[100k + (100+1)100]$ Using $I_E = (\beta+1) I_B$

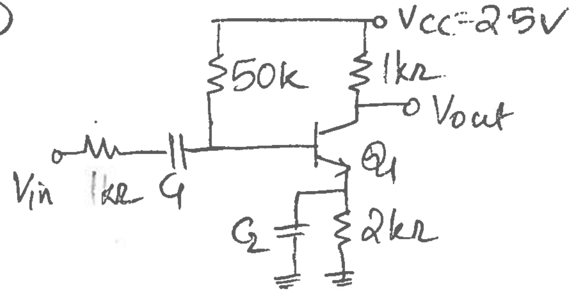
or $\boxed{I_B = 16.35\mu A}$ $\Rightarrow I_C = 100 \times 16.35\mu A$

or $\boxed{I_C = 1.635mA}$

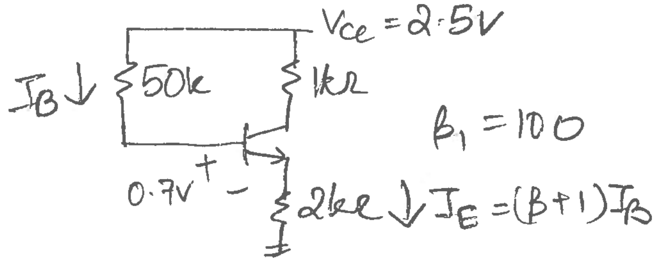
$\therefore g_{m_1} = \frac{I_C}{V_T} = \frac{1.635mA}{25mV}$ or $\boxed{g_{m_1} = 0.0654\,\Omega^{-1}}$

$$\therefore |A_V| = \frac{1k}{100 + \frac{1}{0.0654}} \Rightarrow \boxed{|A_V| = 8.674}$$

(b)



DC Analysis (C_1, C_2 open)



$\beta_1 = 100$

$$V_{CC} - 0.7V - I_B \times 50k - 101 \times I_B \times 2k = 0$$

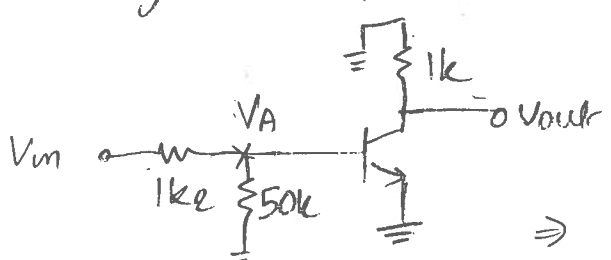
or $$I_B = \frac{2.5 - 0.7}{50k + 202k} = \underline{\underline{7.143\mu A}}$$

$$\therefore I_C = \beta \times I_B = \underline{\underline{714.3\mu A}}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.7143mA}{25mV} = \underline{\underline{0.028\Omega^{-1}}}$$

$r_\pi = \frac{\beta}{g_m}$ or $\underline{\underline{r_\pi = 3.57k}}$

AC Analysis : (C_1, C_2 short)



$$|A_V| = \frac{|V_{out}|}{|V_{in}|} = \frac{|V_{out}|}{|V_A|} \cdot \frac{|V_A|}{|V_{in}|}$$

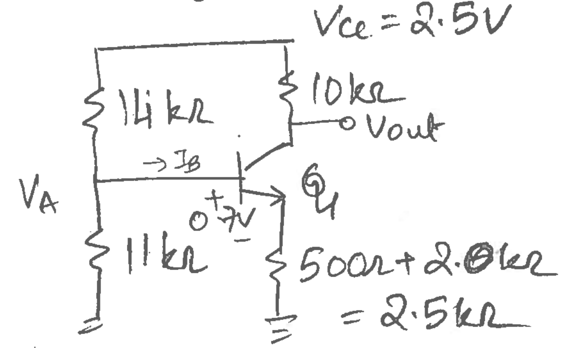
$$\Rightarrow \frac{|V_A|}{|V_{in}|} = \frac{50k \| r_\pi}{1k + 50k \| r_\pi} = 0.769$$

$$\frac{|V_{out}|}{|V_A|} = g_m \times 1k = 28$$

$$\therefore |A_V| = 28 \times 0.769 \Rightarrow \boxed{|A_V| = 21.532}$$

(10)

(c) DC Analysis :



