ECEN326: Electronic Circuits
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Lecture 5: Frequency Response

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Announcements

• HW5 due 11/2

• Reading
  • Razavi Chapter 11
Agenda

- Frequency Response Concepts
- High-Frequency Models of Transistors
- Frequency Response Analysis Procedure
- CE and CS Stages
- CB and CG Stages
- CC and CD (Follower) Stages
- Cascode Stages
- Differential Pairs
- Additional Examples
As frequency of operation increases, the amplifier gain decreases.

This lecture analyzes this frequency response issue.
Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.
Example: Human Voice II

Path traveled by the human voice to the voice recorder

- Mouth
- Air
- Recorder

Path traveled by the human voice to the human ear

- Mouth
- Air
- Ear
- Skull

➢ Since the paths are different, the results will also be different.
Gain Roll-off: Simple Low-pass Filter

In this simple example, as frequency increases the impedance of $C_1$ decreases and the voltage divider consists of $C_1$ and $R_1$ attenuates $V_{in}$ to a greater extent at the output.

\[
V_o(s) = \frac{Z_C}{Z_R + Z_C} V_{in}(s) = \frac{1}{sC} V_{in}(s) = \frac{1}{1 + sRC} V_{in}(s)
\]

\[
H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}
\]

for sinusoidal steady-state response $s = j\omega$

\[
H(j\omega) = \frac{1}{1 + j\omega RC}
\]
Gain Roll-off: Common Source

The capacitive load, $C_L$, is the culprit for gain roll-off since at high frequency, it will “steal” away some signal current and shunt it to ground.

\[ V_{out} = -g_m V_{in} \left( R_D \parallel \frac{1}{C_L s} \right) \]

\[ H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R_D}{1 + s R_D C_L} \]

This circuit has a pole at

\[ |\omega_p| = \frac{1}{R_D C_L} \]
Frequency Response of the CS Stage

At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease.

A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB (half-power). In this single-pole circuit, this is also the pole frequency.

Recall the Power is proportional to $(Voltage)^2$

To find the half - power (-3dB) point relative to the low - frequency gain

$$\left| \frac{V_{out}}{V_{in}} \right|^2 = \frac{(g_m R_D)^2}{(R_D C_L \omega)^2 + 1} = \frac{(g_m R_D)^2}{2}$$

Solving for $\omega$

$$\omega = \frac{1}{R_D C_L}$$
Example: Relationship between Frequency Response and Step Response

\[ |H(s = j\omega)| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}} \]

\[ V_{out}(t) = V_0 \left( 1 - \exp \left( \frac{-t}{R_1 C_1} \right) \right) u(t) \]

- The relationship is such that as \( R_1 C_1 \) increases, the bandwidth drops and the step response becomes slower.

CH 11 Frequency Response
When we hit a zero, $\omega_{zj}$, the Bode magnitude rises with a slope of $+20\text{dB/dec}$. When we hit a pole, $\omega_{pj}$, the Bode magnitude falls with a slope of $-20\text{dB/dec}$.
Example: Bode Plot

The circuit only has one pole (no zero) at \(1/(R_D C_L)\), so the slope drops from 0 to -20dB/dec as we pass \(\omega_{p1}\).

\[
H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R_D}{1 + sR_D C_L}
\]

This circuit has a pole at

\[
|\omega_p| = \frac{1}{R_D C_L}
\]
Pole Identification Example I

- Circuit transfer functions can be well approximated by considering that if a node in the signal path has a small-signal resistance $R_j$ and capacitance $C_j$ in parallel to an AC ground, then it contributes a pole of magnitude $(R_jC_j)^{-1}$

$$|\omega_{p1}| = \frac{1}{R_SC_{in}}$$

$$\lambda = 0$$

$$|\omega_{p2}| = \frac{1}{R_DC_L}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_mR_D}{\sqrt{\left(1 + \frac{\omega^2}{\omega_{p1}^2}\right)\left(1 + \frac{\omega^2}{\omega_{p2}^2}\right)}}$$
Pole Identification Example II

\[
\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \frac{g_m R_D}{\sqrt{1 + \omega^2/\omega_p^2(1 + \omega^2/\omega_{p2}^2)}}
\]

\[
|\omega_{p1}| = \frac{1}{\sqrt{R_S \parallel \frac{1}{g_m}}) C_{\text{in}}}
\]

\[
|\omega_{p2}| = \frac{1}{R_D C_L}
\]
The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.

