ECEN326: Electronic Circuits
Spring 2022

Lecture 2: Transistor Models & Single-Stage Amplifiers

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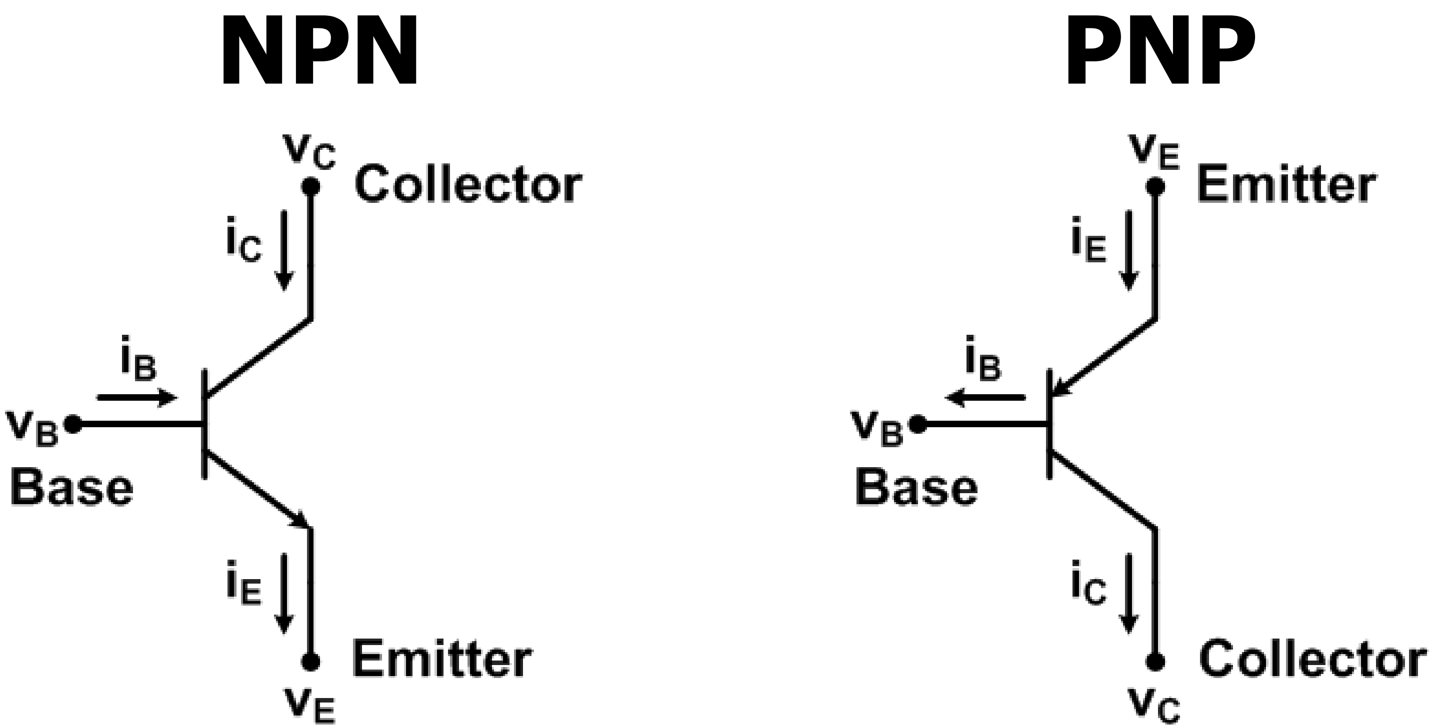
Announcements

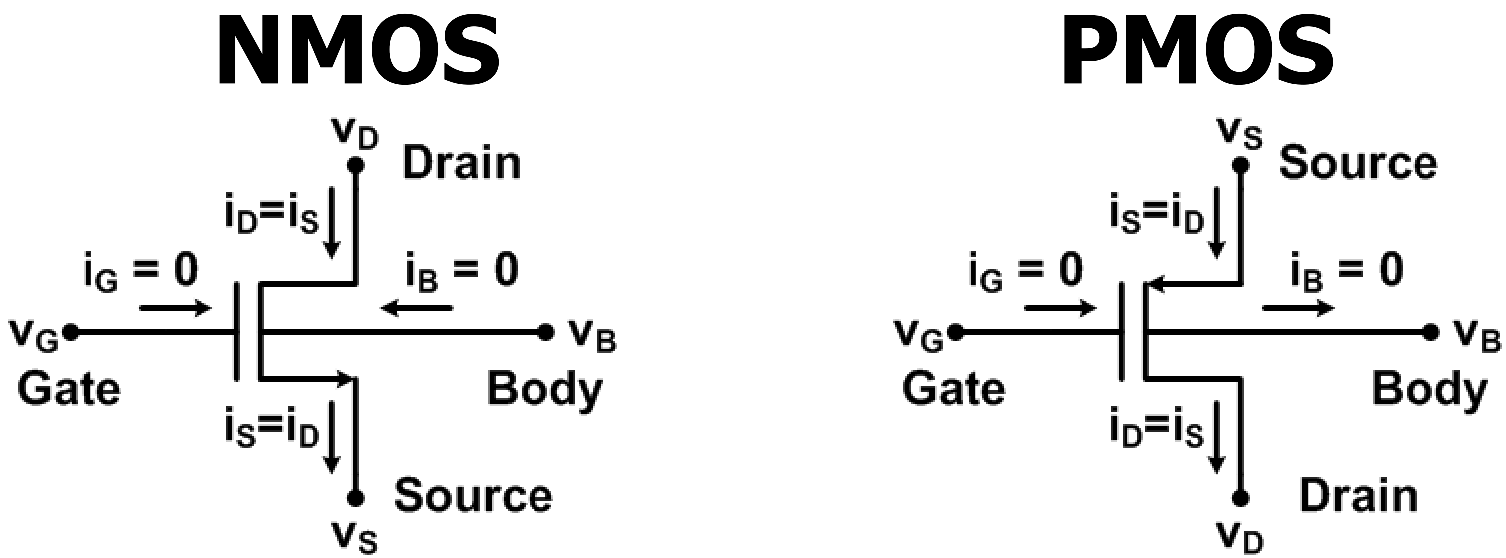
- Lab
  - Lab 1 report is due this week
  - Lab 2 report due next week
    - Use the Lab 2 updated specs
- HW
  - HW1 due today
  - HW2 due Feb 8
- Reading
  - Razavi Chapters 5 & 7

BJT Circuit Symbols



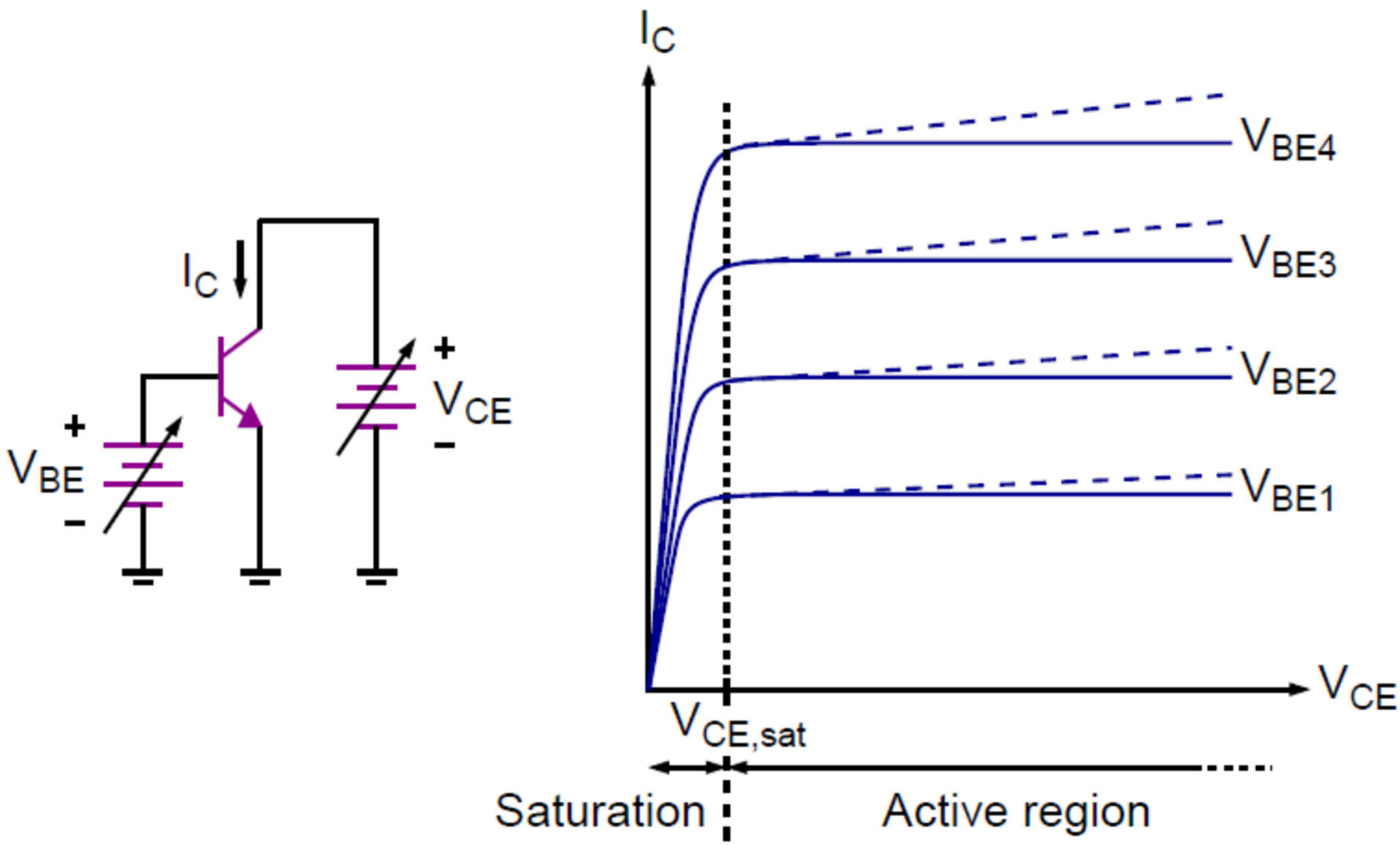
- BJTs are 3 terminal devices
  - Collector, Base, & Emitter
- 2 complementary BJT devices: NPN & PNP

MOSFET Circuit Symbols



- MOSFETs are 4-terminal devices
  - Drain, Gate, Source, & Body
- Body terminal generally has small impact in normal operation modes, thus device is generally considered a 3-terminal device
  - Drain, Gate, and Source are respectively similar to the Collector, Base, and Emitter of the BJT
- 2 complementary MOSFETS: NMOS, PMOS

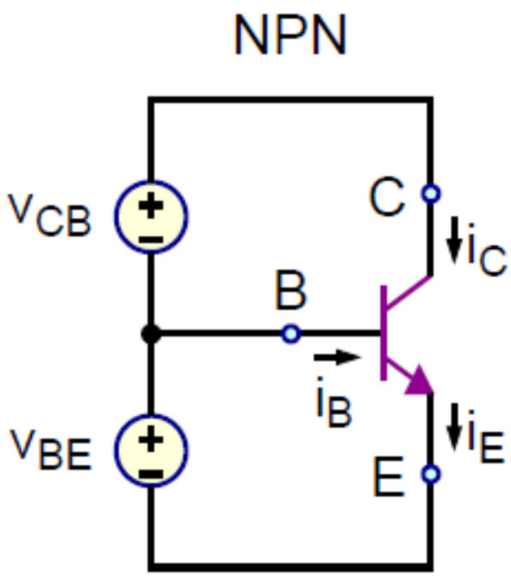
NPN "Large-Signal" (DC) Output Characteristic



- For analog applications, we generally desire the BJT to operate in the "Active" region

$$V_{CE} \geq V_{CE,sat} = 0.3V$$

NPN "Large-Signal" (DC) Model



- Exact equation

$$V_{CE} \geq V_{CE,sat}$$

$$i_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$$

$$i_C = I_S e^{V_{BE}/V_T}$$

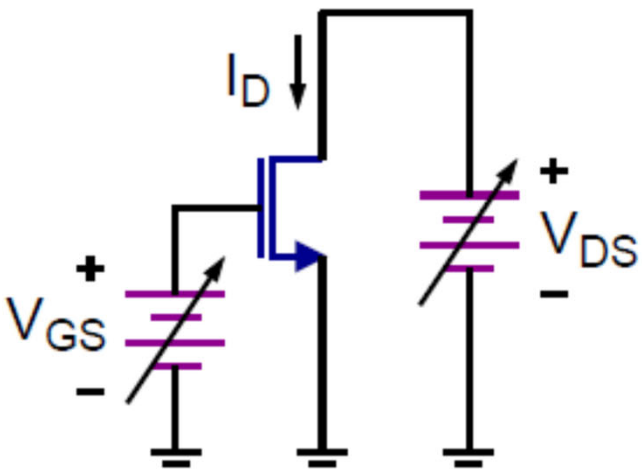
- However, V_{BE} doesn't change much over large values of I_C. So, for hand calculations we assume a fixed V_{BE} and use the following

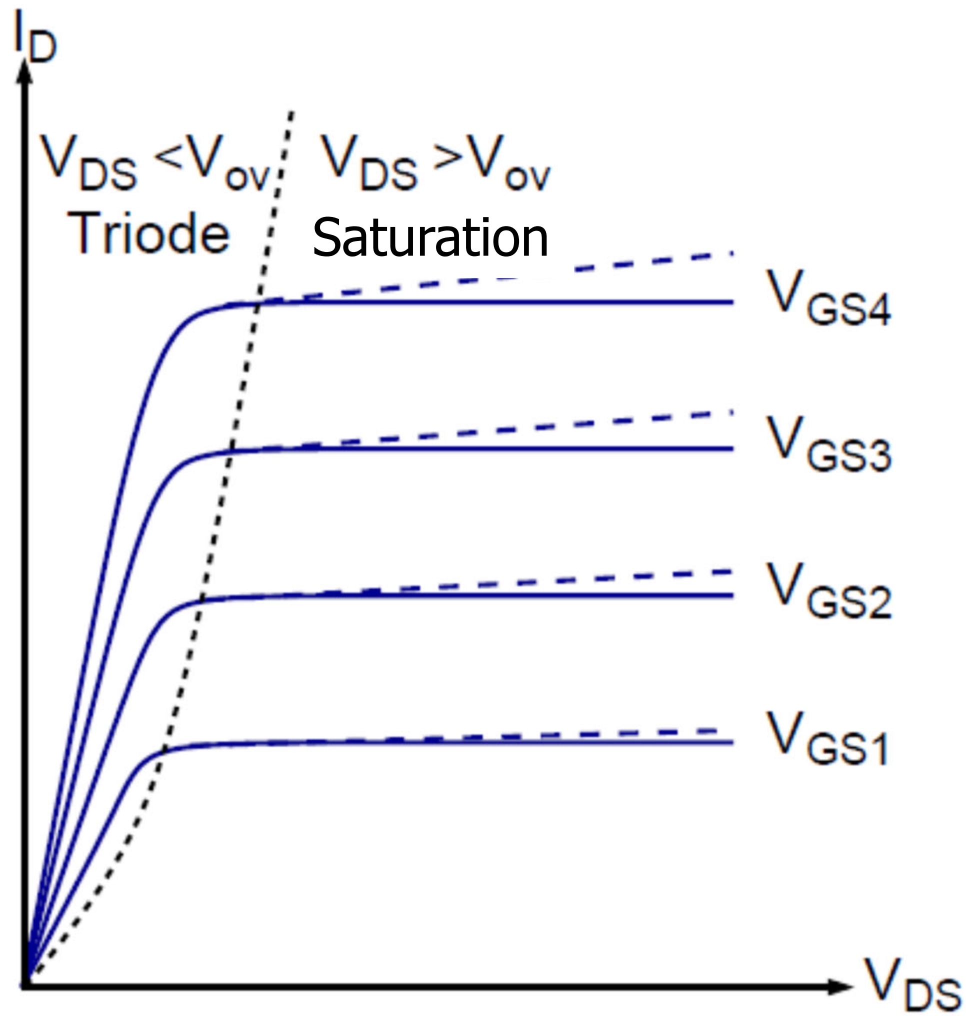
$$V_{BE} = 0.7 \text{ V}$$

$$I_C = \beta I_B$$

$$I_E = I_C + I_B$$

NMOS "Large-Signal" (DC) Output Characteristic

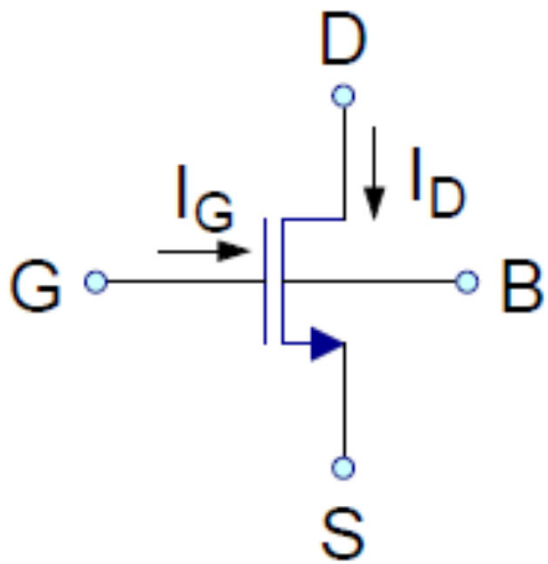




- For analog applications, we generally desire the MOSFET to operate in the "Saturation" region