Given $\beta = 100$
$V_{BE} = 0.7V$

$V_A = 0.7 + (\beta+1) I_B \times 2.5k$

$V_A = 0.7 + 2.5k \times 101 \times I_B$

$$I_B = \frac{V_{cc} - V_A}{14k} - \frac{V_A}{11k} = \frac{2.5}{14k} - \frac{0.7}{14k} - \frac{2.5k \times 101 \, I_B}{14k} - \frac{0.7}{11k} - \frac{2.5k \times 101 \, I_B}{11k}$$

$$\Rightarrow I_B \left[1 + \frac{2.5 \times 101}{14} + \frac{2.5 \times 101}{11} \right] = \frac{2.5}{14k} - \frac{1.4}{11k} = 51.29\mu$$

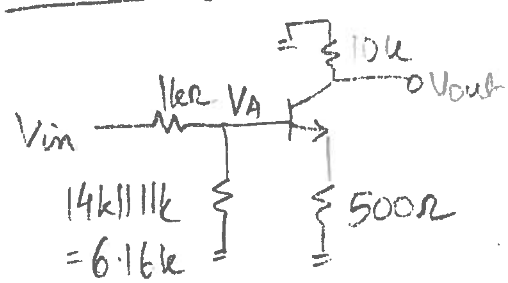
$I_B \times 41.99 = 51.29\mu$ or $\boxed{I_B = 1.2215 \mu A}$

$I_c = \beta I_B \Rightarrow \boxed{I_c = 122.15 \mu A}$

$$g_m = \frac{I_c}{V_T} = \frac{0.12215 mA}{25 mV} = \underline{0.00489 \Omega^{-1}}$$

$$r_\pi = \frac{\beta}{g_m} = \underline{20.5 k\Omega}$$

AC Analysis :



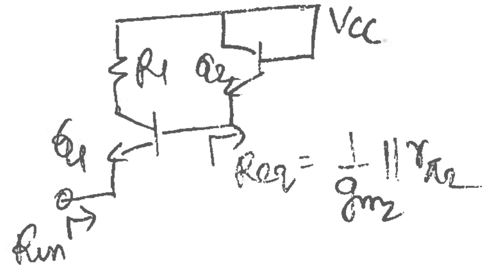
$$|A_v| = \frac{|V_{out}|}{|V_{in}|} = \frac{|V_{out}|}{|V_A|} \cdot \frac{|V_A|}{|V_{in}|}$$

$$\frac{|V_A|}{|V_{in}|} = \frac{6.16k || r_\pi + (\beta+1) 0.5k}{1k + 6.16k || (r_\pi + (\beta+1) 0.5k)}$$

$$\frac{|V_A|}{|V_{in}|} = 0.86. \qquad \frac{|V_{out}|}{|V_A|} = \frac{10k}{0.5k + \frac{1}{g_m}} = 14.2$$

$$\Rightarrow |A_v| = 14.2 \times 0.86 \Rightarrow \boxed{|A_v| = 12.2}$$

5.56) (a)

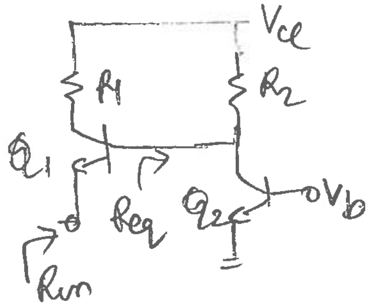


$$R_{in} = \frac{r_{\pi_1} + R_{eq}}{\beta_1 + 1}$$

or

$$\boxed{R_{in} = \frac{r_\pi + \frac{1}{g_{m2}} \| r_{\pi 2}}{\beta_1 + 1}} = r_{e1} + \frac{r_{e2}}{\beta_1 + 1}$$

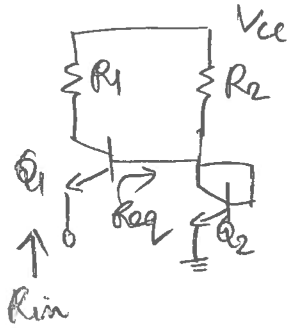
(b)



$$R_{eq} = R_2$$

$$R_{in} = \frac{r_{\pi_1} + R_{eq}}{\beta_1 + 1} \Rightarrow \boxed{R_{in} = \frac{r_{\pi_1} + R_2}{\beta_1 + 1}} = r_{e_1} +$$