The circuit above creates a problem since neither terminal of $C_F$ is grounded.

While we could always derive the transfer function from the small-signal model, there is a useful “Miller’s Theorem” which can be used to approximate the circuit’s poles.
Miller’s Theorem

If $A_v$ is the gain from node 1 to 2, then a floating impedance $Z_F$ can be converted to two grounded impedances $Z_1$ and $Z_2$.

$I_1$ should be the same in both circuits

$$I_1 = \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$Z_1 = \left(\frac{V_1}{V_1 - V_2}\right)Z_F = \left(\frac{1}{1 - \frac{V_2}{V_1}}\right)Z_F = \frac{Z_F}{1 - A_v}$$

where $A_v = \frac{V_2}{V_1}$

$I_2$ should be the same in both circuits

$$I_2 = \frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2}$$

$$Z_2 = \left(\frac{V_2}{V_2 - V_1}\right)Z_F = \left(\frac{1}{1 - \frac{V_1}{V_2}}\right)Z_F = \frac{Z_F}{1 - \frac{1}{A_v}}$$
Miller Multiplication

With Miller’s theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

\[
Z_{in} = \frac{1}{j \omega C_F (1 - A_v)} = \frac{1}{j \omega C_F (1 - (-A_o))} = \frac{1}{j \omega C_F (1 + A_o)}
\]

Equivalent to an input cap that is the original \(C_F\) multiplied by \((1 + A_o)\)

Following a similar procedure, the output cap is the original \(C_F\) multiplied by \(\left(1 + \frac{1}{A_o}\right)\)
Example: Miller Theorem

\[ A_v = -g_m R_D \]

\[ \omega_{in} = \frac{1}{R_S \left(1 + g_m R_D \right) C_F} \]

\[ \omega_{out} = \frac{1}{R_D \left(1 + \frac{1}{g_m R_D} \right) C_F} \]

- Note, this is only a (often good) approximation of the transfer function
- Uses only the low-frequency gain
- Neglects a zero

\[ \left| \frac{V_{out}}{V_{in}} \right| \approx \frac{g_m R_D}{\sqrt{\left(1 + \omega^2/\omega_{in}^2\right) \left(1 + \omega^2/\omega_{out}^2\right)}} \]
The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.
In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and output capacitances are needed.

\[ \omega_{p,in} = \frac{1}{R_i C_i} = 2\pi(20 \text{Hz}) \]

\[ \omega_{p,out} = \frac{1}{\left(\frac{1}{g_{m1}}\right)C_L} = 2\pi(20 \text{kHz}) \]

\[ C_i = 79.6 \text{nF} \]

\[ C_L = 39.8 \text{nF} \]

\[ R_i = 100 \text{K}\Omega \]

\[ g_m = \frac{1}{200\Omega} \]
Capacitive Coupling vs. Direct Coupling

- Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.
- This technique allows independent bias conditions between stages. Direct coupling does not.

Allows for high $V(R_D)$ (gain), while also allowing a high output stage gate bias for good output swing.

Due to direct coupling, must trade-off $A_{V1}$ for output stage biasing/swing.
Typical Frequency Response

Often due to AC coupling

Often due to load/parasitic capacitors
Agenda

• Frequency Response Concepts
• **High-Frequency Models of Transistors**
• Frequency Response Analysis Procedure
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At high frequency, capacitive effects come into play

- $C_\mu$ and $C_{je}$ are the junction capacitances
- $C_b$ represents the base charge to generate the non-uniform charge profile required for proper operation (Chapter 4)
Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely $C_{CS}$. 
Example: Capacitance Identification

$V_{CC}$

$R_C$

$V_{out}$

$V_{b1}$

$Q_2$

$V_{in}$

$Q_1$

$V_{out}$

$V_{CC}$

$R_C$

$V_{out}$

$V_{b}$

$Q_2$

$C_{π2}$

$C_{μ2}$

$C_{CS2}$

$V_{in}$

$Q_1$

$C_{π1}$

$C_{μ1}$

$C_{CS1}$

CH 11 Frequency Response
For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.
The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{\text{gate}}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form $C_{\text{GS}}$ and $C_{\text{GD}}$.

