ECEN326: Electronic Circuits
Fall 2022

Lecture 6: Frequency Response

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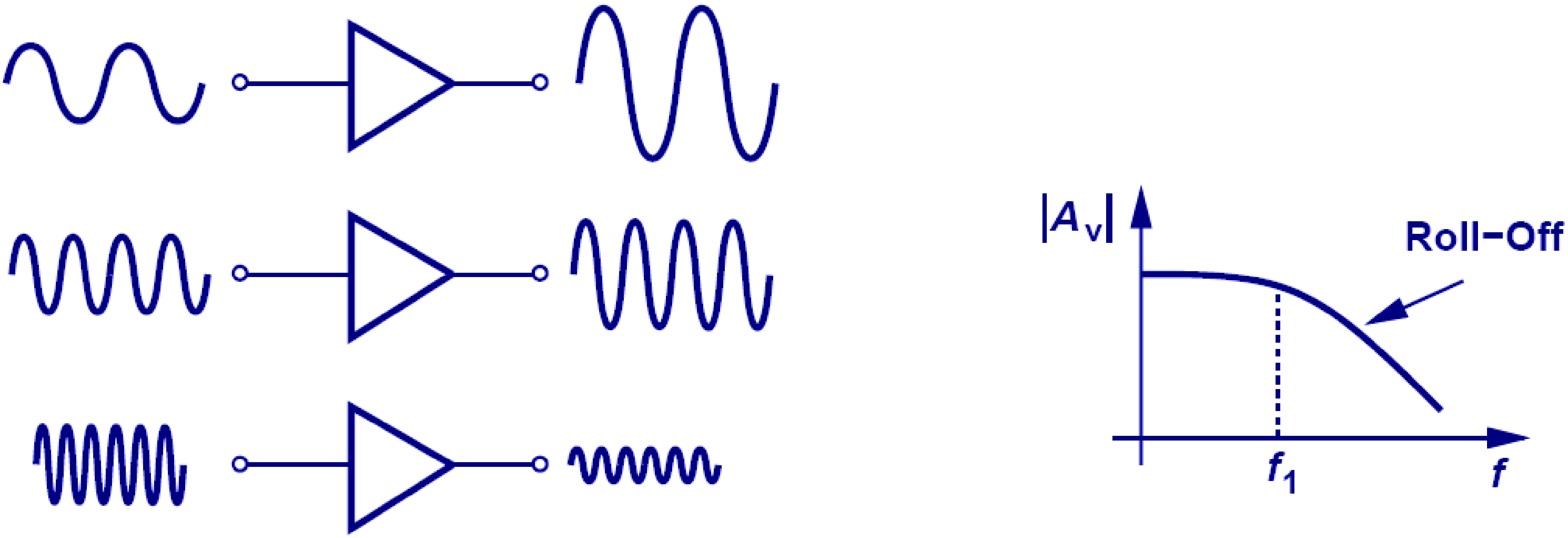
Announcements

- HW5 Due Mar 29
- Exam 2 Mar 31
  - 9:35 – 11:00 (10 extra minutes)
  - Closed book w/ one standard note sheet
  - 8.5”x11” front & back
  - Bring your calculator
  - Emphasis will be on Lectures 4-6
  - Sample Exam2s posted on website
- Reading
  - Razavi Chapter 11

Agenda

- Frequency Response Concepts
- High-Frequency Models of Transistors
- Frequency Response Analysis Procedure
- CE and CS Stages
- CB and CG Stages
- CC and CD (Follower) Stages
- Cascode Stages
- Differential Pairs
- Additional Examples

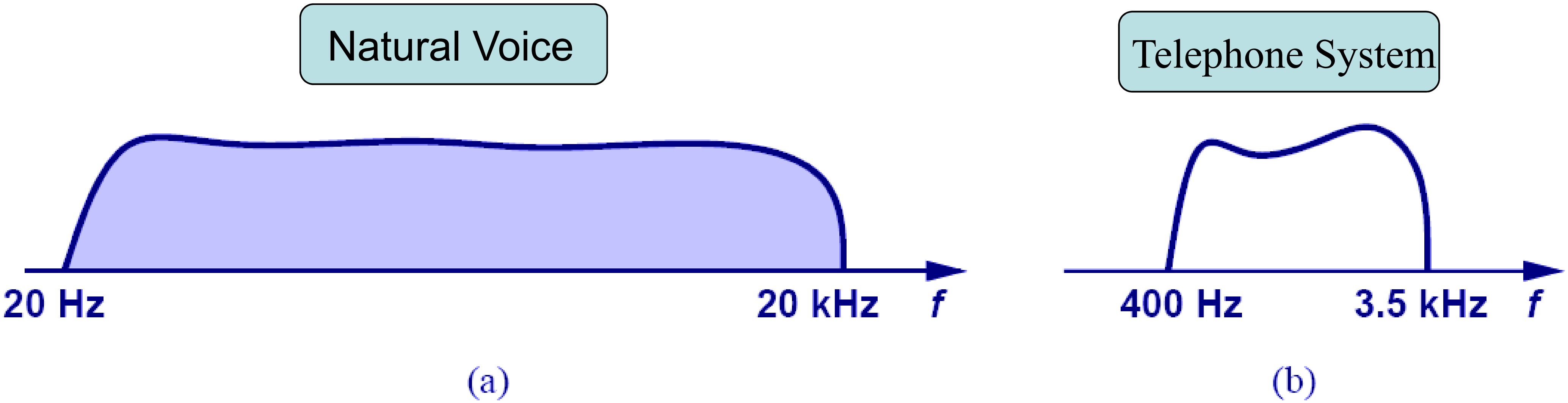
High Frequency Roll-off of Amplifier



(a) (b)

- As frequency of operation increases, the amplifier gain decreases
- This lecture analyzes this frequency response issue

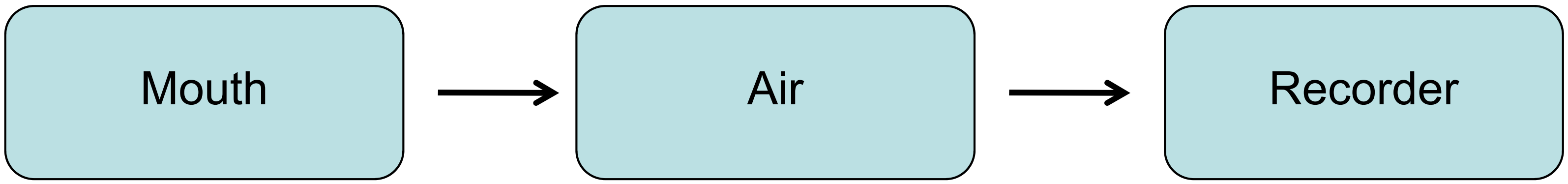
Example: Human Voice I



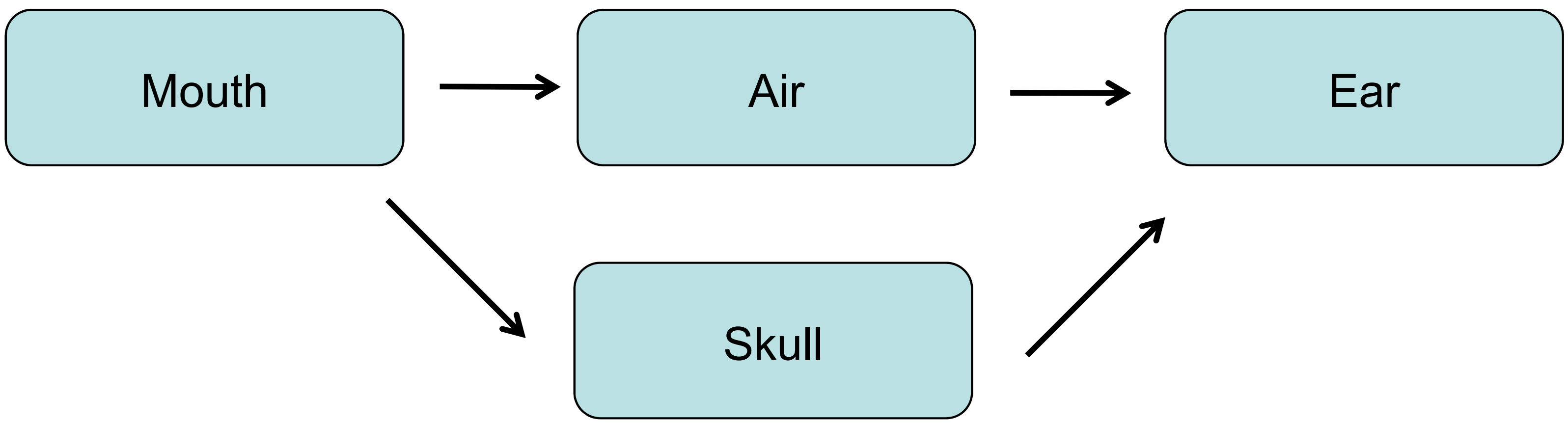
- Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder

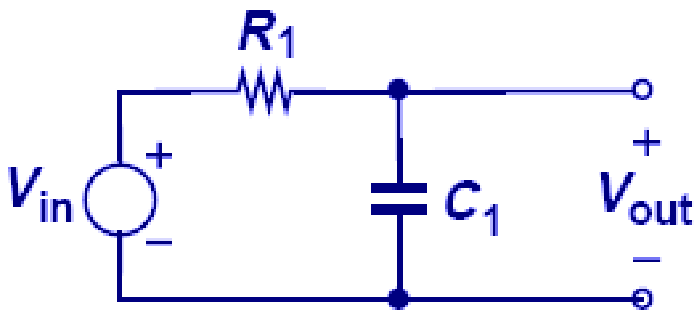


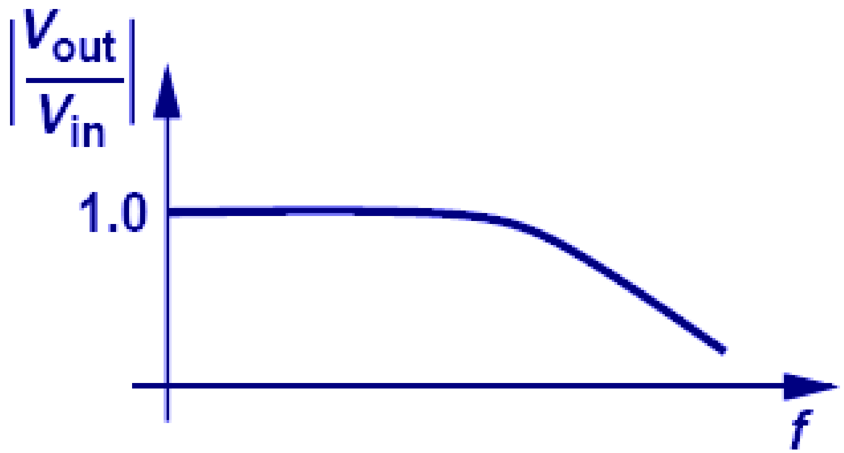
Path traveled by the human voice to the human ear



- **Since the paths are different, the results will also be different.**

Gain Roll-off: Simple Low-pass Filter





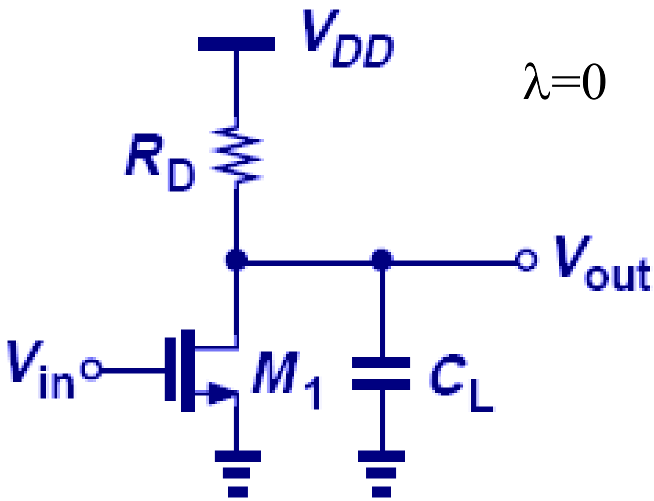
$$V_o(s) = \frac{Z_C}{Z_R + Z_C} V_{in}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}(s) = \frac{1}{1 + sRC} V_{in}(s) \Rightarrow H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}$$

for sinusoidal steady-state response $s = j\omega$

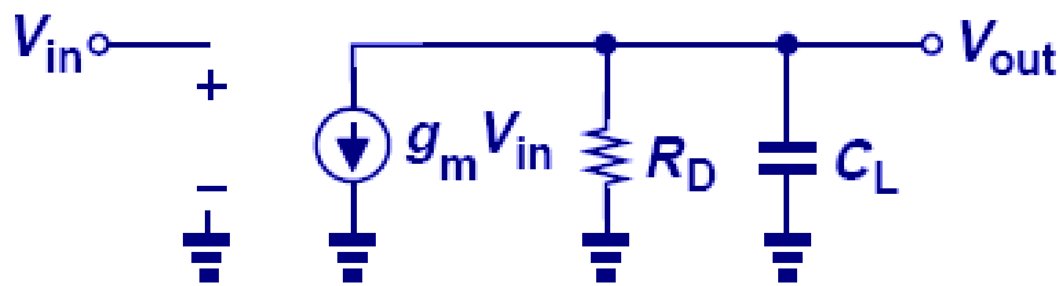
$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

- **In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.**

Gain Roll-off: Common Source



$\lambda=0$



$$V_{out} = -g_m V_{in} \left(R_D \,||\, \frac{1}{C_L s} \right)$$

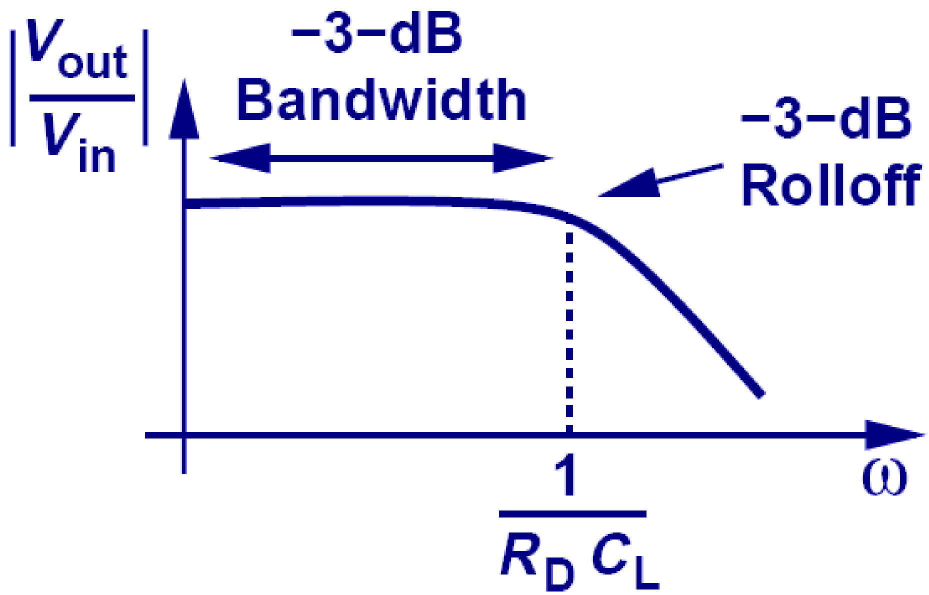
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R_D}{1 + sR_D C_L}$$

This circuit has a pole at

$$|\omega_{\mathrm{p}}| = \frac{1}{R_D C_L}$$

- **The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will "steal" away some signal current and shunt it to ground.**

Frequency Response of the CS Stage



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

Recall the Power is proportional to $(\text{Voltage})^2$

To find the half - power (-3dB) point relative to the low - frequency gain

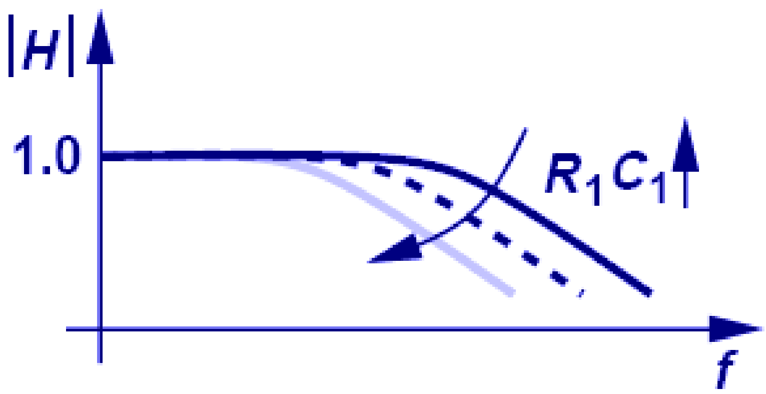
$$\left|\frac{V_{out}}{V_{in}}\right|^2 = \frac{(g_m R_D)^2}{(R_D C_L \omega)^2 + 1} = \frac{(g_m R_D)^2}{2}$$

Solving for ω

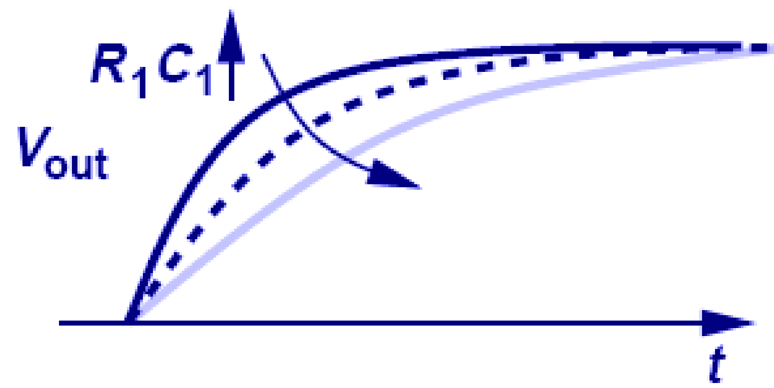
$$\omega = \frac{1}{R_D C_L}$$

- **At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease.**
- **A special frequency is $\omega=1/(R_D C_L)$, where the gain drops by 3dB (half-power). In this single-pole circuit, this is also the pole frequency.**

Example: Relationship between Frequency Response and Step Response



(a)



(b)

$$\left|H(s=j\omega)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0\left(1-\exp\frac{-t}{R_1C_1}\right)u(t)$$

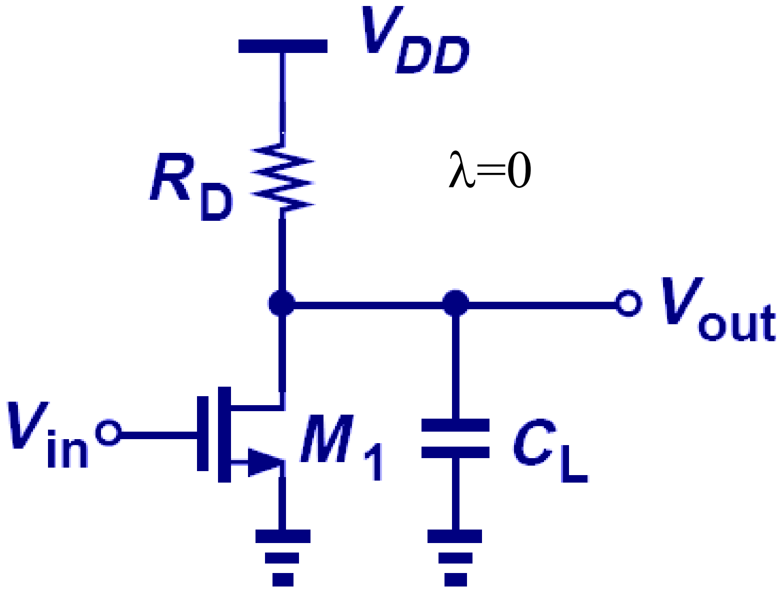
- The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.*

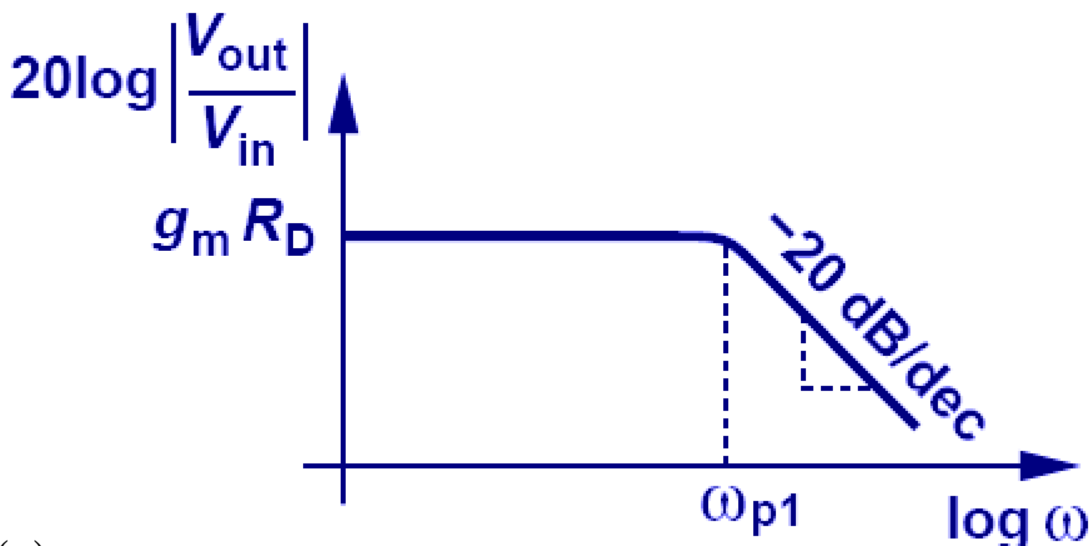
Bode Plot

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- **When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.**
- **When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example: Bode Plot





$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{-g_m R_D}{1+sR_D C_L}$$

This circuit has a pole at

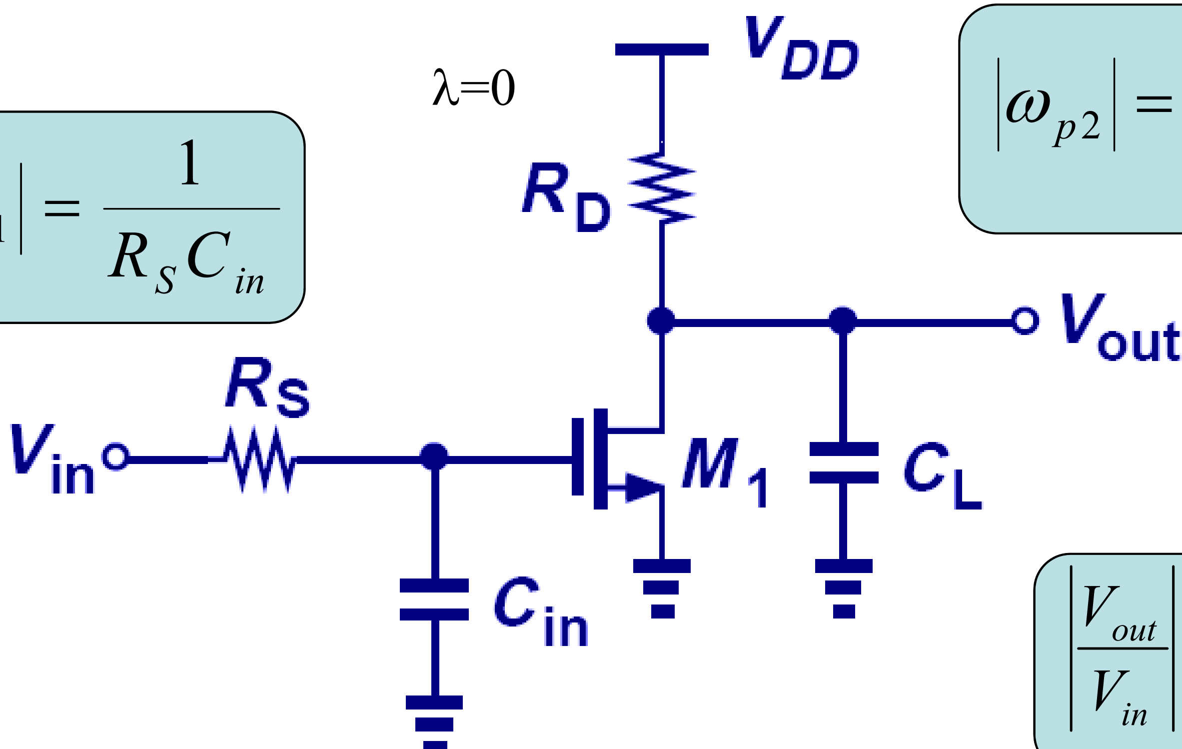
$$|\omega_p|=\frac{1}{R_D C_L}$$

- **The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.**

Pole Identification Example I

- Circuit transfer functions can be well approximated by considering that if a node in the signal path has a small-signal resistance R_j and capacitance C_j in parallel to an AC ground, then it contributes a pole of magnitude $(R_jC_j)^{-1}$

$$\left|\omega_{p1}\right| = \frac{1}{R_S C_{in}}$$

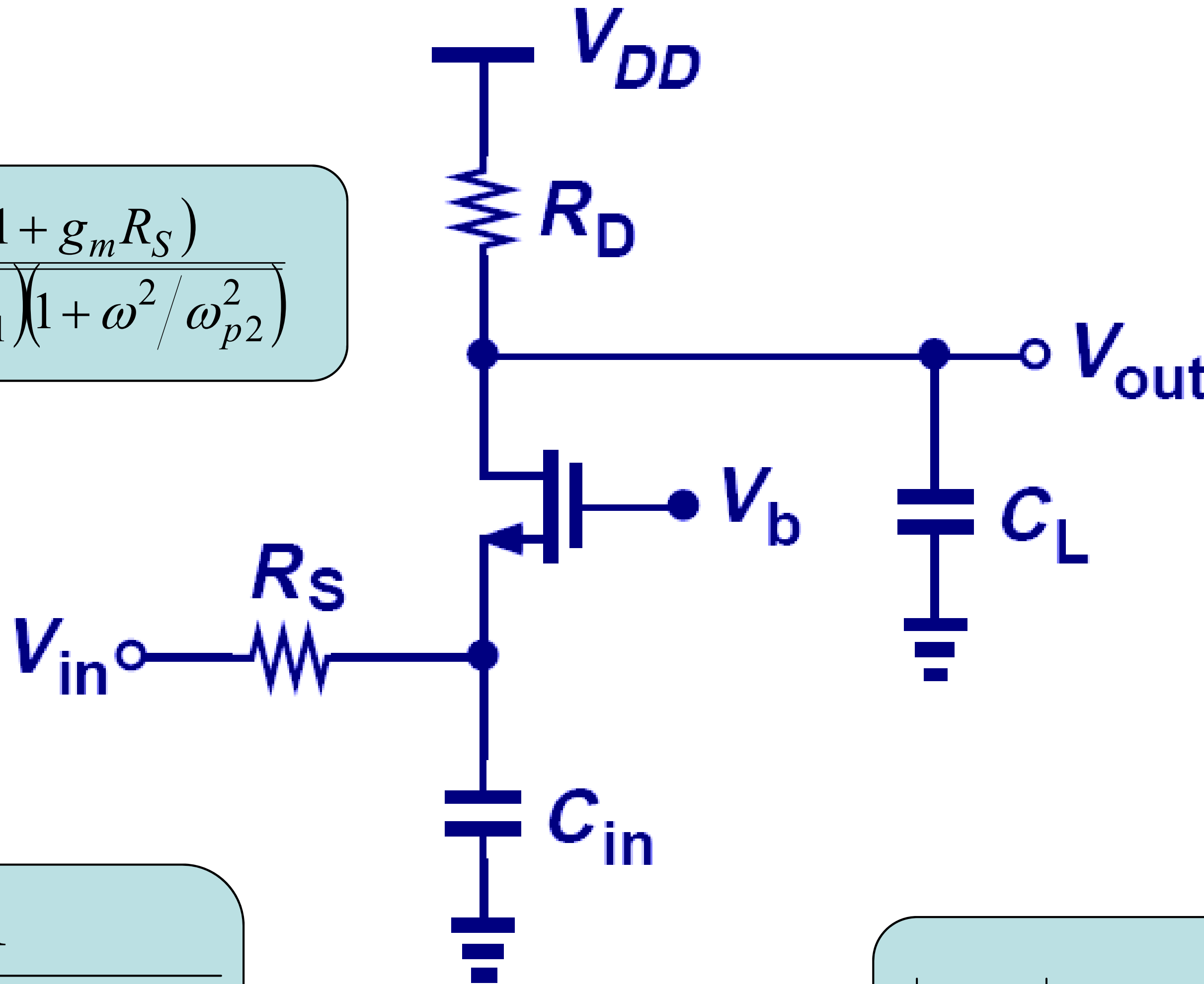


$$\left|\omega_{p2}\right| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

Pole Identification Example II

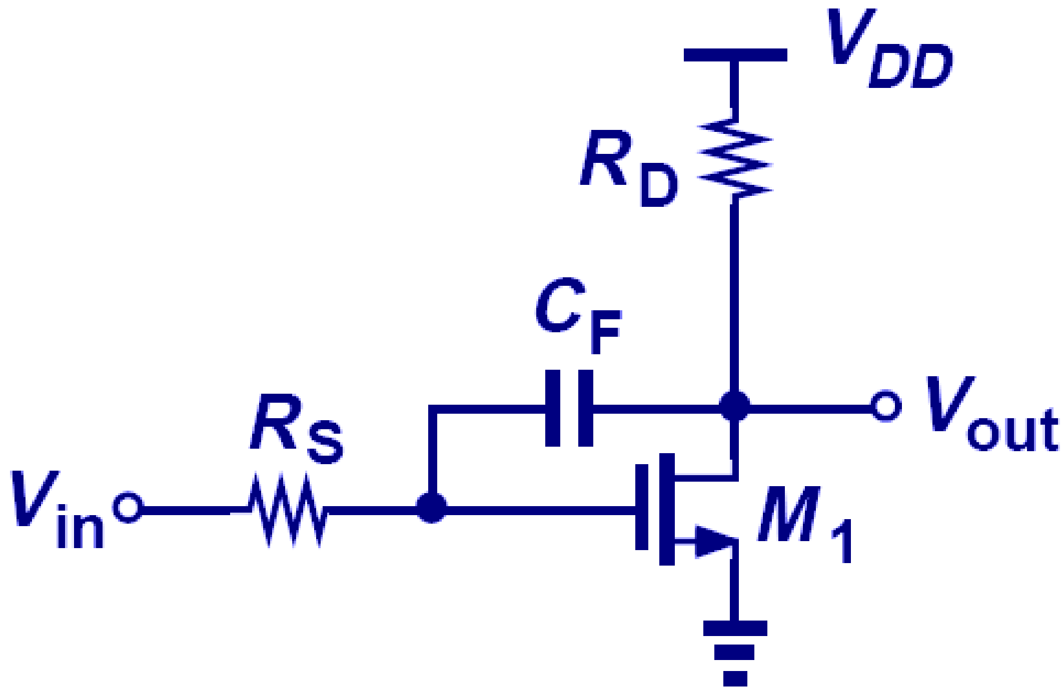
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D / (1+g_m R_S)}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

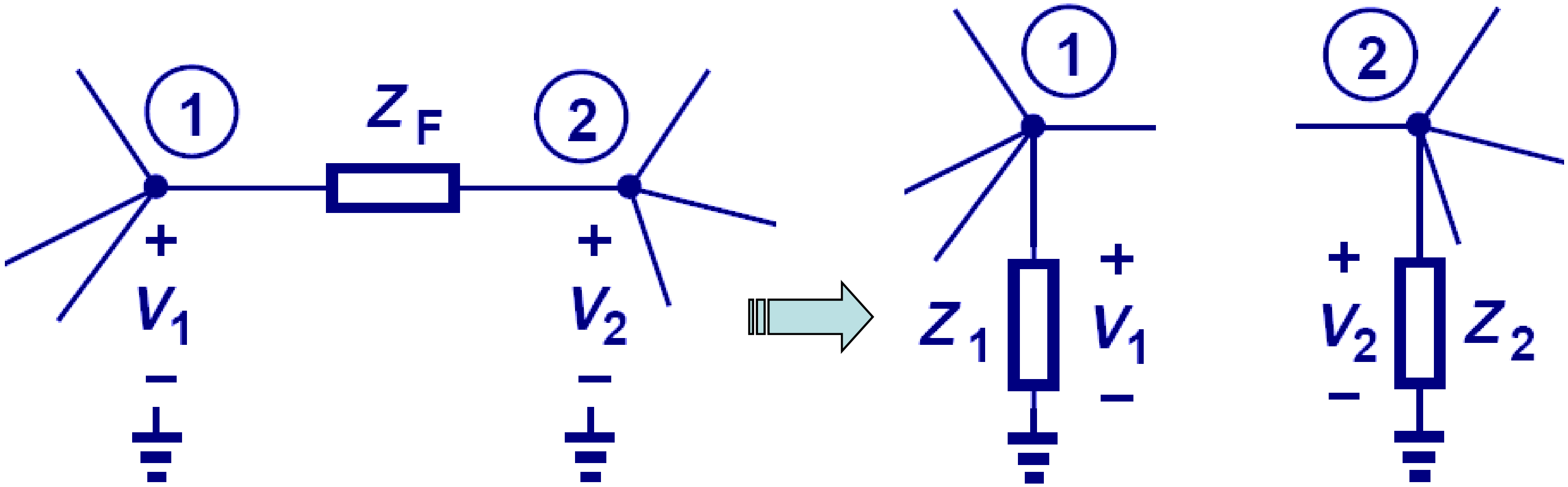
Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.
- While we could always derive the transfer function from the small-signal model, there is a useful "Miller's Theorem" which can be used to approximate the circuit's poles

Miller's Theorem

- If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.