$$V_{DS} \geq V_{OV} = V_{GS} - V_{tn}$$

NMOS "Large-Signal" (DC) Model



$$V_{DS} \geq V_{OV} \Rightarrow \text{Saturation}$$

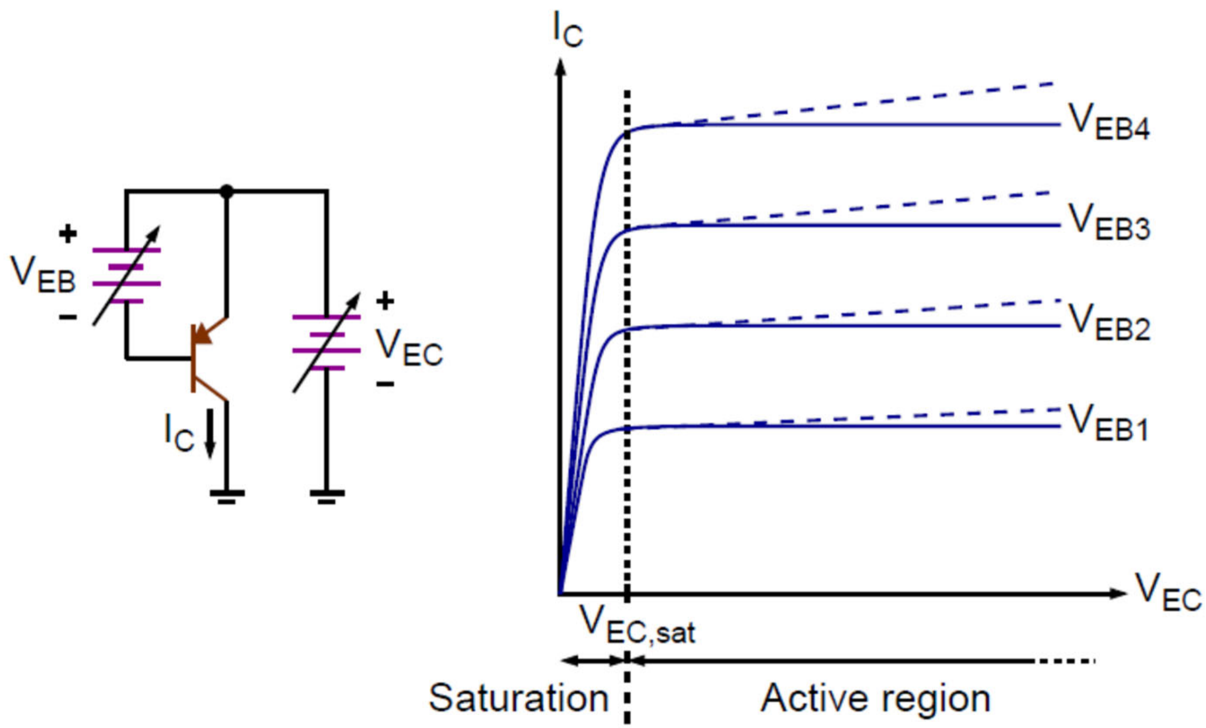
$$V_{OV} = V_{GS} - V_{tn}$$

$$I_G = 0$$

$$I_D = \frac{k'_n}{2}\frac{W}{L}V_{OV}^2$$

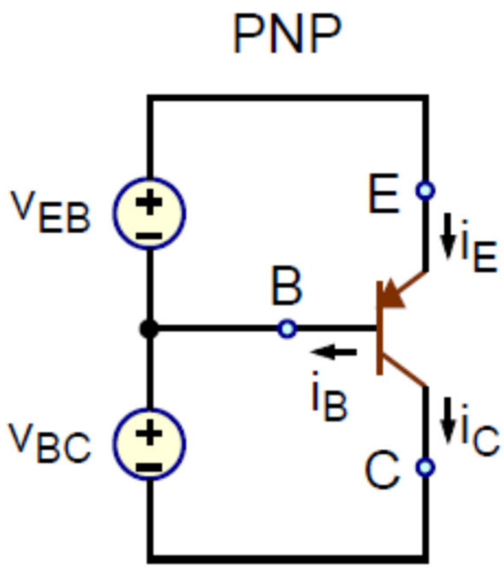
where $k_n^{'} = \mu_n C_{ox}$

PNP "Large-Signal" (DC) Output Characteristic



- Similar operation to NPN transistor, except
  - I_C flows out of the device
  - Flip the terminal voltages in calculations
- For analog applications, we generally desire the BJT to operate in the "Active" region $V_{EC} \geq V_{EC,sat} = 0.3V$

PNP "Large-Signal" (DC) Model



- Exact equation

$$V_{EC} \geq V_{EC,sat}$$

$$i_B = \frac{I_S}{\beta} e^{V_{EB}/V_T}$$

$$i_C = I_S e^{V_{EB}/V_T}$$

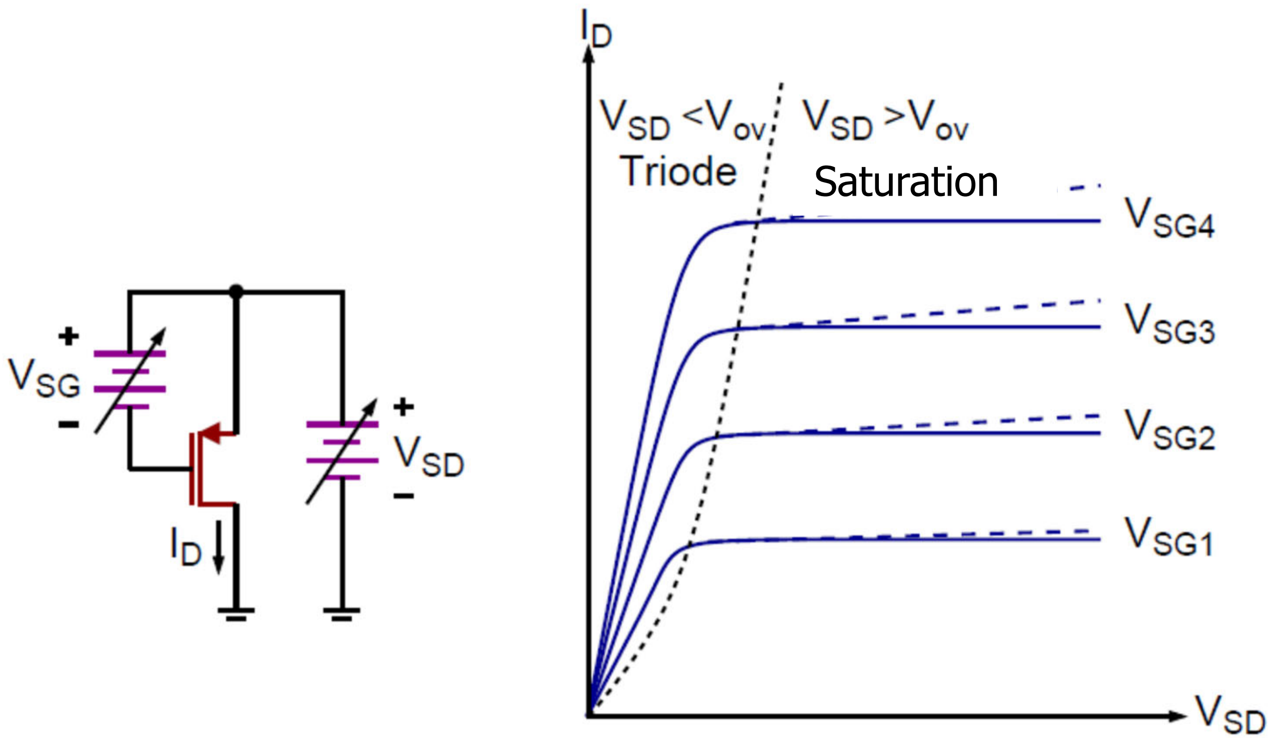
- However, V_{EB} doesn't change much over large values of I_C. So, for hand calculations we assume a fixed V_{EB} and use the following

$$V_{EB} = 0.7 \text{ V}$$

$$I_C = \beta I_B$$

$$I_E = I_C + I_B$$

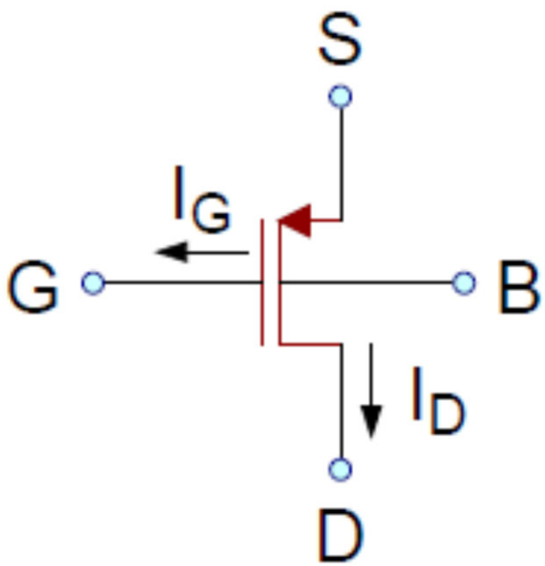
PMOS "Large-Signal" (DC) Output Characteristic



- Similar operation to NMOS transistor, except
  - I_D flows out of the device
  - Flip the terminal voltages in calculations
- For analog applications, we generally desire the BJT to operate in the "Saturation" region

$$V_{SD} \geq V_{OV} = V_{SG} - |V_{tp}|$$

PMOS "Large-Signal" (DC) Model



$$V_{SD} \geq V_{ov} \Rightarrow \text{Saturation}$$

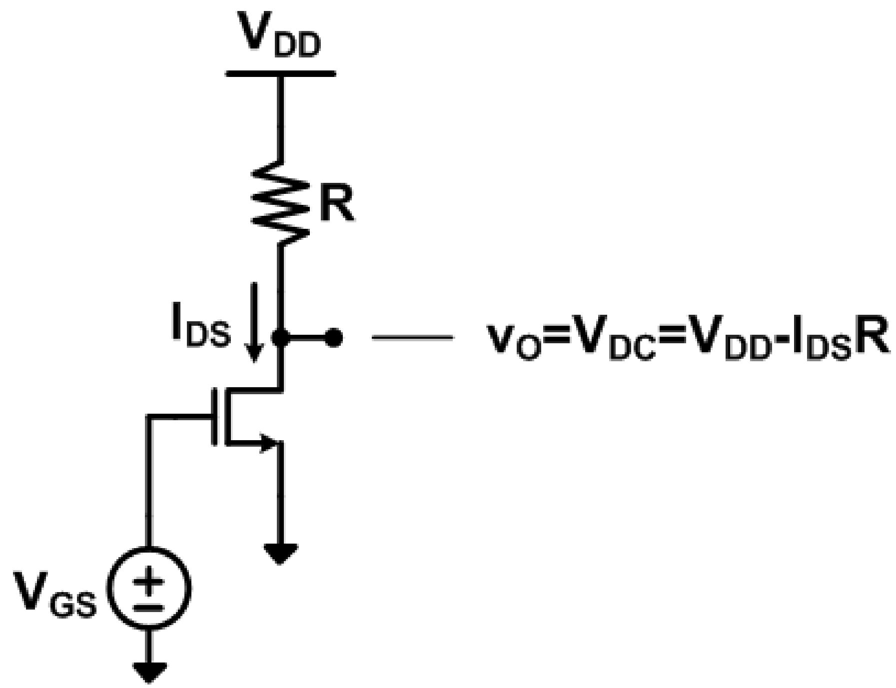
$$V_{ov} = V_{SG} - |V_{tp}|$$

$$I_G = 0$$

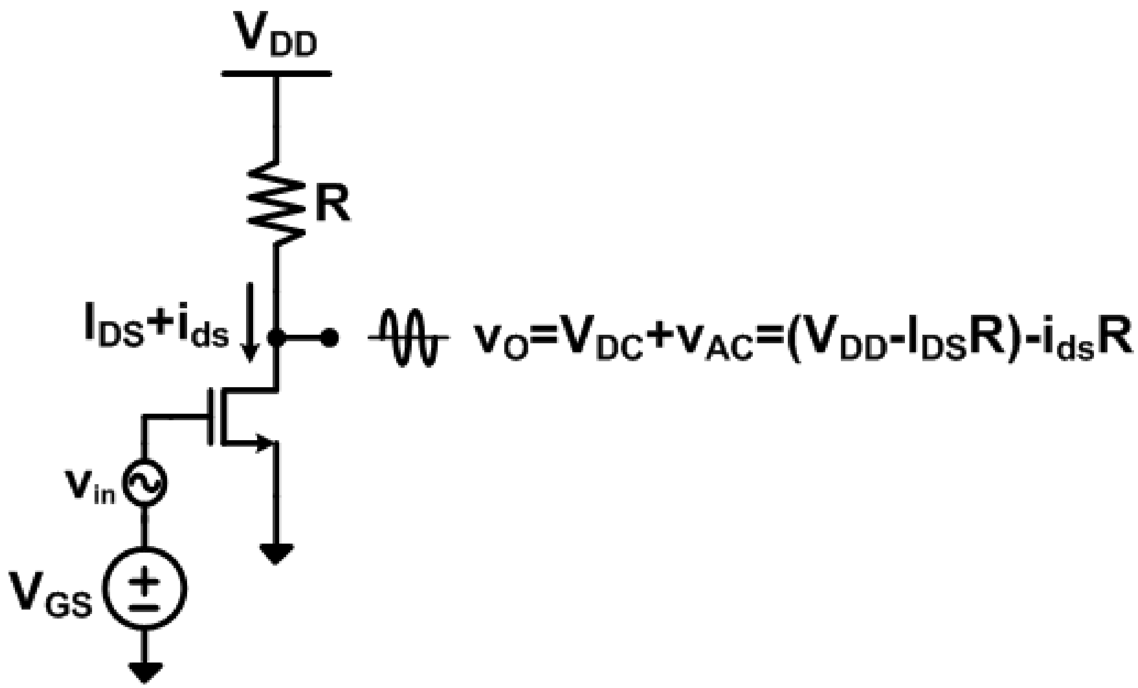
$$I_D = \frac{k_p'}{2}\frac{W}{L}V_{ov}^2$$