Assuming bulk is an AC ground
Example: Capacitance Identification
Transit Frequency

Transit frequency, $f_T$, is defined as the frequency where the current gain from input to output drops to 1.

Neglecting $C_\mu$

$$I_{out} = g_m V_{in} = g_m I_{in} Z_{in}$$

$$Z_{in} = \frac{1}{C_\pi s r_\pi}$$

$$\frac{I_{out}}{I_{in}} = \frac{g_m r_\pi}{r_\pi C_\pi s + 1} = \frac{\beta}{r_\pi C_\pi s + 1}$$

Setting the magnitude equal to 1 at $\omega_T$ yields

$$r_\pi^2 C_\pi^2 \omega_T^2 = \beta^2 - 1 \approx \beta^2$$

$$\omega_T \approx \frac{\beta}{r_\pi C_\pi} = g_m$$

Neglecting $C_{GD}$

$$I_{out} = g_m V_{in} = g_m I_{in} Z_{in}$$

$$Z_{in} = \frac{1}{C_{GS} s}$$

$$\frac{I_{out}}{I_{in}} = \frac{g_m}{C_{GS}}$$

Setting the magnitude equal to 1 at $\omega_T$ yields

$$C_{GS}^2 \omega_T^2 = g_m^2$$

$$\omega_T = \frac{g_m}{C_{GS}}$$
Example: Transit Frequency Calculation

The transit frequency increases dramatically as the channel length is shrunk, allowing for much faster transistors with CMOS scaling.

Note, this neglects some advanced device physics (carrier velocity saturation) which slows this rate of frequency increase.

\[ \omega_T = \frac{g_m}{C_{GS}} \]

\[ g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \]

\[ C_{GS} = \frac{2}{3} W L C_{ox} \] (talked about in 474)

\[ \omega_T = 2 \pi f_T = \frac{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}{\frac{2}{3} W L C_{ox}} \]

\[ f_T = \frac{3 \mu_n (V_{GS} - V_{TH})}{4 \pi L^2} \]

\[ L = 65nm \]

\[ V_{GS} - V_{TH} = 100mV \]

\[ \mu_n = 400cm^2/(V\cdot s) \]

\[ f_T = 226GHz \]
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The frequency response refers to the magnitude of the transfer function.

Bode’s approximation simplifies the plotting of the frequency response if poles and zeros are known.

In general, it is possible to associate a pole with each node in the signal path.

Miller’s theorem helps to decompose floating capacitors into grounded elements.

Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.
High Frequency Circuit Analysis Procedure

- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode’s rules or exact analysis.
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The input AC coupling forms a high-pass filter which should be designed for a certain minimum cut-off frequency.
In order to increase the midband gain, a capacitor $C_b$ is placed in parallel with $R_s$. 

\[
\frac{V_{out}}{V_X} = -\frac{R_D}{R_s} \left[ \frac{1}{C_b s} + \frac{1}{g_m} \right] = -\frac{g_m R_D (R_s C_b s + 1)}{R_s C_b s + g_m R_s + 1}
\]

The pole frequency

\[
|\alpha_p| = \frac{1 + g_m R_S}{R_s C_b}
\]

should also be designed to set the minimum cut-off frequency.

CH 11 Frequency Response
Unified Model for CE and CS Stages

- \( V_{in} \) to \( V_{out} \) for CE stage with \( Q_1 \) and feedback components \( R_C \) and \( R_S \)
- \( V_{in} \) to \( V_{out} \) for CS stage with \( M_1 \) and feedback components \( R_D \) and \( R_S \)

Component labels:
- \( V_{CC} \)
- \( V_{DD} \)
- \( C_{XY} \)
- \( R_{Thev} \)
- \( C_{in} \)
- \( g_m \)
- \( C_{out} \)
- \( R_D \)
Unified Model Using Miller’s Theorem

**CE Stage**

\[ V_{Thev} = V_{in} \frac{r_\pi}{r_\pi + R_S} \]

\[ R_{Thev} = R_S r_\pi \]

\[ C_X = C_\mu \left( 1 + g_m R_C \right) \]

\[ C_Y = C_\mu \left( 1 + \frac{1}{g_m R_C} \right) \]

\[ C_{in} = C_\pi \]

\[ C_{out} = C_{CS} \]

