ECEN 326
Electronic Circuits

Stability

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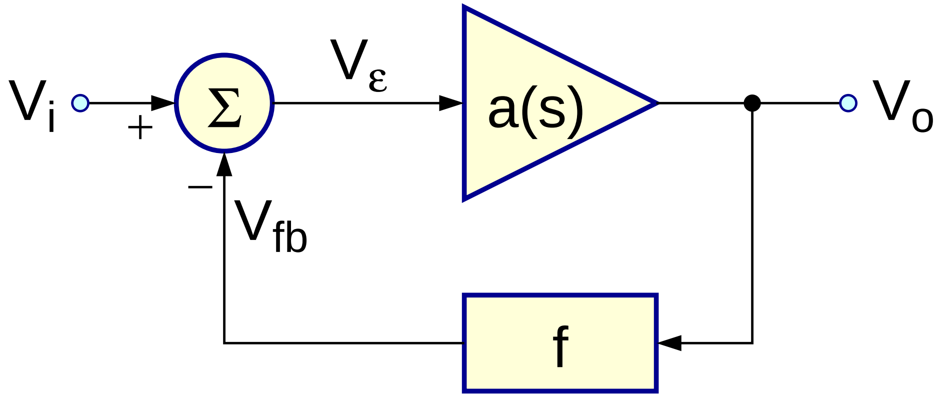
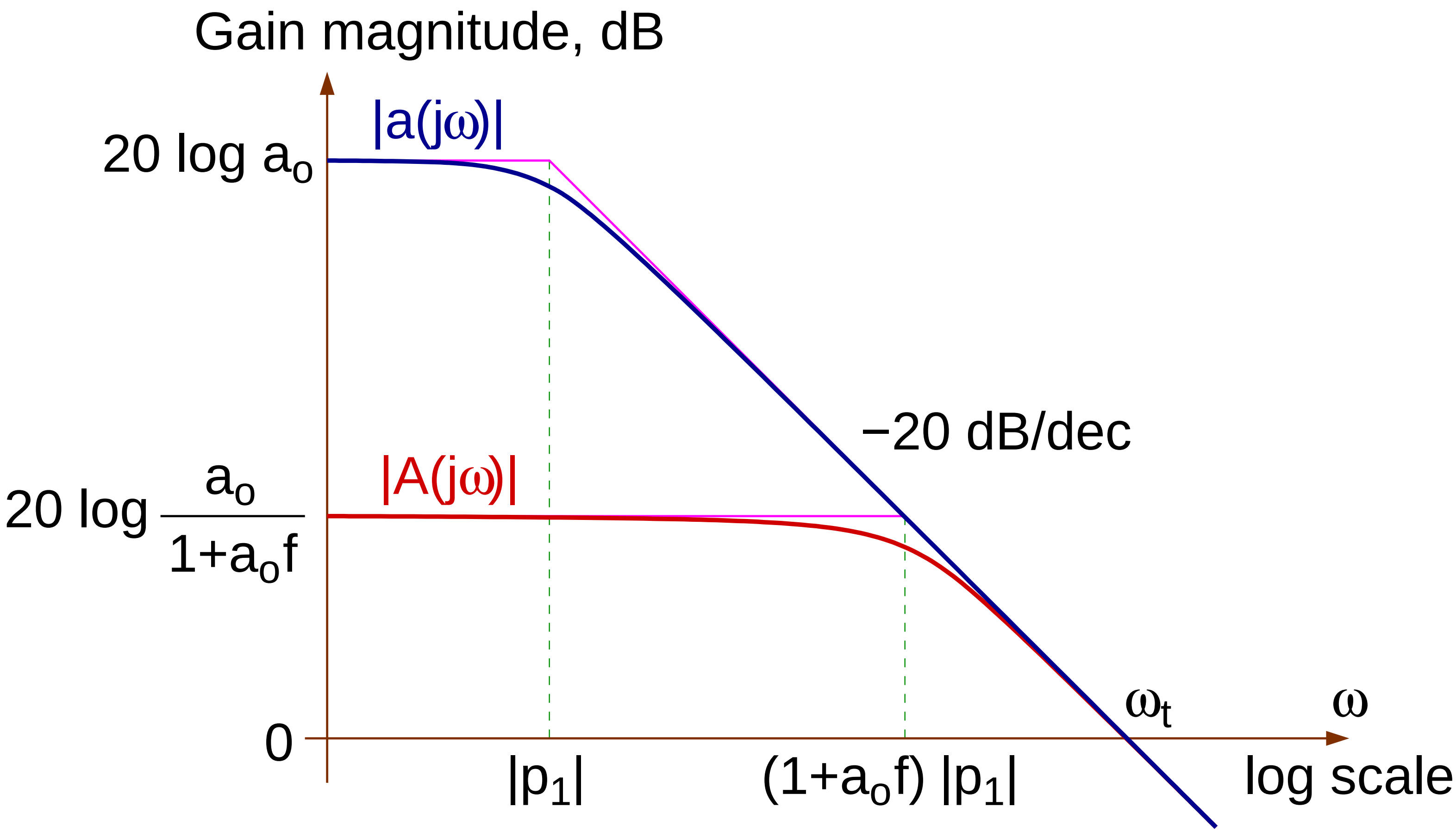
Ideal Configuration