I_1 should be the same in both circuits

$$I_1 = \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$Z_1 = \left(\frac{V_1}{V_1 - V_2}\right) Z_F = \left(\frac{1}{1 - \frac{V_2}{V_1}}\right) Z_F = \frac{Z_F}{1 - A_v}$$

$$\text{where } A_v = \frac{V_2}{V_1}$$

$$\boxed{Z_1 = \frac{Z_F}{1 - A_v}}$$

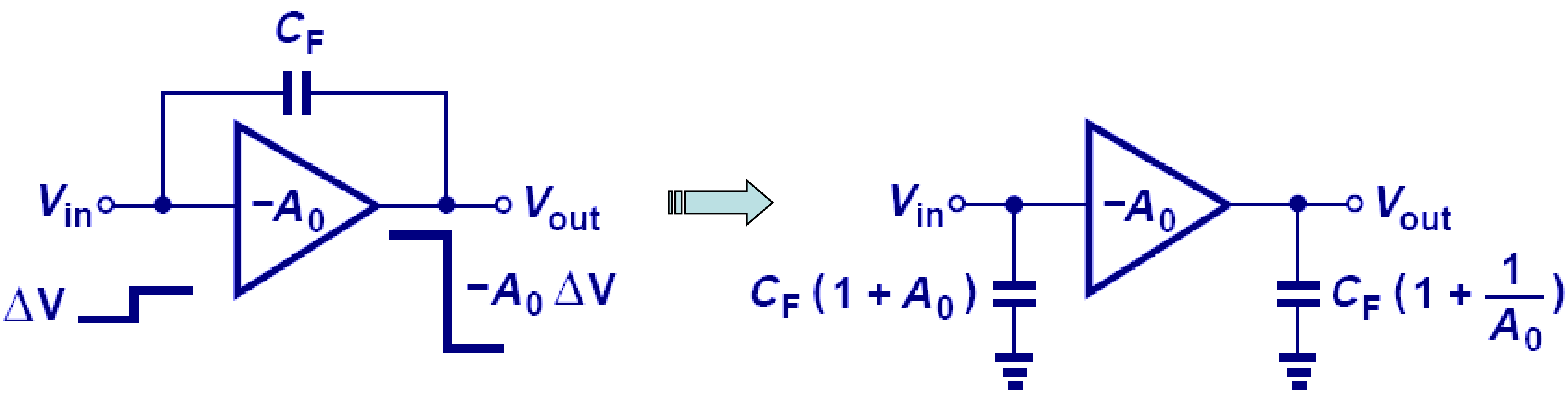
$$\boxed{Z_2 = \frac{Z_F}{1 - 1/A_v}}$$

I_2 should be the same in both circuits

$$I_2 = \frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2}$$

$$Z_2 = \left(\frac{V_2}{V_2 - V_1}\right) Z_F = \left(\frac{1}{1 - \frac{V_1}{V_2}}\right) Z_F = \frac{Z_F}{1 - \frac{1}{A_v}}$$

Miller Multiplication



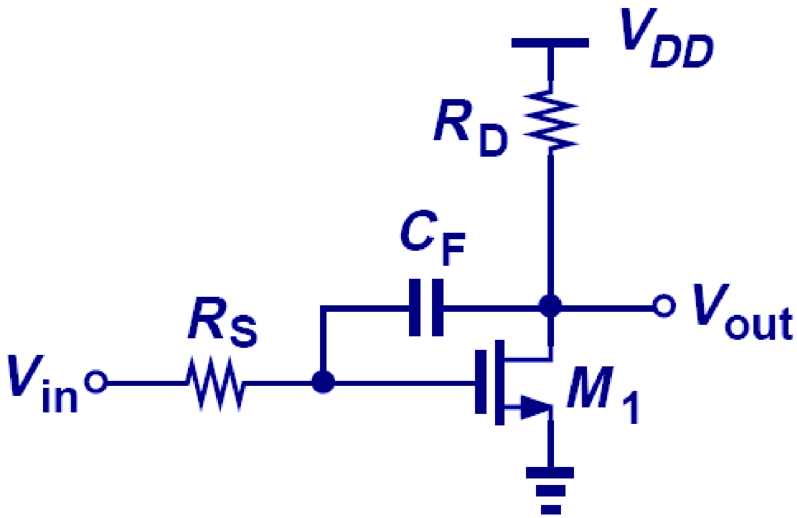
$$Z_{in} = \frac{1}{j\omega C_F(1-A_v)} = \frac{1}{j\omega C_F(1-(-A_o))} = \frac{1}{j\omega C_F(1+A_o)}$$

Equivalent to an input cap that is the original C_F multiplied by $(1+A_o)$

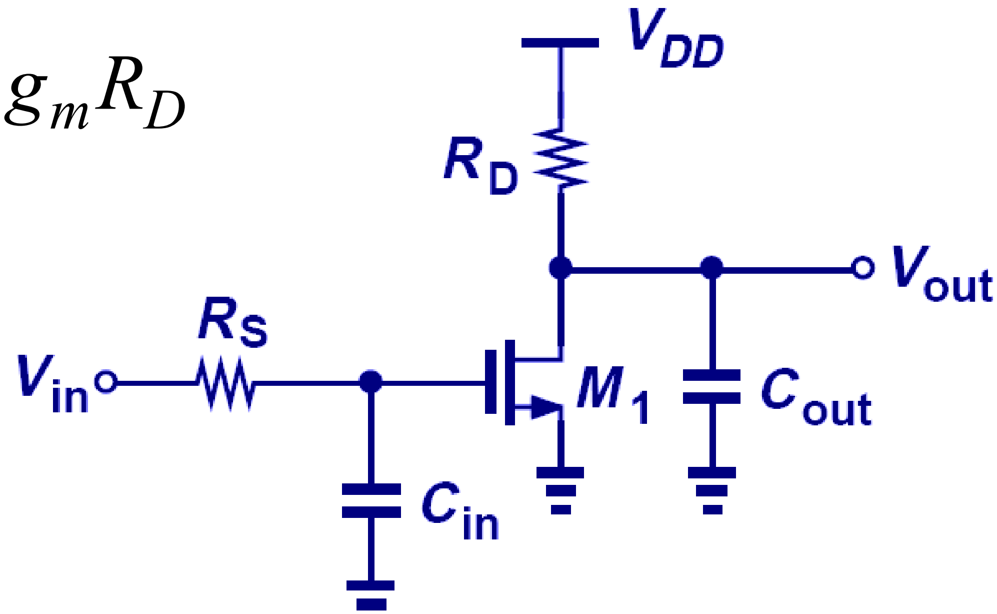
Following a similar procedure, the output cap is the original C_F multiplied by $\left(1+\frac{1}{A_o}\right)$

- **With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.**

Example: Miller Theorem



$$A_v = -g_m R_D$$



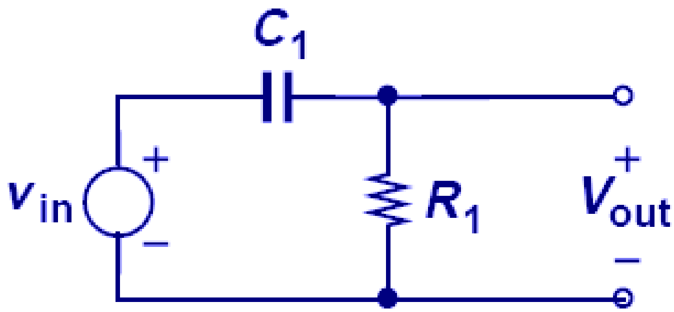
$$\omega_{in} = \frac{1}{R_S\left(1 + g_m R_D\right)C_F}$$

$$\omega_{out} = \frac{1}{R_D\left(1 + \dfrac{1}{g_m R_D}\right)C_F}$$

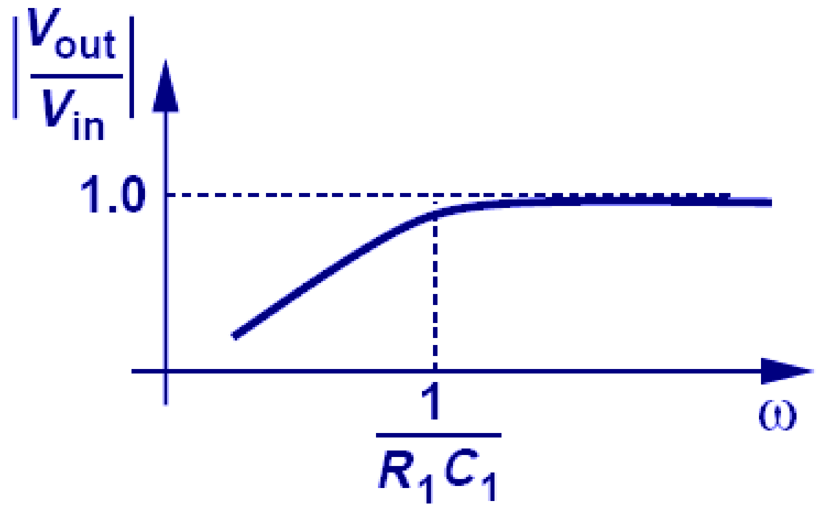
- Note, this is only a (often good) approximation of the transfer function
  - Uses only the low-frequency gain
  - Neglects a zero

$$\left|\frac{V_{out}}{V_{in}}\right| \approx \frac{g_m R_D}{\sqrt{\left(1 + \omega^2 / \omega_{in}^2\right)\left(1 + \omega^2 / \omega_{out}^2\right)}}$$

High-Pass Filter Response



(a)



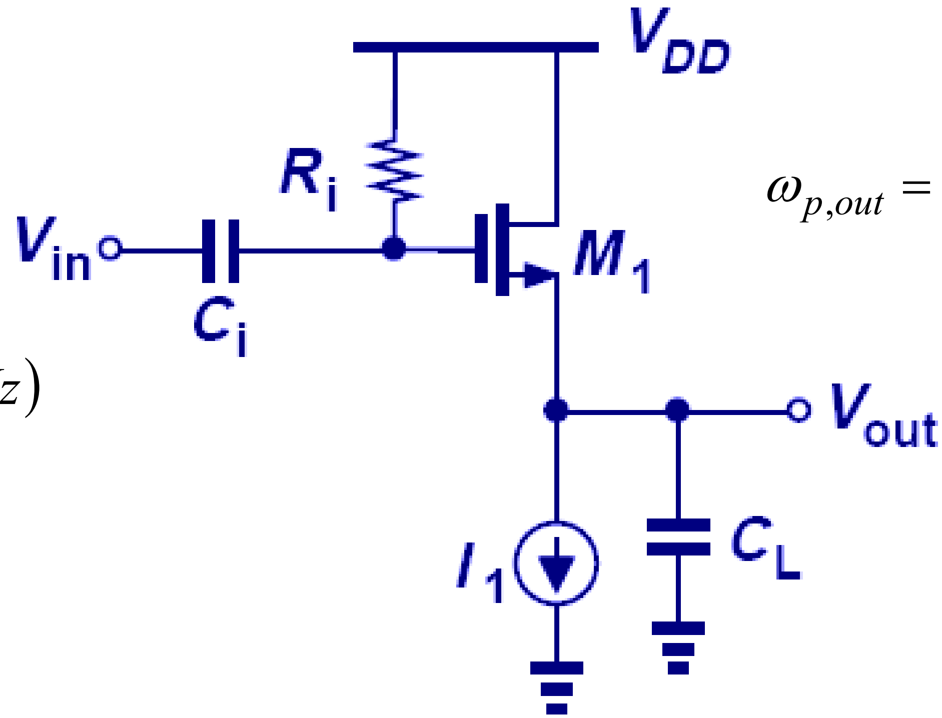
(b)

$$V_o(s) = \frac{Z_{R_1}}{Z_{R_1} + Z_{C_1}} V_{in}(s) = \frac{R_1}{R_1 + \frac{1}{sC_1}} V_{in}(s) = \frac{sR_1C_1}{1 + sR_1C_1} V_{in}(s)$$

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1C_1\omega}{\sqrt{R_1^2C_1^2\omega_1^2 + 1}}}$$

- **The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.**

Example: Audio Amplifier



$$\omega_{p,in} = \frac{1}{R_i C_i} = 2\pi(20Hz)$$

$$\omega_{p,out} = \frac{1}{\left(\frac{1}{g_{m1}}\right) C_L} = 2\pi(20kHz)$$

$$C_i = 79.6nF$$

$$C_L = 39.8nF$$

$$R_i = 100K\Omega$$

$$g_m = 1/200\Omega$$

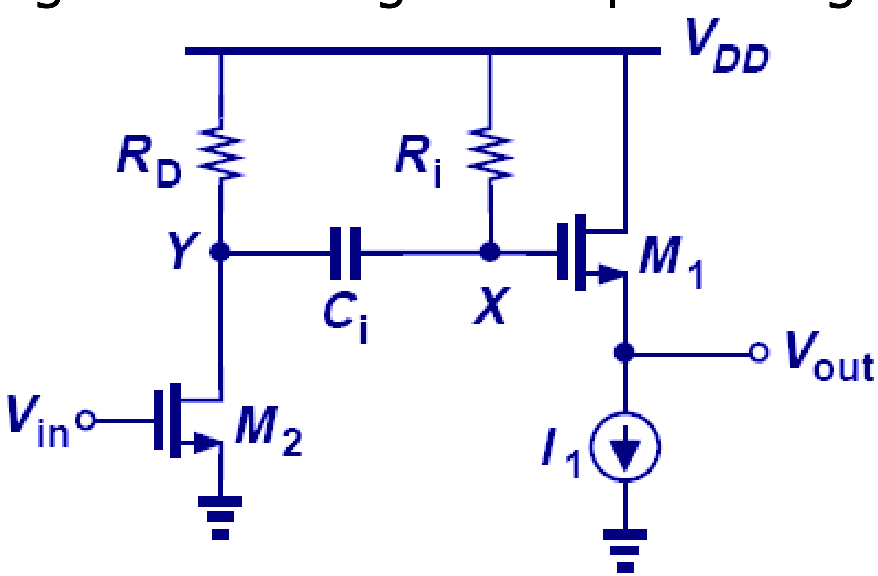
- **In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and output capacitances are needed.**

Capacitive Coupling vs. Direct Coupling

- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

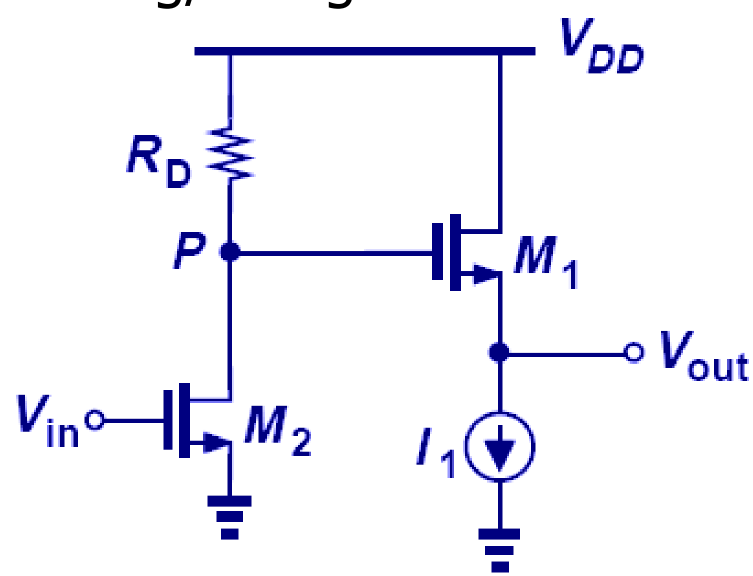
Allows for high $V(R_D)$ (gain), while also allowing a high output stage gate bias for good output swing

Due to direct coupling, must trade-off A_{V1} for output stage biasing/swing



(a)

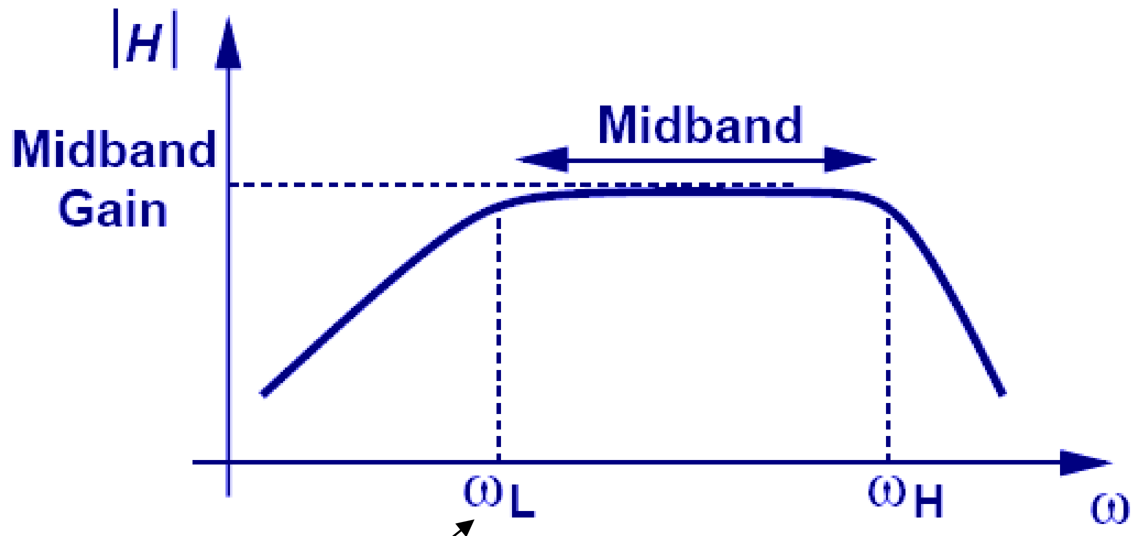
Capacitive Coupling



(b)

Direct Coupling

Typical Frequency Response



Lower Corner

Upper Corner

Often due to AC coupling

Often due to load/parasitic capacitors

Agenda

- Frequency Response Concepts
- High-Frequency Models of Transistors
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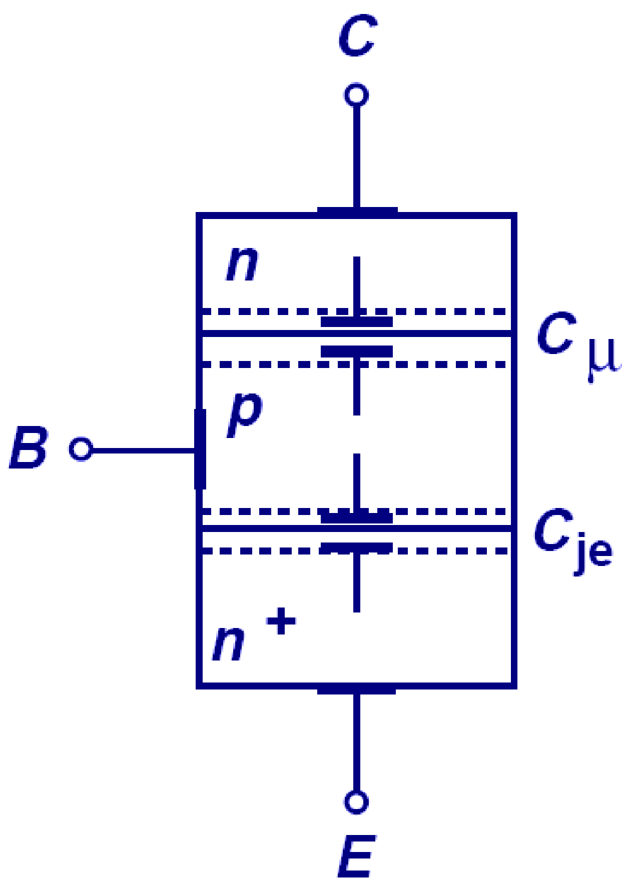
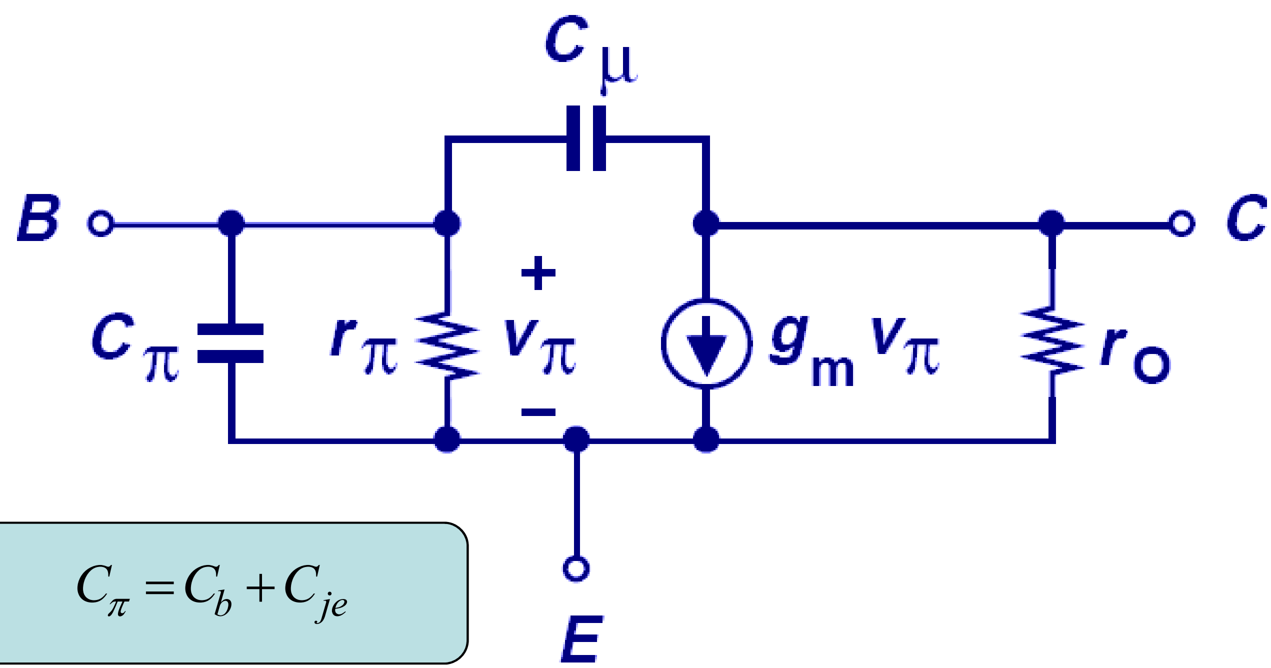
High-Frequency Bipolar Model

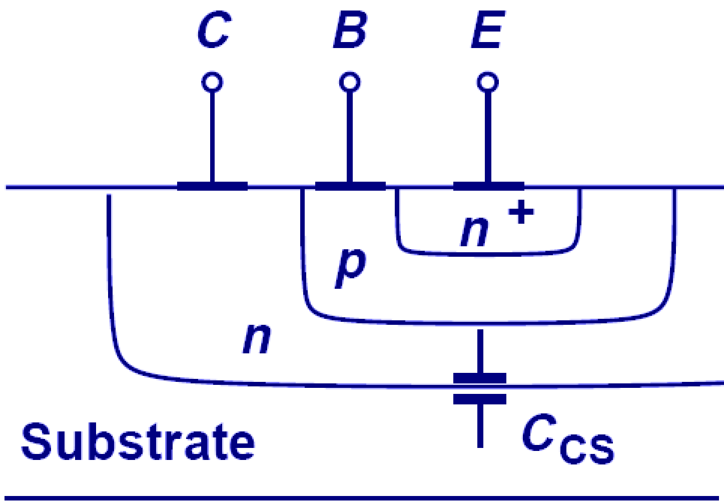


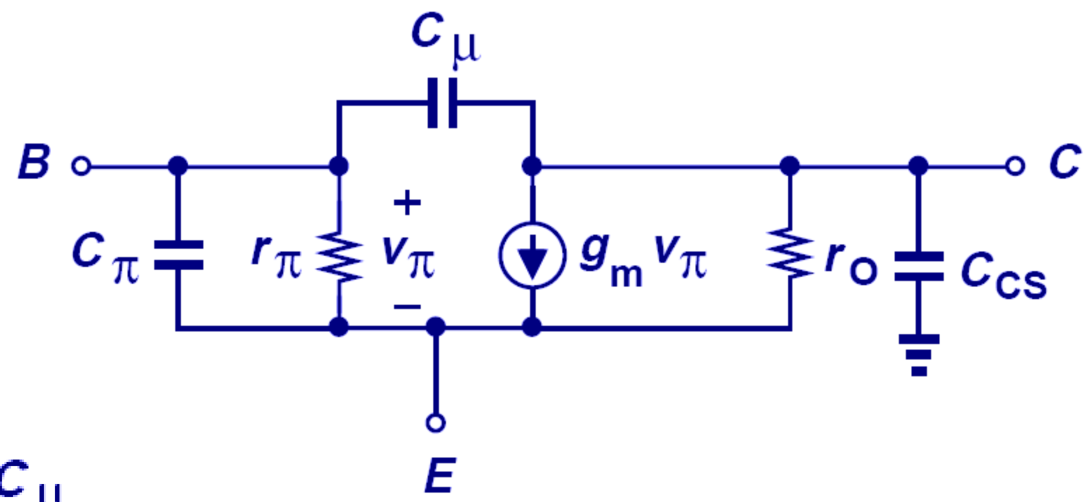


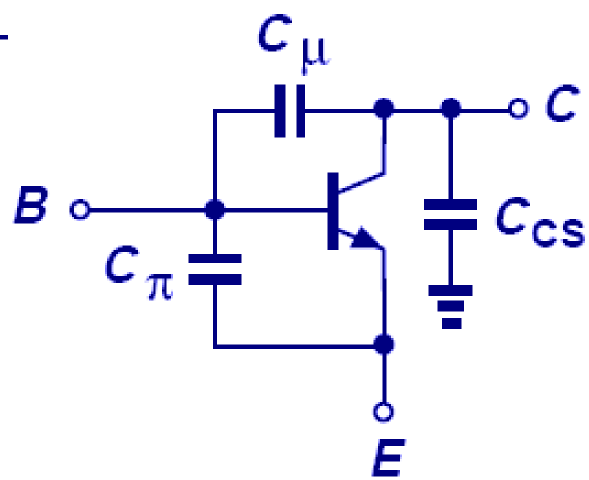
$$C_\pi = C_b + C_{je}$$

- At high frequency, capacitive effects come into play
- C_μ and C_{je} are the junction capacitances
- C_b represents the base charge to generate the non-uniform charge profile required for proper operation (Chapter 4)