**CS Stage**

\[ V_{Thev} = V_{in} \]

\[ R_{Thev} = R_S \]

\[ C_X = C_{GD} \left( 1 + g_m R_D \right) \]

\[ C_Y = C_{GD} \left( 1 + \frac{1}{g_m R_D} \right) \]

\[ C_{in} = C_{GS} \]

\[ C_{out} = C_{DB} \]
Unified Model Using Miller’s Theorem

\[ |\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + (1 + g_m R_L)C_{XY}]} \]

\[ |\omega_{p,out}| = \frac{1}{R_L \left[ C_{out} + \left(1 + \frac{1}{g_m R_L} \right)C_{XY} \right]} \]
Example: CE Stage

The input pole is the bottleneck for speed.

\[ |\omega_{p,\text{in}}| = 2\pi \times (516 \text{MHz}) \]

\[ |\omega_{p,\text{out}}| = 2\pi \times (1.59 \text{GHz}) \]
Example: Half Width CS Stage

\[ g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \]

In 474 we will learn that all MOS caps are \( \propto W \)

\[ C_{GS} = \frac{2}{3} W L C_{ox} + W C_{ov} \]
\[ C_{GD} = W C_{ov} \]
\[ C_{DB} = A_D C_j + P_D C_{jsw} \propto W \]

- LF gain, \( g_m R_L \) reduces by 1/2
- Assuming \( g_m R_L \) is still high:
- The input pole increases by \( \sim 4X \)
- The output pole increases by \( \sim 2x \)
  - Constant gain-bandwidth product!

\[ \omega_{p,\text{in}} = \frac{1}{R_S \left[ \frac{C_{in}}{2} + \left( 1 + \frac{g_m R_L}{2} \right) \frac{C_{XY}}{2} \right]} \]
\[ \omega_{p,\text{out}} = \frac{1}{R_L \left[ \frac{C_{out}}{2} + \left( 1 + \frac{2}{g_m R_L} \right) \frac{C_{XY}}{2} \right]} \]
Direct Analysis of CE and CS Stages

- For a detailed direct small-signal analysis, see Razavi 11.4.4

\[
\frac{V_{out}(s)}{V_{Thev}} = \frac{(C_{XY} s - g_m)R_L}{as^2 + bs + 1}
\]

where

\[
a = R_{Thev} R_L \left( C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out} \right)
\]

\[
b = (1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L \left( C_{XY} + C_{out} \right)
\]

To find the 2 poles, we can write

\[
as^2 + bs + 1 = \left( \frac{s}{\omega_{p1}} + 1 \right) \left( \frac{s}{\omega_{p2}} + 1 \right) = \frac{s^2}{\omega_{p1} \omega_{p2}} + \left( \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right)s + 1
\]

Using a "dominant pole" approximation \( \omega_{p1} \ll \omega_{p2} \)

\[
as^2 + bs + 1 = \frac{s^2}{\omega_{p1} \omega_{p2}} + \frac{s}{\omega_{p1}} + 1
\]

\[
\omega_{p1} = \frac{1}{b} \quad \text{and} \quad \omega_{p2} = \frac{b}{a}
\]

Direct analysis yields different pole locations and an extra zero.
Direct Analysis of CE and CS Stages w/ Dominant Pole Approximation

\[
|\omega_z| = \frac{g_m}{C_{XY}}
\]

\[
|\omega_{p1}| = \frac{1}{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}
\]

\[
|\omega_{p2}| = \frac{1}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}
\]

- \(\omega_{p1}\) will be lower due to the additional term
- \(\omega_{p2}\) is at a much higher frequency due to “pole splitting”
  - Discussed more when we talk about stability
Example: CE and CS Direct Analysis (Dominant Pole Approximation)

\[ R_{\text{Thev}} = R_S, \quad R_L = r_{O1} \parallel r_{O2} \]

\[ C_{\text{in}} = C_{GS1}, \quad C_{XY} = C_{GD1} \]

\[ C_{\text{out}} = C_{DB1} + C_{DB2} + C_{GD2} \]

\[ \omega_{p1} \approx \frac{1}{[1 + g_{m1}(r_{O1} \parallel r_{O2})]C_{XY}R_S + R_SC_{\text{in}} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{\text{out}})} \]

\[ \omega_{p2} \approx \frac{1}{R_S(r_{O1} \parallel r_{O2})(C_{\text{in}}C_{XY} + C_{\text{out}}C_{XY} + C_{\text{in}}C_{\text{out}})} \]
Example: Comparison Between Different Methods