$$a(s) = \frac{V_o}{V_\varepsilon}(s) = \frac{a_o}{1 - \dfrac{s}{p_1}}$$

$$A(s) = \frac{V_o}{V_i}(s) = \frac{a(s)}{1 + a(s)f} = \frac{\dfrac{a_o}{1 + a_o f}}{1 - \dfrac{s}{(1 + a_o f)p_1}}$$

Gain-Bandwidth



Instability and the Nyquist Criterion

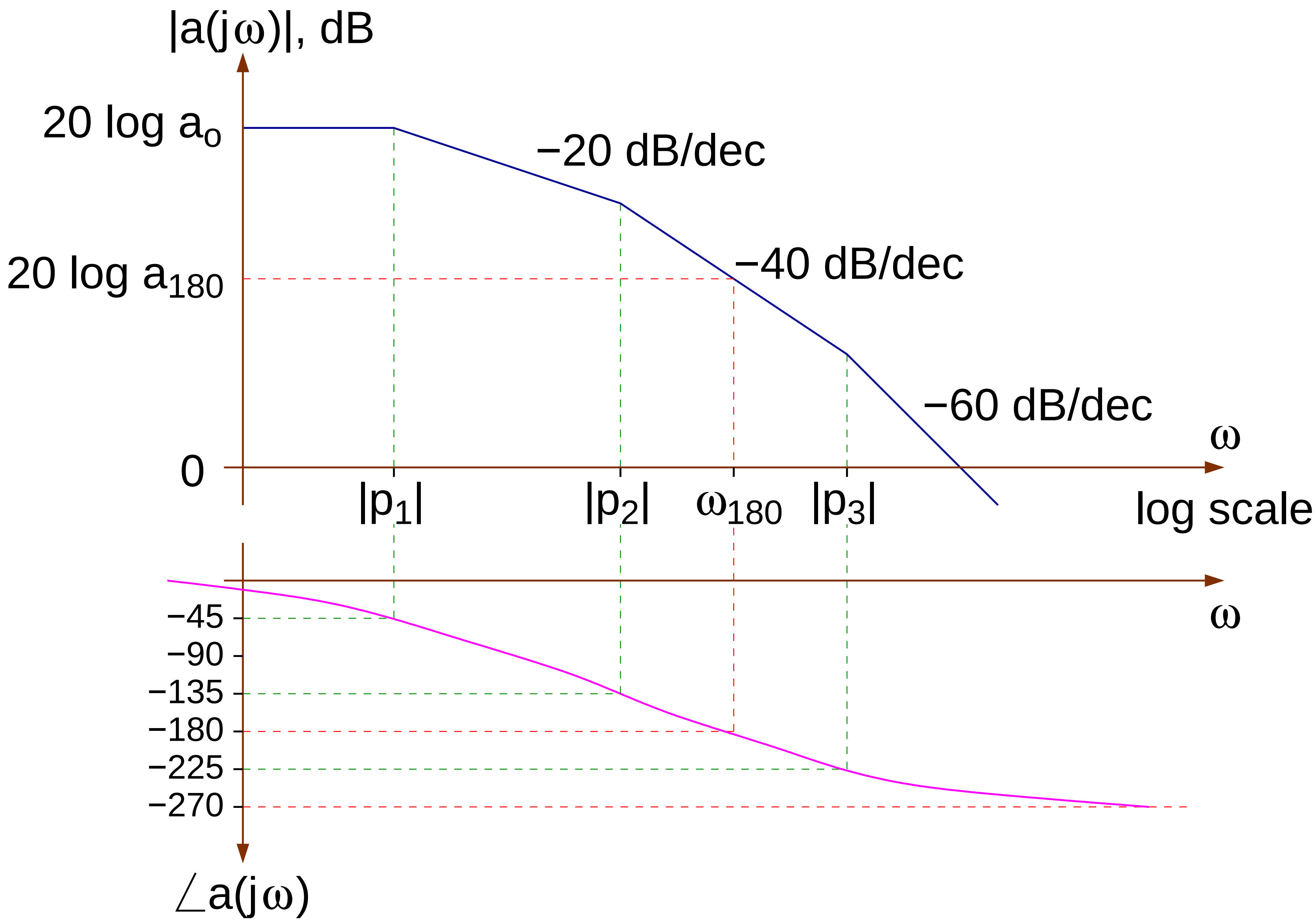
Transfer function of a 3-pole amplifier:

$$\mathbf{a(s)} = \frac{\mathbf{a_o}}{\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_1}}\right)\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_2}}\right)\left(\mathbf{1} - \frac{\mathbf{s}}{\mathbf{p_3}}\right)}$$