High-Frequency Model of Integrated Bipolar Transistor

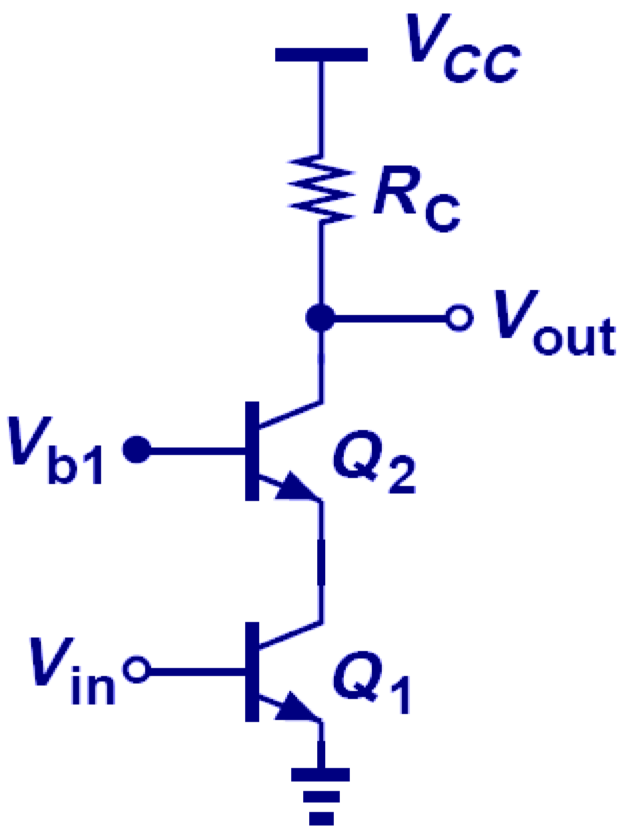


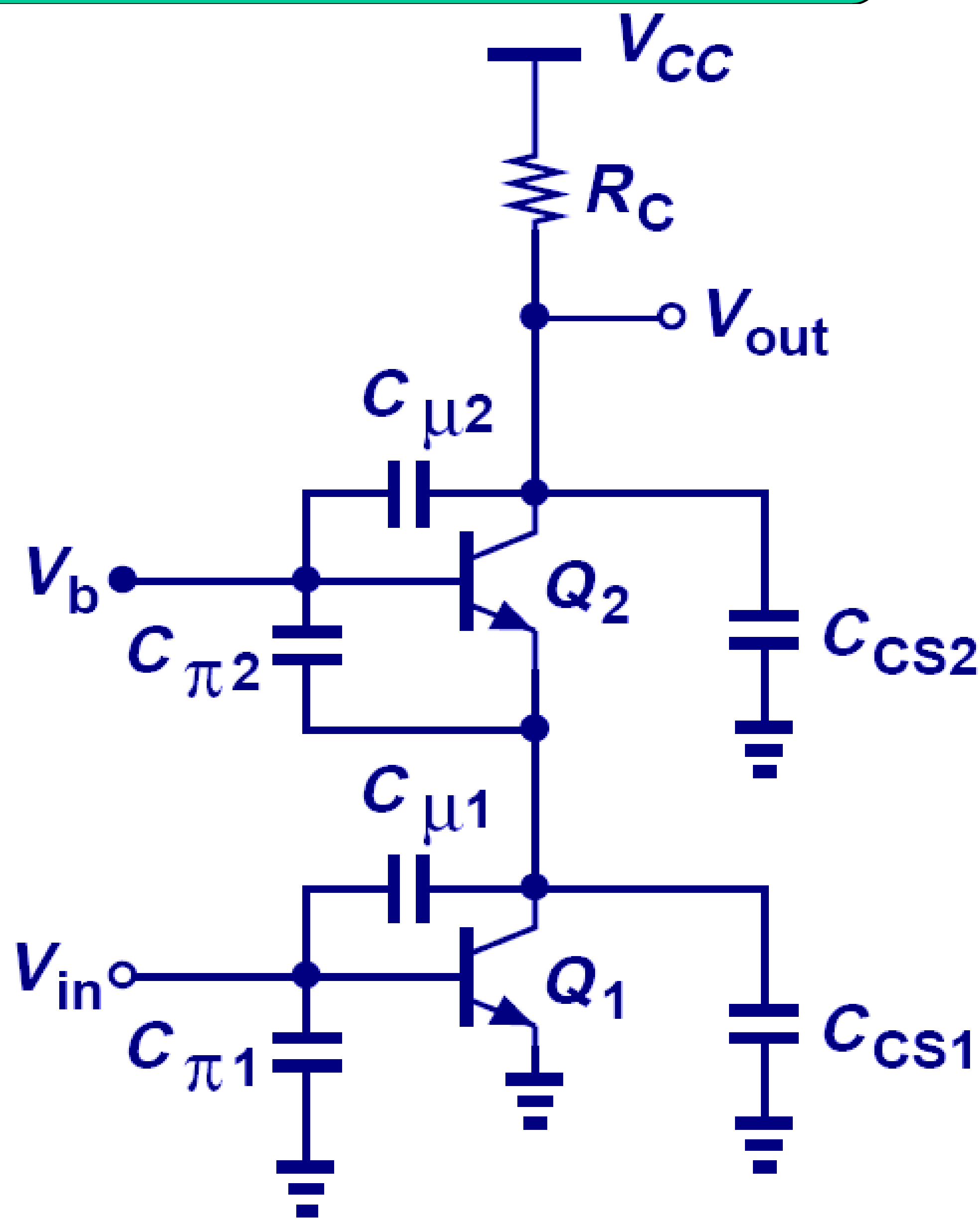




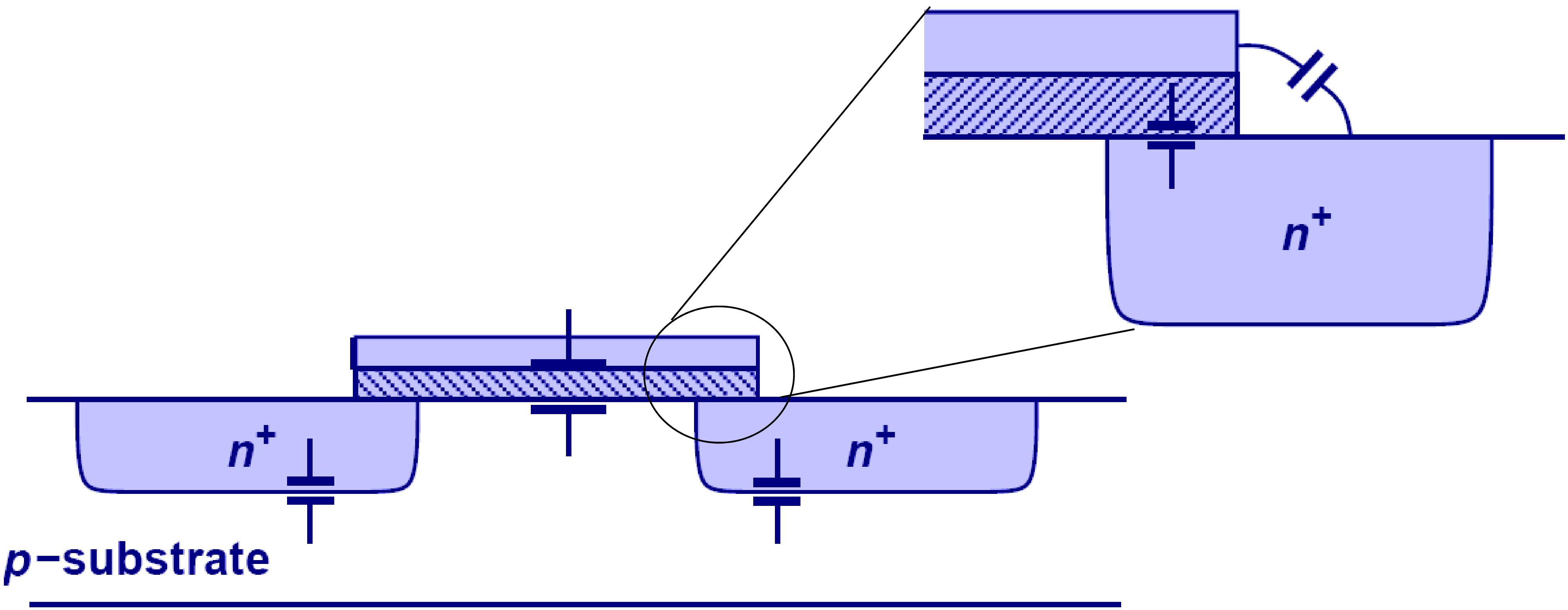
- Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS}.

Example: Capacitance Identification



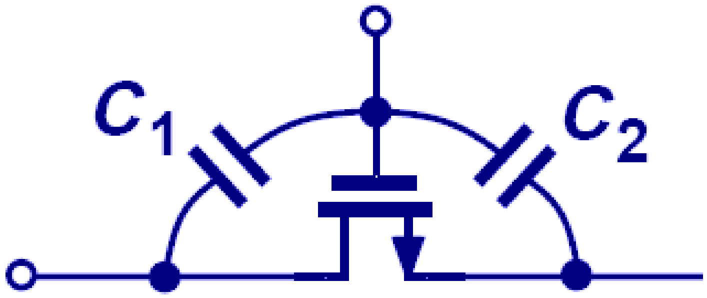


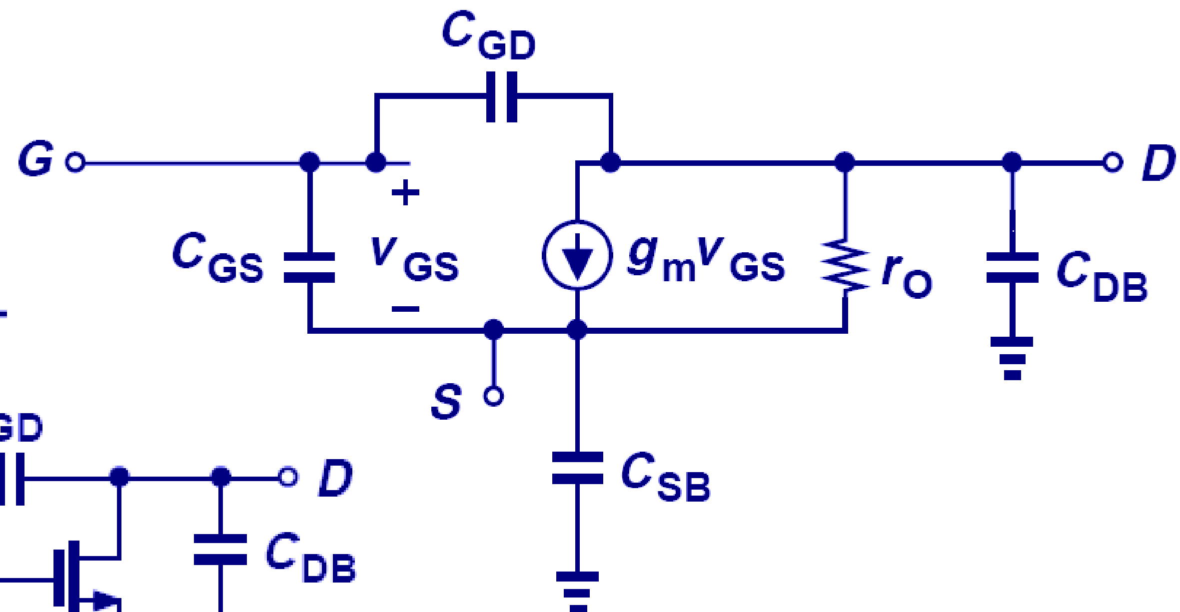
MOS Intrinsic Capacitances

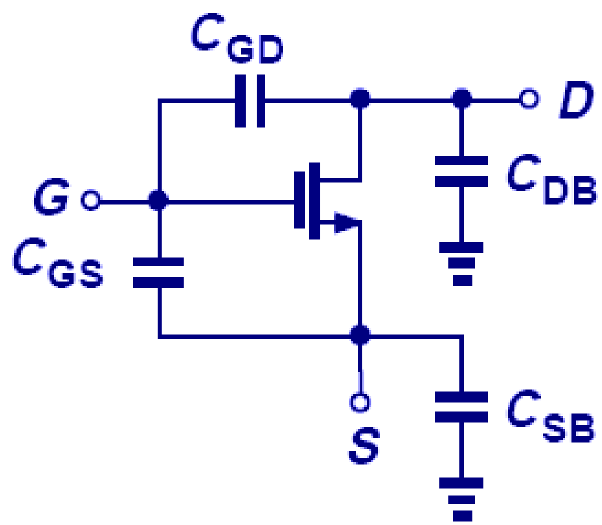


- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model



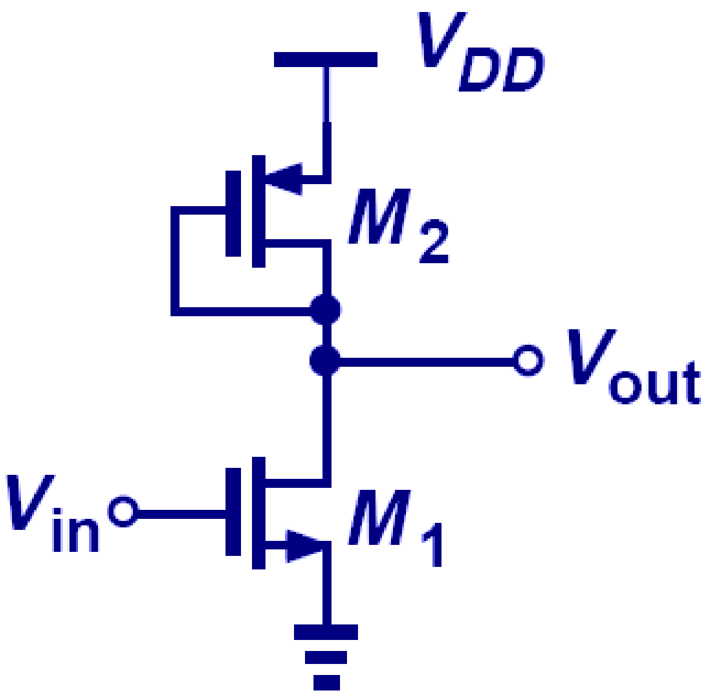


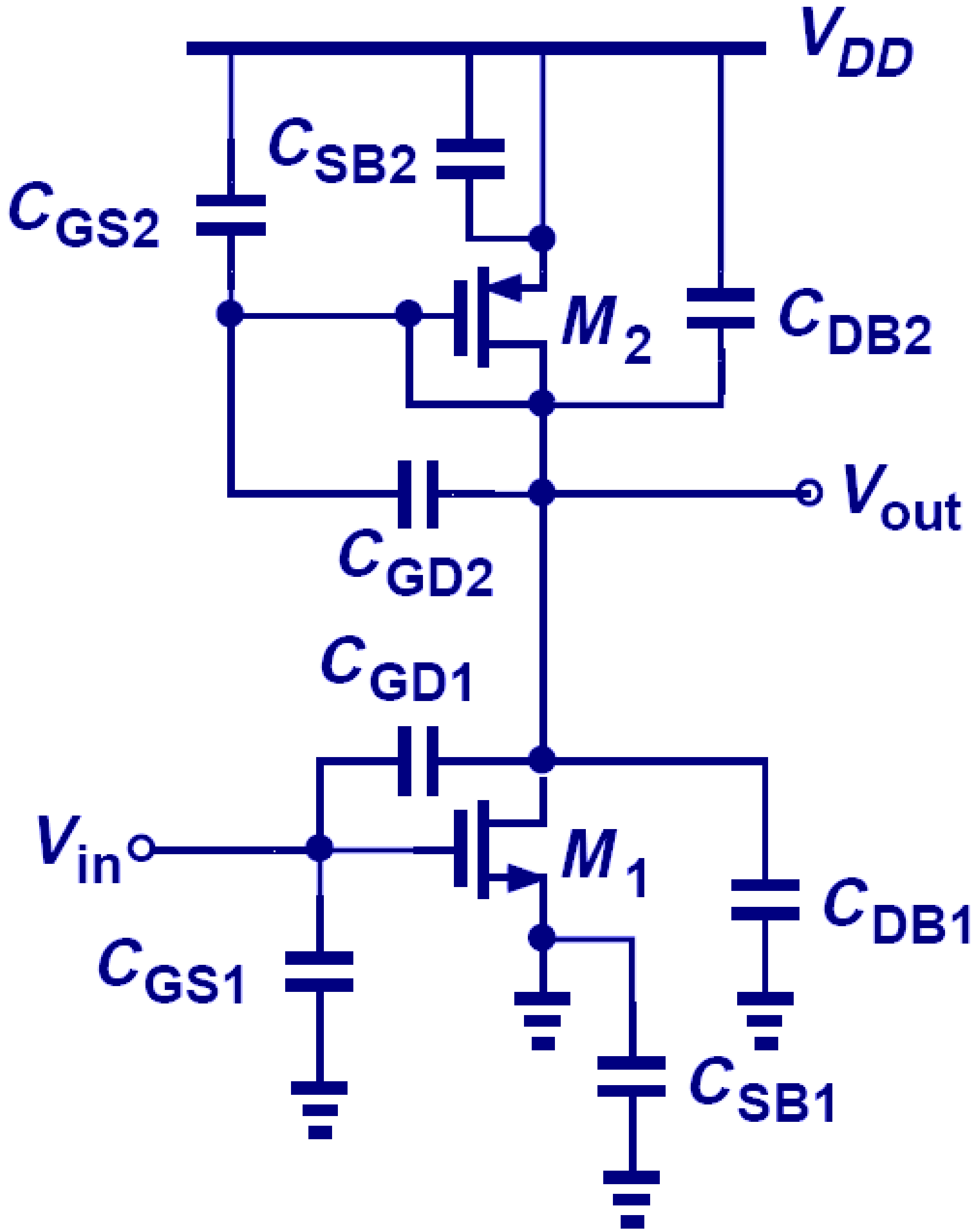


Assuming bulk is an AC ground

- **The gate oxide capacitance is often partitioned between source and drain. In saturation, C_2 ~ C_{gate}, and C_1 ~ 0. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD}.**

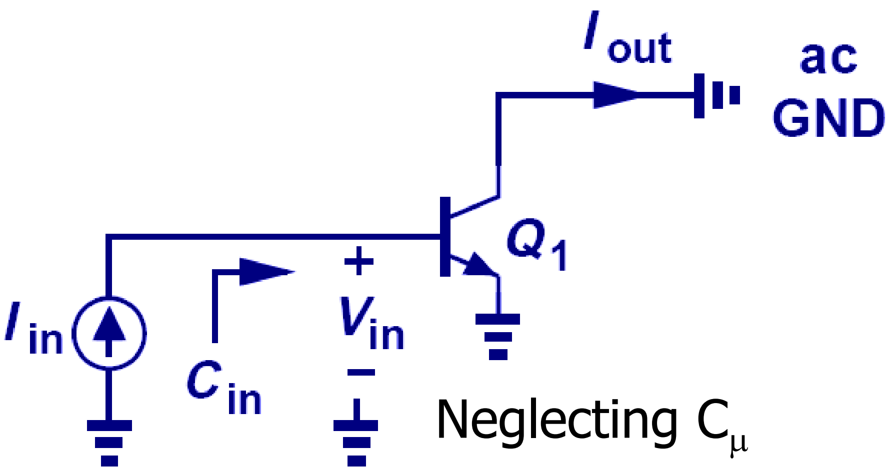
Example: Capacitance Identification





Transit Frequency

- Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.



$$I_{out} = g_m V_{in} = g_m I_{in} Z_{in}$$

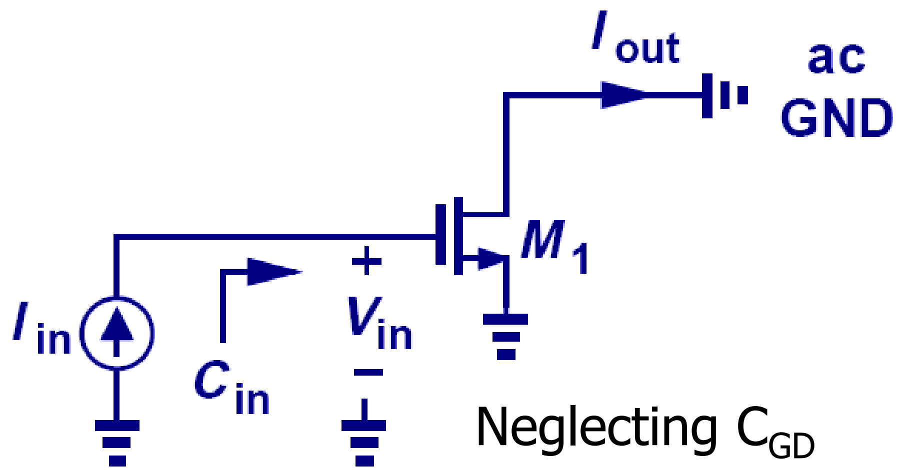
$$Z_{in} = \frac{1}{C_\pi s} \Big\| r_\pi$$

$$\frac{I_{out}}{I_{in}} = \frac{g_m r_\pi}{r_\pi C_\pi s + 1} = \frac{\beta}{r_\pi C_\pi s + 1}$$

Setting the magnitude equal to 1 at ω_T yields

$$r_\pi^2 C_\pi^2 \omega_T^2 = \beta^2 - 1 \approx \beta^2$$

$$\boxed{\omega_T \approx \frac{\beta}{r_\pi C_\pi} = \frac{g_m}{C_\pi}}$$



$$I_{out} = g_m V_{in} = g_m I_{in} Z_{in}$$

$$Z_{in} = \frac{1}{C_{GS} s}$$

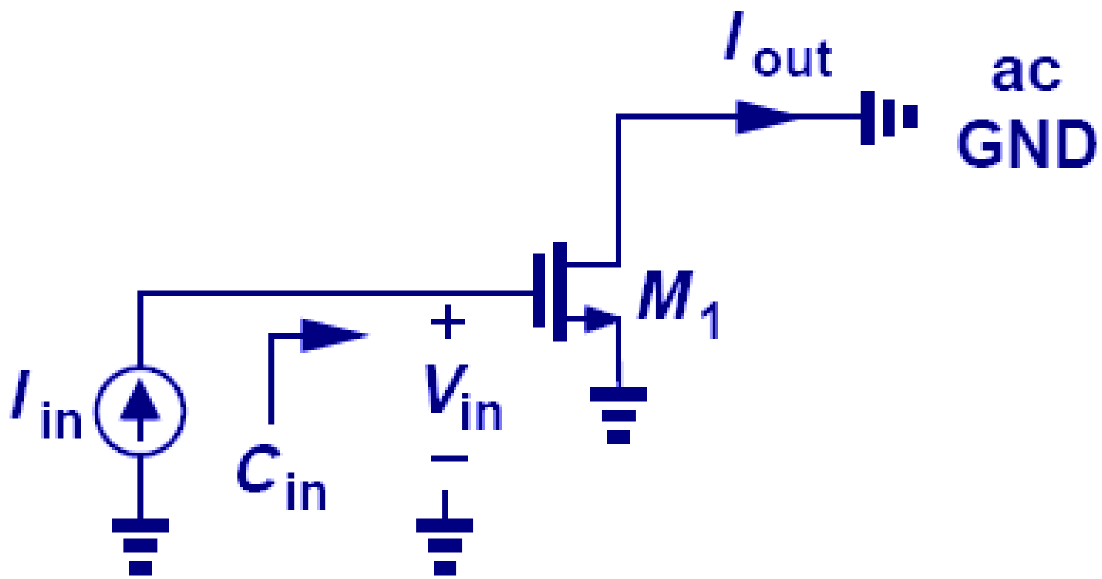
$$\frac{I_{out}}{I_{in}} = \frac{g_m}{C_{GS} s}$$

Setting the magnitude equal to 1 at ω_T yields

$$C_{GS}^2 \omega_T^2 = g_m^2$$

$$\boxed{\omega_T = \frac{g_m}{C_{GS}}}$$

Example: Transit Frequency Calculation



$$\omega_T = \frac{g_m}{C_{GS}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})$$

$$C_{GS} = \frac{2}{3} W L C_{ox} \quad \text{(talked about in 474)}$$

$$\omega_T = 2\pi f_T = \frac{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})}{\frac{2}{3} W L C_{ox}}$$

$$f_T = \frac{3\mu_n (V_{GS} - V_{TH})}{4\pi L^2}$$

$$L = 65nm$$
$$V_{GS} - V_{TH} = 100mV$$
$$\mu_n = 400cm^2/(V.s)$$
$$f_T = 226GHz$$

- The transit frequency increases dramatically as the channel length is shrunk, allowing for much faster transistors with CMOS scaling
- Note, this neglects some advanced device physics (carrier velocity saturation) which slows this rate of frequency increase

Agenda

- Frequency Response Concepts
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Analysis Summary

- The frequency response refers to the magnitude of the transfer function.
- Bode's approximation simplifies the plotting of the frequency response if poles and zeros are known.
- In general, it is possible to associate a pole with each node in the signal path.
- Miller's theorem helps to decompose floating capacitors into grounded elements.
- Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.

High Frequency Circuit Analysis Procedure

- Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).
- Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).
- Include transistor capacitances.
- Merge capacitors connected to AC grounds and omit those that play no role in the circuit.
- Determine the high-frequency poles and zeros.
- Plot the frequency response using Bode's rules or exact analysis.