Simple Miller theorem analysis vastly overestimates the output pole at a lower frequency

**Miller’s**

- $|\omega_{p,in}| = 2\pi \times (571 \text{MHz})$
- $|\omega_{p,out}| = 2\pi \times (428 \text{MHz})$

**Exact**

- $|\omega_{p,in}| = 2\pi \times (264 \text{MHz})$
- $|\omega_{p,out}| = 2\pi \times (4.53 \text{GHz})$

**Dominant Pole**

- $|\omega_{p,in}| = 2\pi \times (249 \text{MHz})$
- $|\omega_{p,out}| = 2\pi \times (4.79 \text{GHz})$

**Circuit Diagram**

- $R_S = 200\Omega$
- $C_{GS} = 250fF$
- $C_{GD} = 80fF$
- $C_{DB} = 100fF$
- $g_m = (150\Omega)^{-1}$
- $\lambda = 0$
- $R_L = 2K\Omega$
Input Impedance of CE and CS Stages

\[ Z_{in} \approx \left[ \frac{1}{C_{\pi} + \left(1 + g_m R_C \right) C_\mu} \right] s \parallel r_\pi \]

At low frequencies: \( r_\pi \)

\( Z_{in} \) has a pole at \( \frac{1}{r_\pi \left( C_{\pi} + \left(1 + g_m R_C \right) C_\mu \right)} \)

\[ Z_{in} \approx \left[ \frac{1}{C_{GS} + \left(1 + g_m R_D \right) C_{GD}} \right] s \]

Approximate as purely capacitive
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As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).
Frequency Response of CB Stage

\[ r_O = \infty \]

\[
\begin{align*}
\omega_{p,X} &= \frac{1}{\left( R_S \parallel \frac{1}{g_m} \right) C_X} \\
C_X &= C_\pi
\end{align*}
\]

\[
\omega_{p,Y} = \frac{1}{R_L C_Y}
\]

\[ C_Y = C_\mu + C_{CS} \]

- No Miller effect
- Input pole is \( \sim f_T \) (very high frequency)
Similar to a CB stage, the input pole is on the order of $f_T$, so rarely a speed bottleneck.
Example: CG Stage Pole Identification

\[ r_O = \infty \]

MOSFET Caps

\[ \omega_{p,X} = \frac{1}{\left( R_S \parallel \frac{1}{g_{m1}} \right) \left( C_{SB1} + C_{GD1} \right)} \]

\[ \omega_{p,Y} = \frac{1}{g_{m2} \left( C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2} \right)} \]

CH 11 Frequency Response
Example: Frequency Response of CG Stage

- Input pole is $\sim f_T$
- Output pole limits bandwidth

$R_S = 200\Omega$
$C_{GS} = 250\, fF$
$C_{GD} = 80\, fF$
$C_{SB} = C_{DB} = 100\, fF$
$g_m = (150\Omega)^{-1}$
$\lambda = 0$
$R_d = 2\, K\Omega$

$\omega_{p,x} = 2\pi \times (5.31 GHz)$
$\omega_{p,y} = 2\pi \times (442 MHz)$
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The following will discuss the frequency response of emitter and source followers using direct analysis, as this circuit typically has 2 poles that are close together.

Emitter follower is treated first and source follower is derived easily by allowing $r_\pi$ to go to infinity.
For detailed analysis, see Razavi 11.6

Assuming that \( r_\pi \gg \frac{1}{g_m} \)

\[
\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_\pi}{g_m} s}{a s^2 + b s + 1}
\]

\[
a = \frac{R_S}{g_m} \left( C_\mu C_\pi + C_\mu C_L + C_\pi C_L \right)
\]

\[
b = R_S C_\mu + \frac{C_\pi}{g_m} + \left( 1 + \frac{R_S}{r_\pi} \right) \frac{C_L}{g_m}
\]

\[
|\omega_z| = \frac{g_m}{C_\pi}
\]

Generally, this yields 2 close poles, necessitating a direct solution approach.