where $k_p' = \mu_p C_{ox}$

Large-Signal “DC” Response

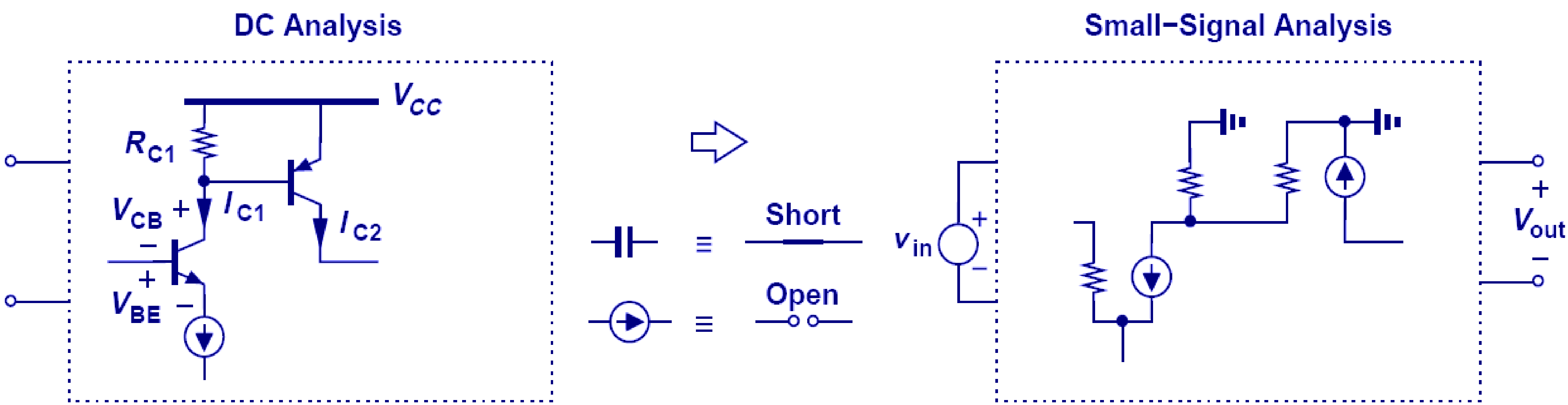


Large-Signal "DC" + Small-Signal "AC" Response



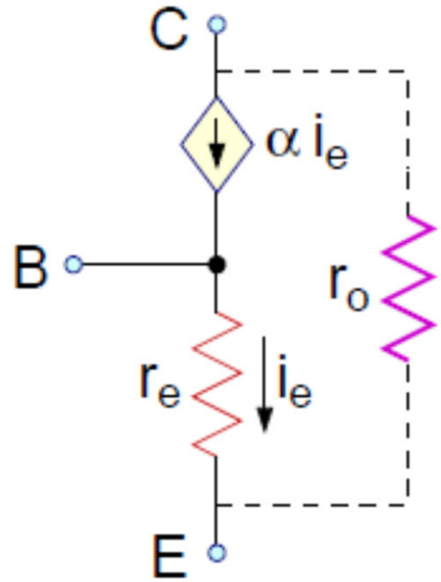
- For small-signal analysis, we "linearize" the response about the DC operating point
- If the signal is small enough, linearity holds and the complete response is the summation of the large-signal "DC" response and the small-signal "AC" response

DC Analysis vs. Small-Signal Analysis



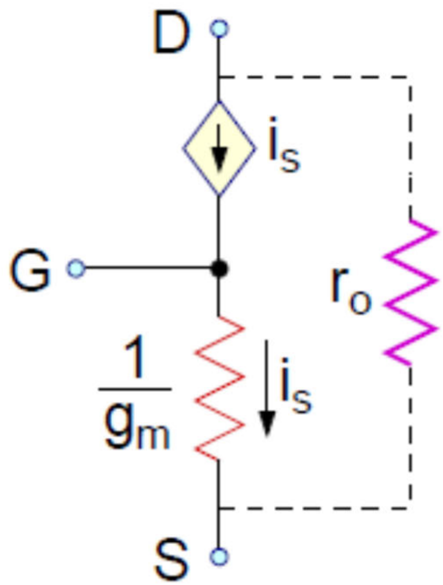
- First, DC analysis is performed to determine operating point and obtain small-signal parameters.
- Second, sources are set to zero and small-signal model is used.

NPN & NMOS Small-Signal T-Models ($r_o = \infty$)



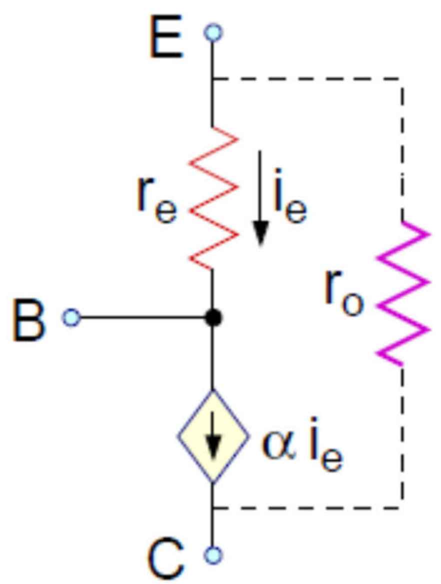
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

$$g_m = \frac{I_C}{V_T}$$



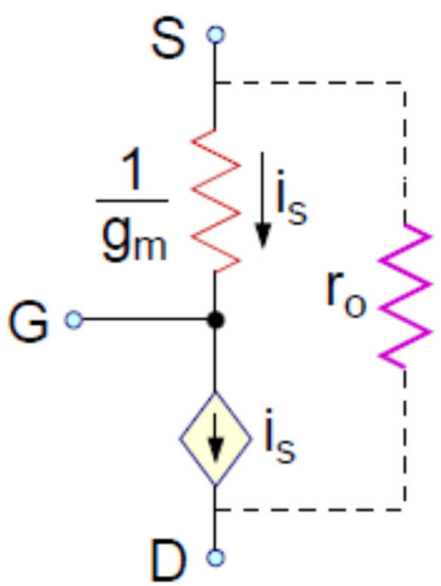
$$g_m = k_n' \frac{W}{L} V_{OV} = \sqrt{2k_n' \frac{W}{L} I_D}$$

Formal PNP & PMOS Small-Signal T-Models ($r_o=\infty$)



$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

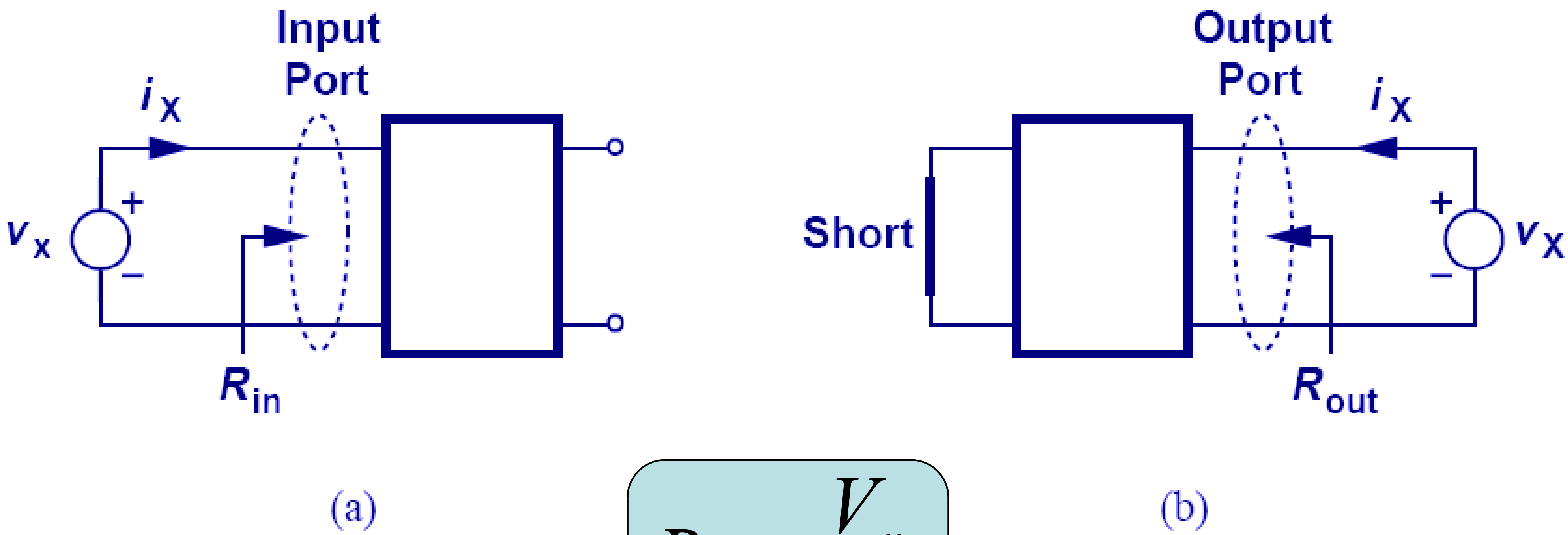
$$g_m = \frac{I_C}{V_T}$$



- While these are the formal models, for both the PNP and PMOS device **you can use the exact same model as the NPN and NMOS device**
  - This is easier to remember
  - Obtained by flipping the current sources in the formal model

$$g_m = k_p' \frac{W}{L} V_{ov} = \sqrt{2k_p' \frac{W}{L} I_D}$$

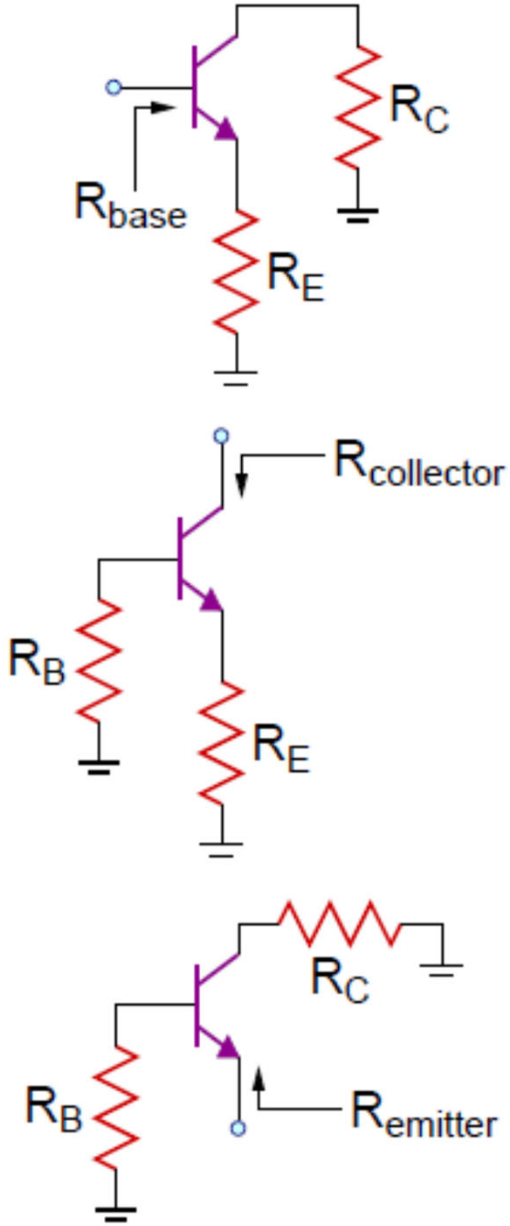
Input/Output Impedances



$$R_x = \frac{V_x}{i_x}$$

- The figure above shows the techniques of measuring input and output impedances.
- Small signal analysis is used

BJT Node AC Resistances ($r_o=\infty$)



$$R_{base} = (\beta + 1)(r_e + R_E)$$

- Using T-model, base AC resistance is the resistors connected from base through the emitter to ground **multiplied by (β+1)**

$$R_{collector} = \infty$$

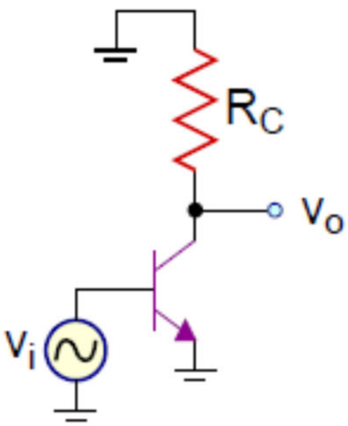
- If r_o is ∞, then collector AC resistance is ∞

$$R_{emitter} = r_e + \frac{R_B}{\beta + 1}$$

- Using T-model, emitter AC resistance is r_e plus the base resistor **divided by (β+1)**

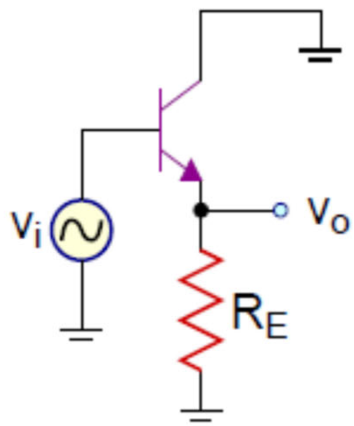
BJT Amplifiers AC Gain ($r_o = \infty$)

CE Amp



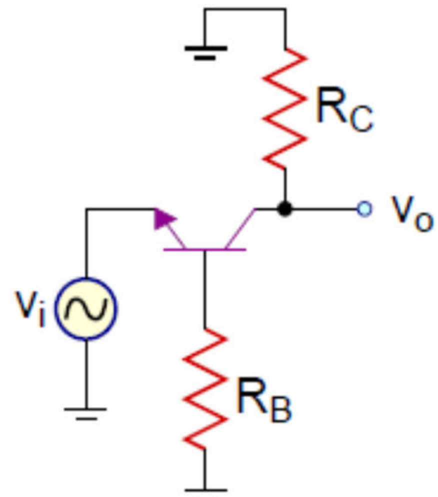
$$\frac{v_o}{v_i} = \frac{-\alpha R_C}{r_e}$$

CC Amp



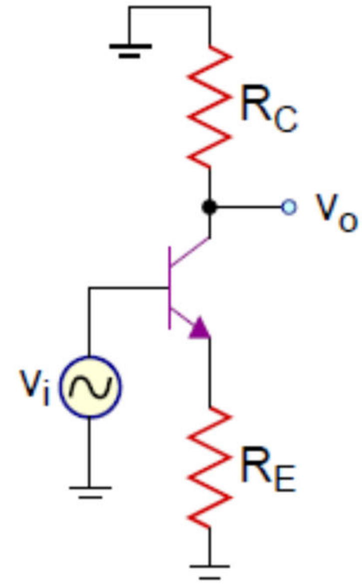
$$\frac{v_o}{v_i} = \frac{R_E}{r_e + R_E}$$

CB Amp



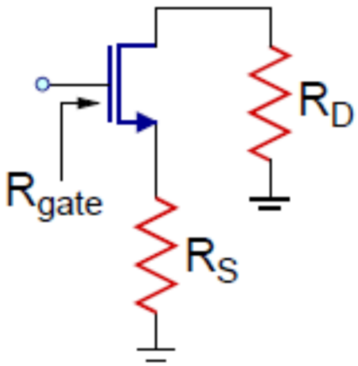
$$\frac{v_o}{v_i} = \frac{\alpha R_C}{R_{emitter}}$$

CE Amp w/ R_E



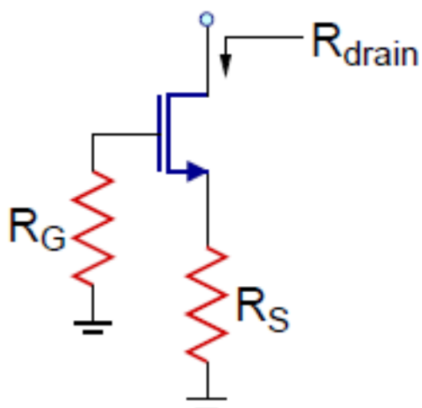
$$\frac{v_o}{v_i} = \frac{-\alpha R_C}{r_e + R_E}$$

MOSFET Node AC Resistances ($r_o=\infty$)