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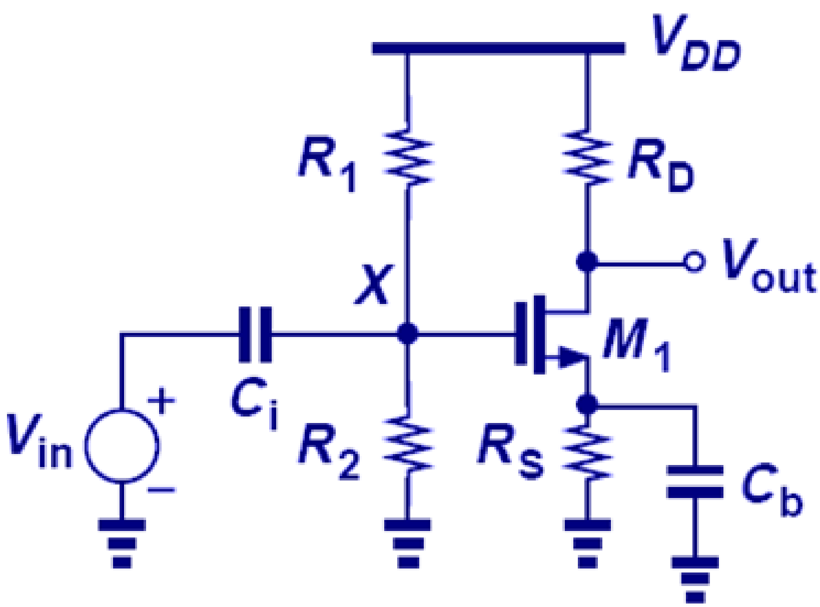
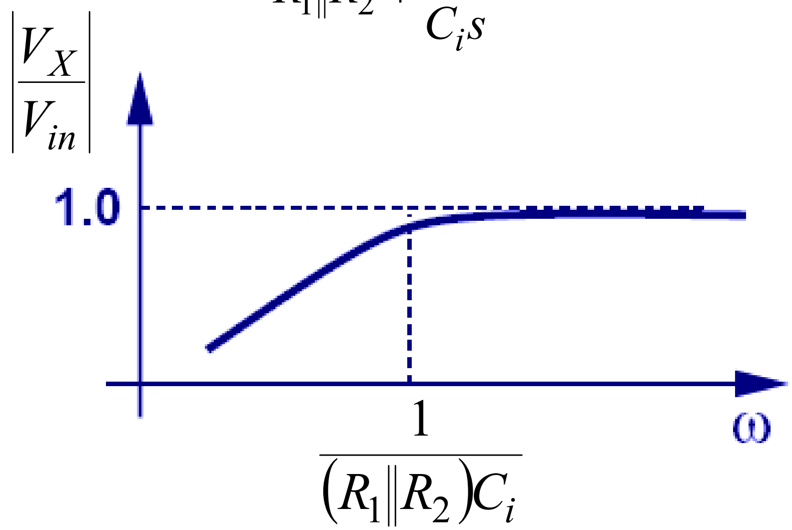
Frequency Response of CS Stage with Bypassed Degeneration – Input AC Coupling



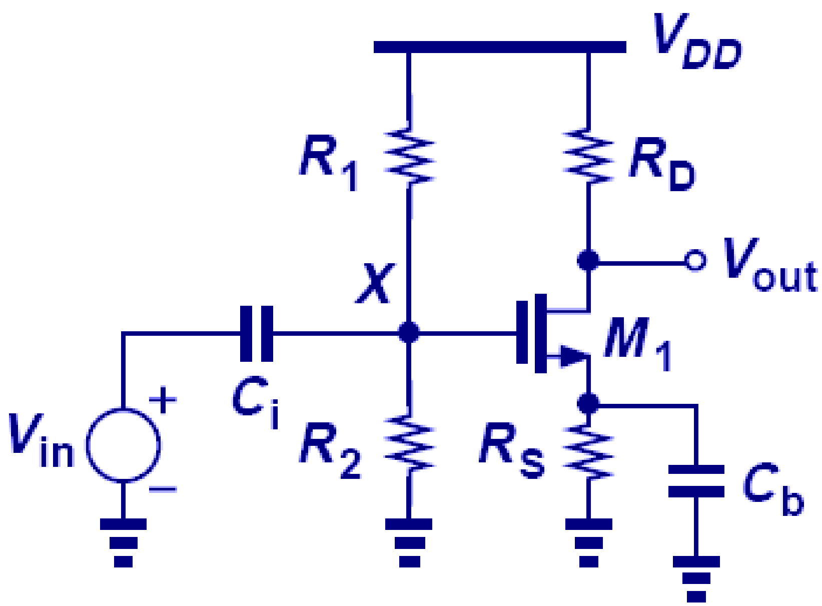
$$\frac{V_{out}}{V_{in}} = \left(\frac{V_X}{V_{in}}\right)\left(\frac{V_{out}}{V_X}\right)$$

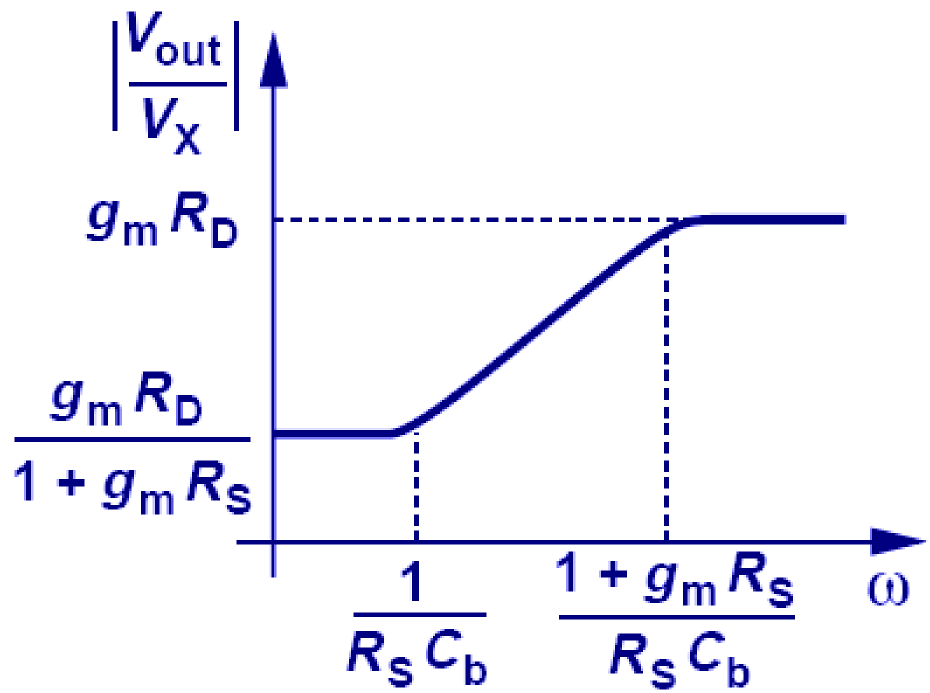
$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \| R_2}{R_1 \| R_2 + \frac{1}{C_i s}} = \frac{(R_1 \| R_2) C_i s}{(R_1 \| R_2) C_i s + 1}$$



- The input AC coupling forms a high-pass filter which should be designed for a certain minimum cut-off frequency

Frequency Response of CS Stage with Bypassed Degeneration – Main Amplifier





- **In order to increase the midband gain, a capacitor C_b is placed in parallel with R_s.**

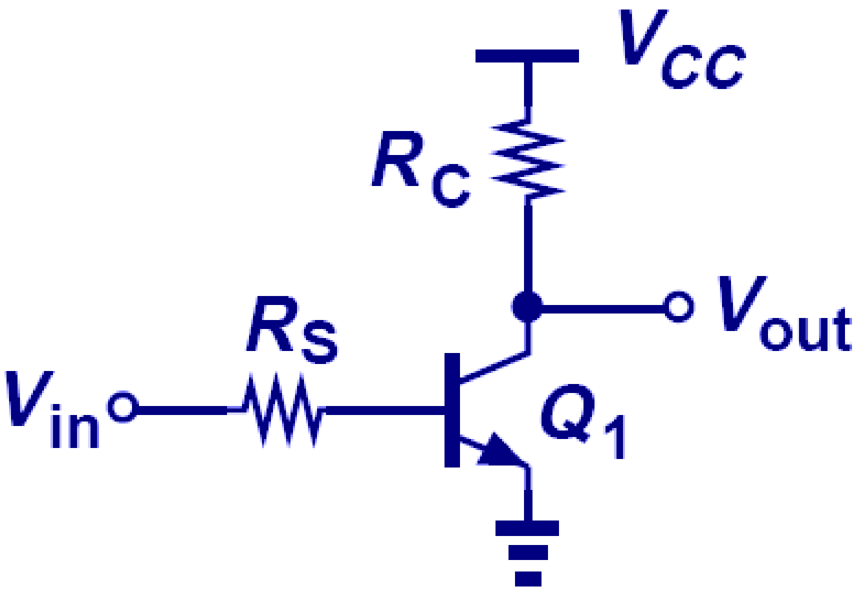
$$\frac{V_{out}}{V_X} = \frac{-R_D}{R_S \| \frac{1}{C_b s} + \frac{1}{g_m}} = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

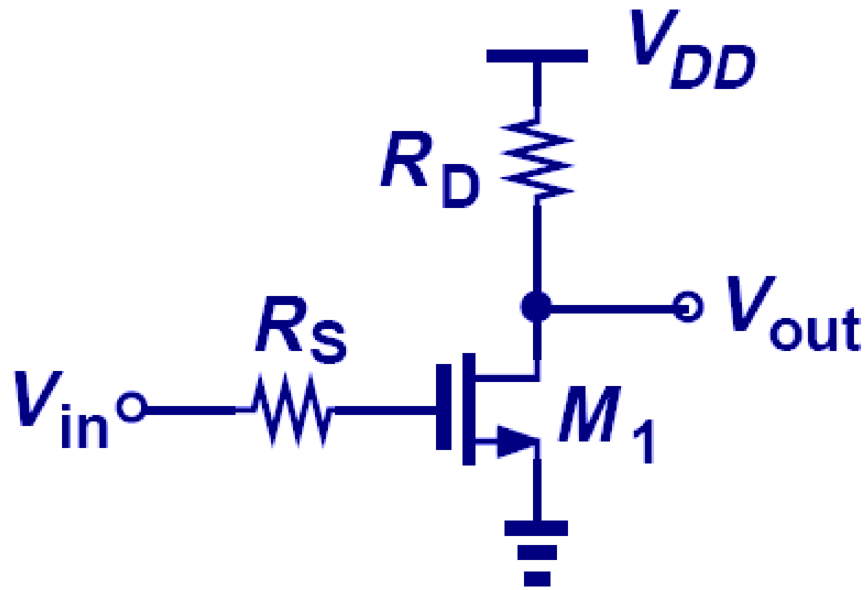
The pole frequency

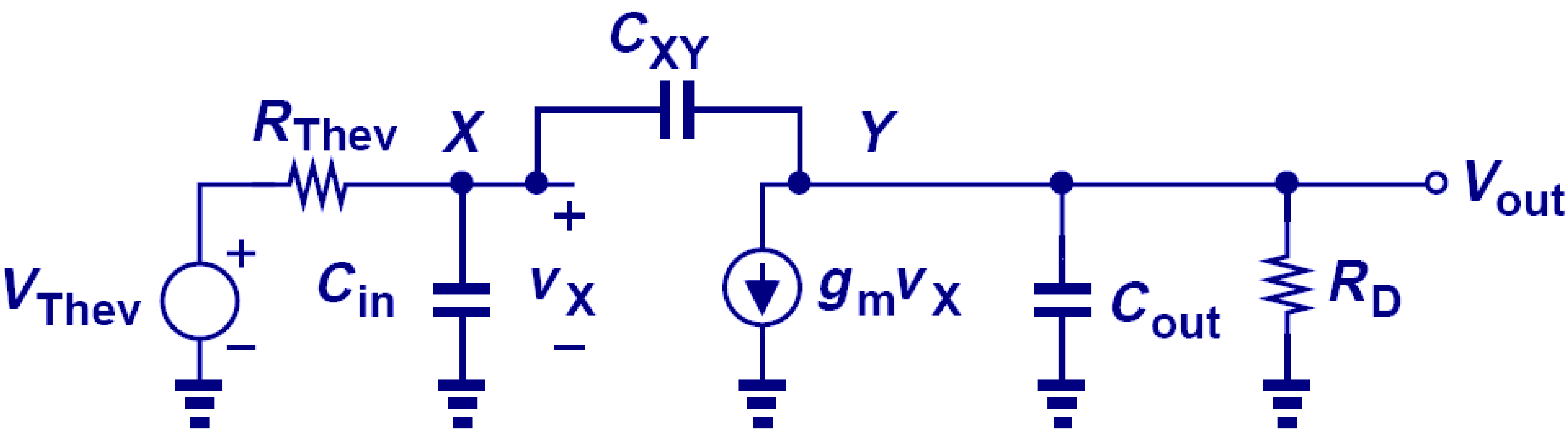
$$|\omega_p| = \frac{1 + g_m R_S}{R_S C_b}$$

should also be designed to set the minimum cut - off frequency

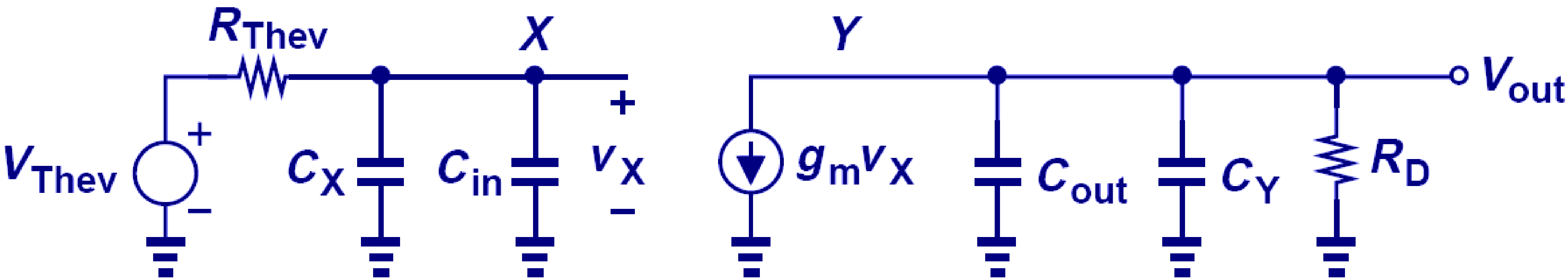
Unified Model for CE and CS Stages







Unified Model Using Miller's Theorem



CE Stage

$$V_{Thev} = V_{in}\frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S \| r_\pi$$

$$C_X = C_\mu (1 + g_m R_C)$$

$$C_Y = C_\mu \left(1 + \frac{1}{g_m R_C}\right)$$

$$C_{in} = C_\pi$$

$$C_{out} = C_{CS}$$

CS Stage

$$V_{Thev} = V_{in}$$

$$R_{Thev} = R_S$$

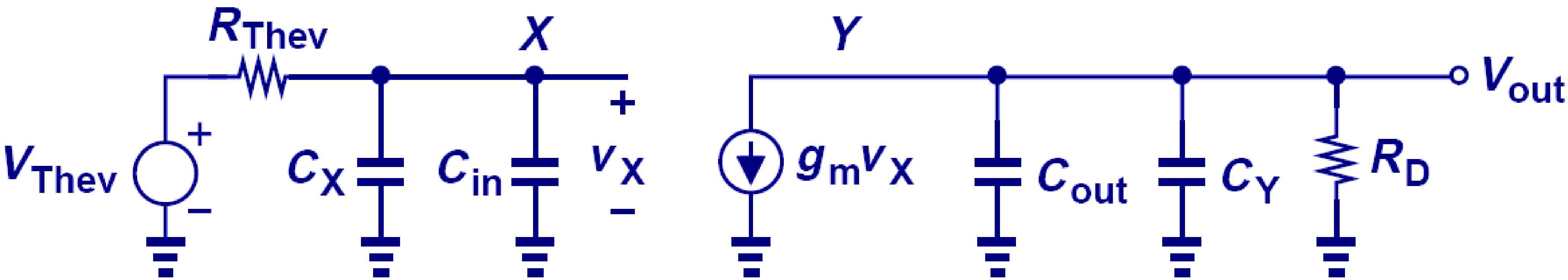
$$C_X = C_{GD}(1 + g_m R_D)$$

$$C_Y = C_{GD}\left(1 + \frac{1}{g_m R_D}\right)$$

$$C_{in} = C_{GS}$$

$$C_{out} = C_{DB}$$

Unified Model Using Miller's Theorem

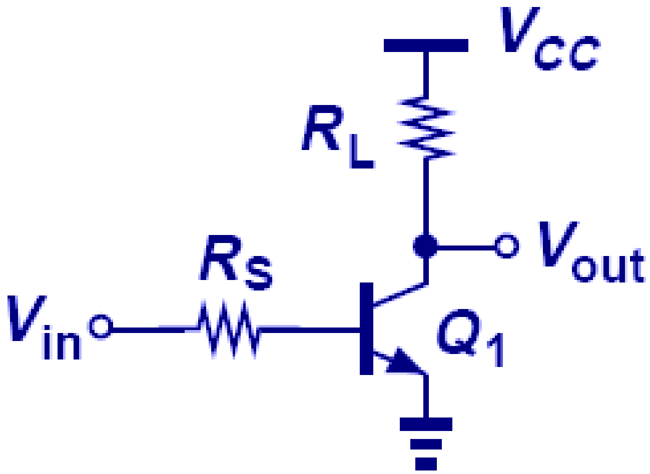


$$\left|\omega_{p,in}\right| = \frac{1}{R_{Thev}\left[C_{in} + \left(1 + g_m R_L\right)C_{XY}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[C_{out} + \left(1 + \frac{1}{g_m R_L}\right)C_{XY}\right]}$$

Example: CE Stage

$R_L = 2k\Omega$

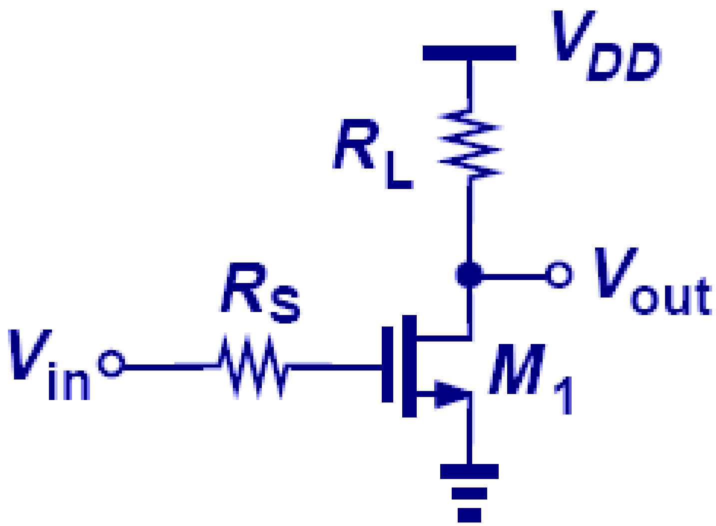


$$R_S = 200\Omega$$
$$I_C = 1mA$$
$$\beta = 100$$
$$C_\pi = 100\,fF$$
$$C_\mu = 20\,fF$$
$$C_{CS} = 30\,fF$$

$$\left|\omega_{p,in}\right| = 2\pi \times (516MHz)$$
$$\left|\omega_{p,out}\right| = 2\pi \times (1.59GHz)$$

- **The input pole is the bottleneck for speed.**

Example: Half Width CS Stage



$$W \downarrow 2X$$

$$g_m = \mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$$

In 474 we will learn that all MOS caps are $\propto W$

$$C_{GS} = \frac{2}{3}WLC_{ox} + WC_{ov}$$

$$C_{GD} = WC_{ov}$$

$$C_{DB} = A_D C_j + P_D C_{jsw} \propto W$$

$$\left|\omega_{p,in}\right| = \frac{1}{R_S\left[\frac{C_{in}}{2} + \left(1 + \frac{g_m R_L}{2}\right)\frac{C_{XY}}{2}\right]}$$

$$\left|\omega_{p,out}\right| = \frac{1}{R_L\left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_m R_L}\right)\frac{C_{XY}}{2}\right]}$$

- LF gain, $g_m R_L$ reduces by 1/2
- Assuming $g_m R_L$ is still high:
- The input pole increases by ~4X
- The output pole increases by ~2x
  - Constant gain-bandwidth product!

Direct Analysis of CE and CS Stages

- For a detailed direct small-signal analysis, see Razavi 11.4.4

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

where

$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})$$

Exact Expression

To find the 2 poles, we can write

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

Using a "dominant pole" approximation $\omega_{p1} << \omega_{p2}$

$$as^2 + bs + 1 = \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{s}{\omega_{p1}} + 1$$

$$\omega_{p1} = \frac{1}{b} \quad \text{and} \quad \omega_{p2} = \frac{b}{a}$$

➤ **Direct analysis yields different pole locations and an extra zero.**

Direct Analysis of CE and CS Stages w/ Dominant Pole Approximation

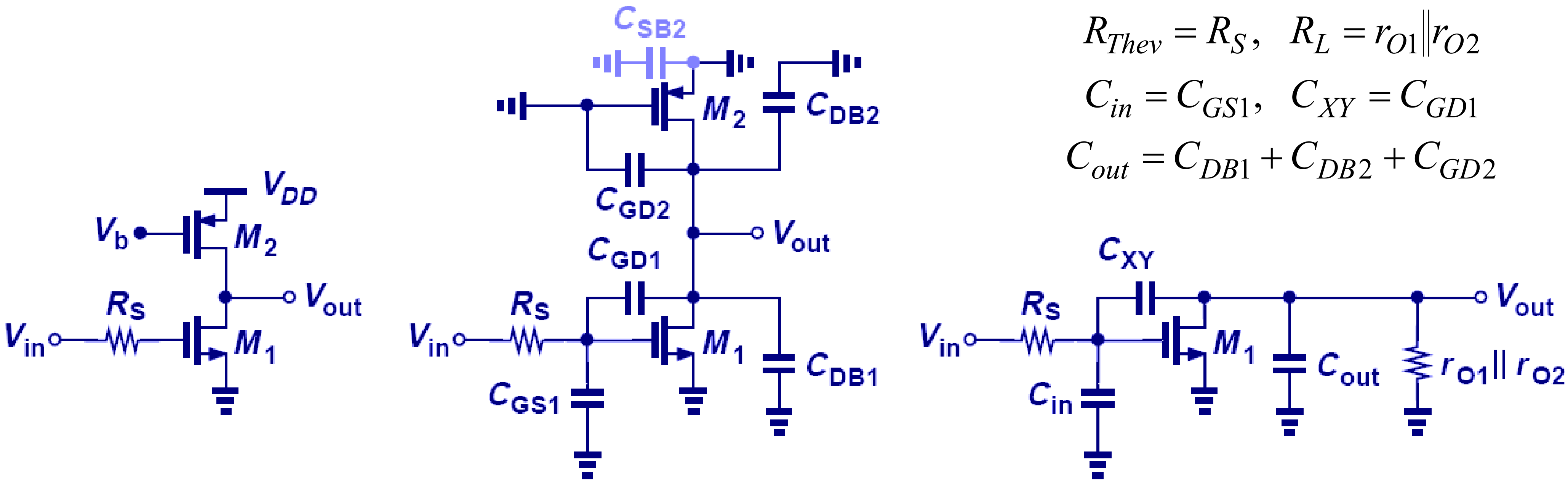
$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$|\omega_{p1}| = \frac{1}{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out})}$$

$$|\omega_{p2}| = \frac{(1+g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out})}{R_{Thev}R_L(C_{in}C_{XY}+C_{out}C_{XY}+C_{in}C_{out})}$$

- ω_{p1} will be lower due to the additional term
- ω_{p2} is at a much higher frequency due to "pole splitting"
  - Discussed more when we talk about stability

Example: CE and CS Direct Analysis (Dominant Pole Approximation)



$$R_{Thev} = R_S, \quad R_L = r_{O1} \| r_{O2}$$

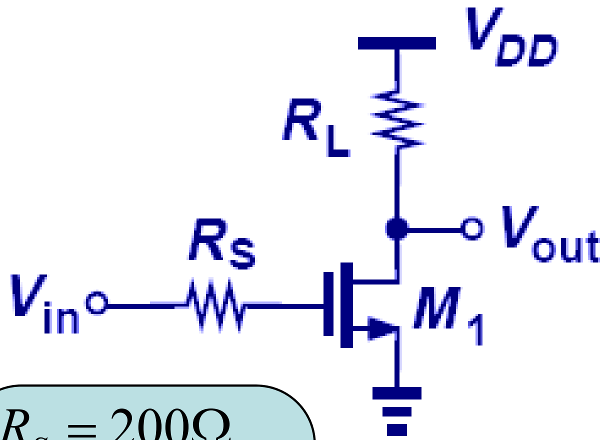
$$C_{in} = C_{GS1}, \quad C_{XY} = C_{GD1}$$

$$C_{out} = C_{DB1} + C_{DB2} + C_{GD2}$$

$$\omega_{p1} \approx \frac{1}{[1 + g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{[1 + g_{m1}(r_{O1} \| r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \| r_{O2})(C_{XY} + C_{out})}{R_S (r_{O1} \| r_{O2})(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

Example: Comparison Between Different Methods



$R_S = 200\Omega$

$C_{GS} = 250\,fF$

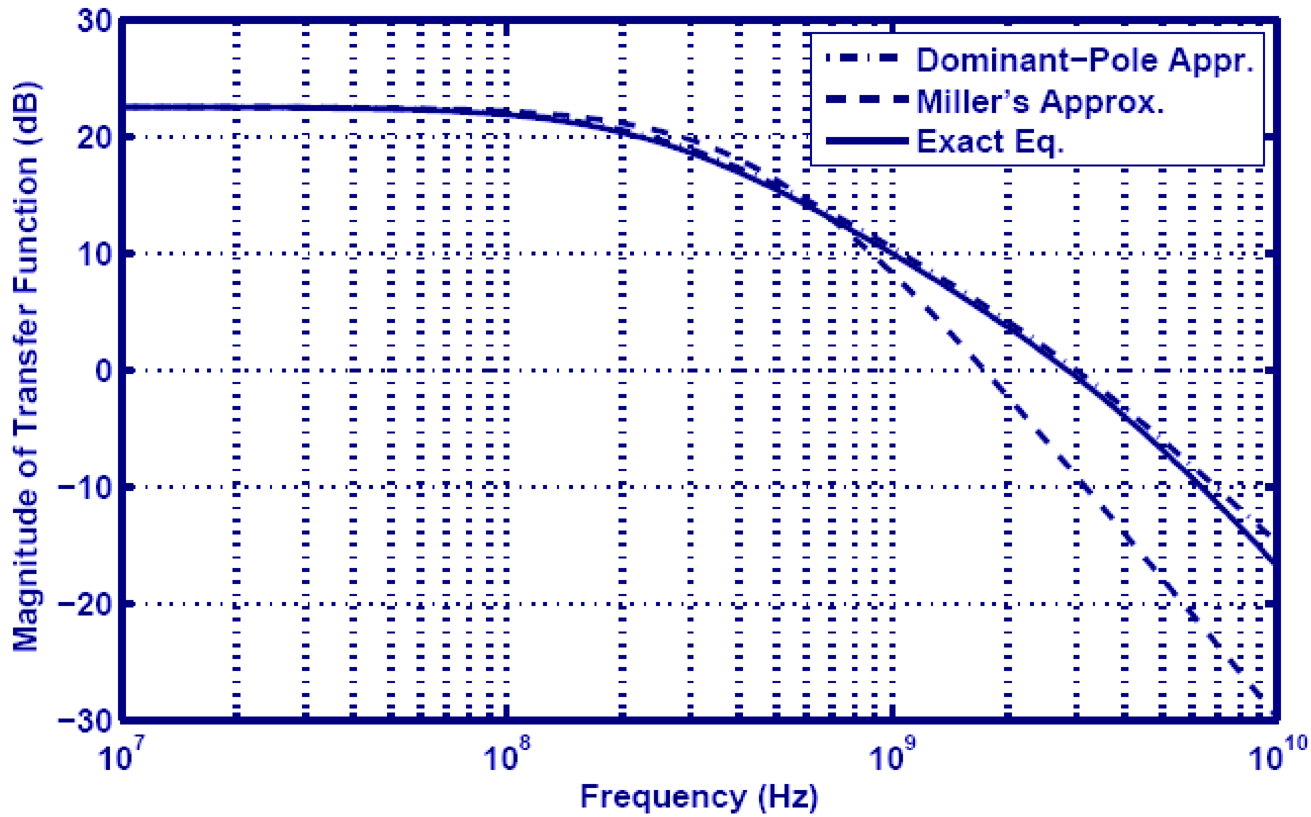
$C_{GD} = 80\,fF$

$C_{DB} = 100\,fF$

$g_m = (150\Omega)^{-1}$

$\lambda = 0$

$R_L = 2K\Omega$



Simple Miller theorem analysis vastly overestimates the output pole at a lower frequency

<u>*Miller's*</u>

$$|\omega_{p,in}| = 2\pi \times (571MHz)$$
$$|\omega_{p,out}| = 2\pi \times (428MHz)$$

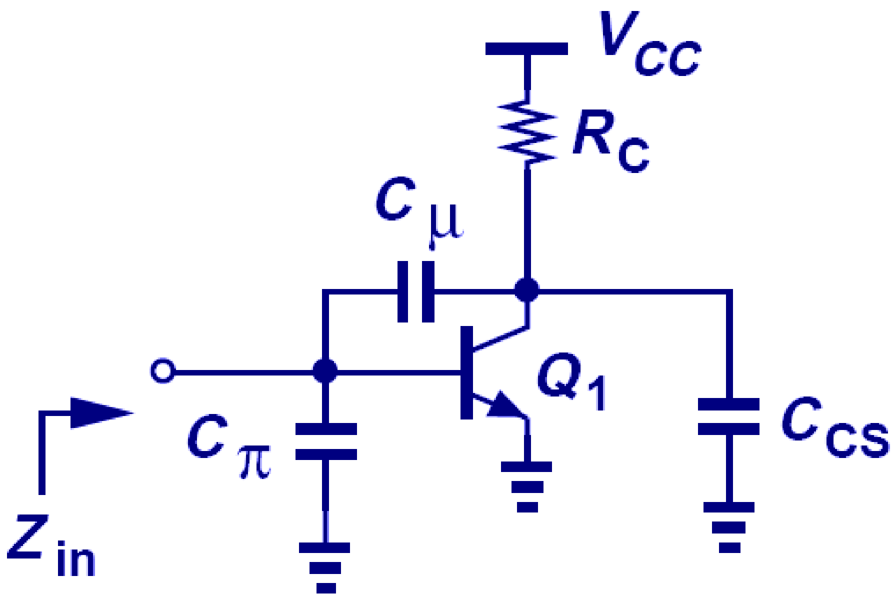
<u>*Exact*</u>

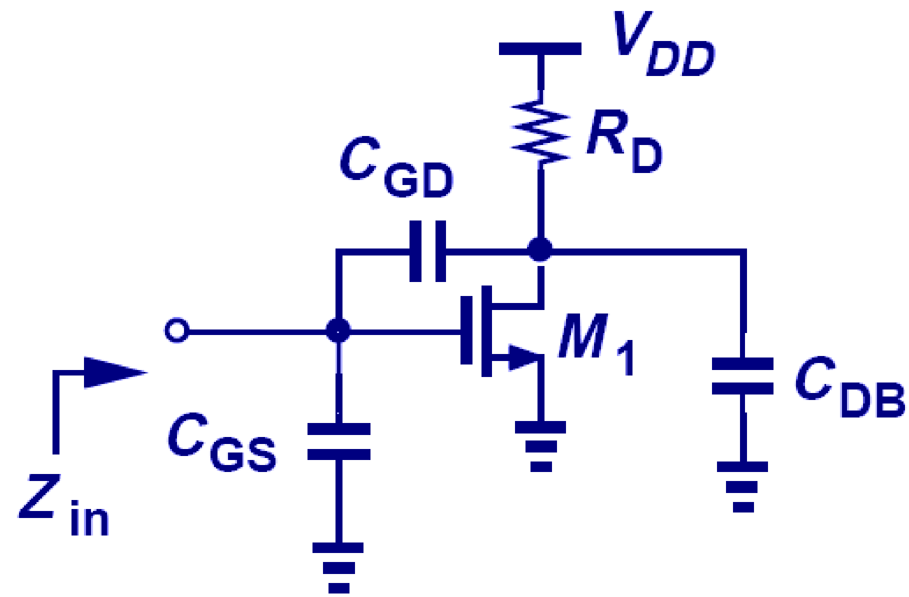
$$|\omega_{p,in}| = 2\pi \times (264MHz)$$
$$|\omega_{p,out}| = 2\pi \times (4.53GHz)$$

<u>*Dominant Pole*</u>

$$|\omega_{p,in}| = 2\pi \times (249MHz)$$
$$|\omega_{p,out}| = 2\pi \times (4.79GHz)$$