CH 11 Frequency Response
Direct Analysis of Source Follower Stage

Taking $r_\pi \to \infty$

$C_\pi \to C_{GS}$

$C_\mu \to C_{GD}$

\[ V_{out} \Bigg/ V_{in} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1} \]

\[ a = \frac{R_S}{g_m} \left( C_{GD}C_{GS} + C_{GD}(C_L + C_{SB}) + C_{GS}(C_L + C_{SB}) \right) \]

\[ b = R_S C_{GD} + \frac{C_{GD} + C_L + C_{SB}}{g_m} \]
Example: Frequency Response of Source Follower

\[ R_S = 200\Omega \]
\[ C_L = 100\, fF \]
\[ C_{GS} = 250\, fF \]
\[ C_{GD} = 80\, fF \]
\[ C_{DB} = 100\, fF \]
\[ g_m = (150\Omega)^{-1} \]
\[ \lambda = 0 \]

\[ \omega_{p1} = 2\pi[-1.79GHz + j(2.57GHz)] \]
\[ \omega_{p2} = 2\pi[-1.79GHz - j(2.57GHz)] \]

2 Complex Conjugate Poles

CH 11 Frequency Response
Example: Source Follower

\[ r_O = \infty \]

\[
\begin{align*}
V_{out} &= \frac{1 + \frac{C_{GS1}}{s}}{V_{in} \frac{g_m}{a s^2 + b s + 1}}
\end{align*}
\]

\[
\begin{align*}
a &= \frac{R_S}{g_{m1}} \left[ C_{GD1} C_{GS1} + (C_{GD1} + C_{GS1})(C_{SB1} + C_{GD2} + C_{DB2}) \right] \\
b &= R_S C_{GD1} + \frac{C_{GD1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}
\end{align*}
\]
Input Capacitance of Emitter/Source Follower

\[ r_O = \infty \]

Using Miller Theorem with

\[ A_v = \frac{g_m R_L}{1 + g_m R_L} \]

\[ C_{in} = C_\mu + \frac{C_\pi}{1 + g_m R_L} \]

\[ C_{in} = C_{GD} + \frac{C_{GS}}{1 + g_m R_L} \]
Example: Source Follower Input Capacitance

\[ C_{in} = C_{GD1} + \frac{1}{1 + g_{m1}(r_{O1} \parallel r_{O2})} C_{GS1} \]
Output Impedance of Emitter Follower

- Need to consider the output resistance of the previous stage, $R_S$

\[
\begin{align*}
V_X &= \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1} \\
I_X &= \frac{r_\pi C_\pi s + \beta + 1}{r_\pi C_\pi s + \beta + 1}
\end{align*}
\]

Low Frequency: \( \frac{r_\pi + R_S}{\beta + 1} = r_e + \frac{R_S}{\beta + 1} \)

High Frequency: \( R_S \)
Output Impedance of Source Follower

\[ \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m} \]

Low Frequency: \( \frac{1}{g_m} \)

High Frequency: \( R_S \)
The plot above shows the output impedance of emitter and source followers. Since a follower’s primary duty is to lower the driving impedance ($R_S > 1/g_m$), the “active inductor” characteristic on the right is usually observed.
Example: Output Impedance

\[
\begin{align*}
V_X &= \frac{(r_{o1} \parallel r_{o2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}} \\
I_X &= (r_{o1} \parallel r_{o2})C_{GS3}s + 1
\end{align*}
\]

Note: This neglects the capacitors from M1 and M2
Agenda

- Frequency Response Concepts
- High-Frequency Models of Transistors
- Frequency Response Analysis Procedure
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- CB and CG Stages
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- Cascode Stages
- Differential Pairs
- Additional Examples
For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.