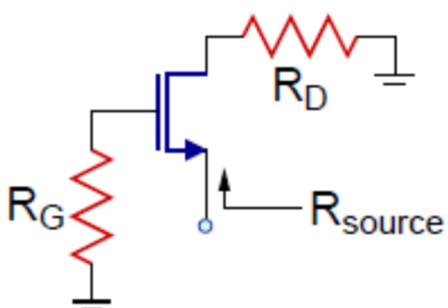
$$R_{gate} = \infty$$

- Gate input impedance is capacitive, which is assumed to be infinite resistance at low frequencies



$$R_{drain} = \infty$$

- If r_o is ∞, then drain AC resistance is ∞

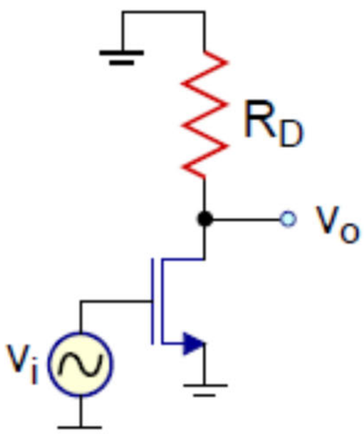


$$R_{source} = \frac{1}{g_m}$$

- Using T-model, source AC resistance is the 1/gm resistor

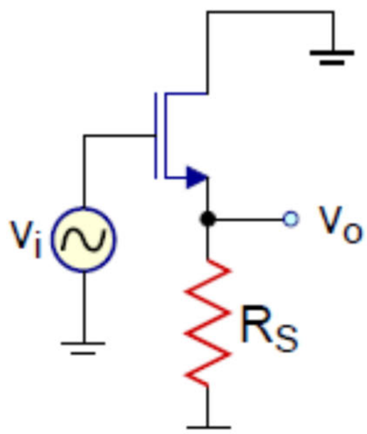
MOSFET Amplifiers AC Gain ($r_o=\infty$)

CS Amp



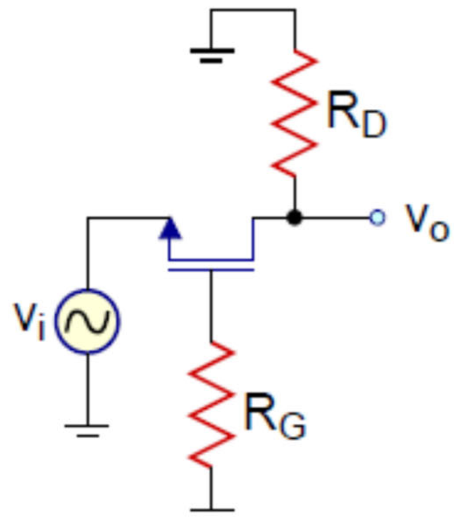
$$\frac{v_o}{v_i} = \frac{-R_D}{\frac{1}{g_m}}$$

CD Amp



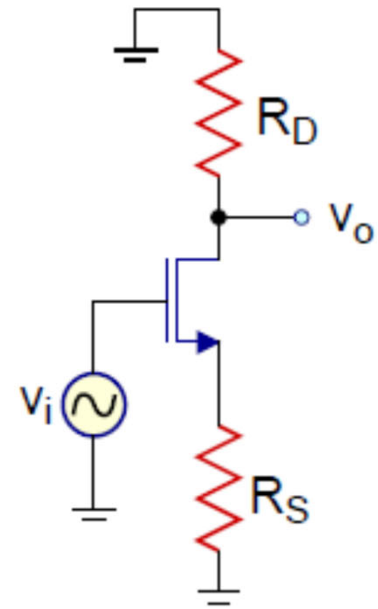
$$\frac{v_o}{v_i} = \frac{R_S}{\frac{1}{g_m} + R_S}$$

CG Amp



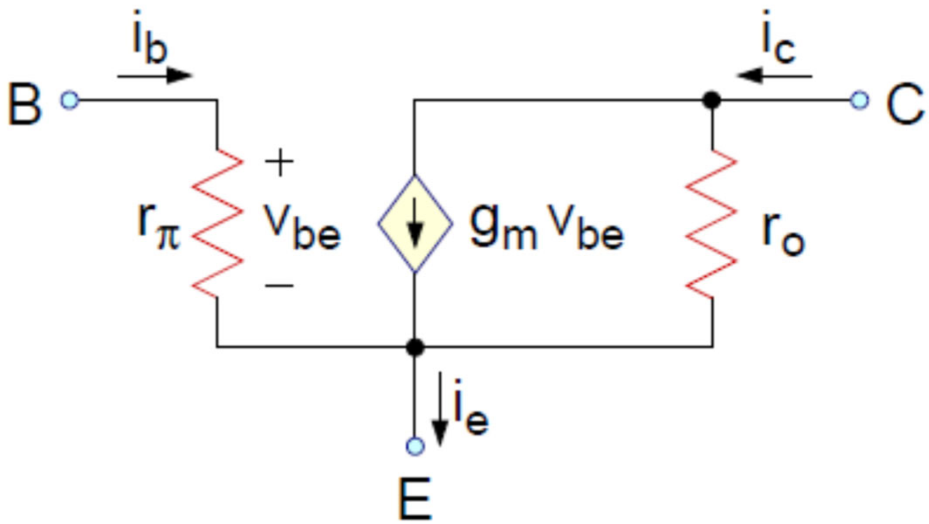
$$\frac{v_o}{v_i} = \frac{R_D}{R_{source}}$$

CS Amp w/ R_S



$$\frac{v_o}{v_i} = \frac{-R_D}{\frac{1}{g_m} + R_S}$$

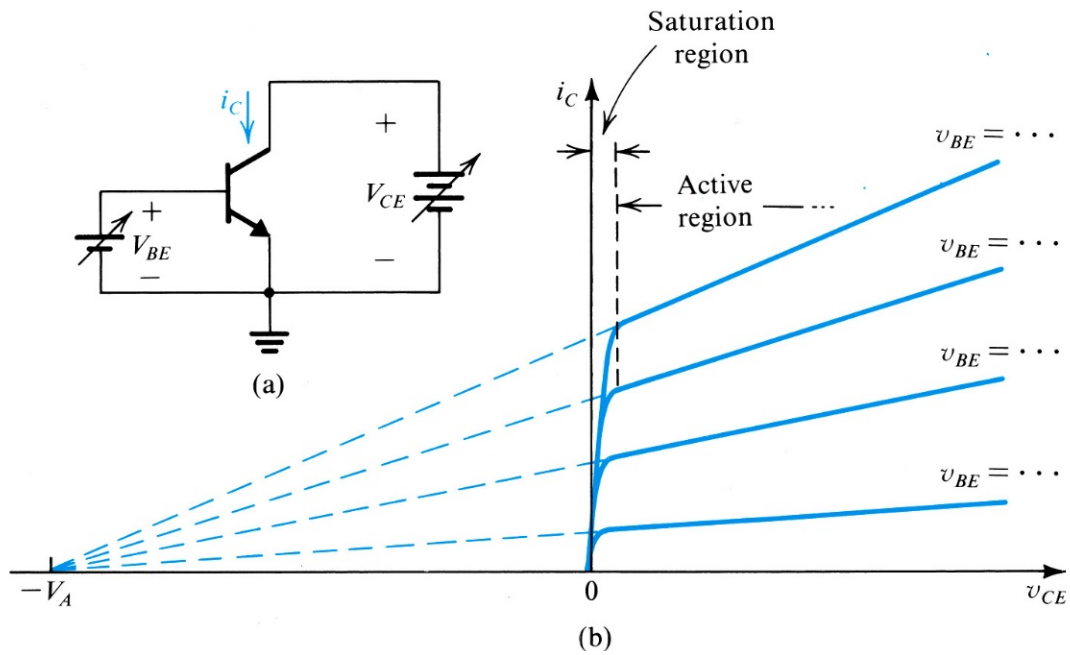
NPN Small-Signal π Model & Introducing Finite r_o



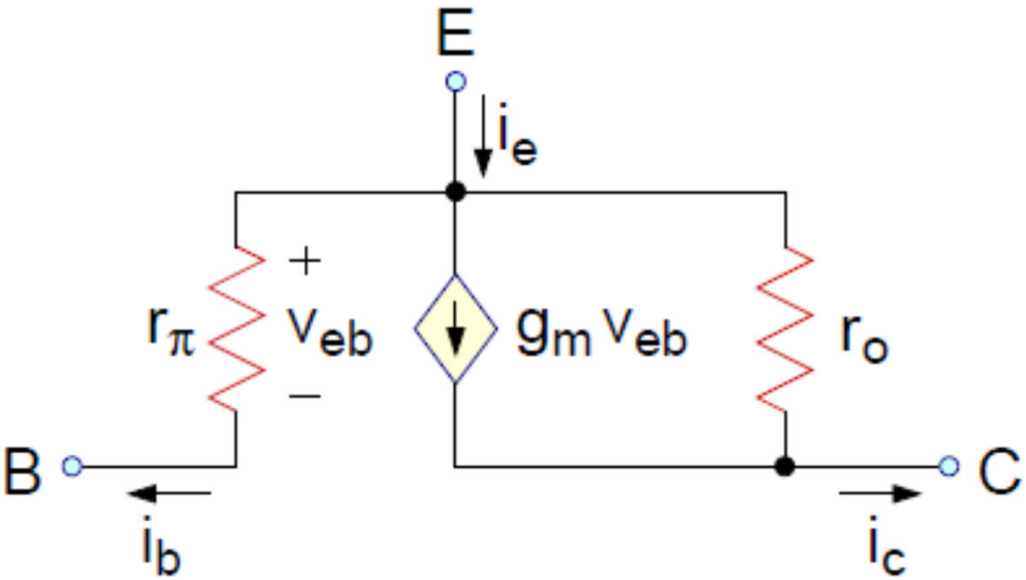
Early Effect → r_o

$$g_m = \frac{I_C}{V_T} \qquad r_o = \frac{V_A}{I_C}$$

$$r_\pi = \frac{\beta}{g_m} \qquad V_T = \frac{kT}{q}$$



PNP Small-Signal π Model w/ Finite r_o



Early Effect $\rightarrow r_o$

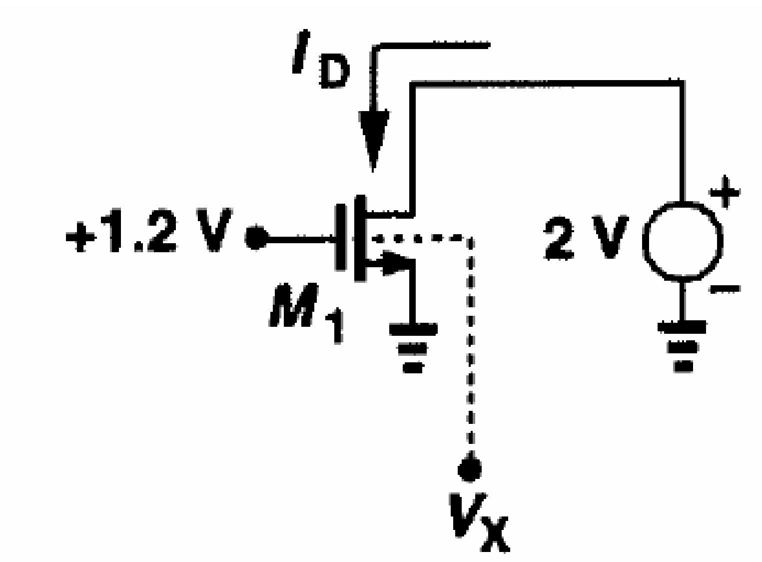
$$g_m = \frac{I_C}{V_T} \qquad r_o = \frac{V_A}{I_C}$$

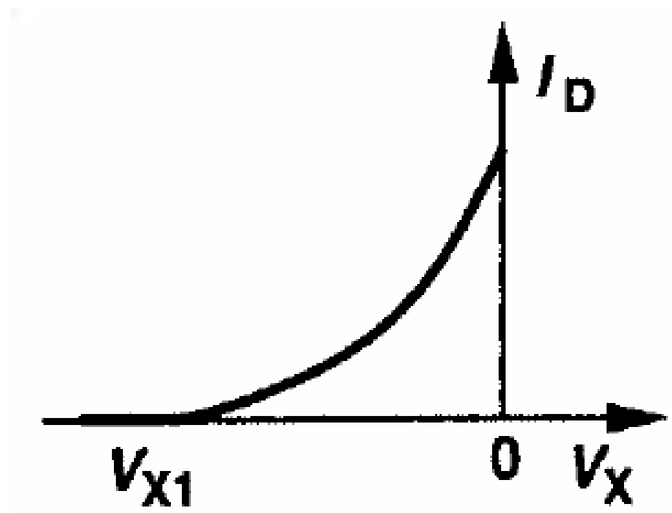
$$r_\pi = \frac{\beta}{g_m} \qquad V_T = \frac{kT}{q}$$

- While this is the formal model, **you can use the exact same model as the NPN device**
  - This is easier to remember
  - Obtained by flipping the small signal v_{eb} and the current source in the formal model

MOSFET – Impact of Body Voltage

- Before we go over the MOSFET π-model, lets consider the impact of the 4th terminal, the Body, on the drain current I_D
- The MOSFET V_{tn} is a function of the Body-Source voltage V_{BS}
  - If the threshold voltage changes, then so does I_D





$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{tn})^2(1 + \lambda V_{DS})$$

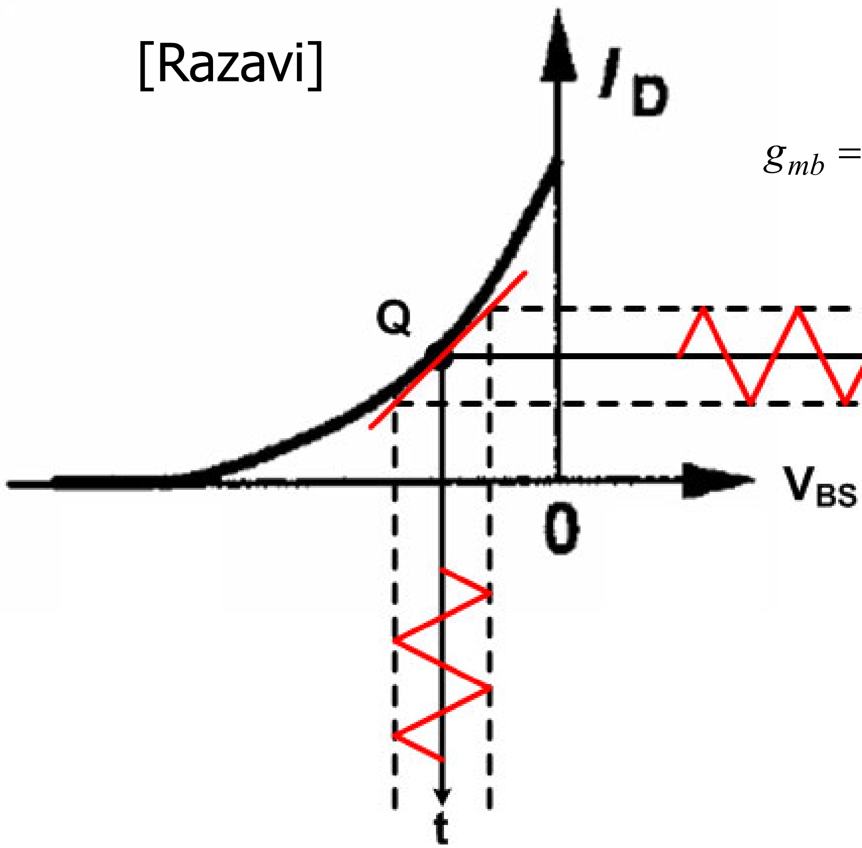
[Razavi]

$$V_{tn} = V_{tn0} + \gamma\left[\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}\right] \Rightarrow V_{tn0}\big|_{V_{SB}=0}$$

Body Transconductance, g_{mb}

- The small-signal drain current changes with V_{BS} modulation due to changes in V_{tn}

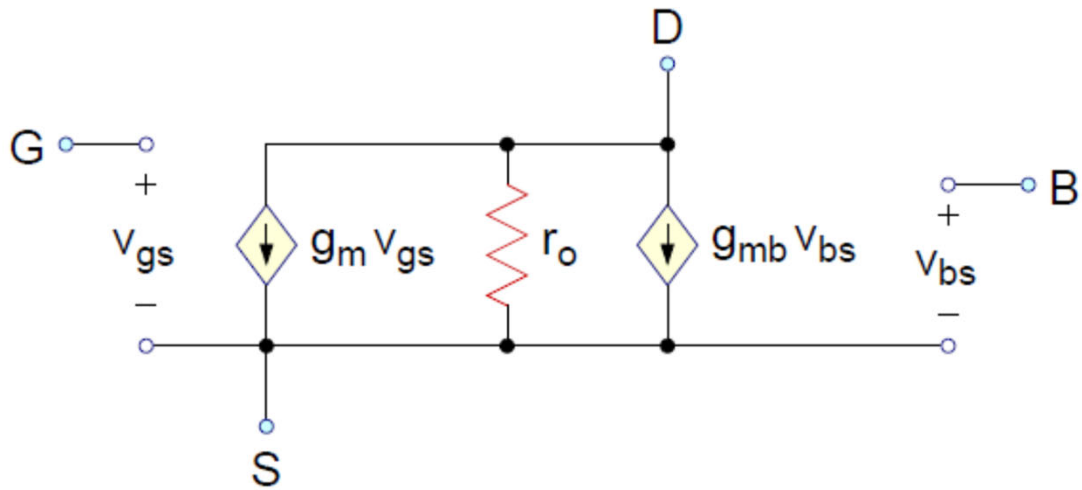
[Razavi]