Nyquist criterion for stability of the amplifier:

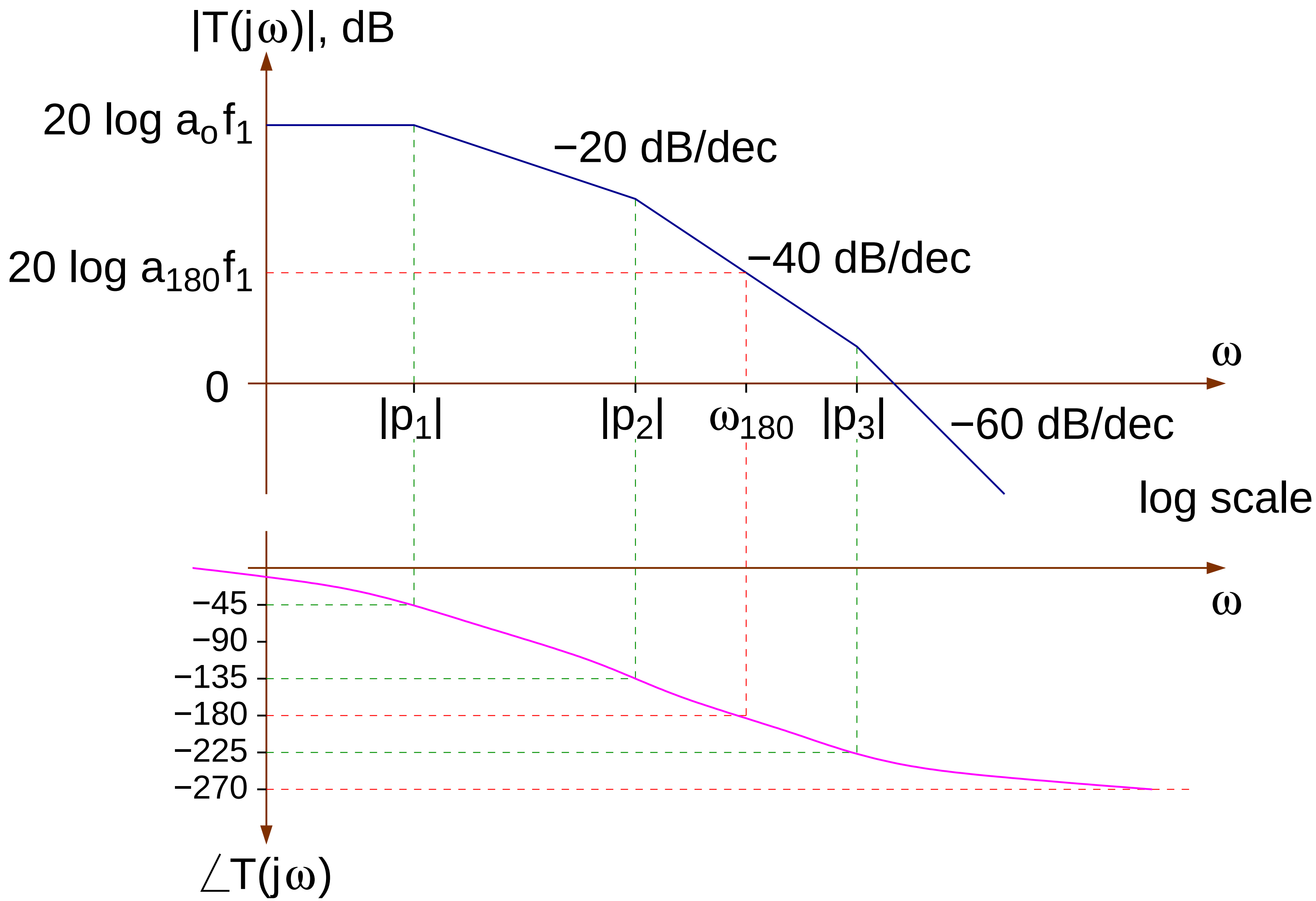
> *Consider a feedback amplifier with a stable* $\mathbf{T(s)}$. *If the Nyquist plot of* $\mathbf{T(j\omega)}$ *encircles the point (-1,0), the feedback amplifier is unstable.*