Assume 
\((W/L)_1 = (W/L)_2\)

\[ A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1 \]

\[ C_x \approx 2C_{XY} \]
Poles of Bipolar Cascode

\[
\omega_{p,X} = \frac{1}{(R_S \parallel r_{\pi_1})(C_{\pi_1} + 2C_{\mu_1})}
\]

\[
\omega_{p,Y} = \frac{1}{g_{m2}} \left( C_{CS1} + C_{\pi_2} + 2C_{\mu_1} \right)
\]

Comparable to \( f_T \)

\[
\omega_{p,out} = \frac{1}{R_L \left( C_{CS2} + C_{\mu_2} \right)}
\]

CH 11 Frequency Response
Poles of MOS Cascode

\[ \omega_{p,X} = \frac{1}{R_S \left[ C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right]} \]

\[ \omega_{p,\text{out}} = \frac{1}{R_L \left( C_{DB2} + C_{GD2} \right)} \]

\[ \omega_{p,Y} = \frac{1}{g_{m2} \left[ C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}} \right) C_{GD1} + C_{SB2} \right]} \]
Example: Frequency Response of Cascode

\[ R_S = 200\Omega \]
\[ C_{GS} = 250\, fF \]
\[ C_{GD} = 80\, fF \]
\[ C_{DB} = 100\, fF \]
\[ g_m = (150\Omega)^{-1} \]
\[ \lambda = 0 \]
\[ R_L = 2K\Omega \]

\[ |\omega_{p,X}| = 2\pi \times (1.95\, GHz) \]
\[ |\omega_{p,Y}| = 2\pi \times (1.73\, GHz) \]
\[ |\omega_{p,\text{out}}| = 2\pi \times (442\, MHz) \]

Compare to simple CS

Exact

\[ |\omega_{p,\text{in}}| = 2\pi \times (264\, MHz) \]
\[ |\omega_{p,\text{out}}| = 2\pi \times (4.53\, GHz) \]

Now output pole sets the bandwidth, and it has increased by 67%
MOS Cascode Example

\[
\omega_{p,X} = \frac{1}{R_S \left[ C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right) C_{GD1}\right]}
\]

\[
\omega_{p,\text{out}} = \frac{1}{R_L \left(C_{DB2} + C_{GD2}\right)}
\]

- Allows for a smaller M₂
  - Improves output pole
  - Lowers poles at nodes X and Y, but they should still be relatively high

\[
\omega_{p,Y} = \frac{1}{g_{m2}} \left[ C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right) C_{GD1} + C_{GD3} + C_{DB3}\right]
\]
I/O Impedance of Bipolar Cascode

\[ V_A = \infty \]

\[ Z_{in} = r_{\pi 1} \parallel \frac{1}{(C_{\pi 1} + 2C_{\mu 1})} \]

\[ (\text{Neglecting } R_S) \]

\[ Z_{out} = R_L \parallel \frac{1}{(C_{\mu 2} + C_{CS2})} \]
I/O Impedance of MOS Cascode

\[ \lambda = 0 \]

\[
Z_{in} = \frac{1}{C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}}s
\]

\[
Z_{out} = R_L \parallel \frac{1}{\left(C_{GD2} + C_{DB2}\right)s}
\]

(Neglecting \(R_S\))

\(C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2}}\right)\)

\(C_{GD2} + C_{DB2}\)
Cascode Frequency Response
Take-Away Points

- Cascode amplifiers offer two good properties
  - High output impedance to serve as a good current source and/or amplifier
  - Reduction of the Miller effect and better high-frequency performance
- Main cost is higher voltage headroom to keep cascode transistor in saturation
  - Impacts maximum output swing and distortion performance
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Bipolar Differential Pair Frequency Response

Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit’s (Slide 43).

\[
\frac{V_{out}(s)}{V_{Thev}} = \frac{(C_{XY} s - g_m)R_L}{as^2 + bs + 1}
\]

where

\[
a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})
\]

\[
b = (1 + g_m R_L)C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L(C_{XY} + C_{out})
\]
Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit’s (Slide 43).
Example: MOS Differential Pair

\[ \omega_{p,x} = \frac{1}{R_S[C_{GS1} + (1 + \frac{g_{m1}}{g_{m3}})C_{GD1}]} \]

\[ \omega_{p,y} = \frac{1 \left[ C_{DB1} + C_{GS3} + \left( 1 + \frac{g_{m3}}{g_{m1}} \right) C_{GD1} + C_{SB3} \right]}{\frac{1}{g_{m3}}} \]

\[ \omega_{p,\text{out}} = \frac{1}{R_D (C_{DB3} + C_{GD3})} \]
Common Mode Frequency Response

\[
\begin{align*}
\frac{\Delta V_{\text{out}}}{\Delta V_{\text{CM}}} &= \frac{\Delta R_D}{1 + 2 \left( \frac{1}{R_S S} \left| \frac{1}{C_S S S} \right| \right)} = \frac{g_m \Delta R_D \left( R_S S C_S S S + 1 \right)}{R_S S C_S S S + 2 g_m R_S S + 1}
\end{align*}
\]