Input Impedance of CE and CS Stages





$$Z_{in} \approx \frac{1}{\left[C_\pi + (1 + g_m R_C) C_\mu\right] s} \parallel r_\pi$$

$$Z_{in} \approx \frac{1}{\left[C_{GS} + (1 + g_m R_D) C_{GD}\right] s}$$

At low frequencies : r_π

Z_{in} has a pole at $\dfrac{1}{r_\pi \left(C_\pi + (1 + g_m R_C) C_\mu\right)}$

Approximate as purely capacitive

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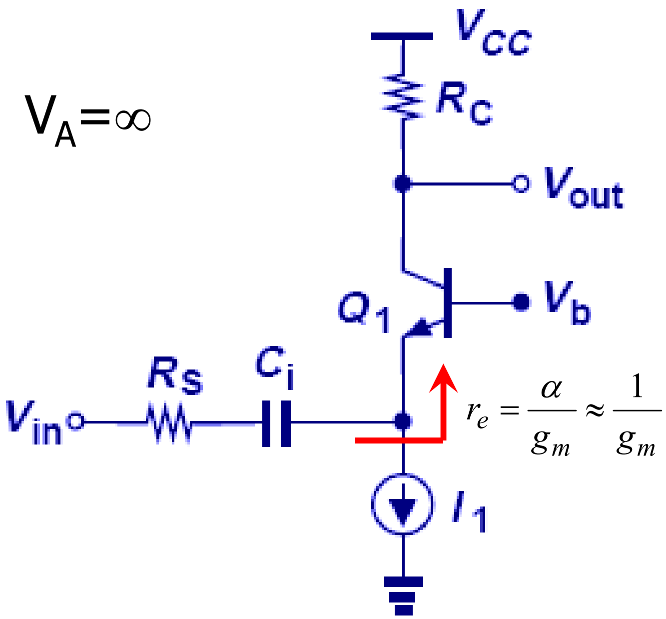
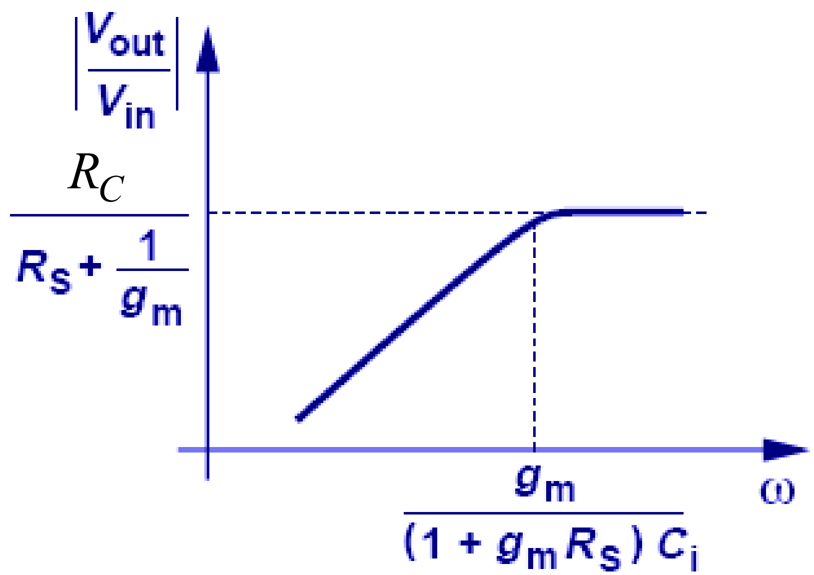
Low Frequency Response of CB and CG Stages

$V_A = \infty$



(a)



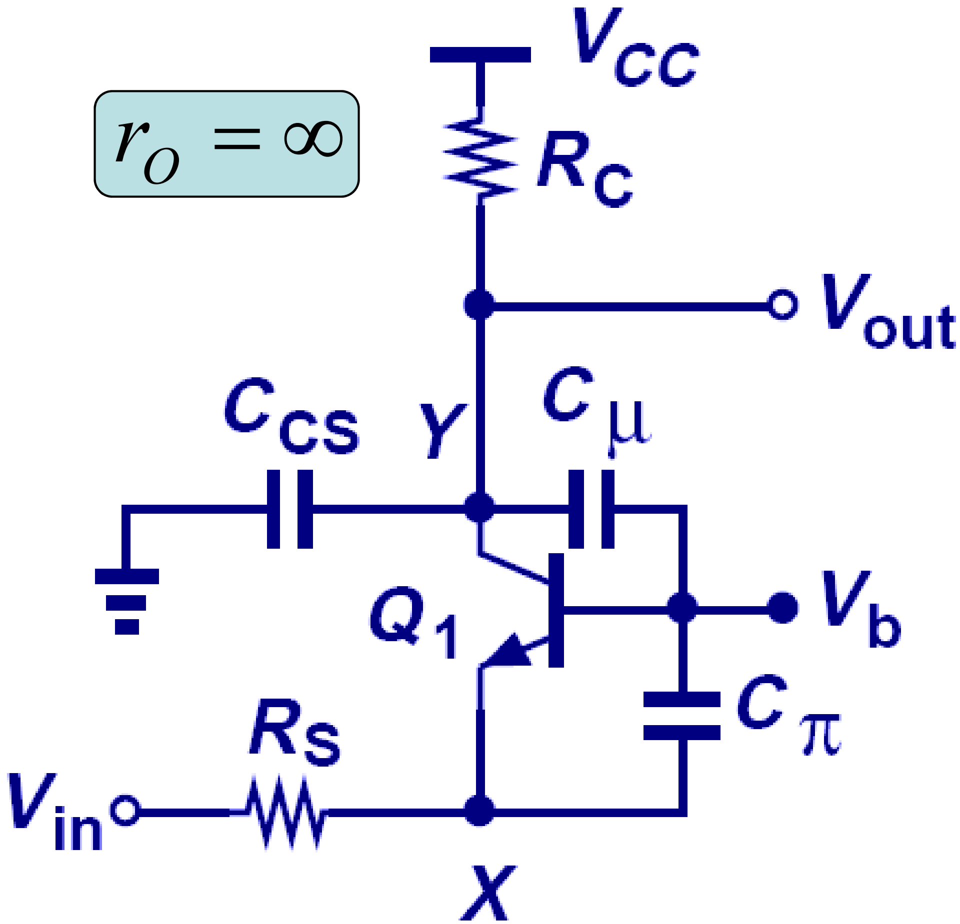
(b)

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m R_C C_i s}{(1 + g_m R_S) C_i s + g_m}$$

- As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).

Frequency Response of CB Stage

$r_O = \infty$



$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

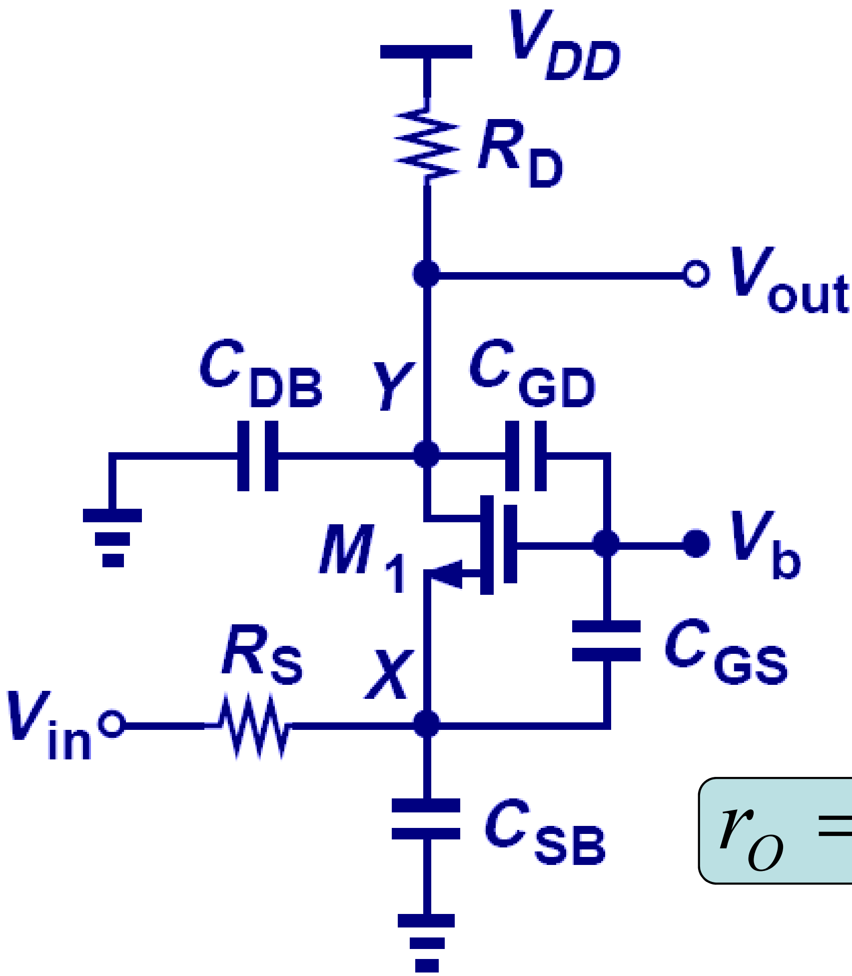
$$C_X = C_\pi$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

- No Miller effect
- Input pole is ~f_T (very high frequency)

Frequency Response of CG Stage



$$r_O = \infty$$

$$\omega_{p,X} = \frac{1}{\left(R_S \,||\, \frac{1}{g_m}\right) C_X}$$

$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_L C_Y}$$

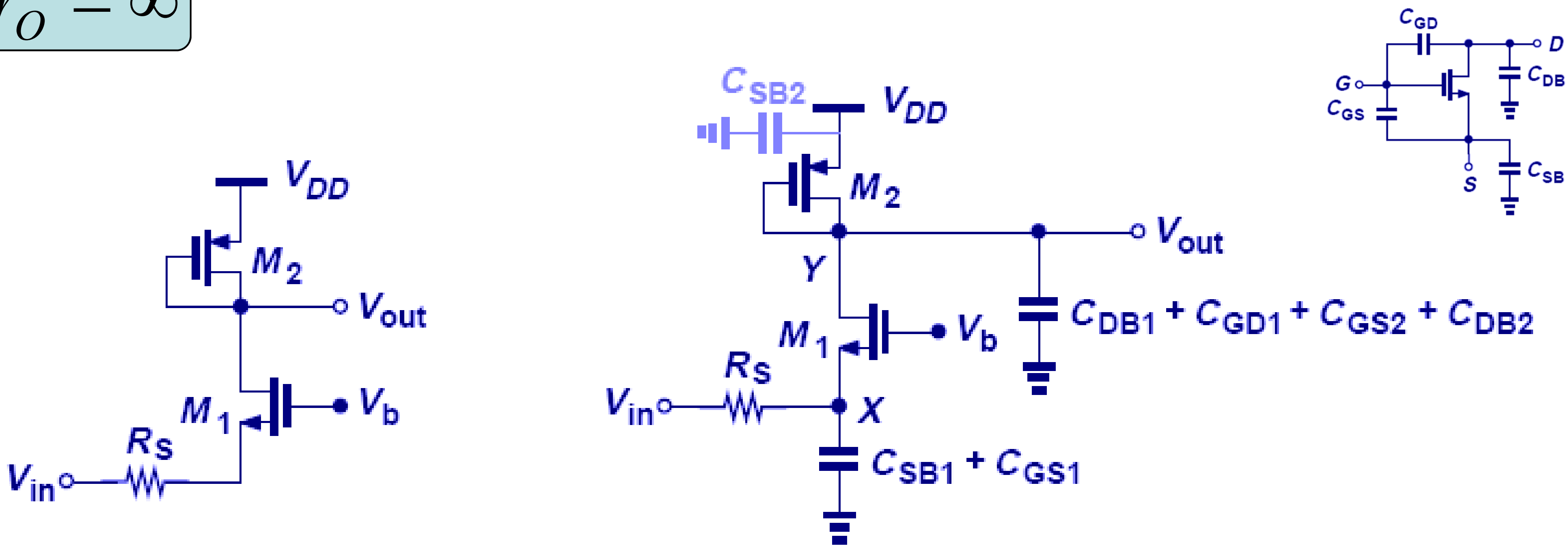
$$C_Y = C_{GD} + C_{DB}$$

- **Similar to a CB stage, the input pole is on the order of f_T, so rarely a speed bottleneck.**

Example: CG Stage Pole Identification

$$r_O = \infty$$

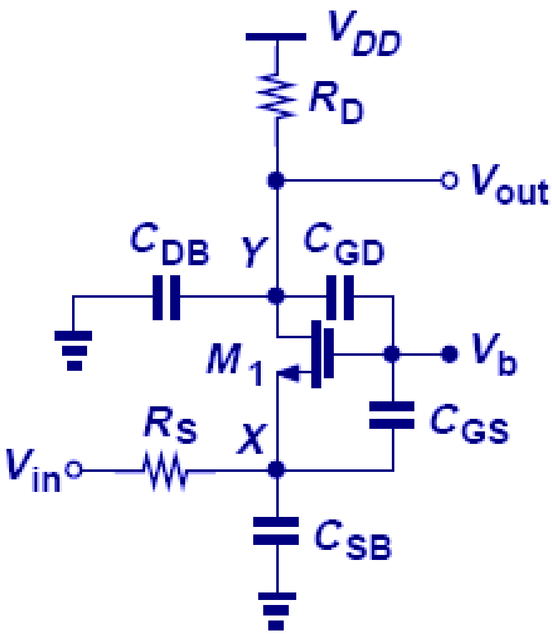
MOSFET Caps

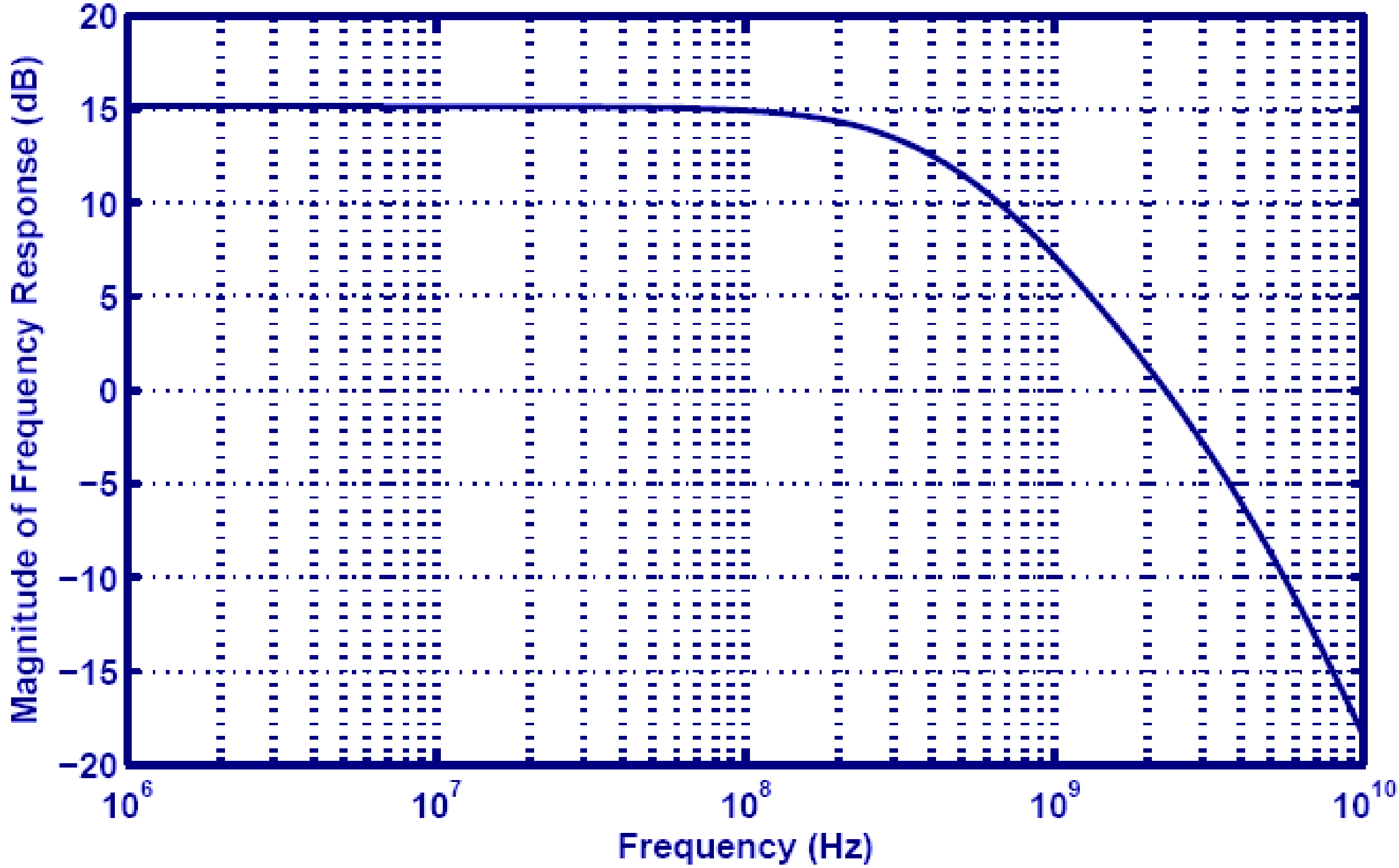


(a) (b)

$$\omega_{p,X} = \frac{1}{\left(R_S \,||\, \frac{1}{g_{m1}}\right)\left(C_{SB1} + C_{GD1}\right)} \qquad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left(C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2}\right)}$$

Example: Frequency Response of CG Stage





$$R_S = 200\Omega$$
$$C_{GS} = 250\,fF$$
$$C_{GD} = 80\,fF$$
$$C_{SB} = C_{DB} = 100\,fF$$
$$g_m = (150\Omega)^{-1}$$
$$\lambda = 0$$
$$R_d = 2K\Omega$$

$$\left|\omega_{p,X}\right| = 2\pi \times (5.31GHz)$$
$$\left|\omega_{p,Y}\right| = 2\pi \times (442MHz)$$

- Input pole is ~f_T
- Output pole limits bandwidth

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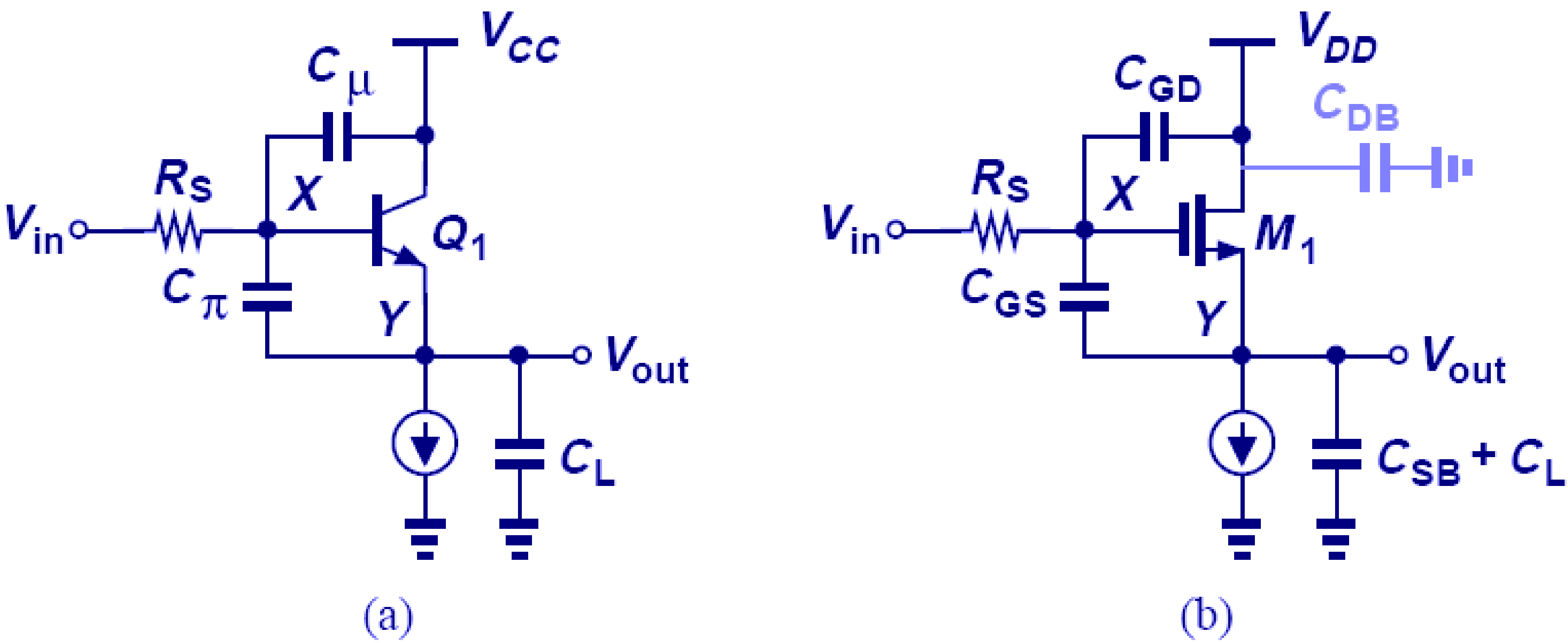
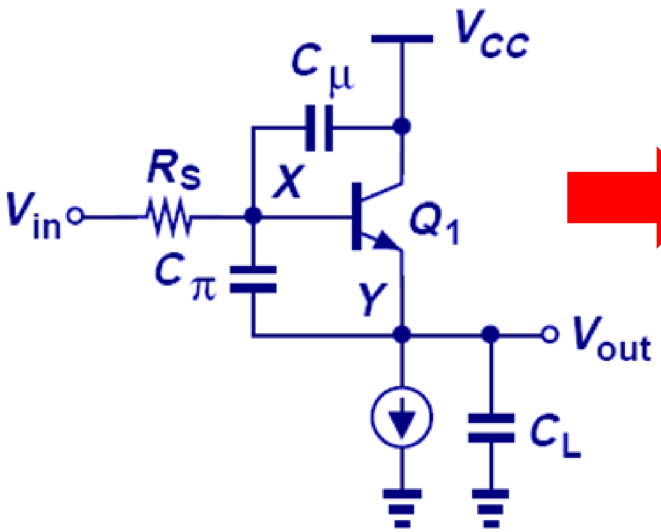
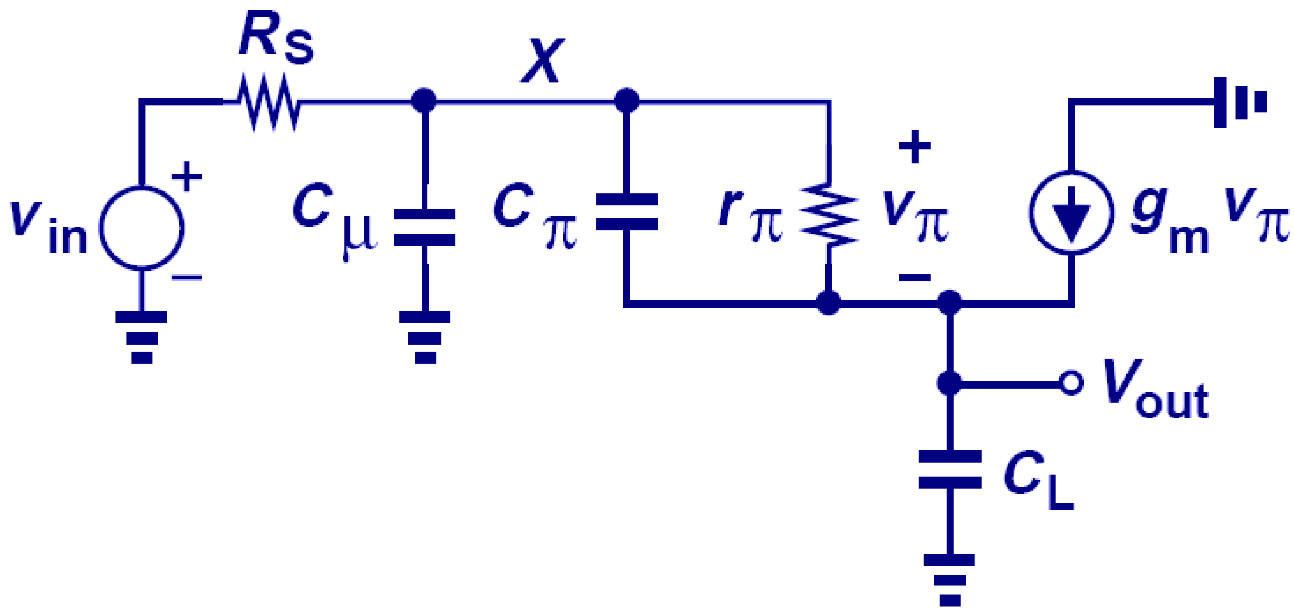
Emitter and Source Followers



- The following will discuss the frequency response of emitter and source followers using direct analysis, as this circuit typically has 2 poles that are close together
- Emitter follower is treated first and source follower is derived easily by allowing r_π to go to infinity

Direct Analysis of Emitter Follower





For detailed analysis, see Razavi 11.6

Assuming that $r_\pi >> \dfrac{1}{g_m}$

$$\frac{V_{out}}{V_{in}} = \frac{1+\dfrac{C_\pi}{g_m}s}{as^2+bs+1}$$

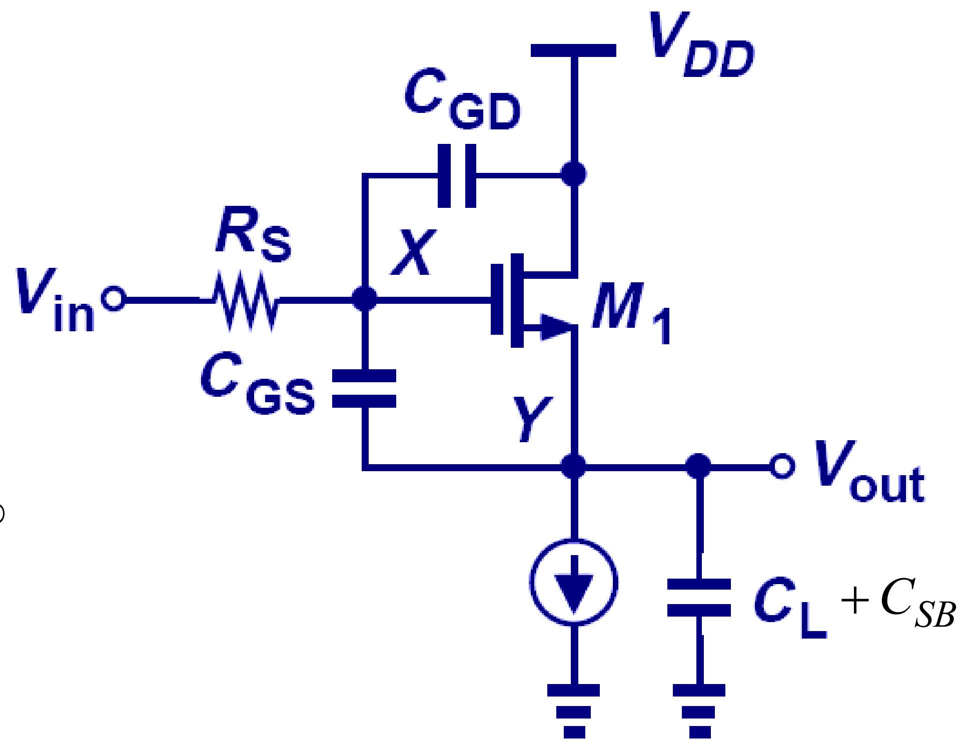
$$a = \frac{R_S}{g_m}\left(C_\mu C_\pi + C_\mu C_L + C_\pi C_L\right)$$

$$b = R_S C_\mu + \frac{C_\pi}{g_m} + \left(1+\frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}$$

$$|\omega_z| = \frac{g_m}{C_\pi}$$

Generally, this yields 2 close poles, necessitating a direct solution approach

Direct Analysis of Source Follower Stage



Taking $r_\pi \to \infty$

$C_\pi \to C_{GS}$

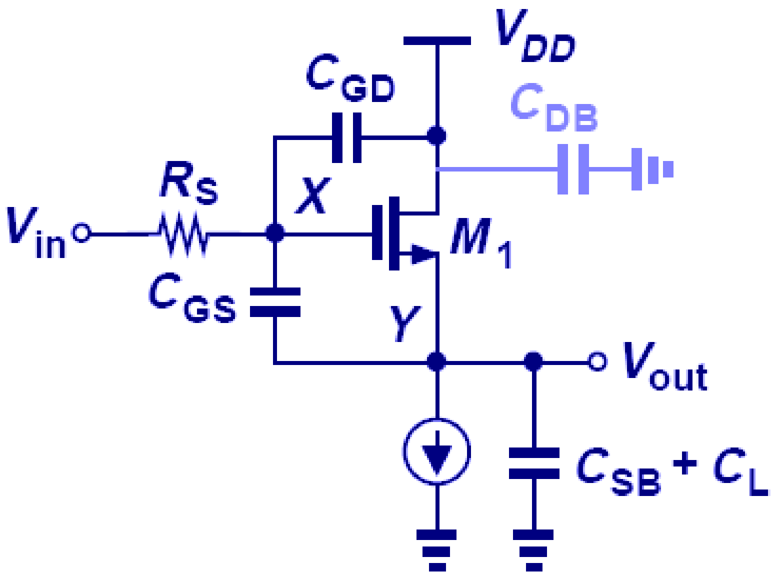
$C_\mu \to C_{GD}$

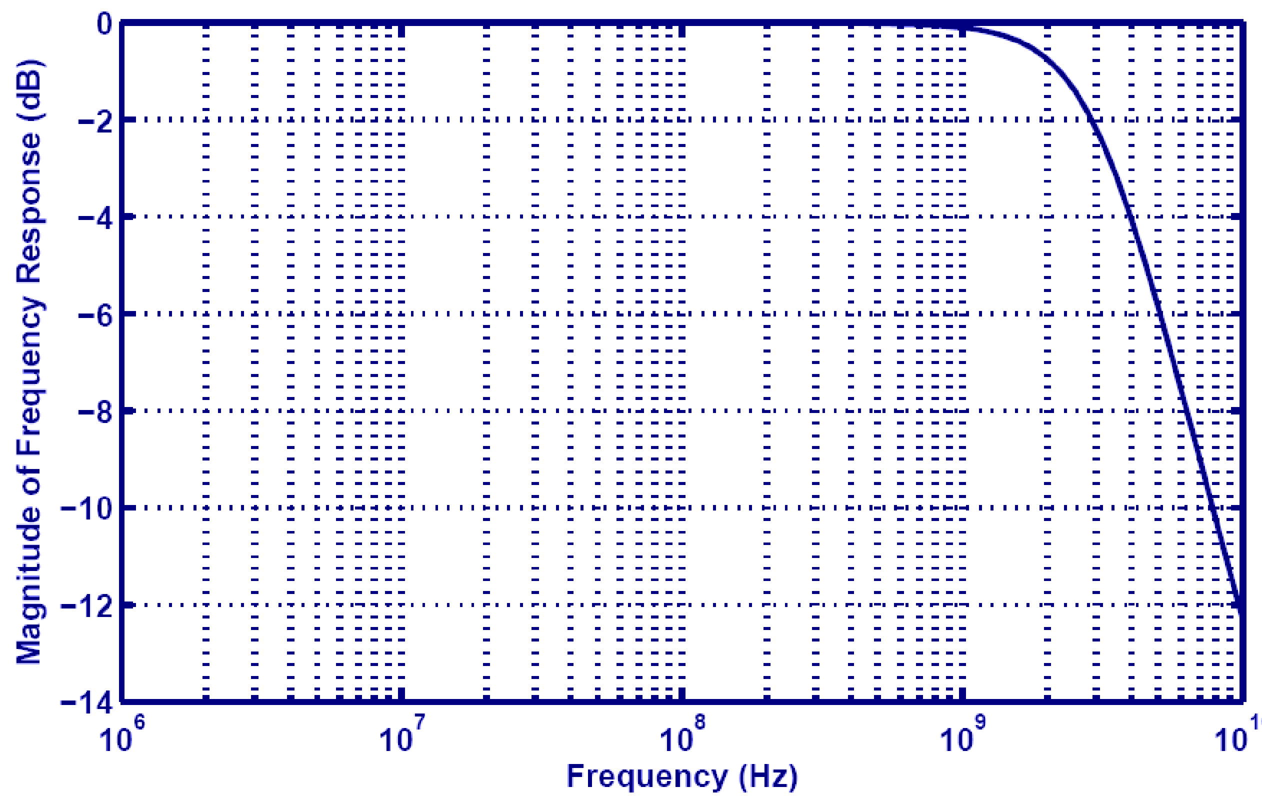
$$\frac{V_{out}}{V_{in}} = \frac{1 + \dfrac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_m}\left(C_{GD}C_{GS} + C_{GD}\left(C_L + C_{SB}\right) + C_{GS}\left(C_L + C_{SB}\right)\right)$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_L + C_{SB}}{g_m}$$