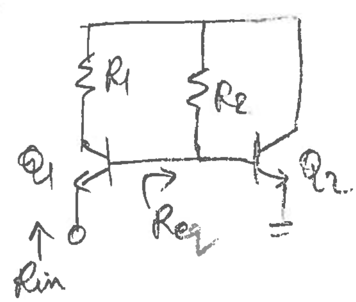
(c)



$$R_{eq} = R_2 \| \frac{1}{g_{m2}} \| r_{\pi_2} \qquad R_{in} = \frac{r_{\pi_1} + R_{eq}}{\beta_1 + 1}$$

$$\boxed{R_{in} = \frac{r_{\pi_1} + R_2 \| \frac{1}{g_{m2}} \| r_{\pi_2}}{\beta_1 + 1}} = r_{e_1} + \frac{R_2 \| r_{e2}}{\beta_1 + 1}$$

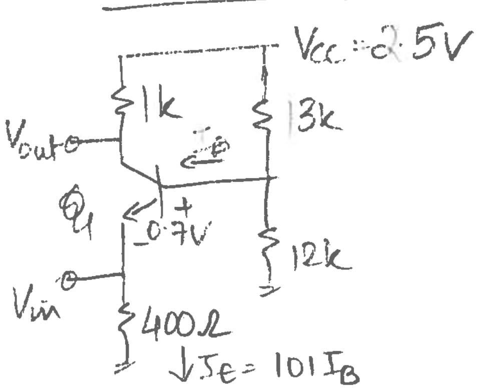
(d)



$$R_{eq} = R_2 \| r_{\pi_2} \qquad R_{in} = \frac{r_{\pi_1} + R_{eq}}{\beta_1 + 1}$$

$$\Rightarrow \boxed{R_{in} = \frac{r_{\pi_1} + R_2 \| r_{\pi_2}}{\beta_1 + 1}} = r_{e_1} + \frac{R_2 \| r_{\pi 2}}{\beta_1 + 1}$$

5.58) (a) DC Analysis:



Given $V_{BE} = 0.7V$ & $\beta = 100$

$$I_B = \frac{\overset{2.5}{V_{cc}''} - 0.7 - (101 I_B \times 0.4k)}{13k} - \left(\frac{0.7 + 101 I_B \times 0.4}{12k}\right)$$

$$\Rightarrow I_B = \frac{1.8}{13k} - \frac{0.7}{12k} - I_B\left[\frac{101 \times 0.4}{13} + \frac{101 \times 0.4}{12}\right]$$

$$\Rightarrow I_B = \frac{80\mu A}{7.474} \Rightarrow \boxed{I_B = 10.7\mu A}$$

$I_C = 100 I_B \Rightarrow \boxed{I_C = 1.07\,mA}$

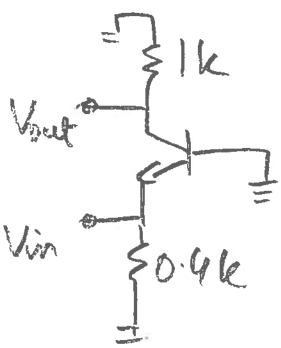
$g_m = \dfrac{I_C}{V_T} = \dfrac{1.07\,mA}{25\,mV} \Rightarrow \boxed{g_m = 0.0428\,\Omega^{-1}}$

$r_\pi = \dfrac{\beta}{g_m} \Rightarrow \boxed{r_\pi = 2.336\,k\Omega}$

$$V_{CE} = 2.5V - \underbrace{1.07m \times 1k}_{1.07V} - \underbrace{\frac{101}{100} \times 1.07m \times 0.4k}_{0.432228V}$$

$\Rightarrow \boxed{V_{CE} = 0.998V}$

(b) AC Analysis:

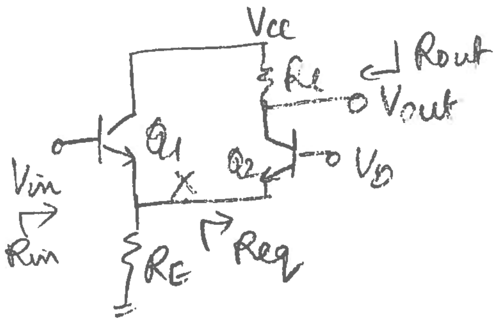


$|A_V| = \dfrac{|V_{out}|}{|V_{in}|} = g_m \times 1k \Rightarrow \boxed{|A_V| = 42.8}$