- \( C_{SS} \) will lower the total impedance between point \( P \) to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.
Tail Node Capacitance Contribution

- Source-Body Capacitance of $M_1$, $M_2$ and $M_3$
- Gate-Drain Capacitance of $M_3$

- $M_3$ is often a large (wide) transistor in order to have a small compliance ($V_{DS}$) voltage
  - Watch out for degraded high-frequency CMRR!
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Example: Capacitive Coupling 
(Low-Frequency Cut-Off)

\[ I_{C1} = 1.75mA \]
\[ I_{C2} = 1.13mA \]

\[ \omega_{L1} = \frac{1}{(r_{\pi1} \parallel R_{B1})C_1} = 2\pi \times (542Hz) \]

The “highest” low-frequency pole \((\omega_{L1} = 542Hz)\) will set the low-frequency cut-off

To find \(\omega_{L2}\):

\[ \frac{V_Y}{V_{Thev}}(s) = \frac{R_{in2}}{R_C + \frac{1}{sC_2} + R_{in2}} = \frac{sC_2R_{in2}}{1 + sC_2(R_C + R_{in2})} \]

\[ R_{in2} = R_{B2} \parallel [r_{\pi2} + (\beta + 1)R_E] \]

\[ \omega_{L2} = \frac{1}{(R_C + R_{in2})C_2} = \pi \times (22.9Hz) \]
Example: IC Amplifier – Low Frequency Design

\[ g_{m1} = g_{m2} = \left(150\Omega\right)^{-1} \]

\[ \omega_{L1} = \frac{1}{\left(\frac{1}{g_{m1}R_{S1}} + \frac{1}{R_{S1}C_1}\right)C_1} = 2\pi \times (37.2MHz) \]

\[ \omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92MHz) \]

\[ R_{in2} = \frac{R_F}{1 - A_{v2}} \]

\[ A_{v2} = -g_{m2}R_{D2} = -\frac{1k\Omega}{150\Omega} = -6.67 \]

\[ R_{in2} = \frac{10k\Omega}{7.67} = 1.30k\Omega \]

The “highest” low-frequency pole (\( \omega_{L1} = 37.2MHz \)) will set the low-frequency cut-off.
Example: IC Amplifier – Midband Design

\[ g_{m1} = g_{m2} = (150\Omega)^{-1} \]

\[ A_{v1} = \frac{v_X}{v_{in}} = -g_{m1} (R_{D1} \ || \ R_{in2}) = -3.77 \]

\[ A_{v2} = -6.67 \]

\[ A_v = A_{v1} A_{v2} = 25.1 = 28.0 \text{dB} \]
Example: IC Amplifier – High Frequency Design

To get an accurate estimate for $\omega_{p1}$ and $\omega_{p2}$, use the dominant pole approximation expressions on Slide 44

$$|\omega_{p1}| = \frac{1}{(1 - A_{v1})C_{GD1}R_S + R_SC_{GS1} + (R_{D1}||R_{in2})(C_{GD1} + C_{out1})} = 2\pi(222MHz)$$

$$|\omega_{p2}| = \frac{(1 - A_{v1})C_{GD1}R_S + R_SC_{GS1} + (R_{D1}||R_{in2})(C_{GD1} + C_{out1})}{R_S(R_{D1}||R_{in2})(C_{GS1}C_{GD1} + C_{GD1}C_{out1} + C_{GS1}C_{out1})} = 2\pi(2.99GHz)$$

where

$$C_{out1} = C_{DB1} + C_{GS2} + C_{GD2}(1 - A_{v2})$$
Example: IC Amplifier – High Frequency Design

\[ |\omega_p| = 2\pi(222\text{MHz}) \]

\[ |\omega_p| = 2\pi(2.99\text{GHz}) \]

\[ |\omega_p| = \frac{1}{R_{D2} \left[ C_{GD2} \left( 1 - \frac{1}{A_{v2}} \right) + C_{DB2} \right]} = 2\pi(829\text{MHz}) \]
Next Time

- Feedback
  - Razavi Chapter 12