Magnitude & Phase



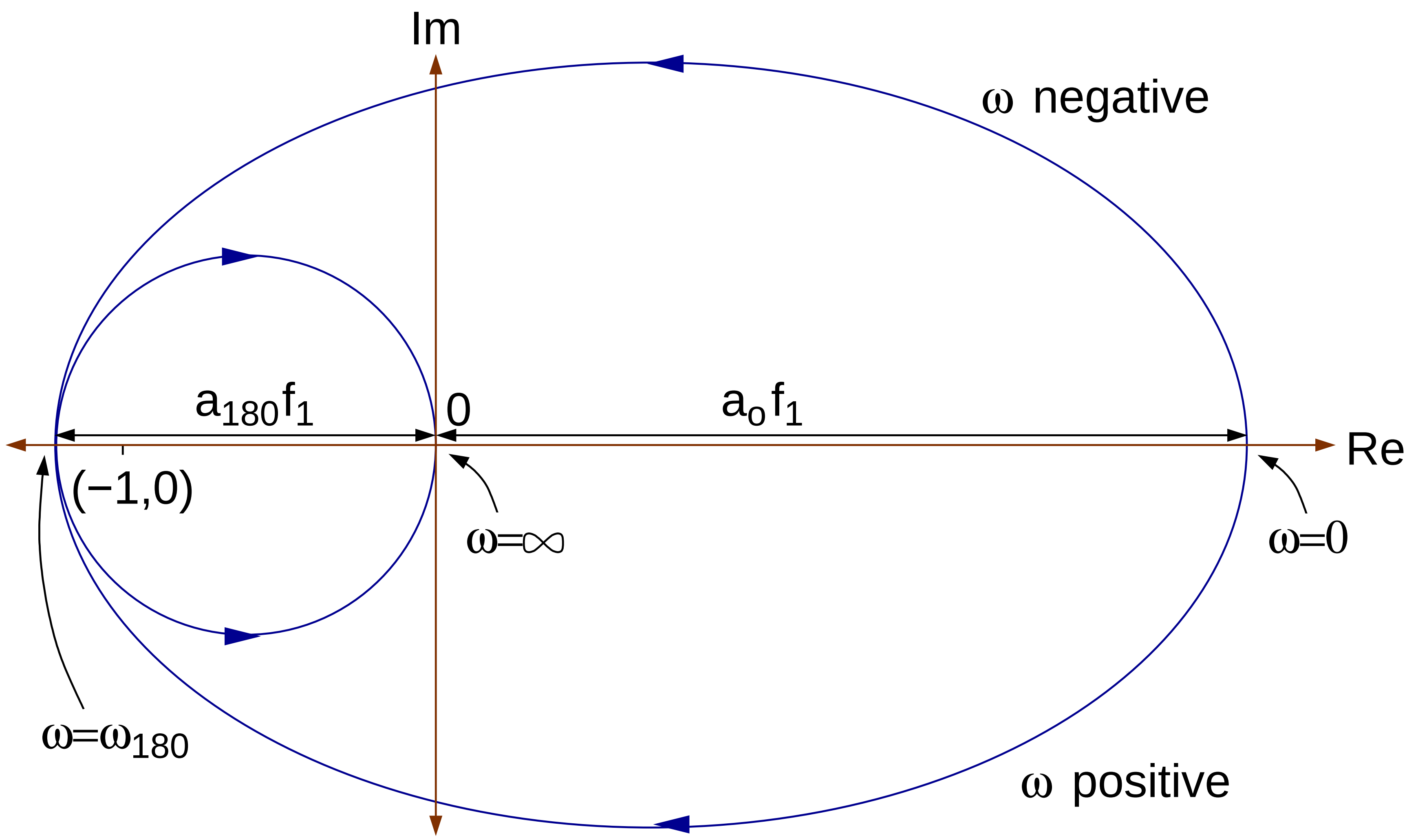
Magnitude & Phase

$T(s) = a(s)f_1$

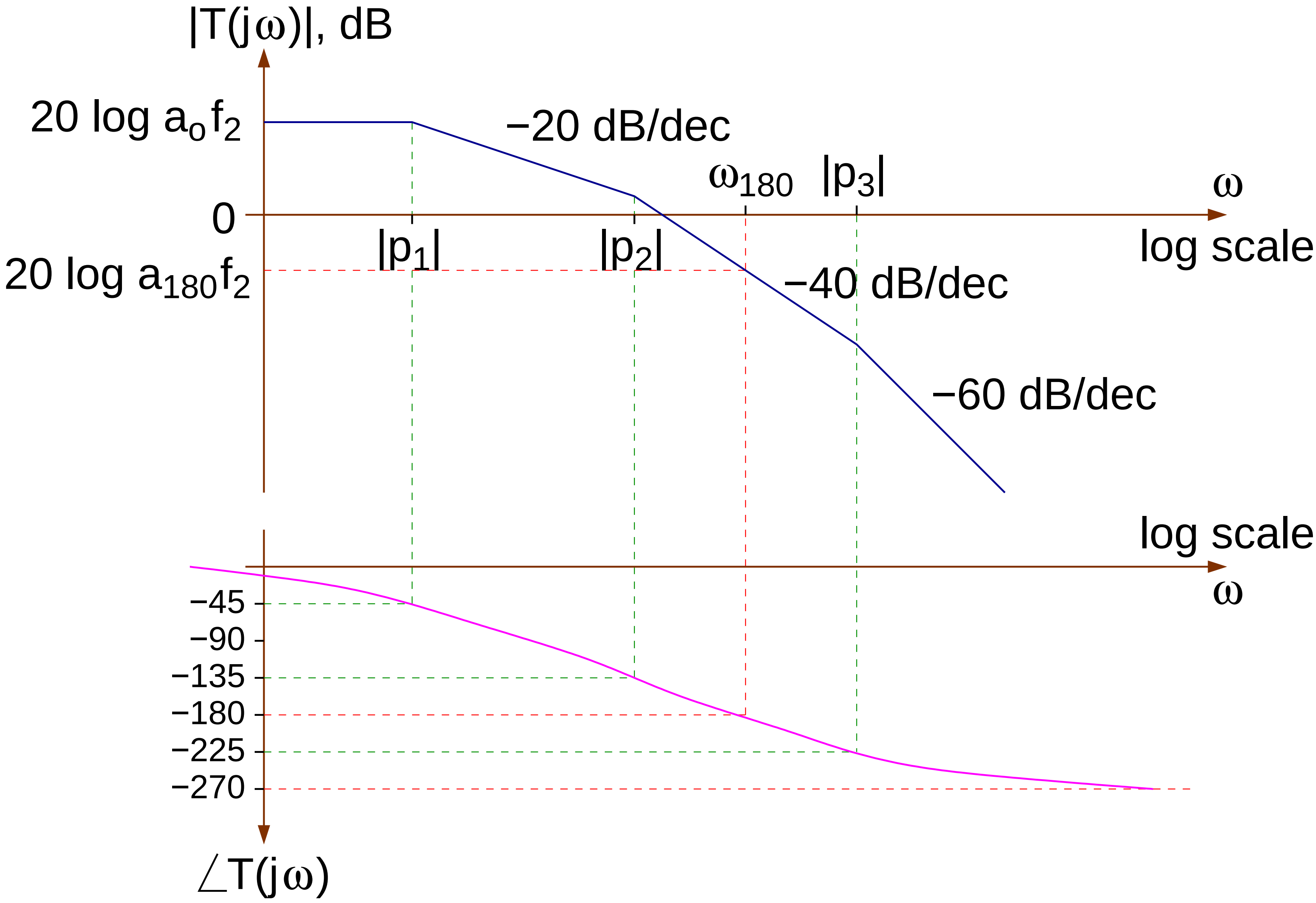


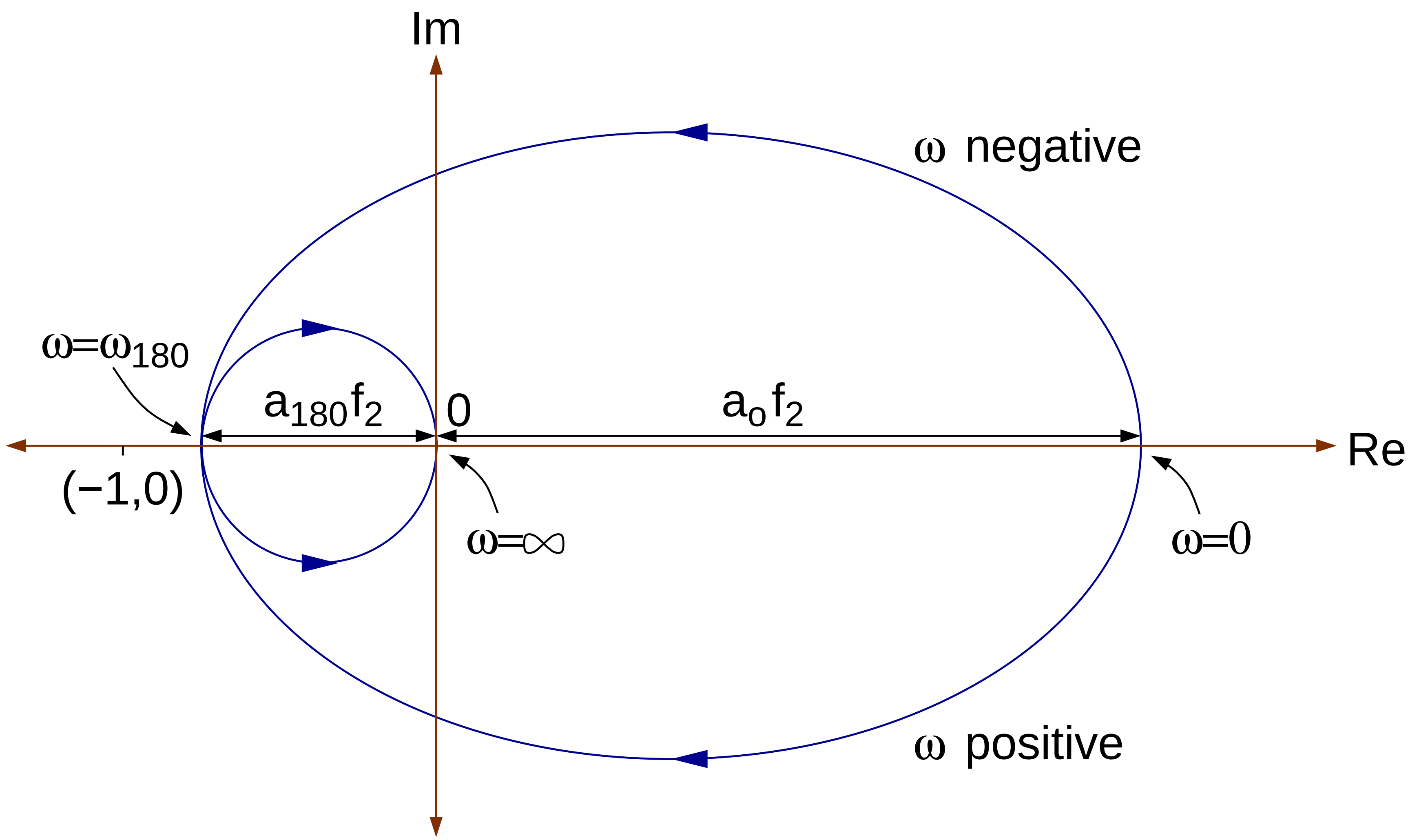
Nyquist Plot

$T(s) = a(s)f_1$



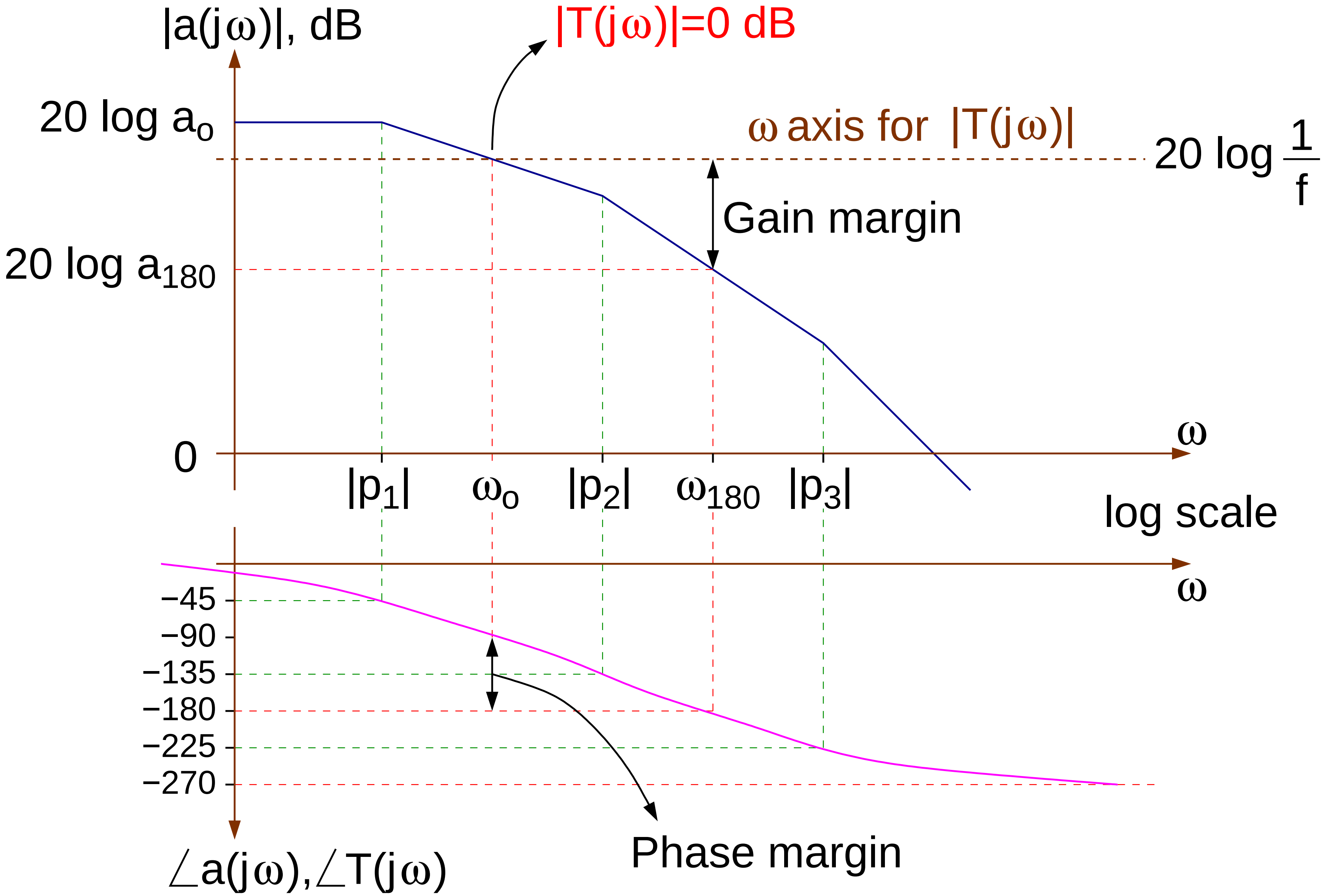
Magnitude & Phase





Im
ω negative
ω=ω180