In Saturation (Neglecting λ Effects)

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{bs}}\right|_Q \approx \mu C_{OX} \left.\frac{W}{L_{eff}}\left[V_{GS} - V_{tn}\right]\right|_Q * \left(-\left.\frac{\partial V_{tn}}{\partial v_{bs}}\right|_Q\right) \cong \frac{\gamma g_m}{2\sqrt{2\phi_F + V_{SB}}}$$

NMOS Small-Signal π Model w/ Finite r_o & Body Transconductance g_{mb}



Channel Length Modulation $\rightarrow r_o$

Body Effect $V_{tn} = V_{tn0} + \gamma\left[\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}\right] \Rightarrow V_{tn0}\big|_{V_{SB}=0} \rightarrow g_{mb}$

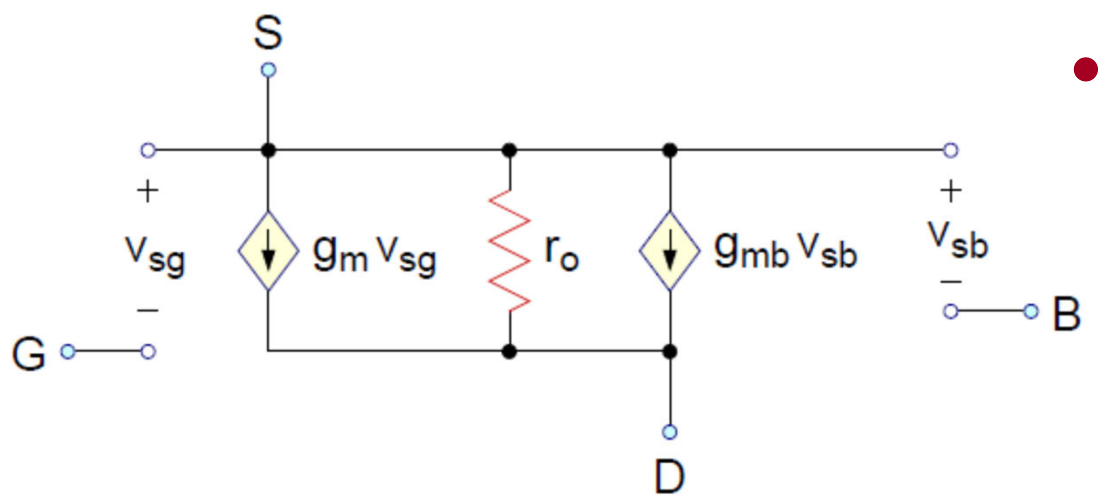
$$g_m = \sqrt{2k_n' \frac{W}{L} I_D} \qquad r_o = \frac{1}{\lambda_n I_D}$$

$$g_{mb} = \chi g_m \qquad k_n' = \mu_n C_{ox}$$

where $\chi = \dfrac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$

- Note, g_{mb} is generally a weak effect (~$0.1g_m$). Thus, we often ignore it.
- In problems/assignments I'll make it clear when we need to consider it

PMOS Small-Signal π Model w/ Finite r_o & Body Transconductance g_{mb}



- While this is the formal model, **you can use the exact same model as the NMOS device**
  - This is easier to remember
  - Obtained by flipping the small signal v_{sg} and v_{sb} and the current sources in the formal model

Channel Length Modulation $\rightarrow$ r_o

Body Effect $V_{tp} = V_{tp0} + \gamma\left[\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}\right] \Rightarrow V_{tp0}\big|_{V_{SB}=0}$ $\rightarrow$ g_{mb}

$$g_m = \sqrt{2k_p' \frac{W}{L} I_D} \qquad r_o = \frac{1}{\lambda_p I_D}$$

$$g_{mb} = \chi g_m \qquad k_p' = \mu_p C_{ox}$$

$$\text{where } \chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$$

- Note, g_{mb} is generally a weak effect (~$0.1g_m$). Thus, we often ignore it.
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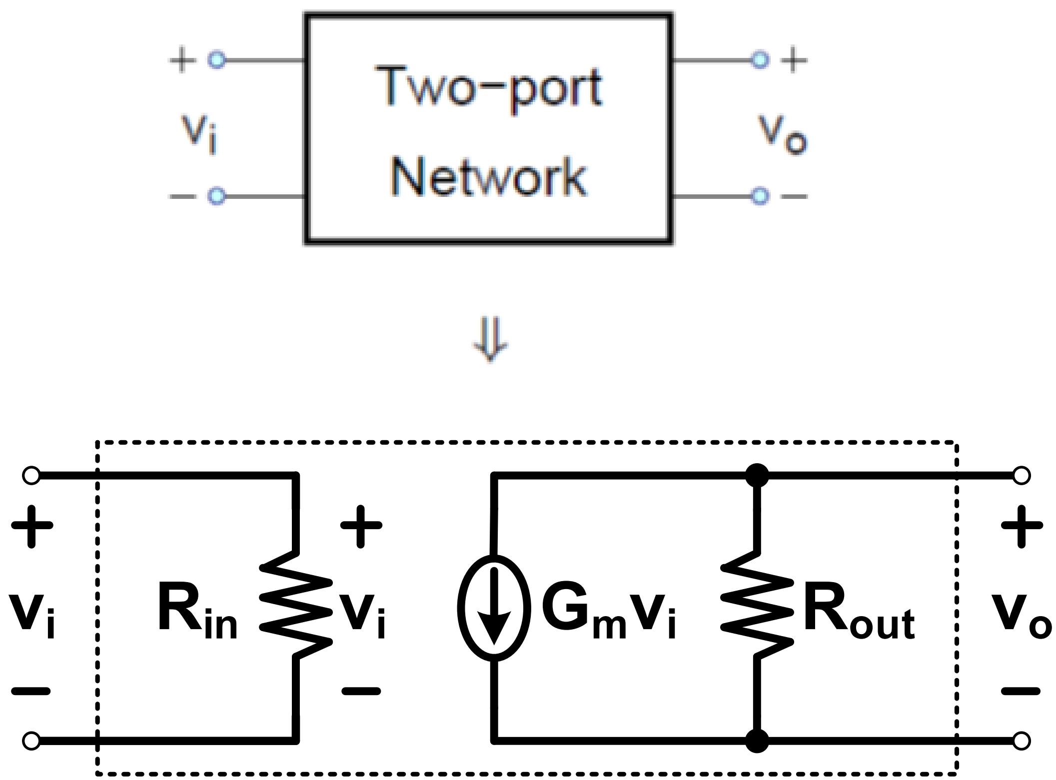
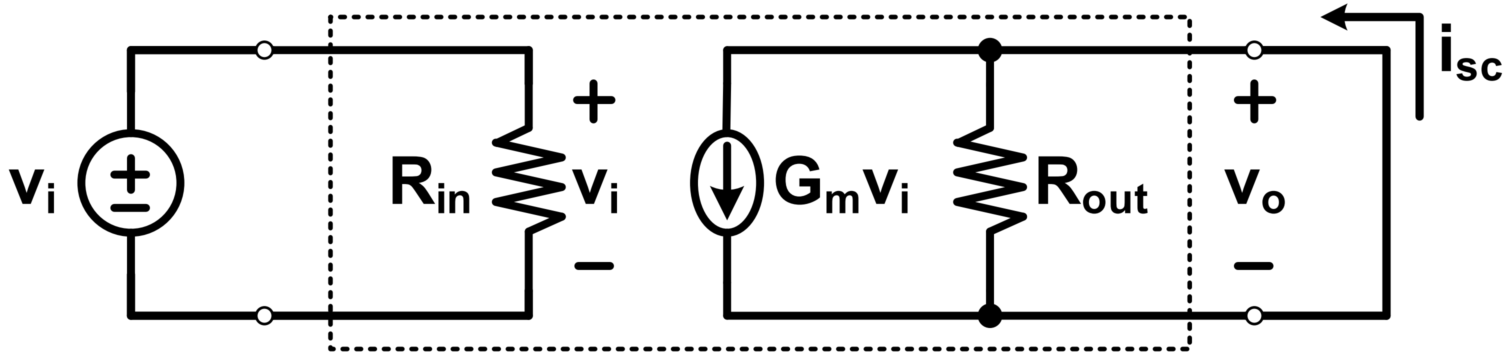
Two-Port Modeling of Amplifiers



$$\frac{v_o}{v_i} = -G_m R_{out}$$

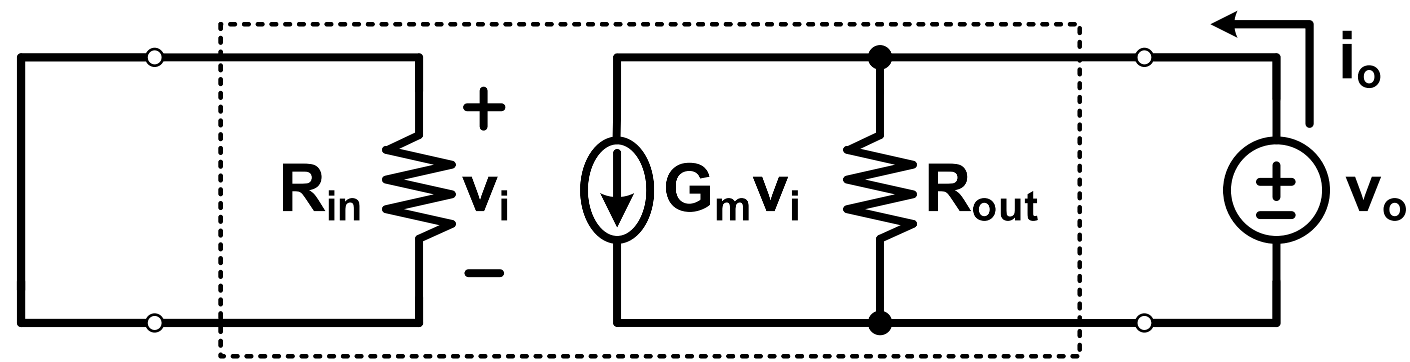
- It is often useful to model transistor amplifiers with a Norton-equivalent model consisting of a transconductance current source and parallel output resistance

Two-Port Modeling – Extracting G_m and R_n



$$G_m = \left.\frac{i_{sc}}{v_i}\right|_{v_o=0}$$

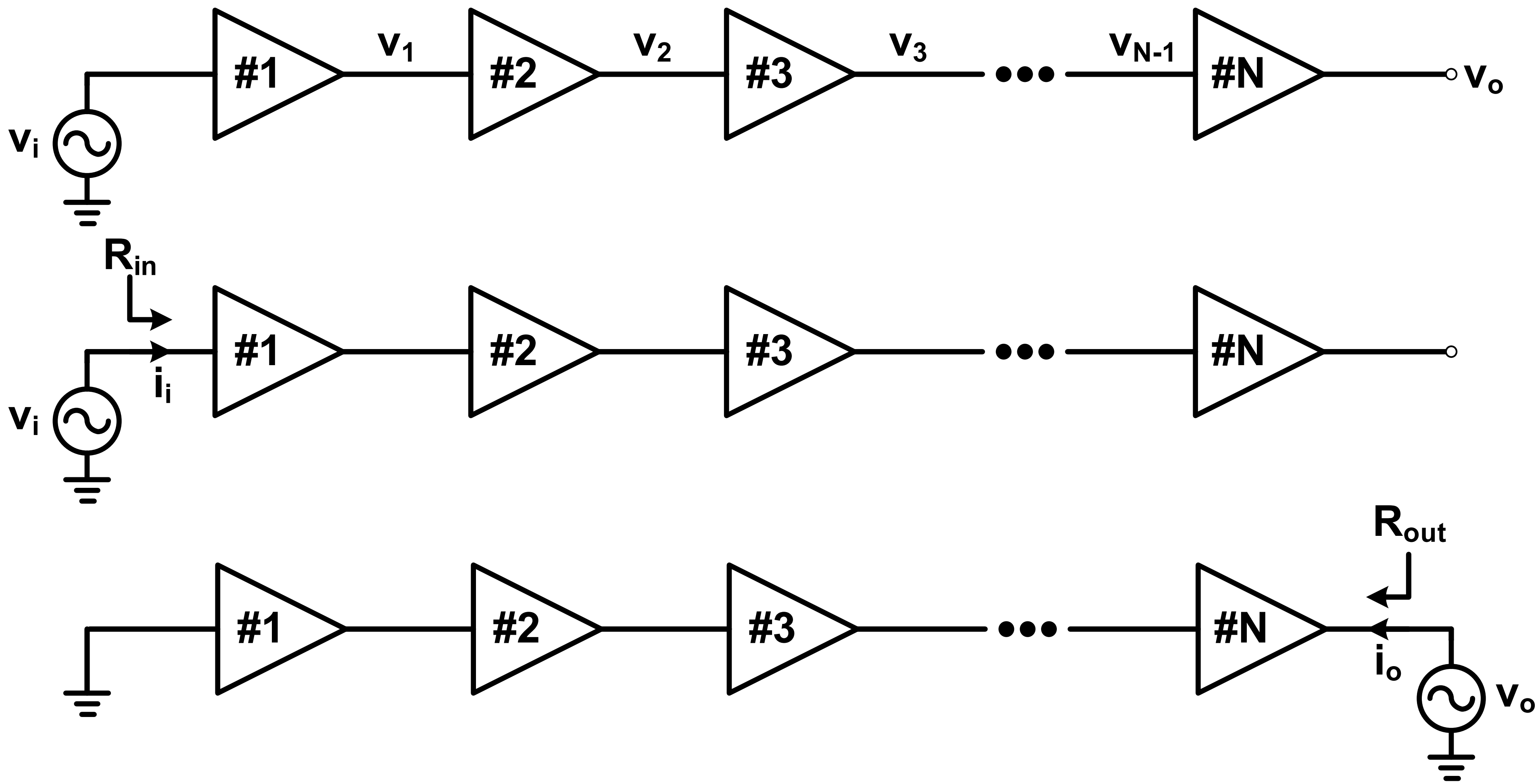
- Short the output, apply an input voltage stimulus, and measure output current