$R_{in} = 0.4k \,||\, \dfrac{1}{g_m} \Rightarrow \boxed{R_{in} = 0.39\,k\Omega}$

$\boxed{R_{out} = 1k\Omega}$

5.73)
(a)



$R_{eq} = \dfrac{1}{g_{m2}} \,||\, r_{\pi 2}$

$R_{in} = r_{\pi 1} + (\beta_1 + 1)\, R_{eq} \,||\, R_E$

$$\boxed{R_{in} = r_{\pi 1} + (\beta_1 + 1)\left[R_E \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}\right]}$$

$= r_{\pi 1} + (\beta_1 + 1)(R_E \,||\, r_{e2})$

$\boxed{R_{out} = R_C}$

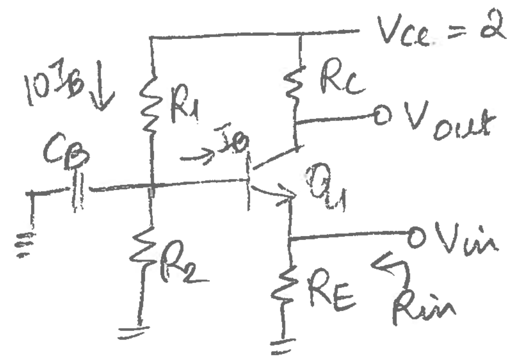
(b) $\dfrac{|V_x|}{|V_{in}|} = \dfrac{R_E \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}}{\frac{1}{g_{m1}} + R_E \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}}$

$\dfrac{|V_{out}|}{|V_x|} = g_{m2} R_C$

$|A_V| = \dfrac{|V_{out}|}{|V_{in}|} = \dfrac{|V_{out}|}{|V_x|} \cdot \dfrac{|V_x|}{|V_{in}|} \Rightarrow \boxed{|A_V| = g_{m2} R_C \dfrac{R_E \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}}{\frac{1}{g_{m1}} + R_E \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}}}$

$= g_{m2} R_C \,(R_E \,||\, r_{e2})$

5.83)



$A_V = 20$

$R_{in} = 50\Omega$ (R_{in} does not depend on R_E)

$I_E R_E = 260mV$

Since R_{in} does not depend on R_E, $R_{in} = \frac{1}{g_{m_1}} = 50\Omega$

$\Rightarrow \boxed{g_{m_1} = 0.02\,\Omega^{-1}} \Rightarrow \frac{I_C}{V_T \to 25mV} = 0.02 \Rightarrow \boxed{I_C = 0.5mA}$

$\Rightarrow \boxed{I_B = 5\mu A} \Rightarrow I_E = 0.505mA$

$I_E R_E = 260mV \Rightarrow R_E = \frac{260mV}{0.505mA} \Rightarrow \boxed{R_E = 514.85\Omega}$

$A_V = g_{m_1} R_C = 20 \Rightarrow R_C = \frac{20}{g_{m_1}} \Rightarrow \boxed{R_C = 1k\Omega}$

$I_{R_1} = 10 I_B \Rightarrow \frac{V_{CC} - V_{BE_1} - 260mV}{R_1} = 10 I_B$

$\Rightarrow R_1 = \frac{2.5 - 0.7 - 0.26}{10 \times 5\mu} \Rightarrow \boxed{R_1 = 30.8k\Omega}$

$I_{R_2} = I_{R_1} - I_B = 9I_B \Rightarrow \frac{V_{BE_1} + 0.26V}{R_2} = 9I_B$

$\Rightarrow R_2 = \frac{0.7 + 0.26}{9 \times 5\mu} \Rightarrow \boxed{R_2 = 21.33k\Omega}$

Assuming the impedance of capacitance $\left|\frac{1}{sC_B}\right| \ll R_1, R_2$ we can write $A_V = \frac{R_C}{\frac{1}{g_{m_1}} + \frac{1}{\frac{sC_B}{\beta_1+1}}}$ Now for this gain to have no dependance on A_V, we can assume $\frac{1}{sC_B(\beta_1+1)} \ll \frac{1}{g_{m_1}}$.

Therefore at the lowest frequency of interest $f = 200Hz$, we can assume,

$$\frac{1}{sC_B(\beta_1+1)} = \frac{1}{10g_{m1}} \Rightarrow \frac{1}{2\pi(200)C_B(101)} = 5$$

$$\Rightarrow C_B = \frac{1}{2\pi \times 200 \times 101 \times 5} \Rightarrow \boxed{C_B = 1.58\mu F}$$