Example: Frequency Response of Source Follower





$$R_S = 200\Omega$$
$$C_L = 100\,fF$$
$$C_{GS} = 250\,fF$$
$$C_{GD} = 80\,fF$$
$$C_{DB} = 100\,fF$$
$$g_m = (150\Omega)^{-1}$$
$$\lambda = 0$$

$$\omega_{p1} = 2\pi[-1.79GHz + j(2.57GHz)]$$
$$\omega_{p2} = 2\pi[-1.79GHz - j(2.57GHz)]$$

2 Complex Conjugate Poles

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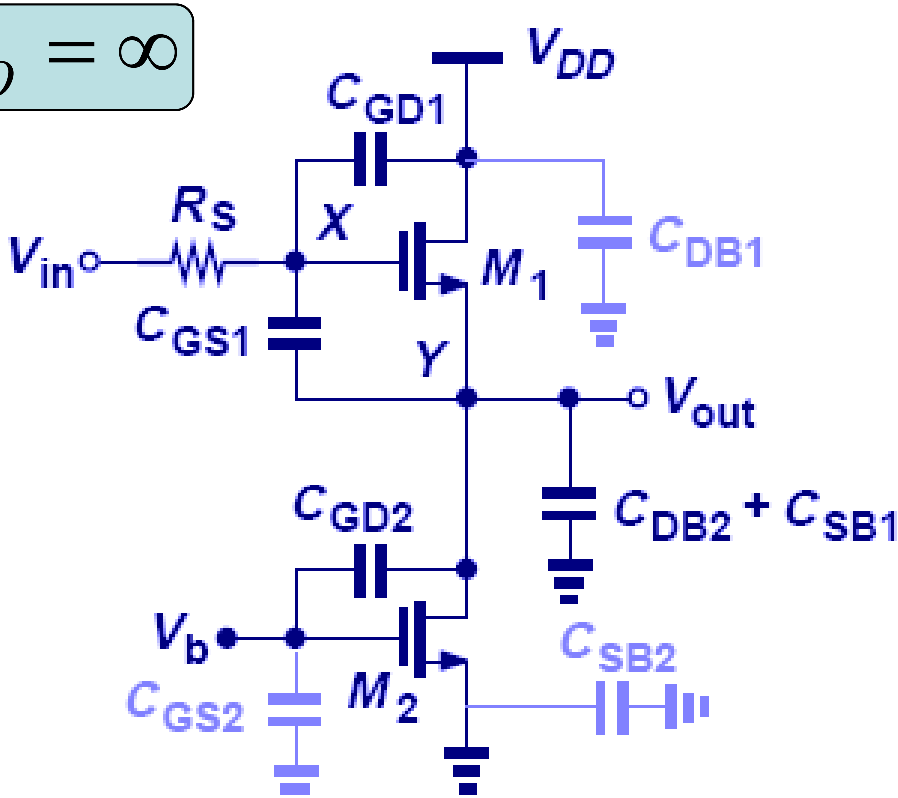
Example: Source Follower

$$r_O = \infty$$



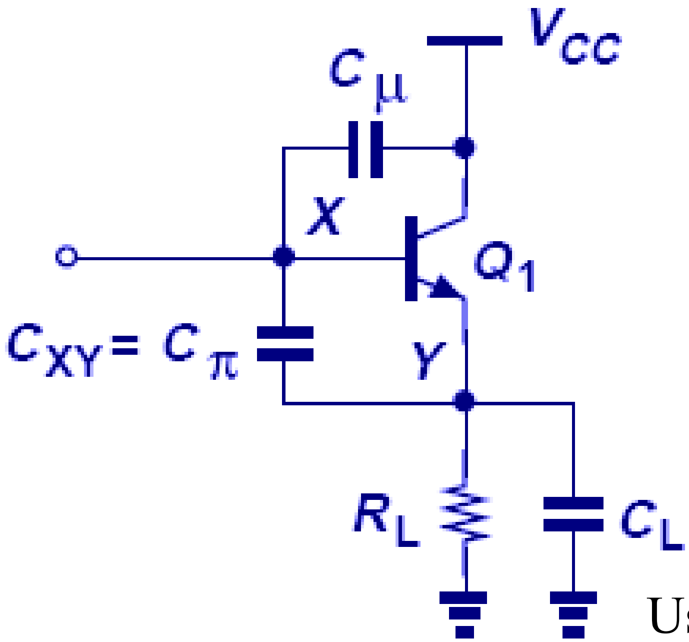
$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS1}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_{m1}} \left[C_{GD1} C_{GS1} + (C_{GD1} + C_{GS1})(C_{SB1} + C_{GD2} + C_{DB2}) \right]$$

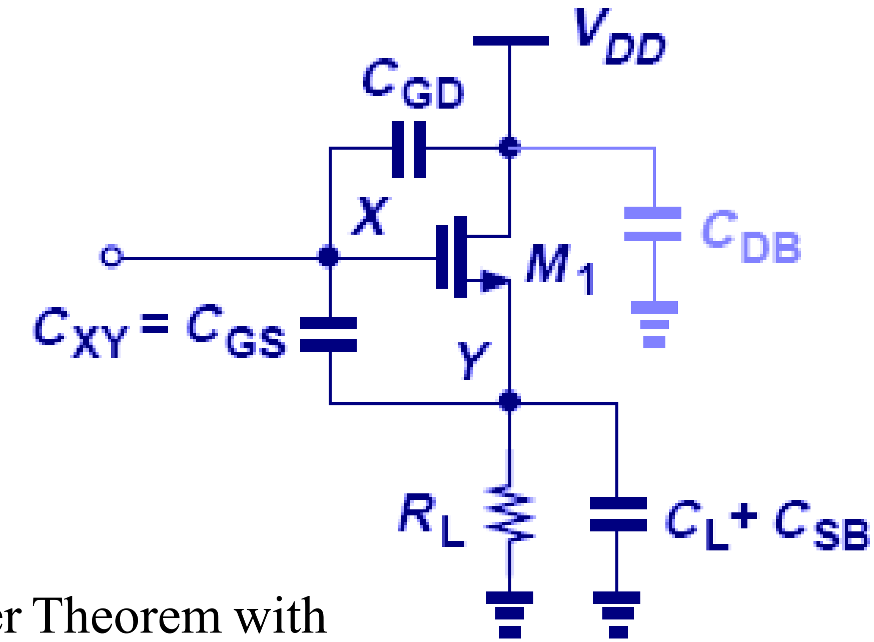
$$b = R_S C_{GD1} + \frac{C_{GD1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}$$

Input Capacitance of Emitter/Source Follower

$$r_O = \infty$$



(a)



(b)

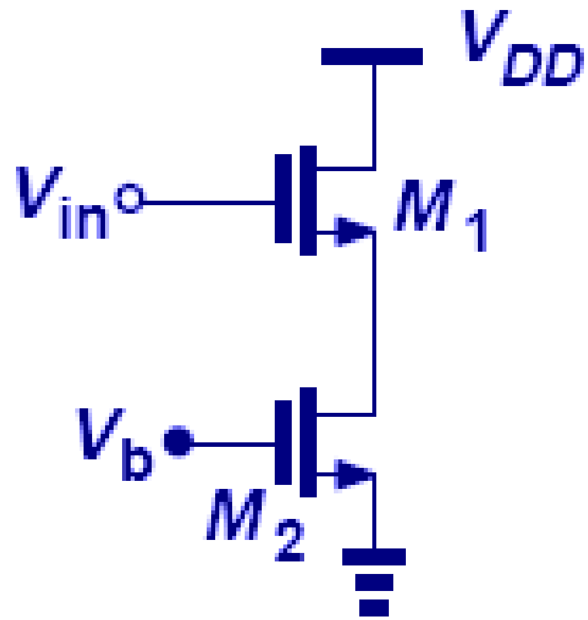
Using Miller Theorem with

$$A_v = \frac{g_m R_L}{1 + g_m R_L}$$

$$C_{in} = C_\mu + \frac{C_\pi}{1 + g_m R_L}$$

$$C_{in} = C_{GD} + \frac{C_{GS}}{1 + g_m R_L}$$

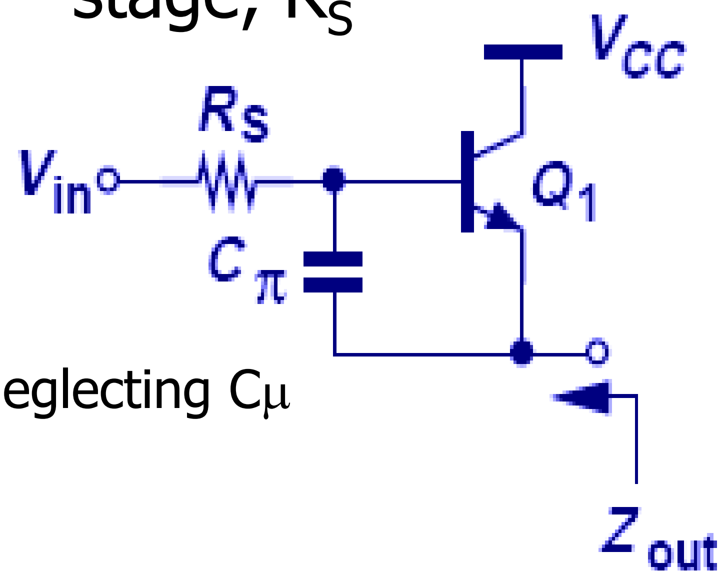
Example: Source Follower Input Capacitance



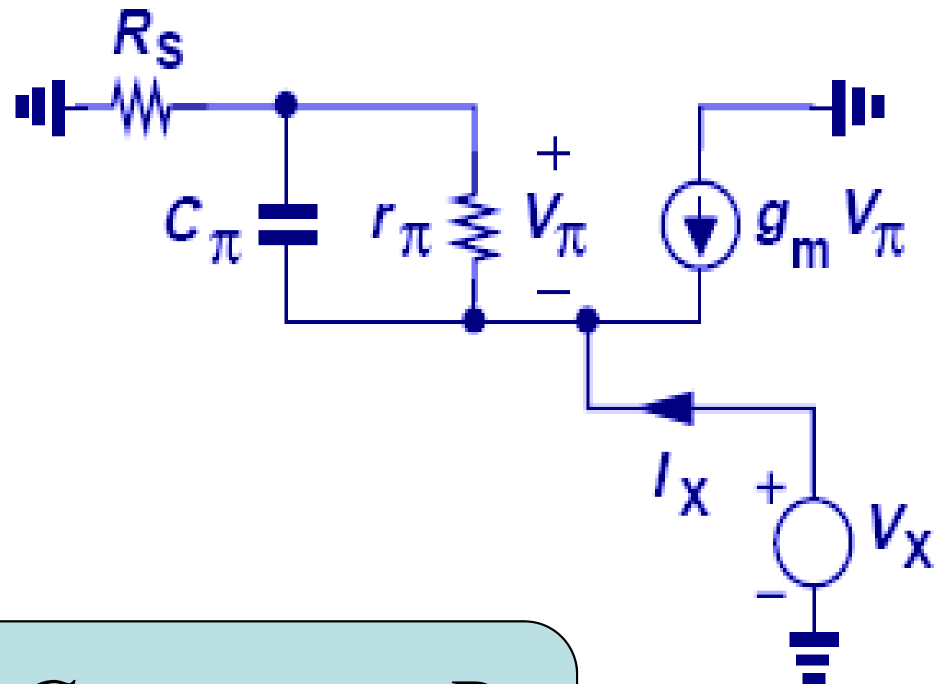
$$C_{in} = C_{GD1} + \frac{1}{1 + g_{m1}\left(r_{O1} \,||\, r_{O2}\right)} C_{GS1}$$

Output Impedance of Emitter Follower

- Need to consider the output resistance of the previous stage, R_S



Neglecting Cμ



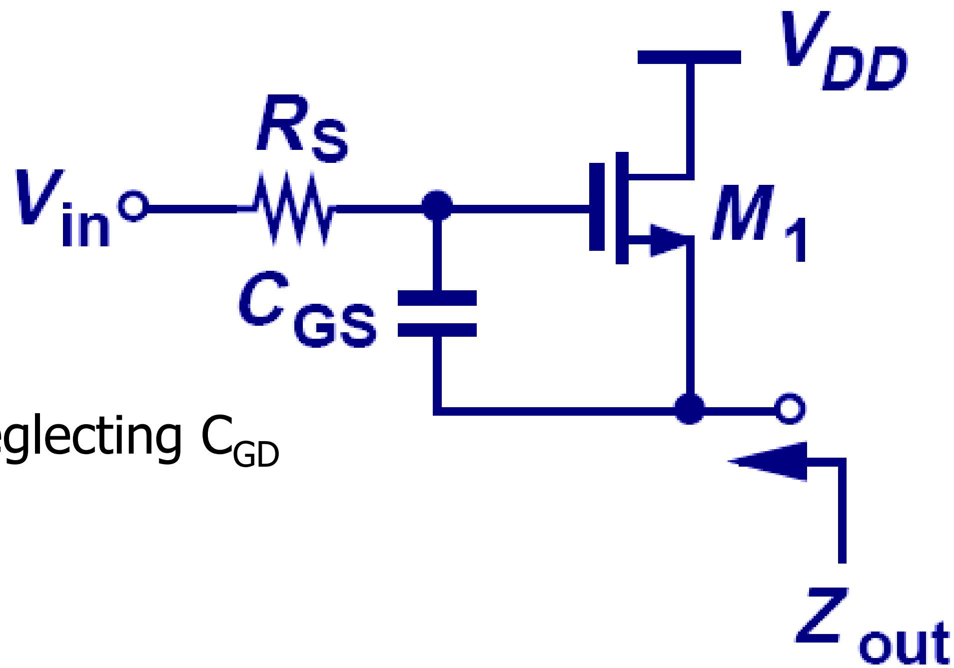
$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

$$\text{Low Frequency}: \frac{r_\pi + R_S}{\beta + 1} = r_e + \frac{R_S}{\beta + 1}$$

$$\text{High Frequency}: R_S$$

Output impedance generally goes up w/ frequency!

Output Impedance of Source Follower



Neglecting C_{GD}

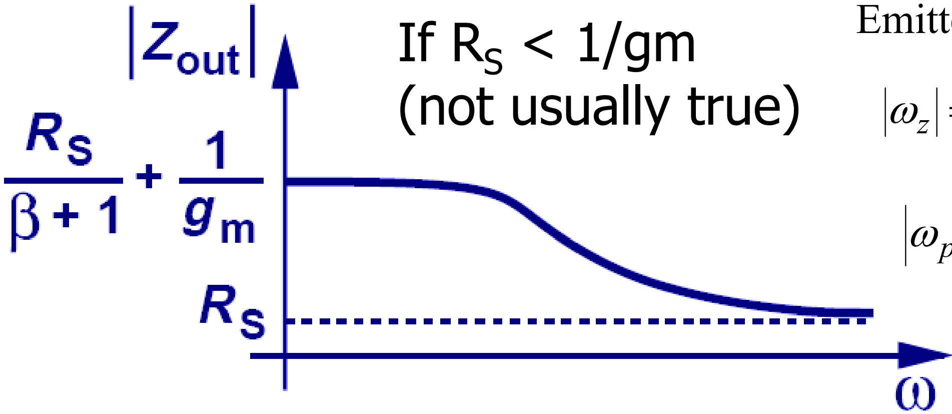
Output impedance generally goes up w/ frequency!

$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

$$\text{Low Frequency}: \frac{1}{g_m}$$

$$\text{High Frequency}: R_S$$

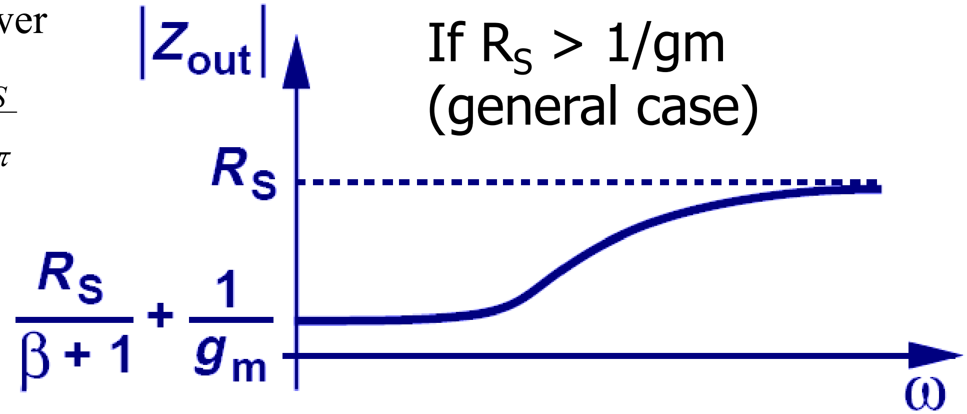
Active Inductor

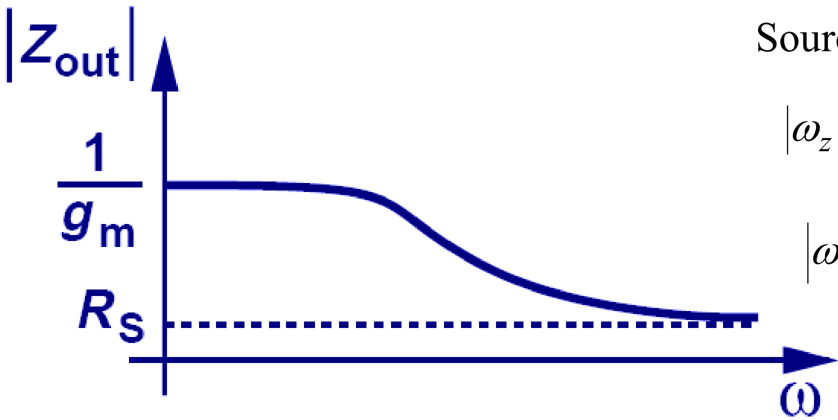


Emitter Follower

$$|\omega_z| = \frac{r_\pi + R_S}{R_S r_\pi C_\pi}$$

$$|\omega_p| = \frac{\beta + 1}{r_\pi C_\pi}$$

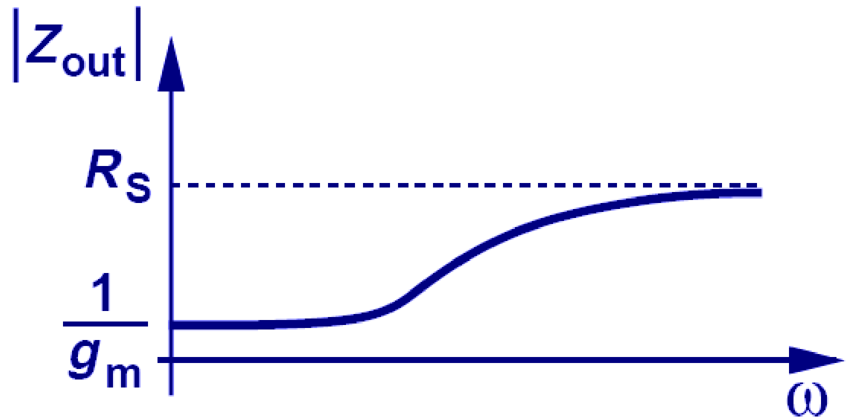




Source Follower

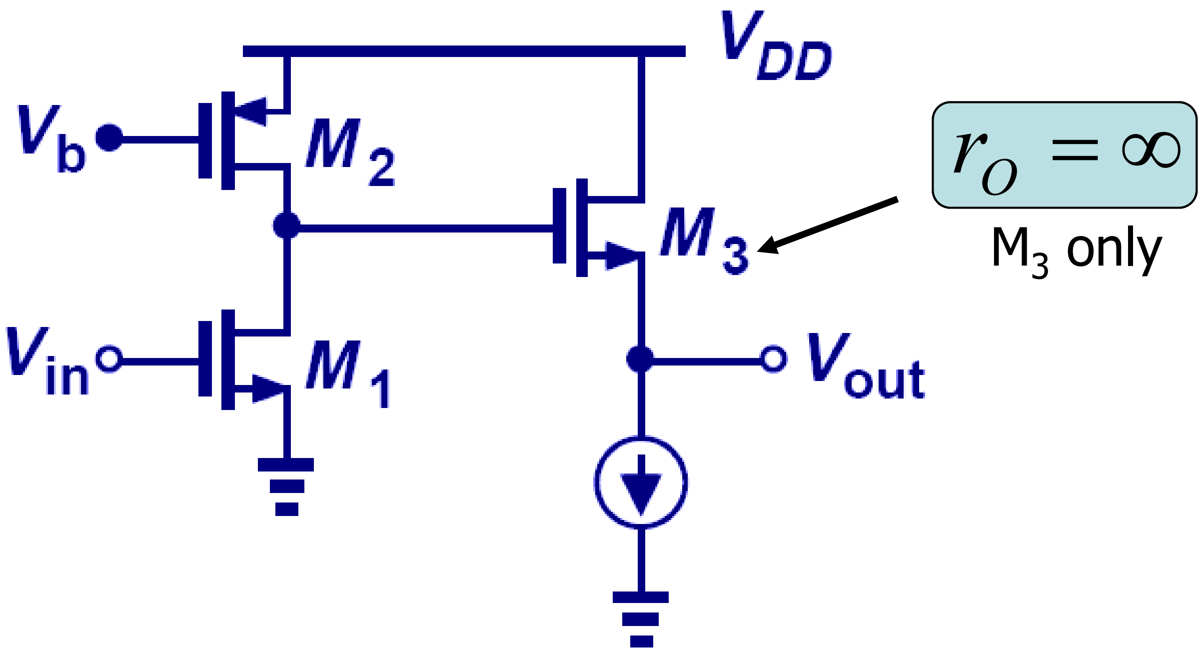
$$|\omega_z| = \frac{1}{R_S C_{GS}}$$

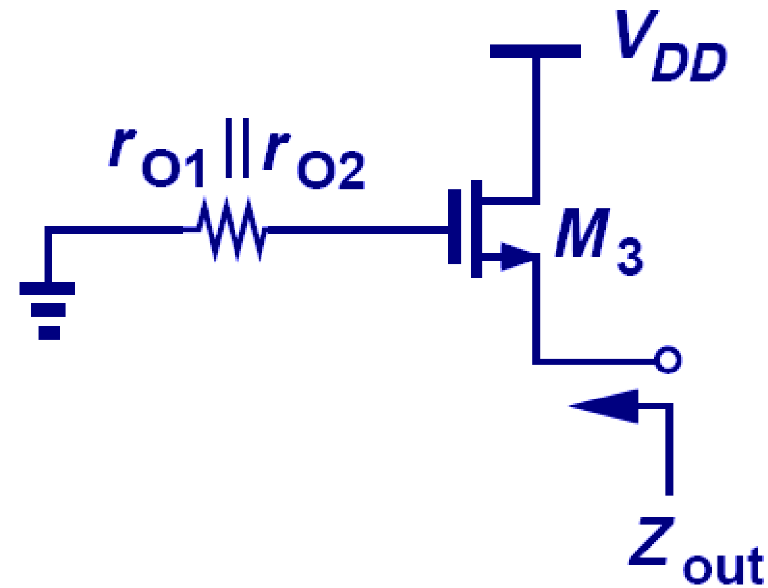
$$|\omega_p| = \frac{g_m}{C_{GS}}$$



- The plot above shows the output impedance of emitter and source followers. Since a follower's primary duty is to lower the driving impedance ($R_S > 1/g_m$), the "active inductor" characteristic on the right is usually observed.