$$R_{out} = \left.\frac{v_o}{i_o}\right|_{v_i=0}$$

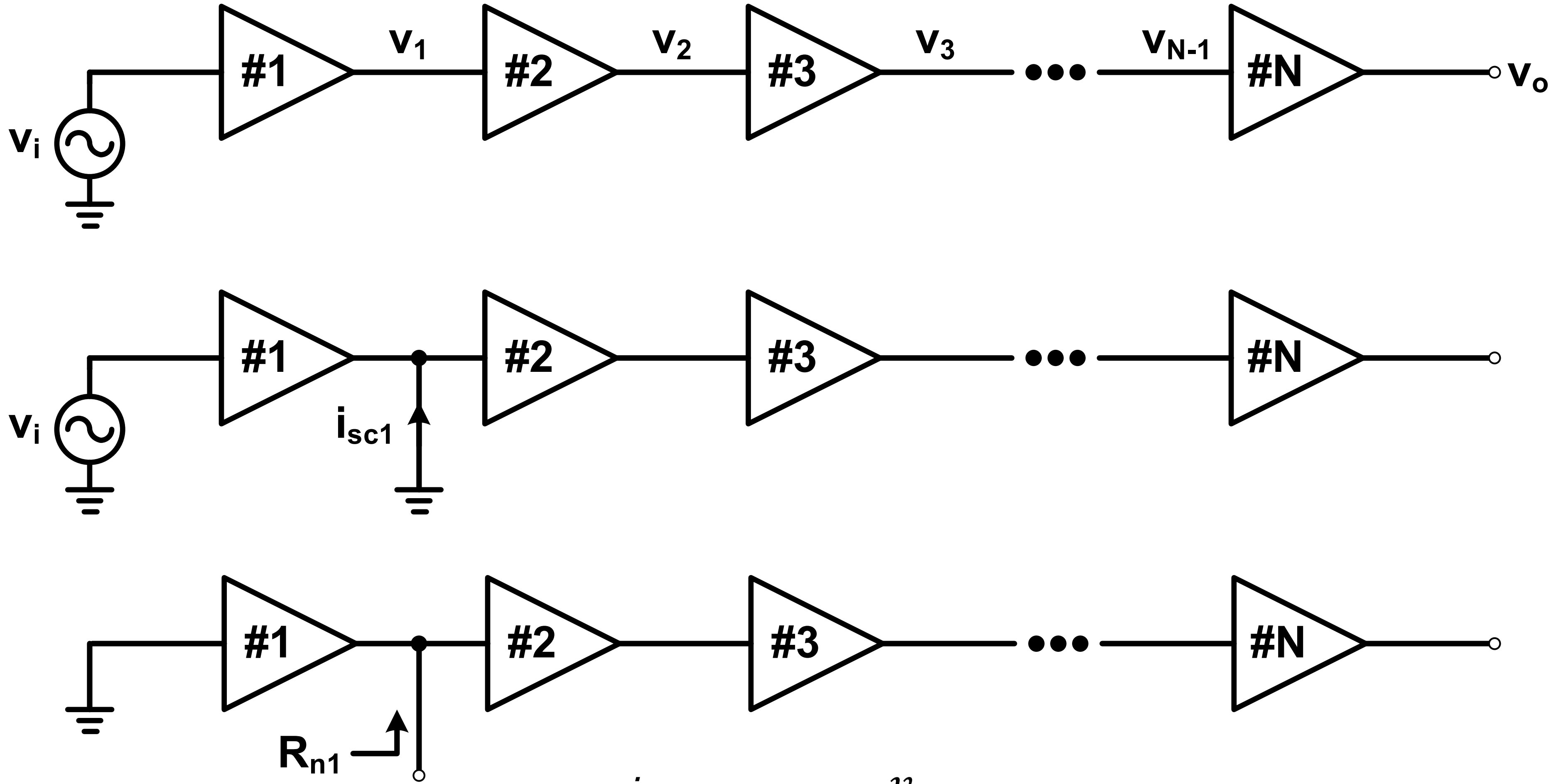
- Short the input, apply an output voltage stimulus, and measure current into output node

Two-Port Modeling – Multi-Stage Amplifiers



$$\frac{v_o}{v_i} = \frac{v_1}{v_i}\frac{v_2}{v_1}\cdots\frac{v_o}{v_{N-1}} \qquad R_{in} = \frac{v_i}{i_i} \qquad R_{out} = \frac{v_o}{i_o}$$

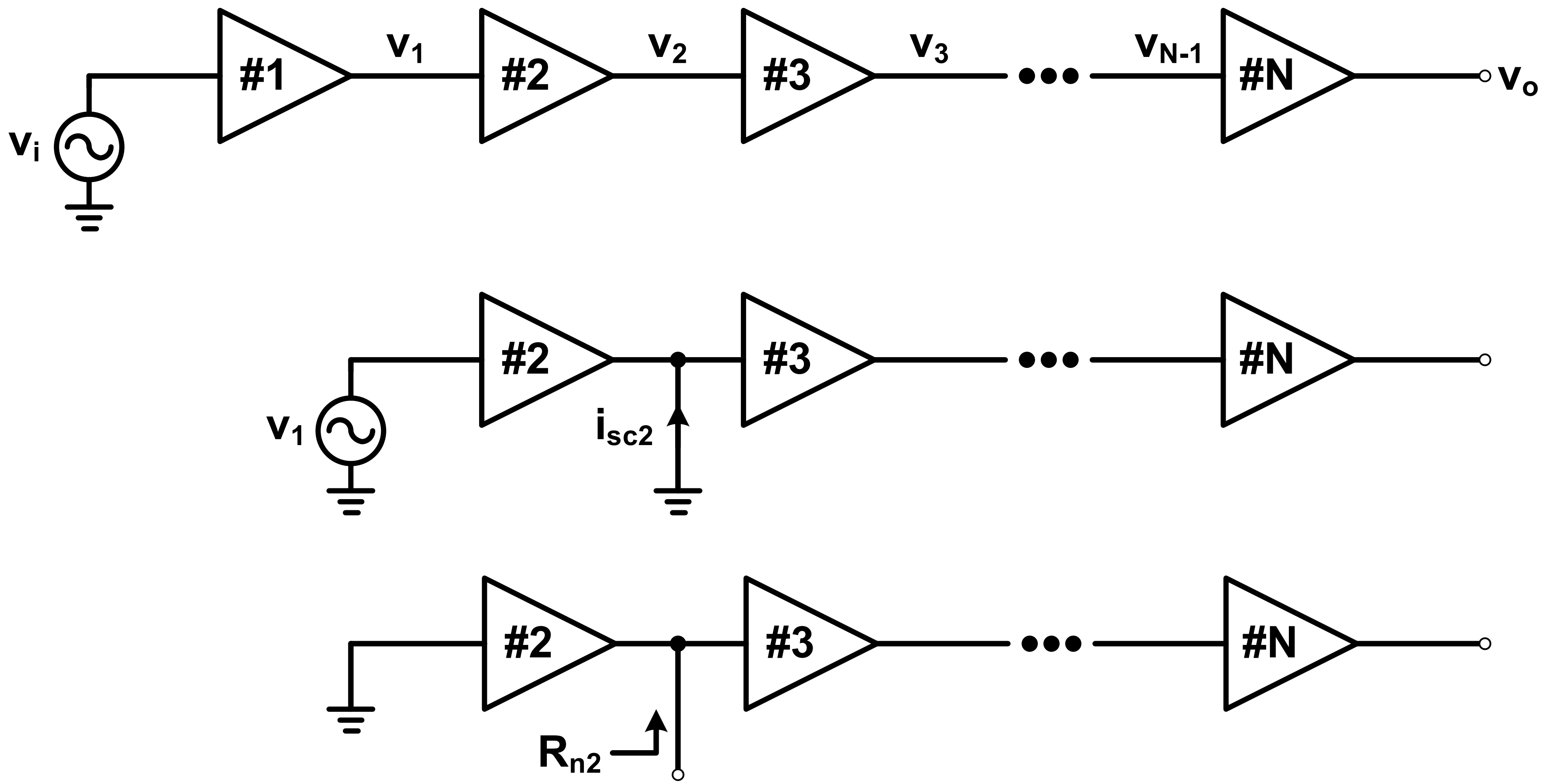
Two-Port Modeling – Multi-Stage Amplifiers



$$G_{m1} = \frac{i_{sc1}}{v_i} \qquad \frac{v_1}{v_i} = -G_{m1}R_{n1}$$

- Note, Rn1 also includes the input resistance of Stage 2

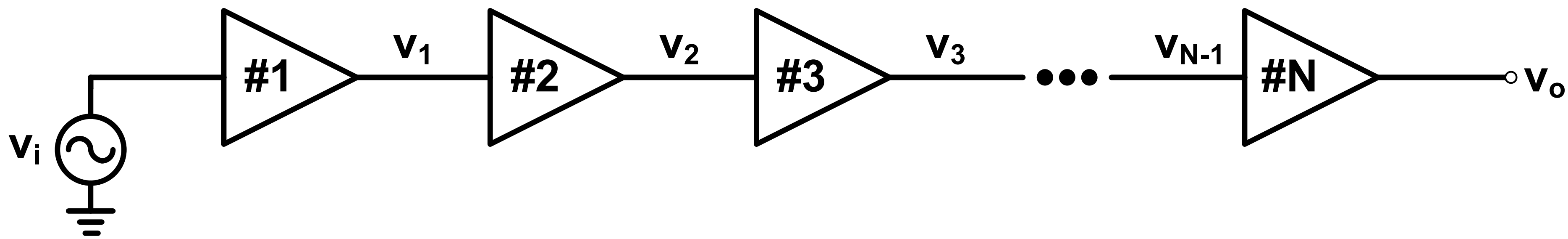
Two-Port Modeling – Multi-Stage Amplifiers

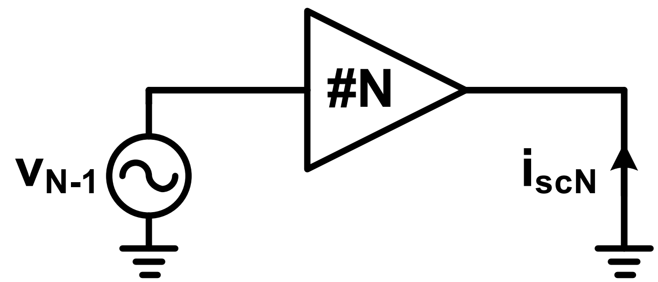


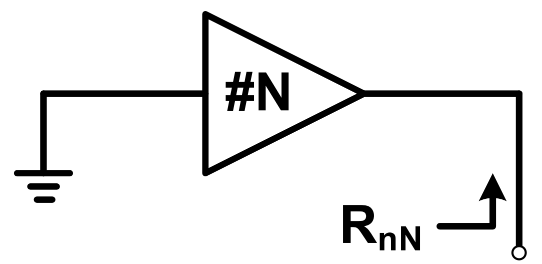
$$G_{m2} = \frac{i_{sc2}}{v_1} \qquad \frac{v_2}{v_1} = -G_{m2}R_{n2}$$

- Repeat procedure stage by stage

Two-Port Modeling – Multi-Stage Amplifiers





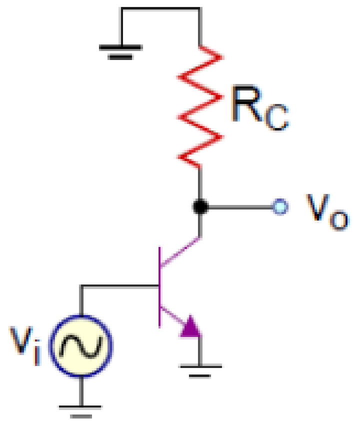


$$G_{mN} = \frac{i_{scN}}{v_{N-1}} \qquad \frac{v_o}{v_{N-1}} = -G_{mN} R_{nN}$$

- Repeat procedure stage by stage
- This procedure will be useful for analyzing multi-stage transistor amplifiers

BJT Amplifiers AC Gain ($r_o=\infty$)

CE Amp

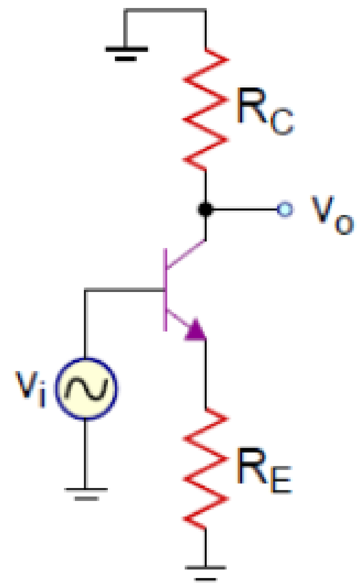


$$\frac{v_o}{v_i} = -g_m R_C = -\frac{\alpha R_C}{r_e}$$

$$G_m = g_m = \frac{\alpha}{r_e}$$

$$R_n = R_C$$

CE Amp w/ R_E



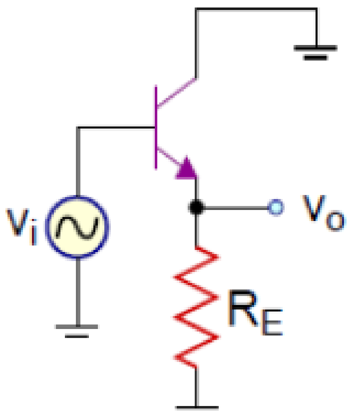
$$\frac{v_o}{v_i} = -\frac{g_m R_C}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha R_C}{r_e + R_E}$$

$$G_m = \frac{g_m}{1 + \frac{g_m R_E}{\alpha}} = \frac{\alpha}{r_e + R_E}$$

$$R_n = R_C$$

BJT Amplifiers AC Gain ($r_o=\infty$)

CC Amp

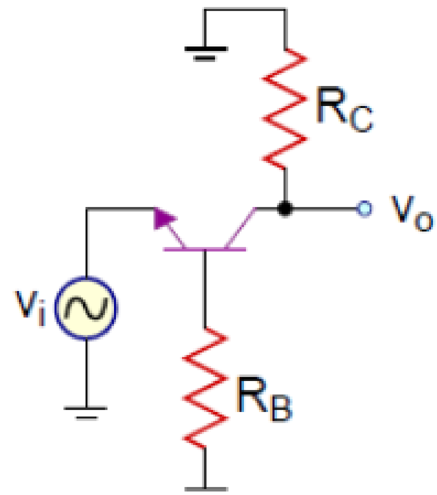


$$\frac{v_o}{v_i} = \frac{R_E}{r_e + R_E}$$

$$G_m = -\frac{1}{r_e}$$

$$R_n = r_e \| R_E$$

CB Amp



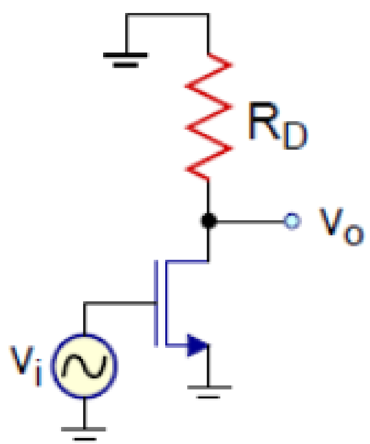
$$\frac{v_o}{v_i} = \frac{g_m R_C}{1 + \frac{g_m R_B}{\beta}} = \frac{\alpha R_C}{R_{emitter}}$$

$$G_m = -\frac{\beta}{r_e(\beta+1) + R_B} = -\frac{\alpha}{R_{emitter}} = -\frac{g_m}{1 + \frac{g_m R_B}{\beta}}$$

$$R_n = R_C$$

MOSFET Amplifiers AC Gain ($r_o=\infty$)

CS Amp

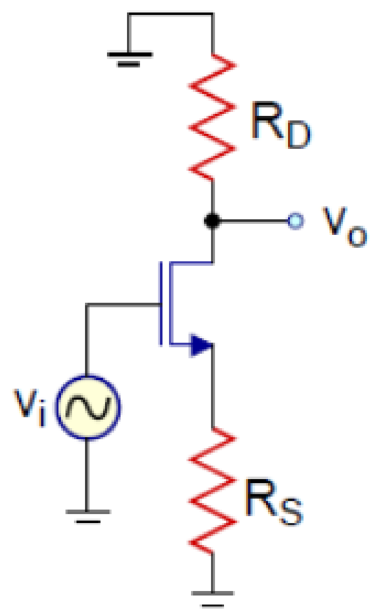


$$\frac{v_o}{v_i} = -g_m R_D$$

$$G_m = g_m$$

$$R_n = R_D$$

CS Amp w/ R_S



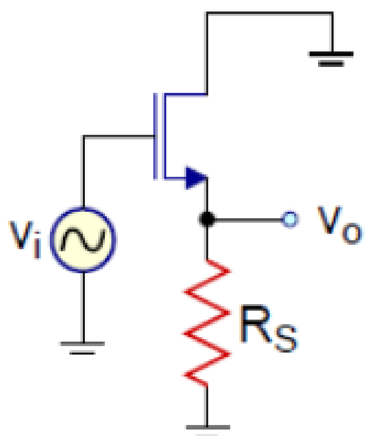
$$\frac{v_o}{v_i} = -\frac{g_m R_D}{1 + g_m R_S}$$

$$G_m = \frac{g_m}{1 + g_m R_S}$$

$$R_n = R_D$$

MOSFET Amplifiers AC Gain ($r_o=\infty$)