a180f2
0
aof2
Re
(−1,0)
ω=∞
ω=0
ω positive

Gain & Phase Margin



Stability Criteria

Nyquist:

$$|\mathbf{T(j\omega_{180})}| = \mathbf{a_{180}f} < \mathbf{1} \quad \Rightarrow \quad \text{Stable}$$

Gain Margin (**GM**):

$$\mathbf{GM} = \mathbf{20\log\frac{1}{|T(j\omega_{180})|}} = -\mathbf{20\log|T(j\omega_{180})|}$$

$$\mathbf{GM} > \mathbf{0} \quad \Rightarrow \quad \text{Stable}$$

Phase Margin (**PM**):

$$\mathbf{PM} = \mathbf{180^\circ} + \angle\mathbf{T(j\omega_o)}$$

$$\mathbf{PM} > \mathbf{0} \quad \Rightarrow \quad \text{Stable}$$

Phase Margin

$$|T(j\omega_o)| = 1 \;\Rightarrow\; |a(j\omega_o)|f = 1 \;\Rightarrow\; |a(j\omega_o)| = \frac{1}{f}$$

$$PM = 45^\circ \;\Rightarrow\; \angle T(j\omega_o) = -135^\circ \;,\; A(j\omega_o) = \frac{a(j\omega_o)}{1 + T(j\omega_o)}$$

$$A(j\omega_o) = \frac{a(j\omega_o)}{1 + e^{-j135^\circ}} = \frac{a(j\omega_o)}{1 - 0.7 - 0.7j}$$