Example: Output Impedance





$$\frac{V_X}{I_X} = \frac{(r_{O1} \| r_{O2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}}$$

Note: This neglects the capacitors from M1 and M2

Agenda

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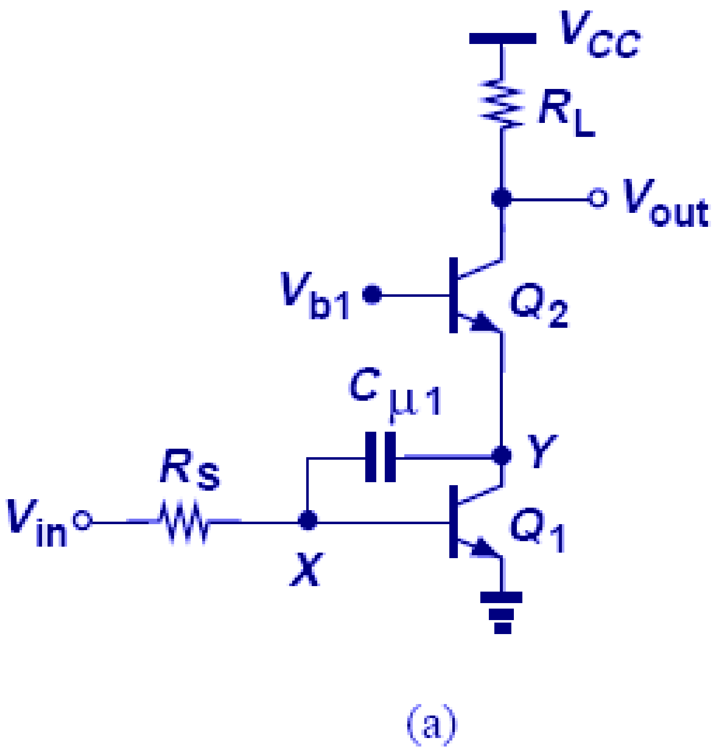
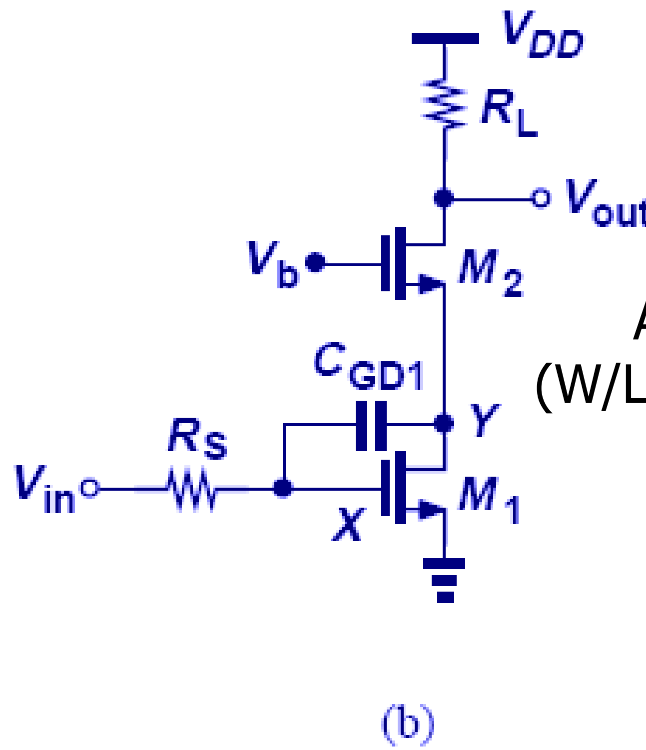
- Additional Examples

Frequency Response of Cascode Stage





Assume
$(W/L)_1 = (W/L)_2$

$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1 \qquad C_x \approx 2C_{XY}$$

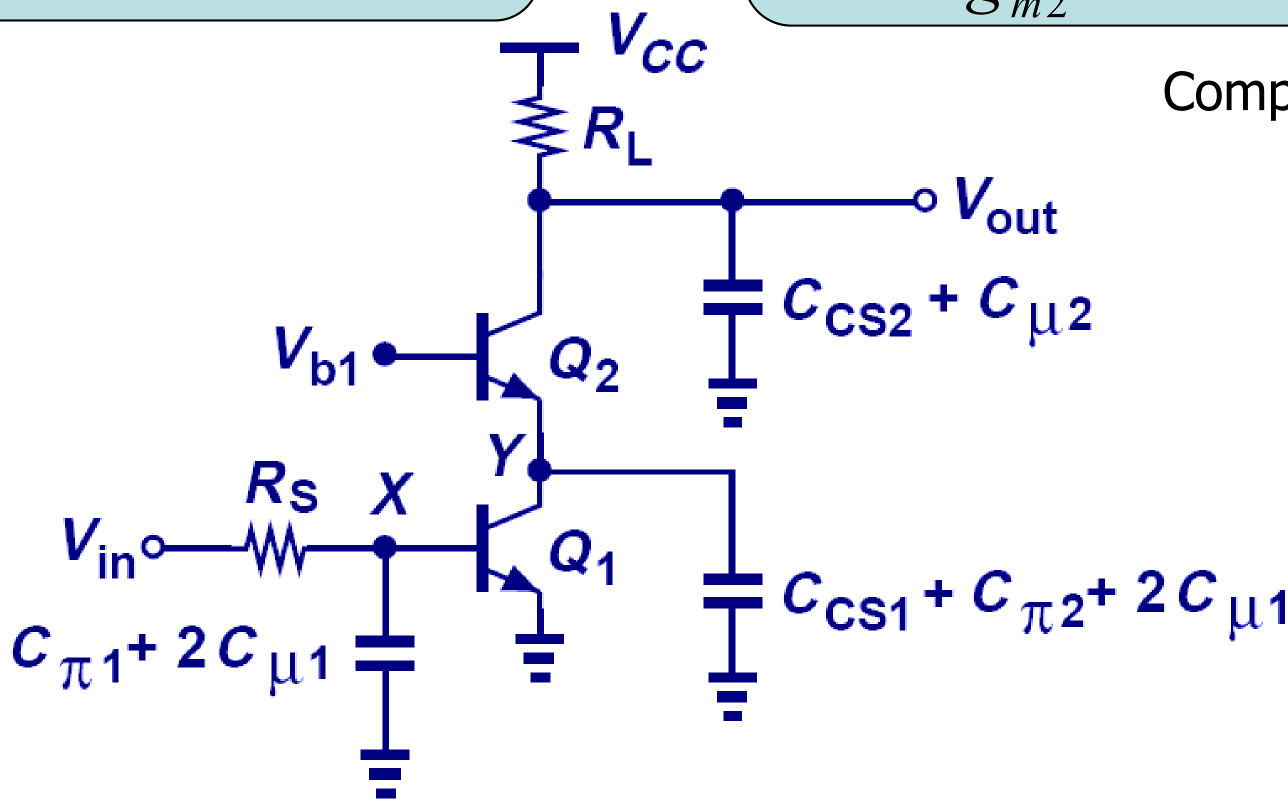
- **For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.**

Poles of Bipolar Cascode

$$\omega_{p,X} = \frac{1}{(R_S \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}(C_{CS1} + C_{\pi 2} + 2C_{\mu 1})}$$

Comparable to f_T

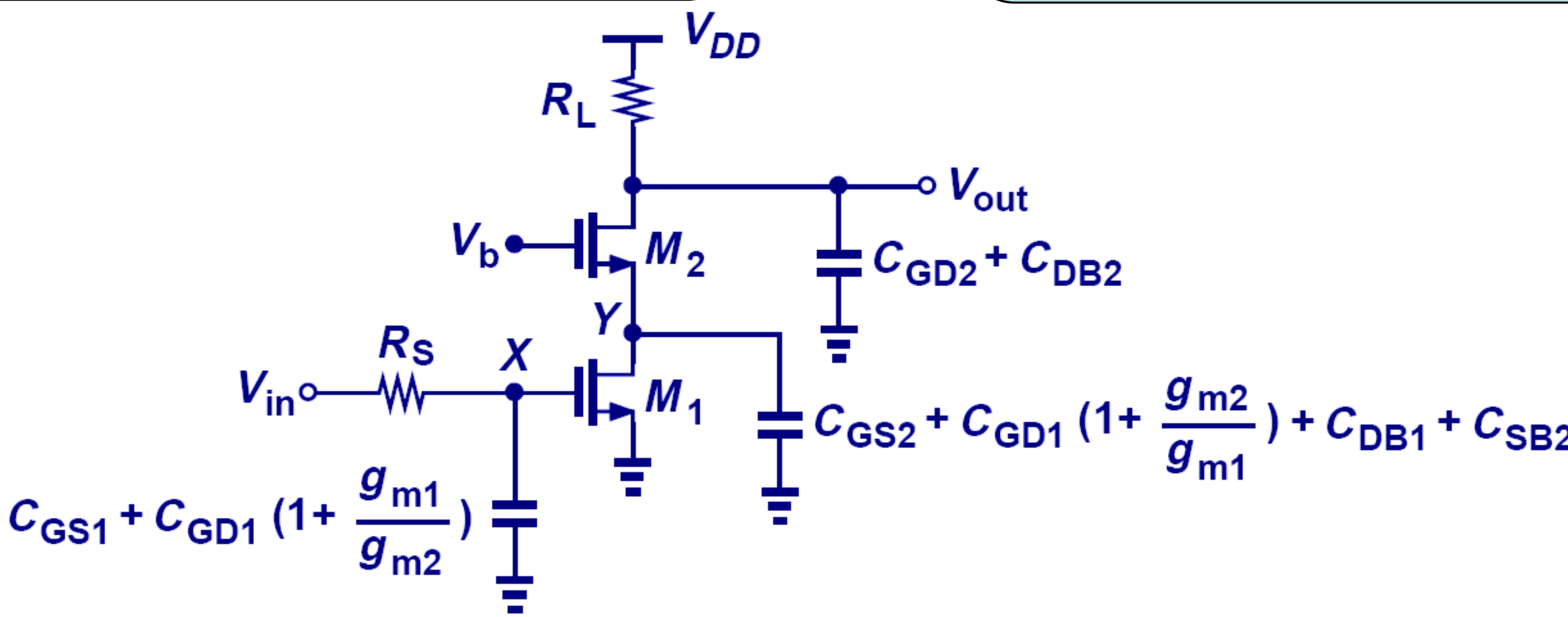


$$\omega_{p,out} = \frac{1}{R_L(C_{CS2} + C_{\mu 2})}$$

Poles of MOS Cascode

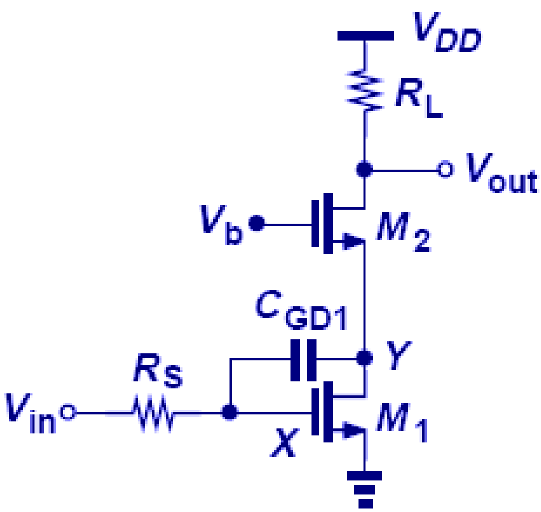
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

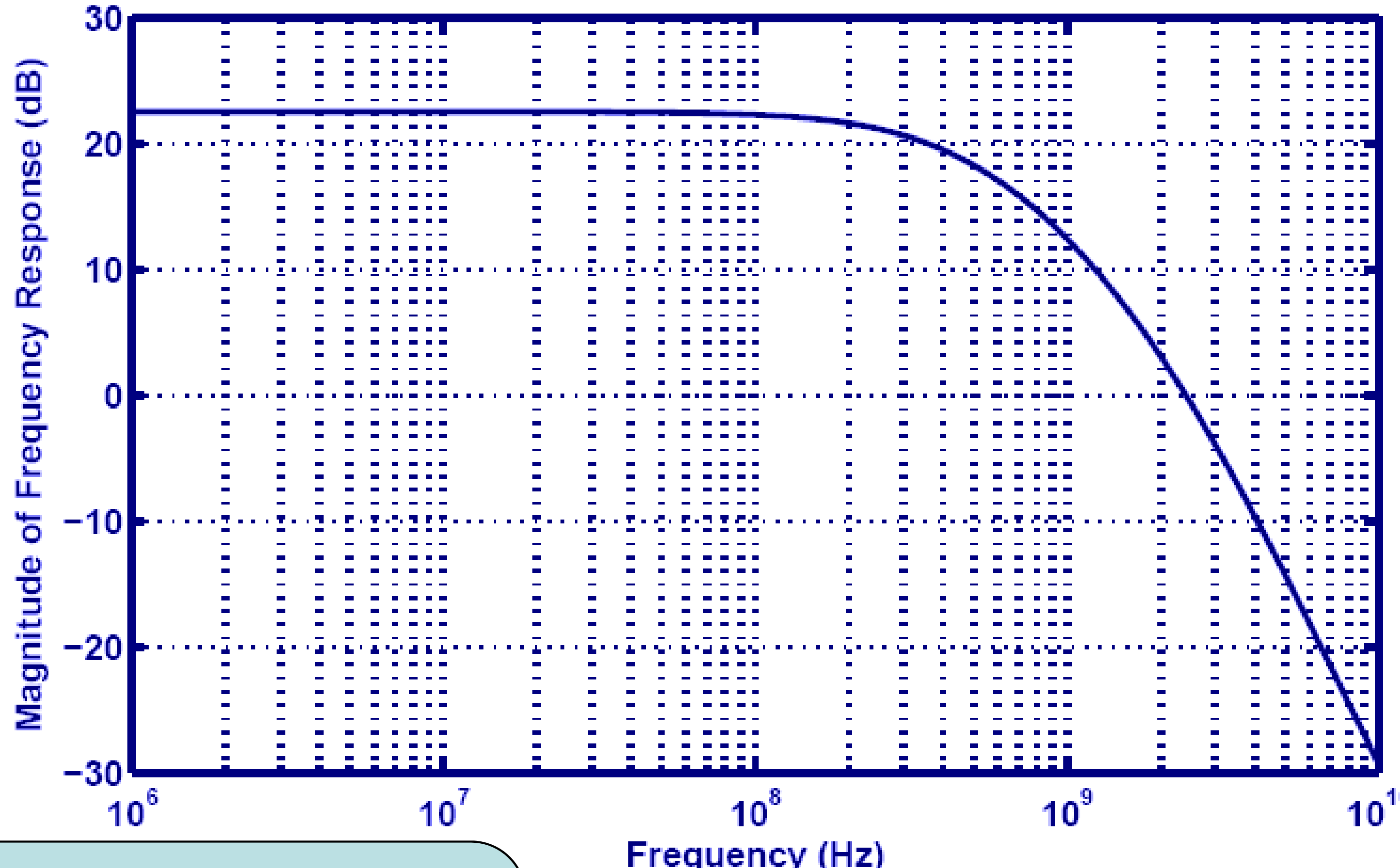
$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{SB2}\right]}$$

Example: Frequency Response of Cascode

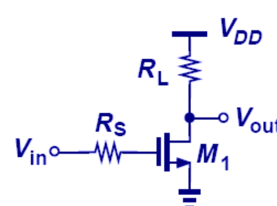




$$R_S = 200\ \Omega$$
$$C_{GS} = 250\ fF$$
$$C_{GD} = 80\ fF$$
$$C_{DB} = 100\ fF$$
$$g_m = (150\ \Omega)^{-1}$$
$$\lambda = 0$$
$$R_L = 2\,K\Omega$$

$$|\omega_{p,X}| = 2\pi \times (1.95GHz)$$
$$|\omega_{p,Y}| = 2\pi \times (1.73GHz)$$
$$|\omega_{p,out}| = 2\pi \times (442MHz)$$

Compare to simple CS *Exact*



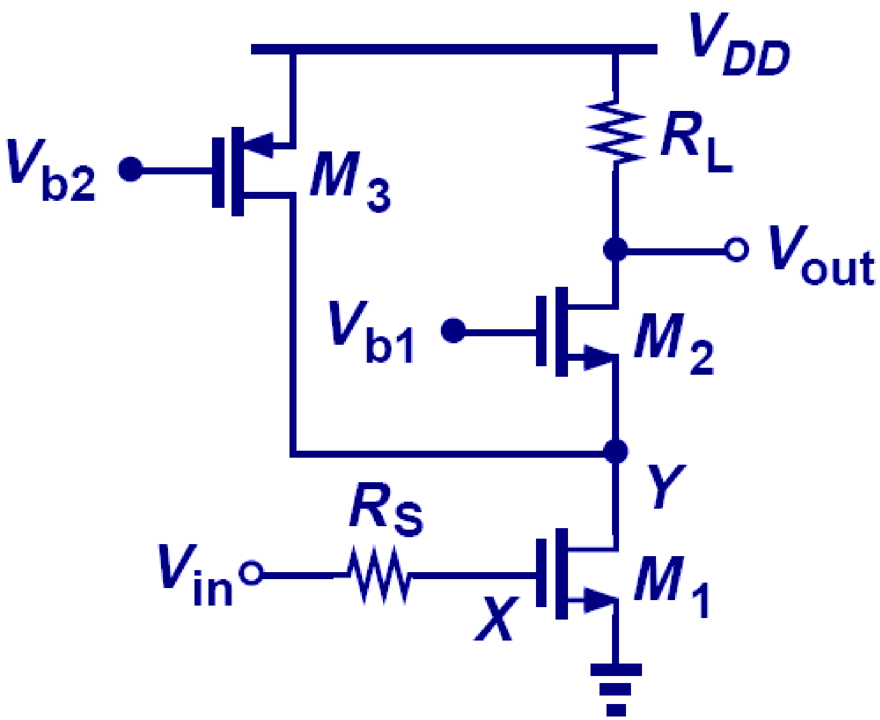
$$|\omega_{p,in}| = 2\pi \times (264MHz)$$
$$|\omega_{p,out}| = 2\pi \times (4.53GHz)$$

Now output pole sets the bandwidth, and it has increased by 67%

MOS Cascode Example

$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$

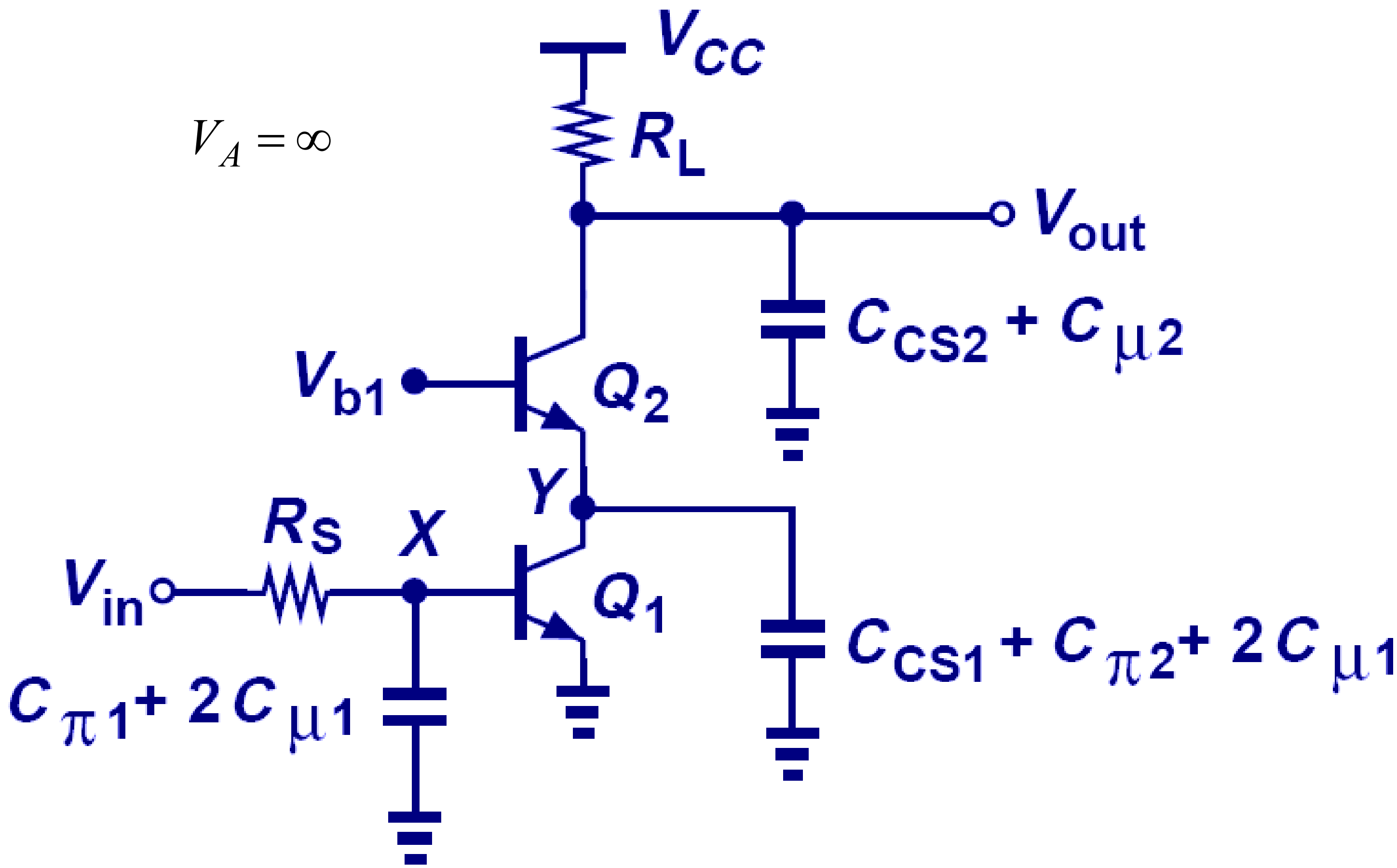


- Allows for a smaller M_2
  - Improves output pole
  - Lowers poles at nodes X and Y, but they should still be relatively high

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{GD3} + C_{DB3}\right]}$$

I/O Impedance of Bipolar Cascode

$V_A = \infty$

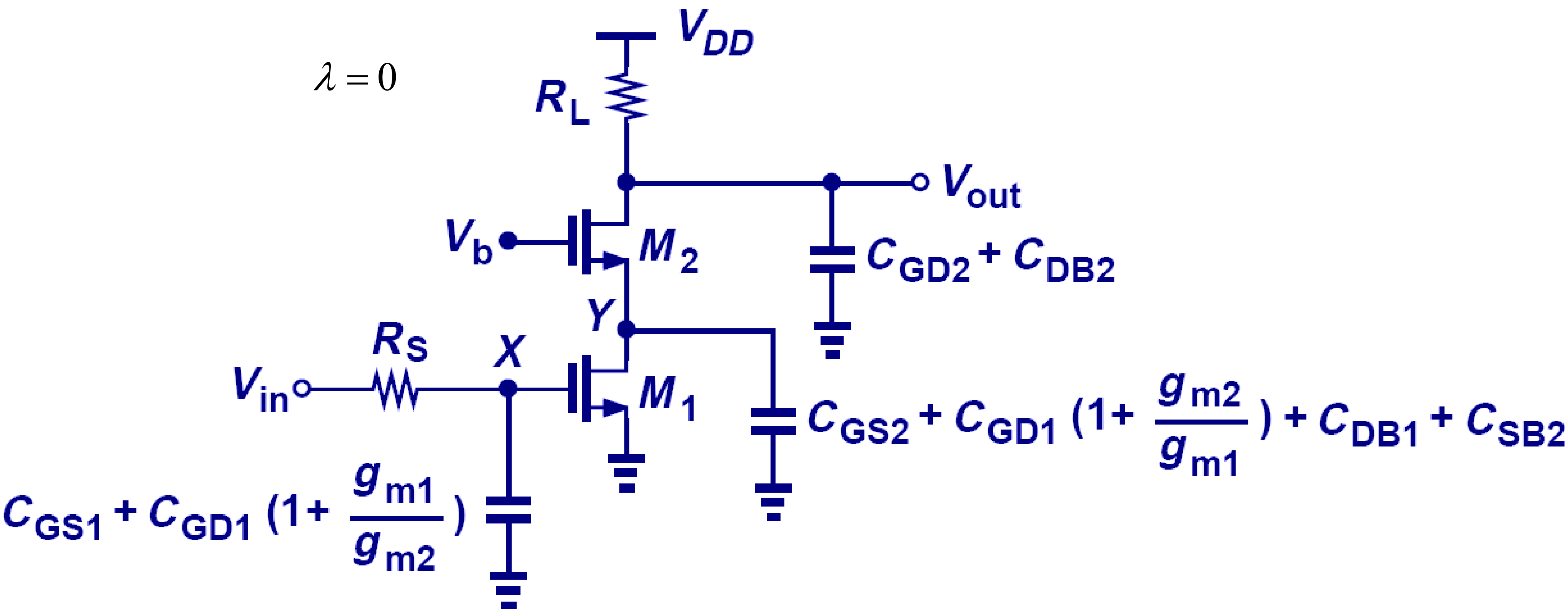


$$Z_{in} = r_{\pi 1} \parallel \frac{1}{\left(C_{\pi 1} + 2C_{\mu 1}\right)s}$$

(Neglecting R_S)

$$Z_{out} = R_L \parallel \frac{1}{\left(C_{\mu 2} + C_{CS2}\right)s}$$

I/O Impedance of MOS Cascode



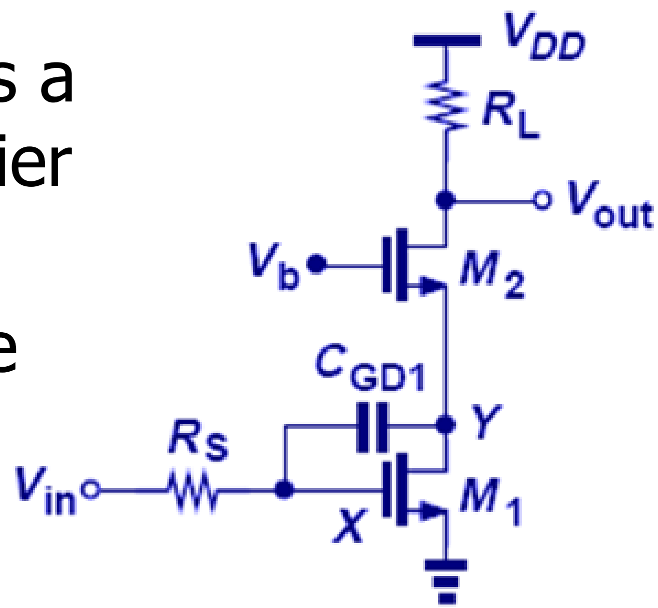
$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]s}$$

$$Z_{out} = R_L \,\|\, \frac{1}{(C_{GD2} + C_{DB2})s}$$

(Neglecting R_S)

Cascode Frequency Response Take-Away Points

- Cascode amplifiers offer two good properties
  - High output impedance to serve as a good current source and/or amplifier
  - Reduction of the Miller effect and better high-frequency performance



- Main cost is higher voltage headroom to keep cascode transistor in saturation
  - Impacts maximum output swing and distortion performance

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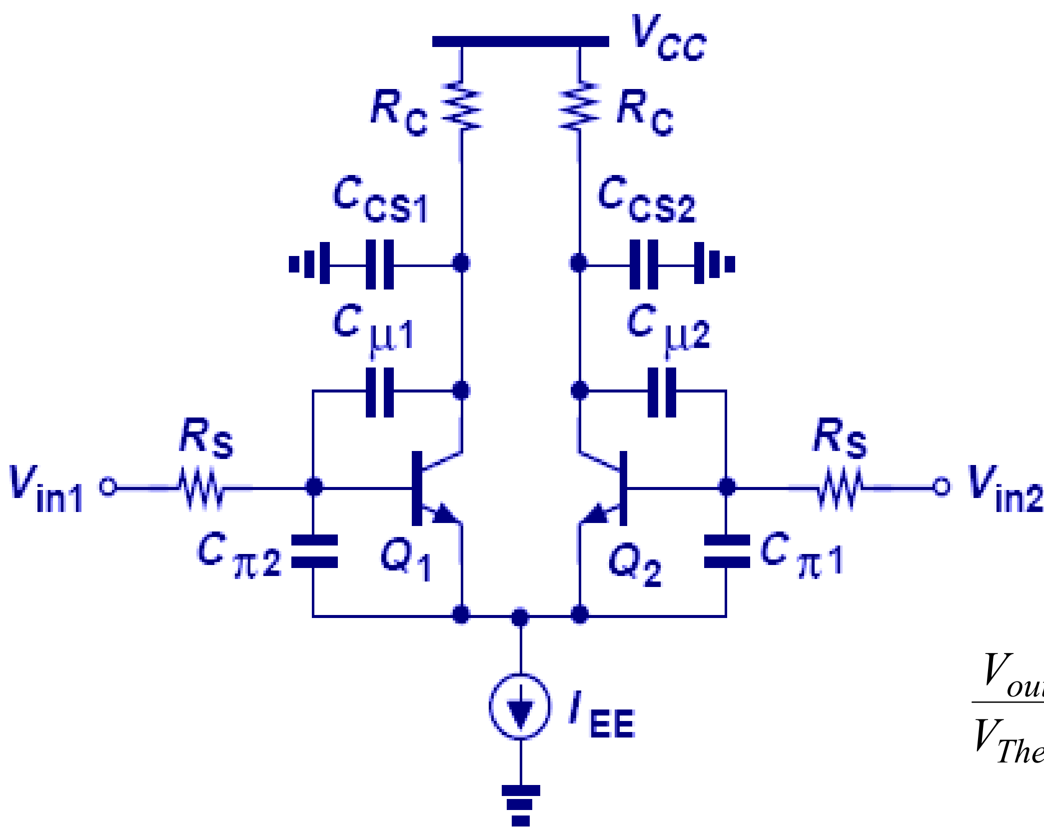
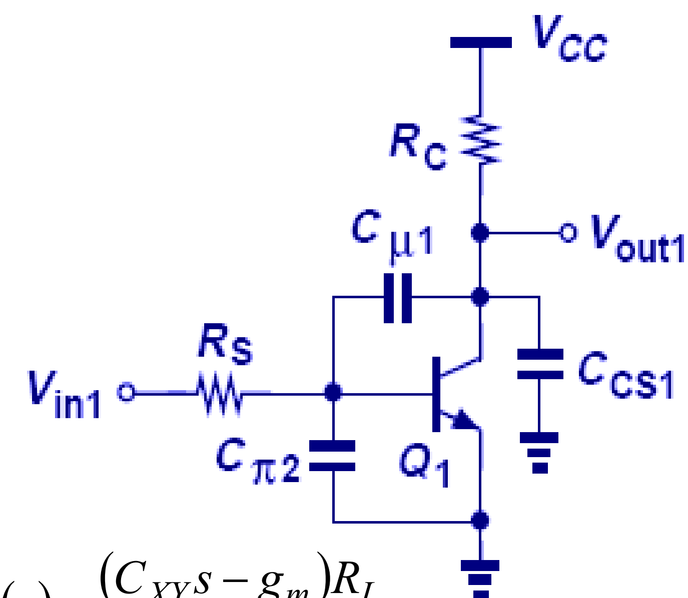
Bipolar Differential Pair Frequency Response



Half Circuit



$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

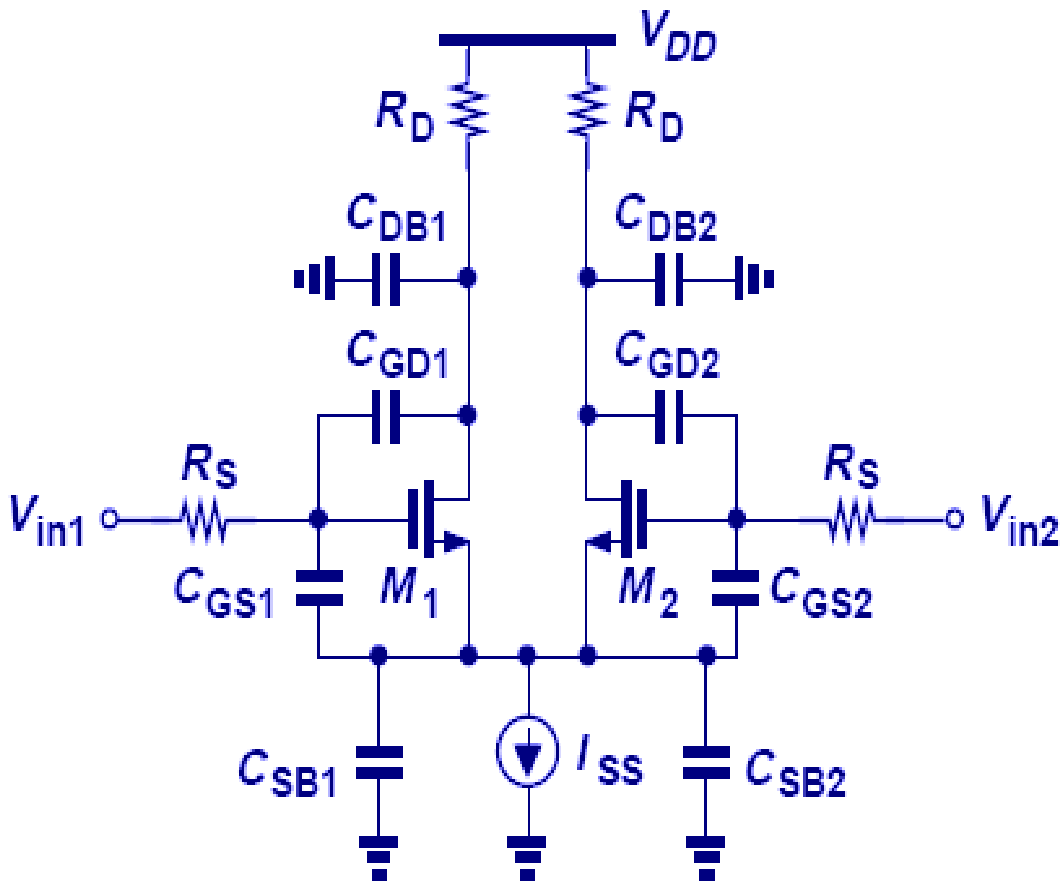
where

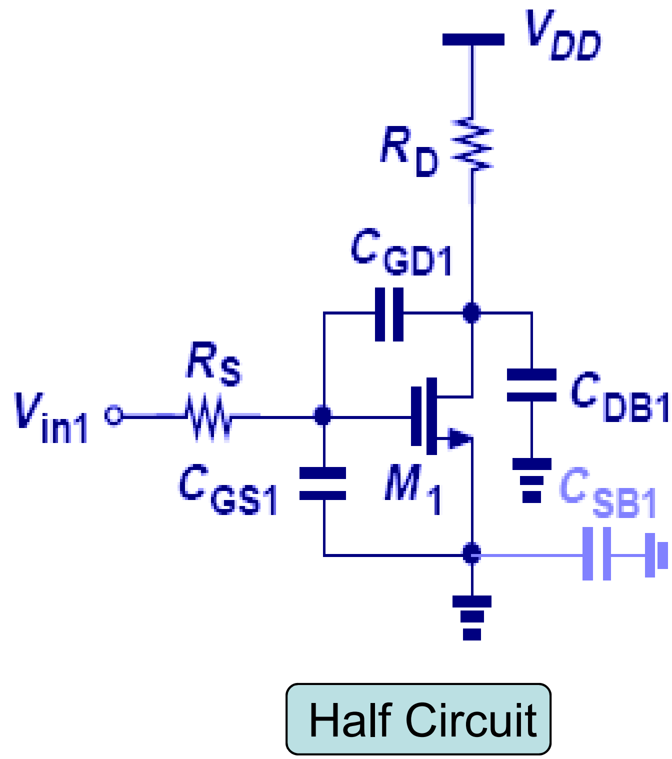
$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1 + g_mR_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})$$

- **Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's (Slide 43).**

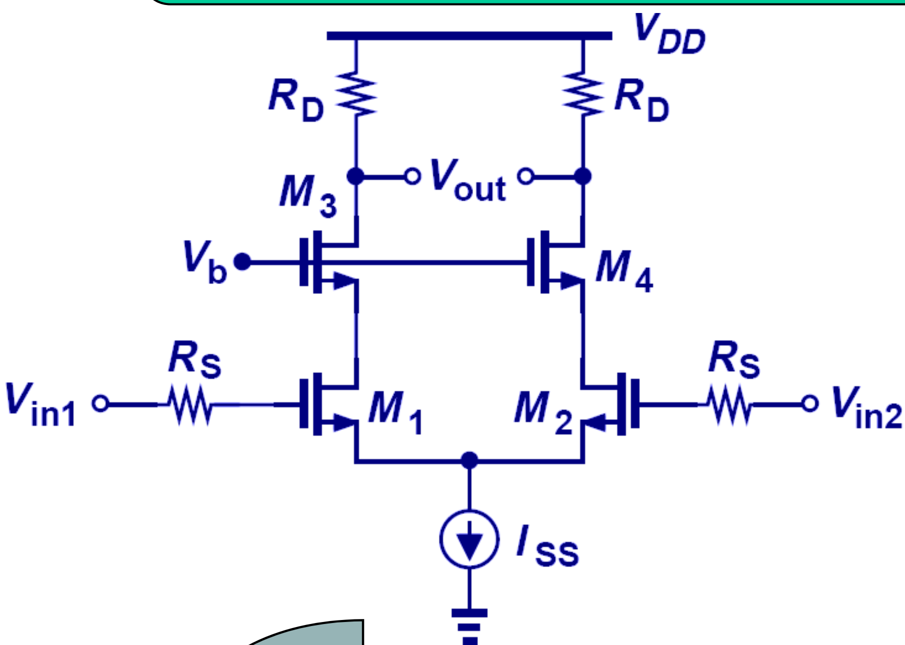
MOS Differential Pair Frequency Response





- Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's (Slide 43).