CD Amp

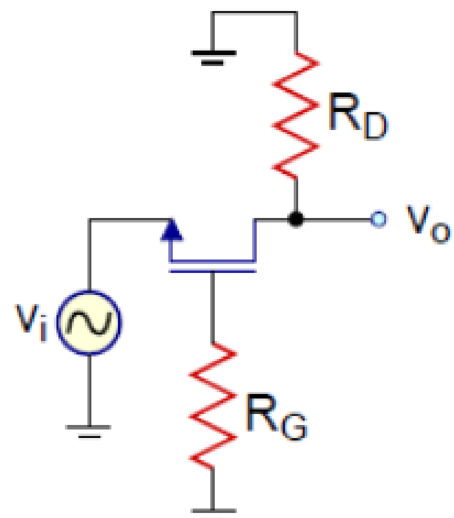


$$\frac{v_o}{v_i} = \frac{g_m R_S}{1 + g_m R_S}$$

$$G_m = -g_m$$

$$R_n = \frac{1}{g_m} \Big\| R_S$$

CG Amp

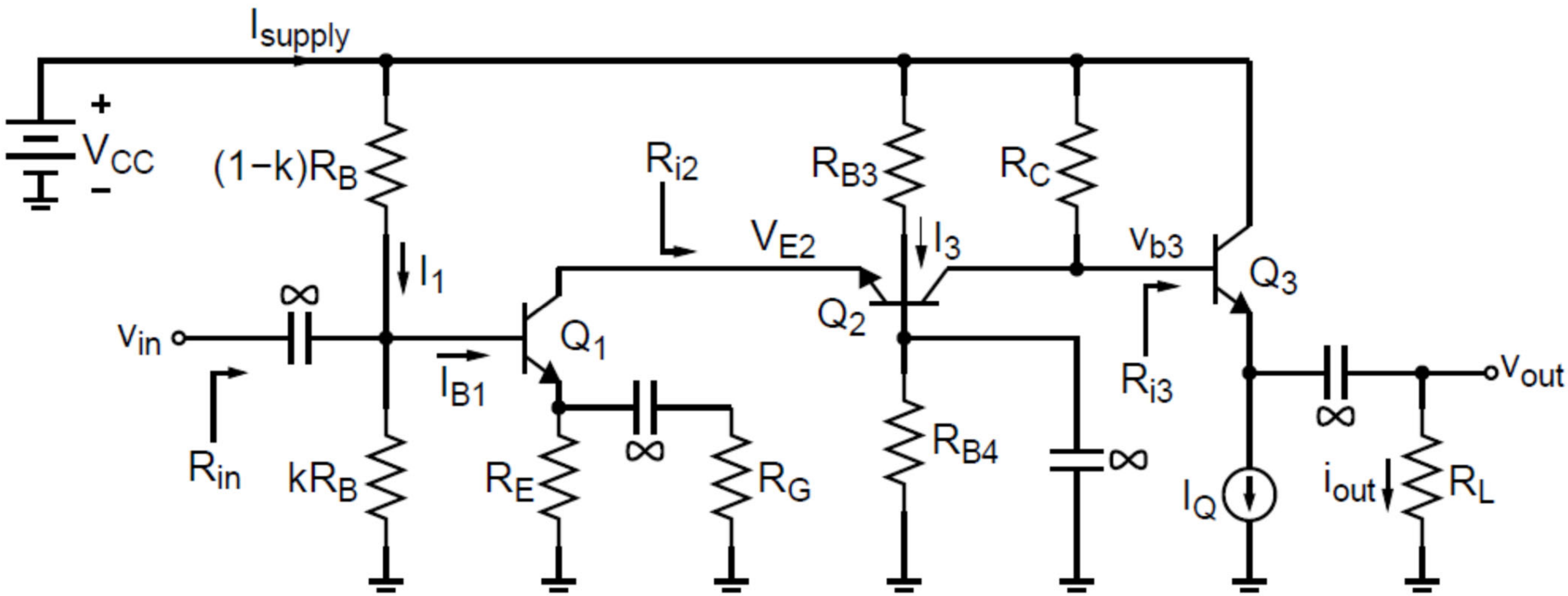


$$\frac{v_o}{v_i} = g_m R_D = \frac{R_D}{R_{source}}$$

$$G_m = -g_m = -\frac{1}{R_{source}}$$

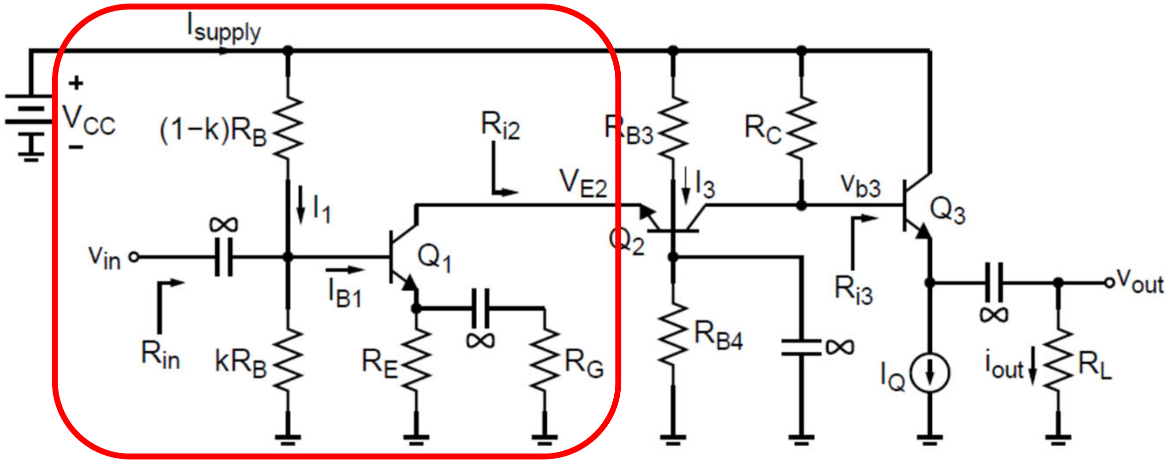
$$R_n = R_D$$

3-Stage BJT Amplifier Example

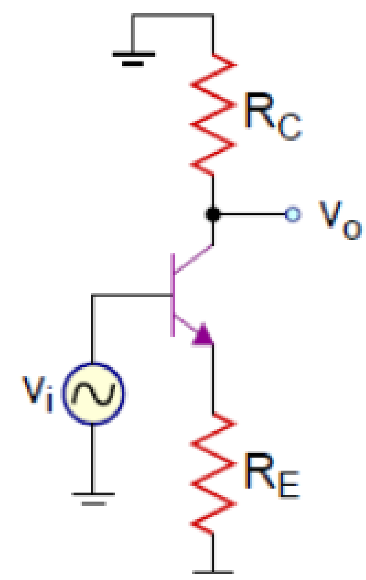


- This multi-stage amplifier consists of common-emitter, common-base, and common-collector amplifier
- The first two common-emitter and common-base stages are commonly used together, and are called a “cascode amplifier”
- The cascode stage provides all the voltage gain of the circuit, while the output common-collector circuit allows driving of a low-resistance load

3-Stage BJT Amplifier Example – 1st Stage Av



CE Amp w/ R_E



- Using the common-emitter amplifier equations, the gain from the input to V_{E2} is

$$A_{v1} = \frac{v_{e2}}{v_{in}} = -G_{m1} R_{n1}$$

$$G_{m1} = \frac{g_{m1}}{1 + \frac{g_{m1}(R_E \| R_G)}{\alpha}} = \frac{\alpha}{r_{e1} + R_E \| R_G}$$

$$R_{n1} = R_{i2} = r_{e2}$$

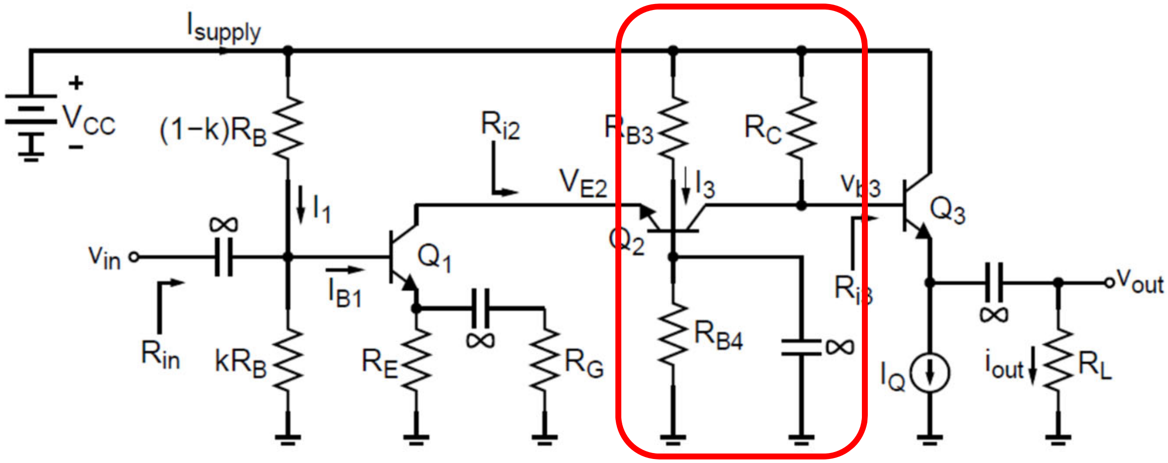
$$A_{v1} = -\frac{\alpha R_{i2}}{r_{e1} + R_E \| R_G}$$

$$\frac{v_o}{v_i} = -\frac{g_m R_C}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha R_C}{r_e + R_E}$$

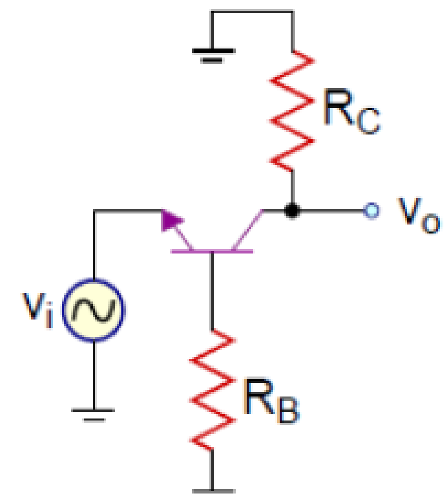
$$G_m = \frac{g_m}{1 + \frac{g_m R_E}{\alpha}} = \frac{\alpha}{r_e + R_E}$$

$$R_n = R_C$$

3-Stage BJT Amplifier Example – 2nd Stage Av



CB Amp



- Using the common-base amplifier equations, the gain from V_{E2} to V_{B3} is

$$A_{v2} = \frac{v_{b3}}{v_{e2}} = -G_{m2}R_{n2}$$

$$G_{m2} = -\frac{\beta}{r_{e2}(\beta+1)} = -\frac{\alpha}{R_{i2}} = -g_{m2}$$

$$R_{n2} = R_C \| R_{i3} = R_C \| [(\beta+1)(r_{e3}+R_L)]$$

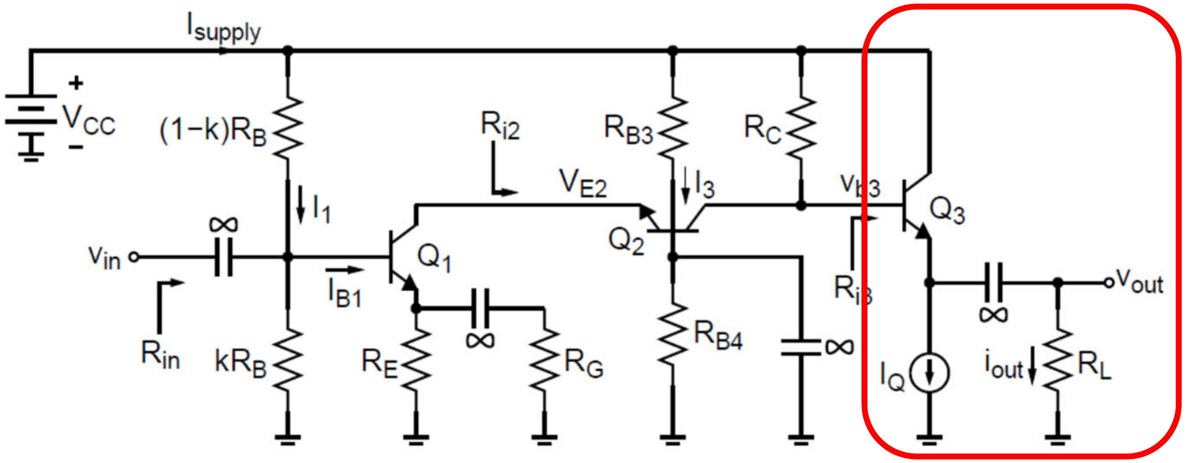
$$A_{v2} = \frac{\alpha(R_C \| [(\beta+1)(r_{e3}+R_L)])}{R_{i2}}$$

$$\frac{v_o}{v_i} = \frac{g_m R_C}{1+\frac{g_m R_B}{\beta}} = \frac{\alpha R_C}{R_{emitter}}$$

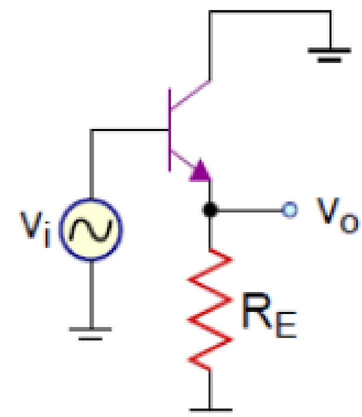
$$G_m = -\frac{\beta}{r_e(\beta+1)+R_B} = -\frac{\alpha}{R_{emitter}} = -\frac{g_m}{1+\frac{g_m R_B}{\beta}}$$

$$R_n = R_C$$

3-Stage BJT Amplifier Example – 3rd Stage Av



CC Amp



- Using the common-collector amplifier equations, the gain from V_{B3} to V_{out} is