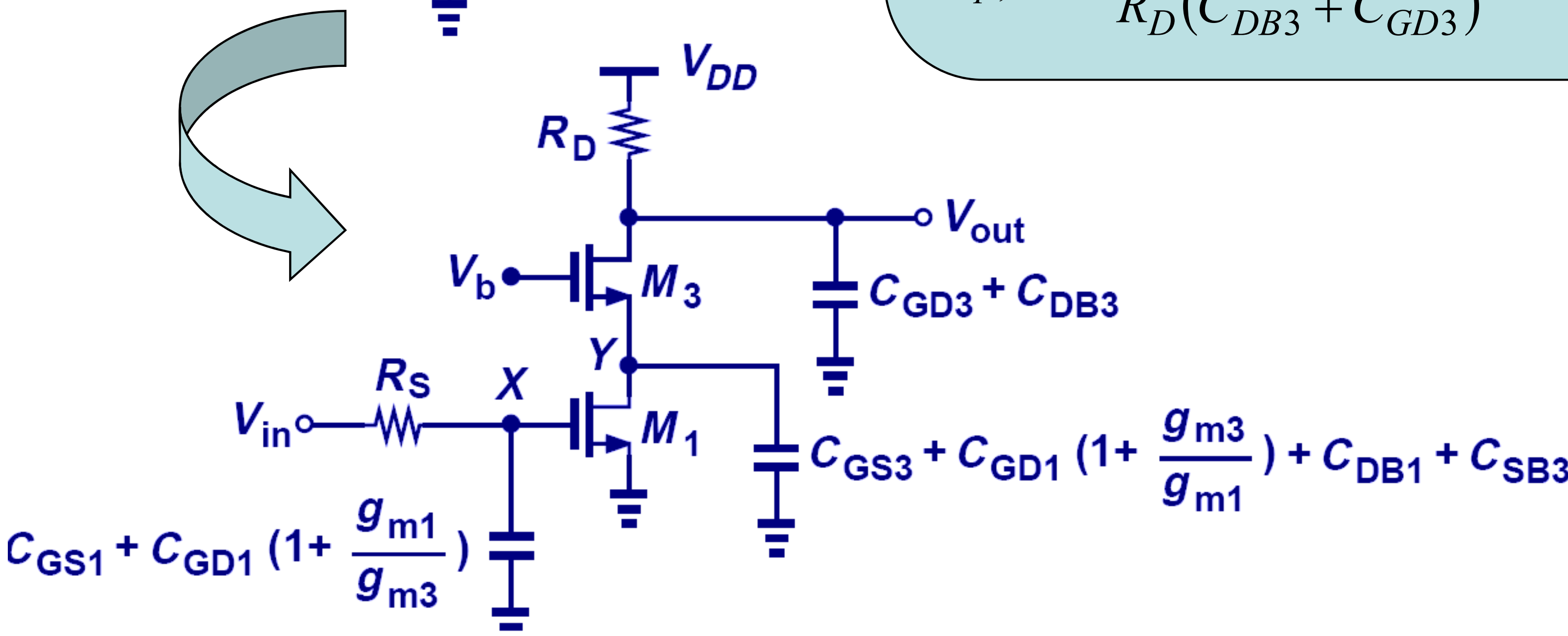
Example: MOS Differential Pair



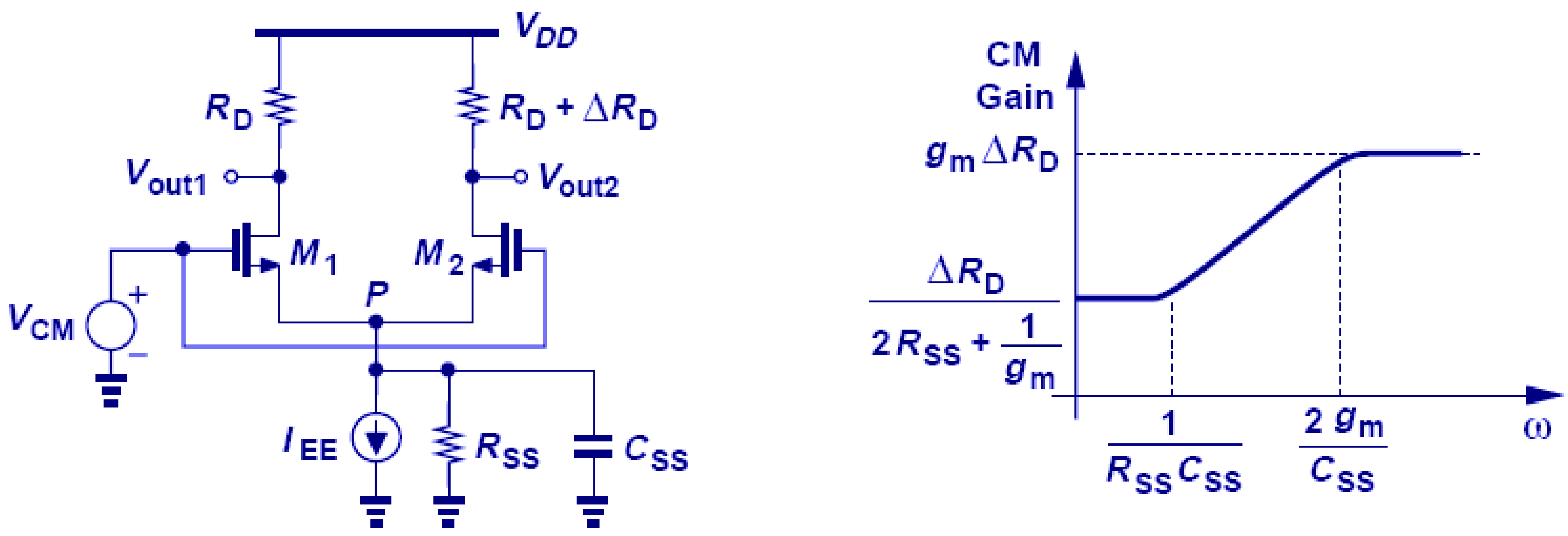
$$\omega_{p,X} = \frac{1}{R_S[C_{GS1} + (1 + g_{m1}/g_{m3})C_{GD1}]}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m3}}\left[C_{DB1} + C_{GS3} + \left(1 + \frac{g_{m3}}{g_{m1}}\right)C_{GD1} + C_{SB3}\right]}$$

$$\omega_{p,out} = \frac{1}{R_D\left(C_{DB3} + C_{GD3}\right)}$$



Common Mode Frequency Response



(a) (b)

$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{\Delta R_D}{\frac{1}{g_m} + 2\left(R_{SS} \| \frac{1}{C_{SS}s}\right)} = \frac{g_m \Delta R_D \left(R_{SS}C_{SS}s + 1\right)}{R_{SS}C_{SS}s + 2g_m R_{SS} + 1}$$

- **C_{ss} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.**

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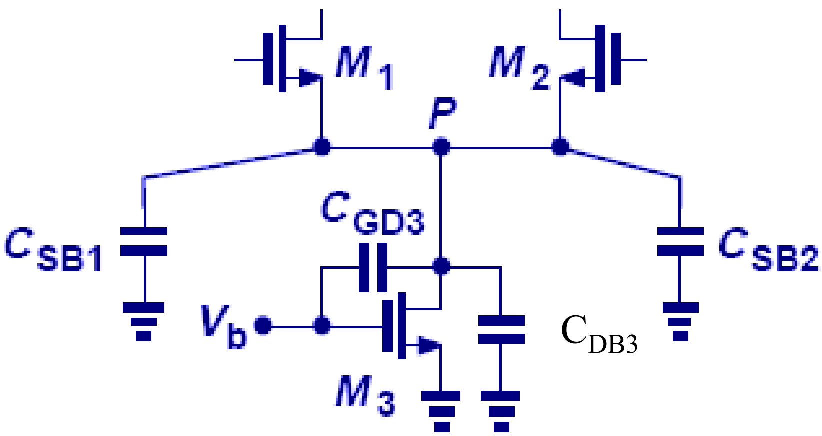
Tail Node Capacitance Contribution



- Source-Body Capacitance of M_1, M_2
- Drain-Body Capacitance of M_3
- Gate-Drain Capacitance of M_3

- M_3 is often a large (wide) transistor in order to have a small compliance (V_{DS}) voltage
  - Watch out for degraded high-frequency CMRR!

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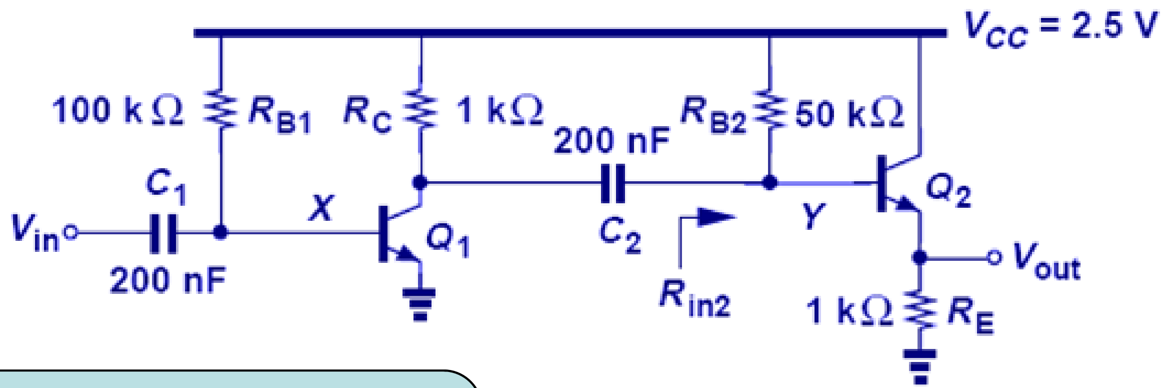
Example: Capacitive Coupling (Low-Frequency Cut-Off)

$$I_{C1} = 1.75mA$$

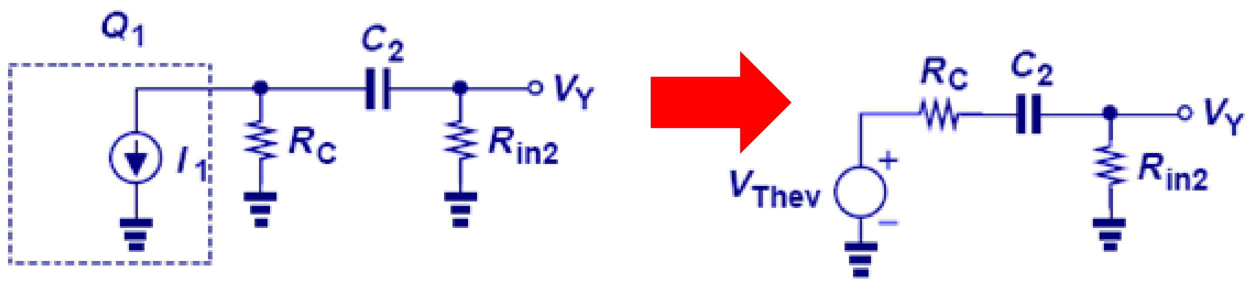
$$I_{C2} = 1.13mA$$



$$\omega_{L1} = \frac{1}{(r_{\pi 1} \| R_{B1})C_1} = 2\pi \times (542Hz)$$

The "highest" low-frequency pole (ω_{L1} = 542Hz) will set the low-frequency cut-off

To find ω_{L2}:



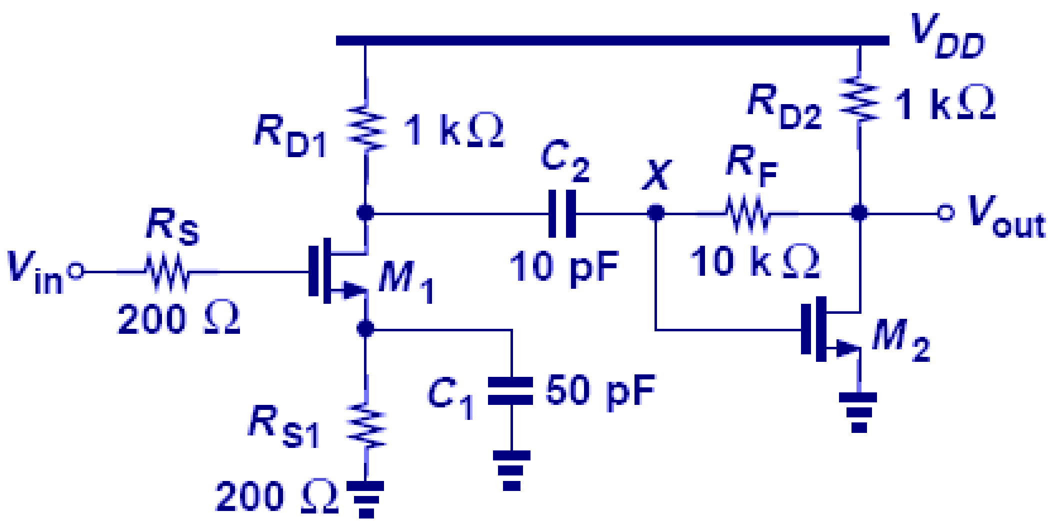
$$R_{in2} = R_{B2} \| [r_{\pi 2} + (\beta + 1)R_E]$$

$$\frac{V_Y}{V_{Thev}}(s) = \frac{R_{in2}}{R_C + \frac{1}{sC_2} + R_{in2}} = \frac{sC_2 R_{in2}}{1 + sC_2(R_C + R_{in2})}$$

$$\omega_{L2} = \frac{1}{(R_C + R_{in2})C_2} = \pi \times (22.9Hz)$$

Example: IC Amplifier – Low Frequency Design

$$g_{m1} = g_{m2} = (150\Omega)^{-1}$$



$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92MHz)$$

$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$A_{v2} = -g_{m2}R_{D2} = -\frac{1k\Omega}{150\Omega} = -6.67$$

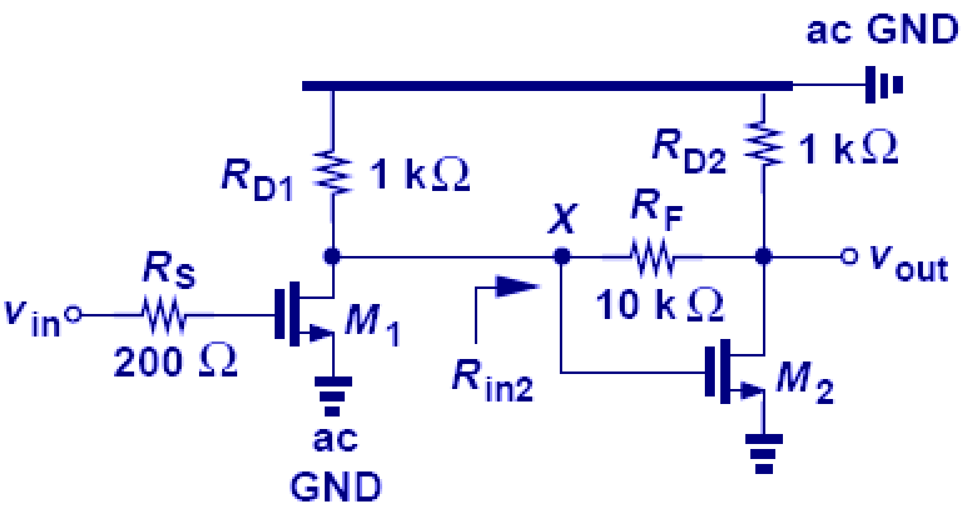
$$R_{in2} = \frac{10k\Omega}{7.67} = 1.30k\Omega$$

$$\omega_{L1} = \frac{1}{\left(\frac{1}{g_{m1}} \| R_{S1}\right)C_1} = \frac{g_{m1}R_{S1} + 1}{R_{S1}C_1} = 2\pi \times (37.2MHz)$$

The "highest" low-frequency pole (ω_{L1} = 37.2MHz) will set the low-frequency cut-off

Example: IC Amplifier – Midband Design

$$g_{m1} = g_{m2} = (150\Omega)^{-1}$$

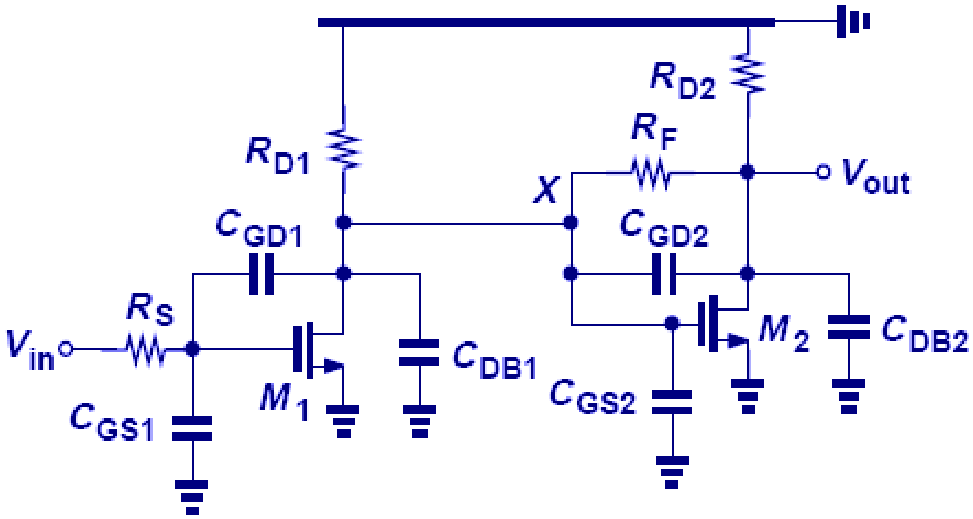


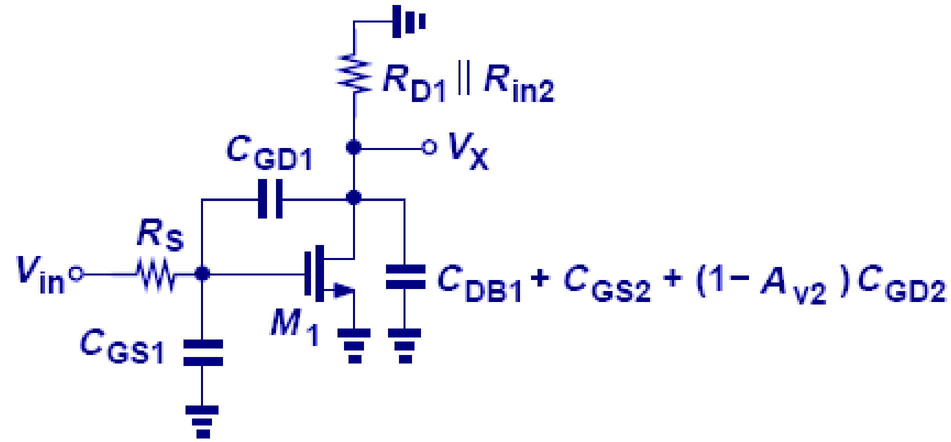
$$A_{v1} = \frac{v_X}{v_{in}} = -g_{m1}(R_{D1} \| R_{in2}) = -3.77$$

$$A_{v2} = -6.67$$

$$A_v = A_{v1}A_{v2} = 25.1 = 28.0dB$$

Example: IC Amplifier – High Frequency Design





To get an accurate estimate for ω_{p1} and ω_{p2}, use the dominant pole approximation expressions on Slide 44

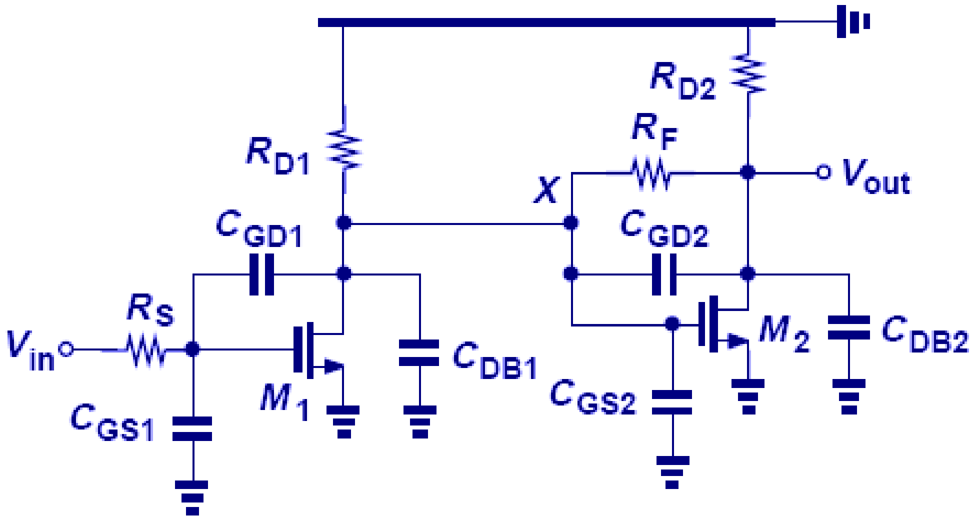
$$|\omega_{p1}| = \frac{1}{(1 - A_{v1})C_{GD1}R_S + R_S C_{GS1} + (R_{D1} \| R_{in2})(C_{GD1} + C_{out1})} = 2\pi(222MHz)$$

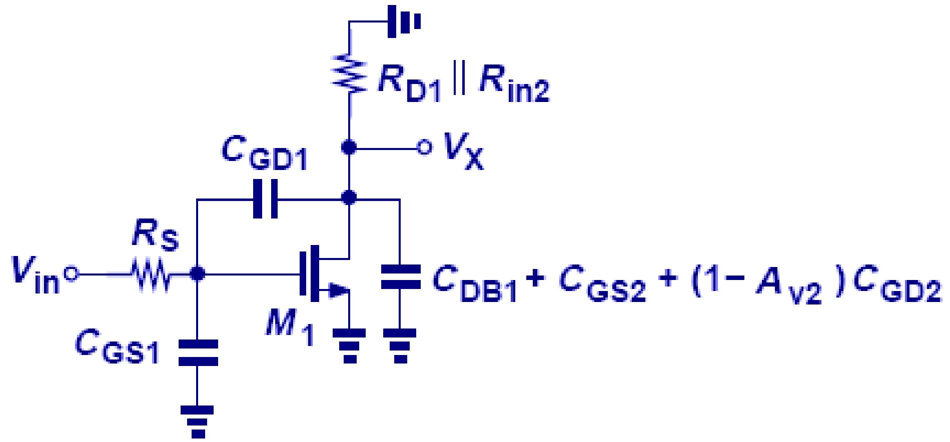
$$|\omega_{p2}| = \frac{(1 - A_{v1})C_{GD1}R_S + R_S C_{GS1} + (R_{D1} \| R_{in2})(C_{GD1} + C_{out1})}{R_S (R_{D1} \| R_{in2})(C_{GS1}C_{GD1} + C_{GD1}C_{out1} + C_{GS1}C_{out1})} = 2\pi(2.99GHz)$$

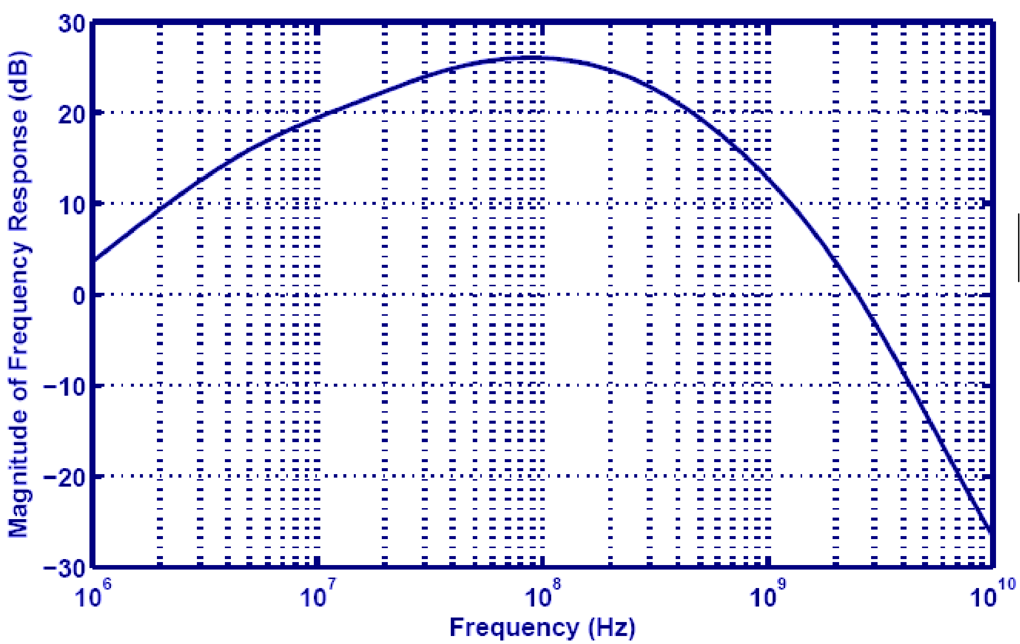
where

$$C_{out1} = C_{DB1} + C_{GS2} + C_{GD2}(1 - A_{v2})$$

Example: IC Amplifier – High Frequency Design







$$|\omega_{p1}| = 2\pi(222MHz)$$

$$|\omega_{p2}| = 2\pi(2.99GHz)$$

$$|\omega_{p3}| = \frac{1}{R_{D2}\left[C_{GD2}\left(1 - \frac{1}{A_{v2}}\right) + C_{DB2}\right]} = 2\pi(829MHz)$$

Next Time

- Feedback
  - Razavi Chapter 12