$$A_{v3} = \frac{v_{out}}{v_{b3}} = -G_{m3}R_{n3}$$

$$G_{m3} = -\frac{1}{r_{e3}}$$

$$R_{n3} = r_{e3} \| R_L$$

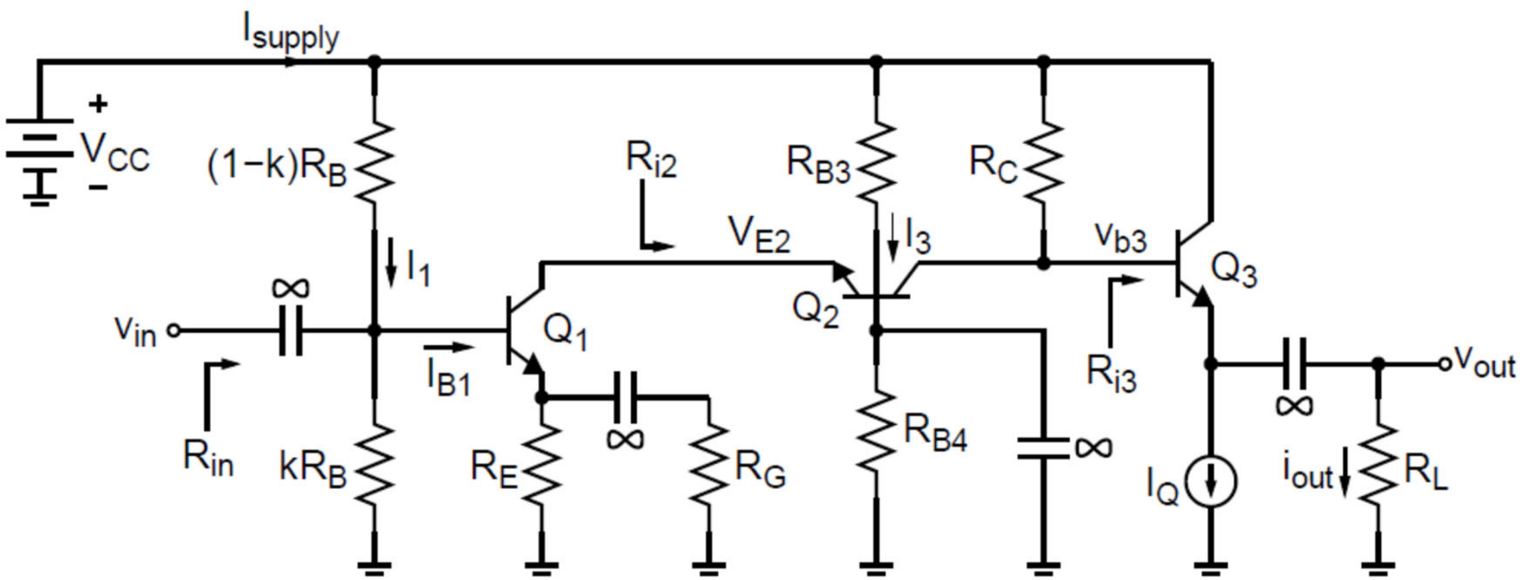
$$A_{v3} = \frac{R_L}{r_{e3} + R_L}$$

$$\frac{v_o}{v_i} = \frac{R_E}{r_e + R_E}$$

$$G_m = -\frac{1}{r_e}$$

$$R_n = r_e \| R_E$$

3-Stage BJT Amplifier Example – Total Av



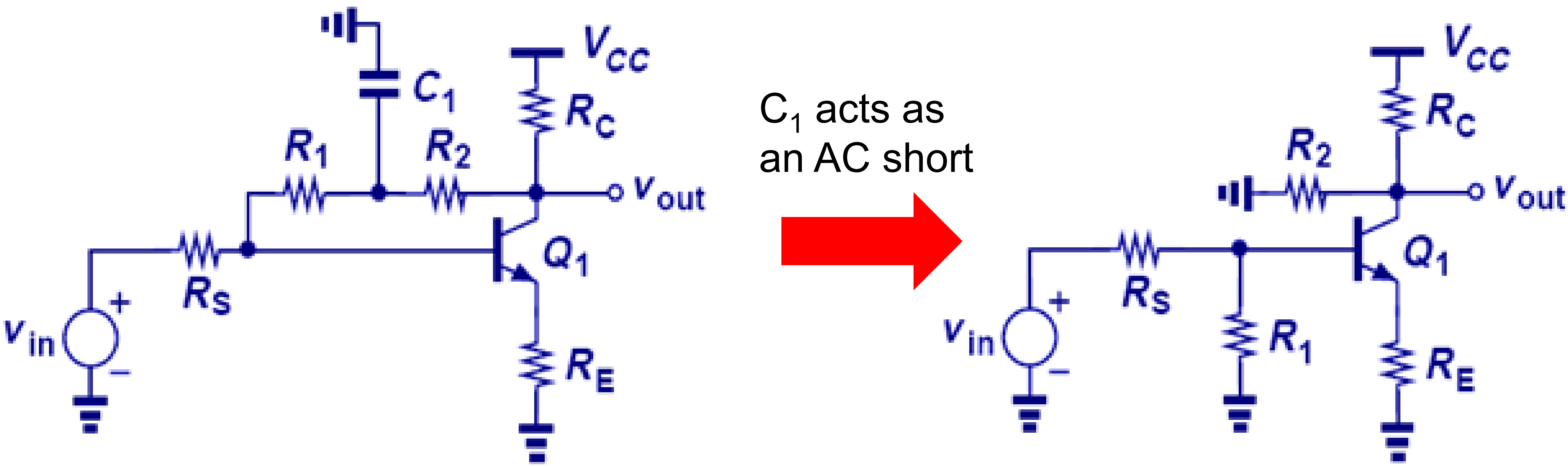
- The total gain is equal to the product of the individual stage gains

$$A_{v,tot} = \frac{v_{out}}{v_{in}} = A_{v1}A_{v2}A_{v3}$$

$$= \left(-\frac{\alpha R_{i2}}{r_{e1} + R_E \| R_G}\right)\left(\frac{\alpha\left(R_C \| R_{i3}\right)}{R_{i2}}\right)\left(\frac{R_L}{r_{e3} + R_L}\right)$$

$$\approx \left(-\frac{R_C \| R_{i3}}{r_{e1} + R_E \| R_G}\right)\left(\frac{R_L}{r_{e3} + R_L}\right)$$

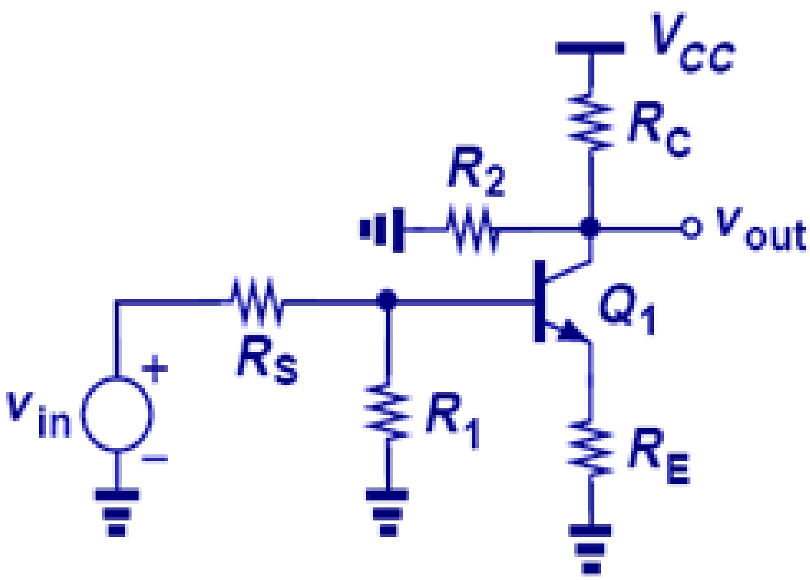
Amplifier Example I

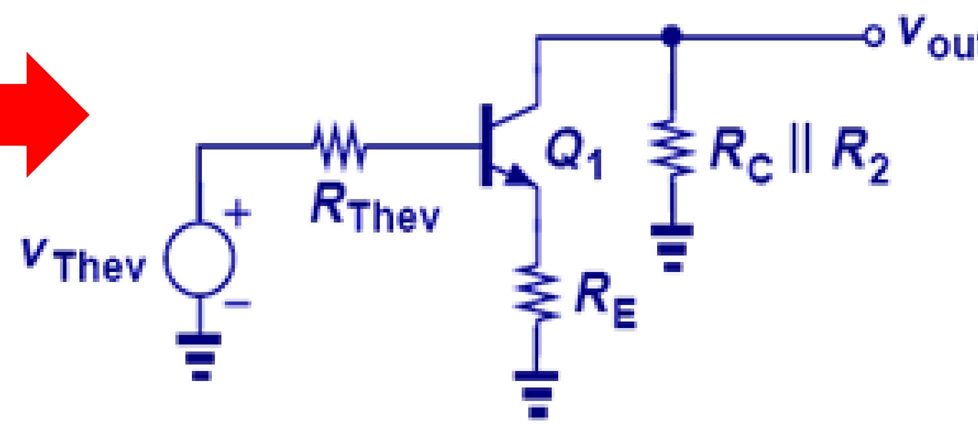


- **The keys in solving this problem are recognizing the AC ground between R_1 and R_2, and Thevenin transformation of the input network**
- **Then the common-emitter amplifer equations can be used**

Amplifier Example I

- Find the input Thevenin equivalent

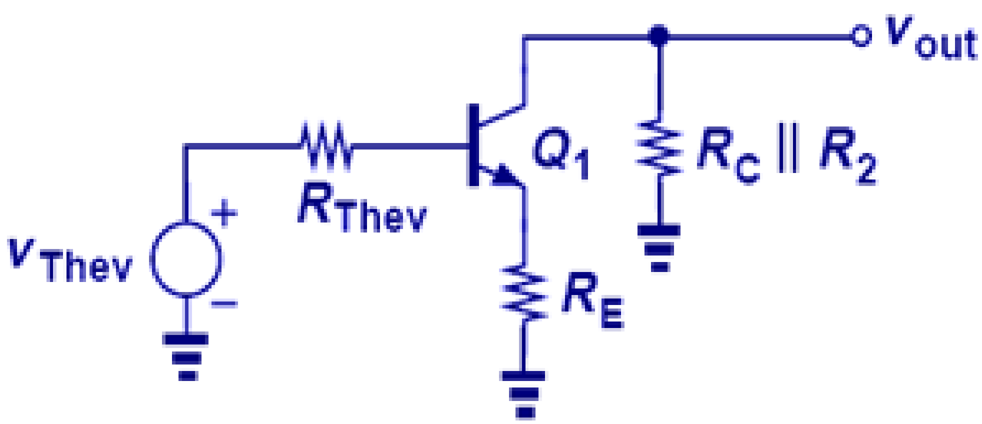




$$v_{Thev} = v_{in}\frac{R_1}{R_S + R_1} \qquad R_{Thev} = R_S \| R_1$$

Amplifier Example I

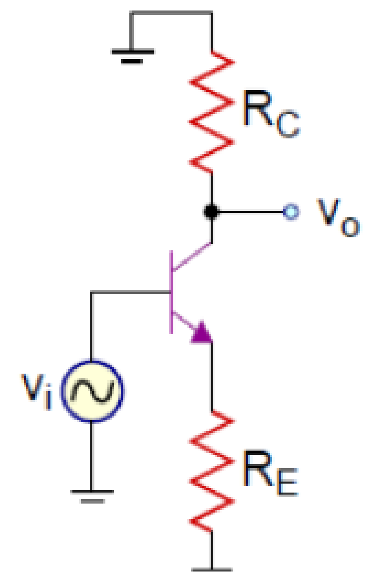
- Use the common-emitter amplifier equations



- As there is a base resistance (R_{Thev}), this must be reflected into the emitter by dividing by (β+1) to use the derived equation

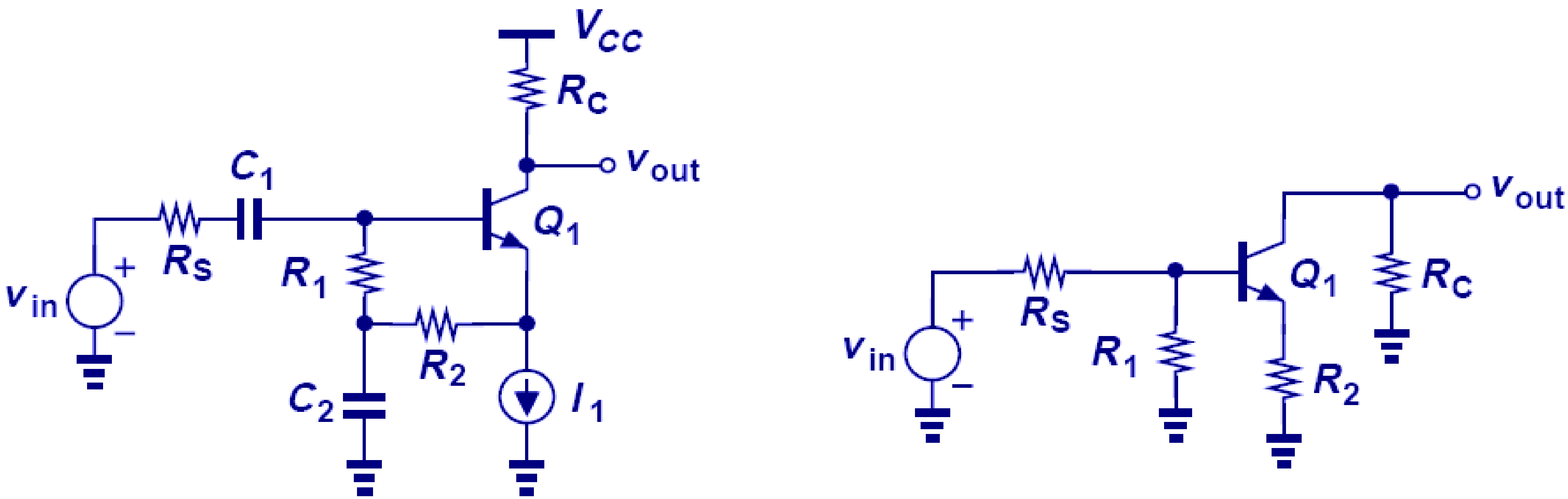
$$\frac{v_o}{v_i} = -\frac{\alpha(R_C \| R_2)}{\frac{R_1 \| R_S}{\beta+1} + r_e + R_E} \cdot \frac{R_1}{R_1 + R_S}$$

CE Amp w/ R_E



$$\frac{v_o}{v_i} = -\frac{g_m R_C}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha R_C}{r_e + R_E}$$

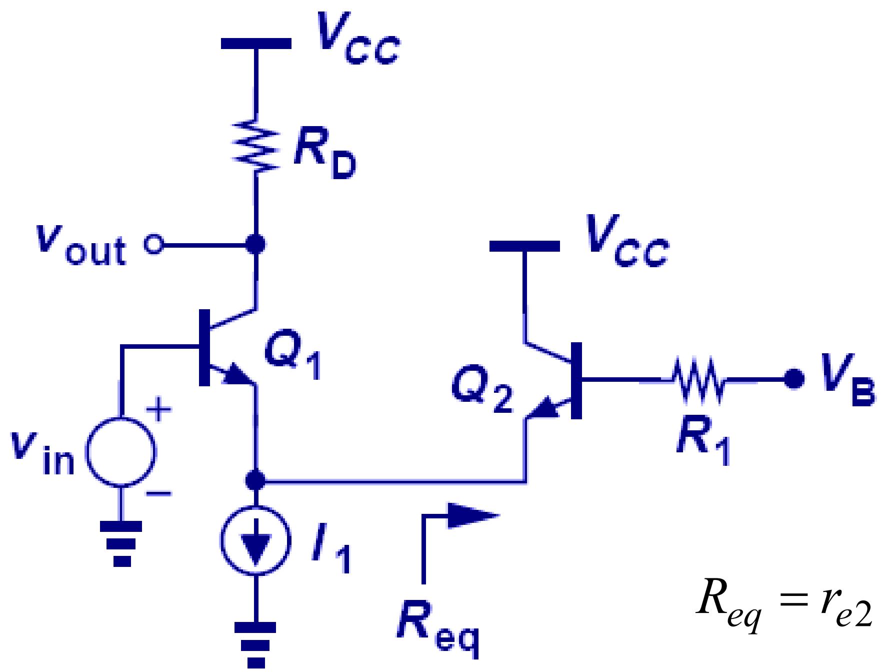
Amplifier Example II

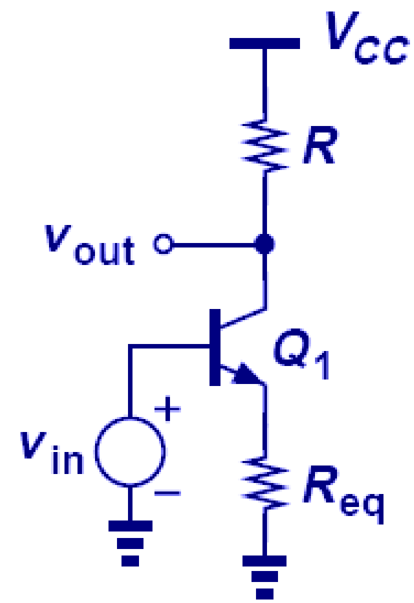


$$\frac{v_o}{v_i} = -\frac{\alpha R_C}{\frac{R_1 \| R_S}{\beta + 1} + r_e + R_2} \cdot \frac{R_1}{R_1 + R_S}$$

- **Again, AC ground/short and Thevenin transformation are needed to transform the complex circuit into a simple stage with emitter degeneration.**

Amplifier Example III





$$R_{eq} = r_{e2} + \frac{R_1}{\beta + 1}$$

$$R_{in} = (\beta + 1)(r_{e1} + R_{eq}) = (\beta + 1)\left(r_{e1} + r_{e2} + \frac{R_1}{\beta + 1}\right) = r_{\pi 1} + r_{\pi 2} + R_1$$

$$\frac{v_o}{v_i} = -\frac{\alpha R_D}{r_{e1} + r_{e2} + \dfrac{R_1}{\beta + 1}}$$

- **The key for solving this problem is first identifying R_{eq}, which is the impedance seen at the emitter of Q_2 in parallel with the infinite output impedance of an ideal current source**

Next Time

- Differential amplifiers
  - Razavi Chapter 10