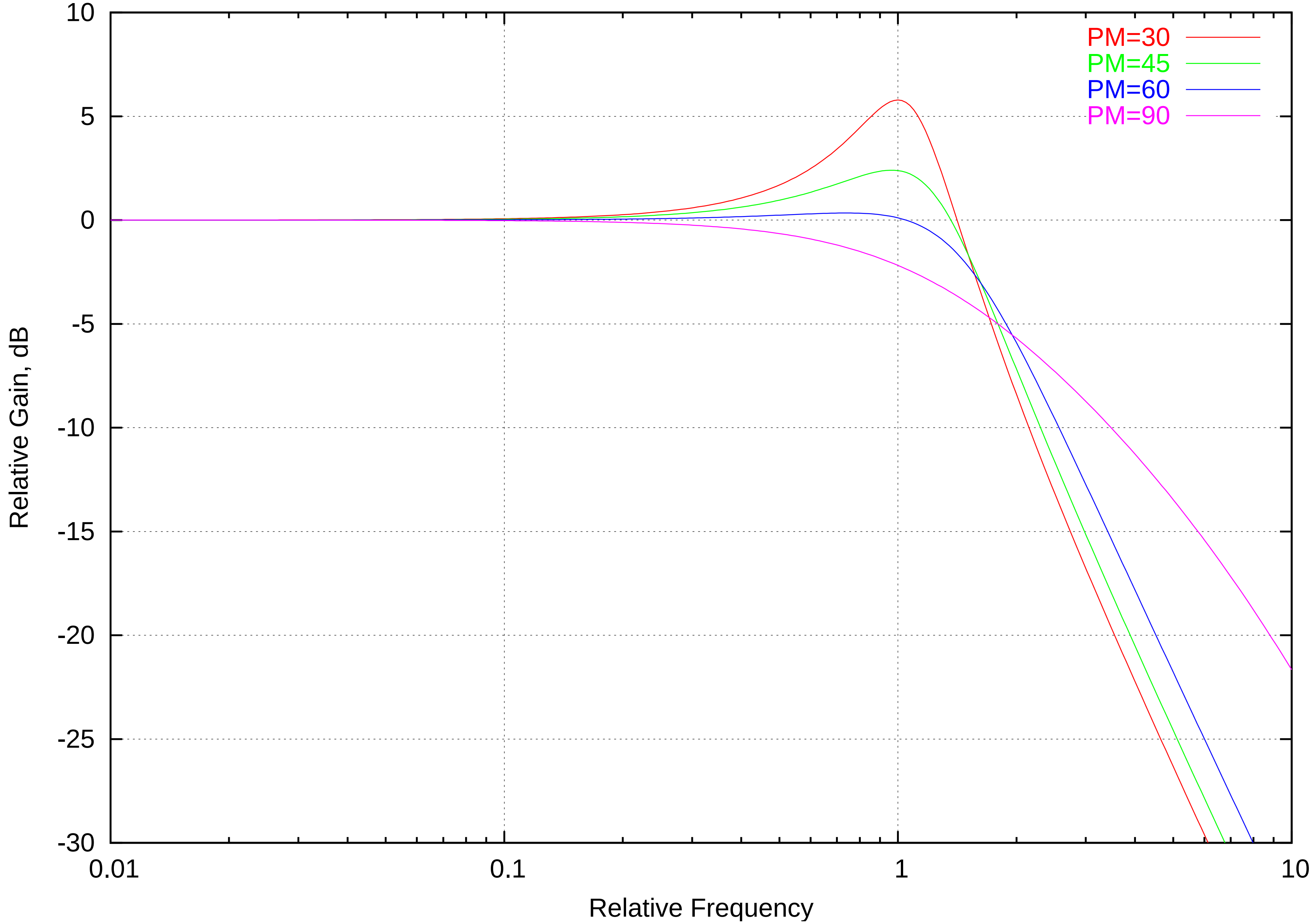
$$|A(j\omega_o)| = \frac{|a(j\omega_o)|}{|0.3 - 0.7j|} = \frac{1}{0.76f} = \frac{1.3}{f}$$

$$PM = 30^\circ \;\Rightarrow\; \angle T(j\omega_o) = -150^\circ \;,\; |A(j\omega_o)| = 1.92/f$$

$$PM = 60^\circ \;\Rightarrow\; \angle T(j\omega_o) = -120^\circ \;,\; |A(j\omega_o)| = 1/f$$

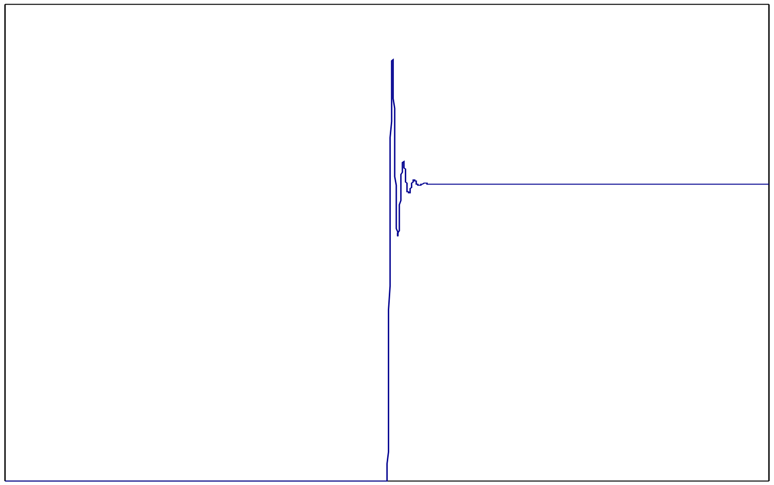
$$PM = 90^\circ \;\Rightarrow\; \angle T(j\omega_o) = -90^\circ \;,\; |A(j\omega_o)| = 0.7/f$$

Closed-Loop Frequency Response

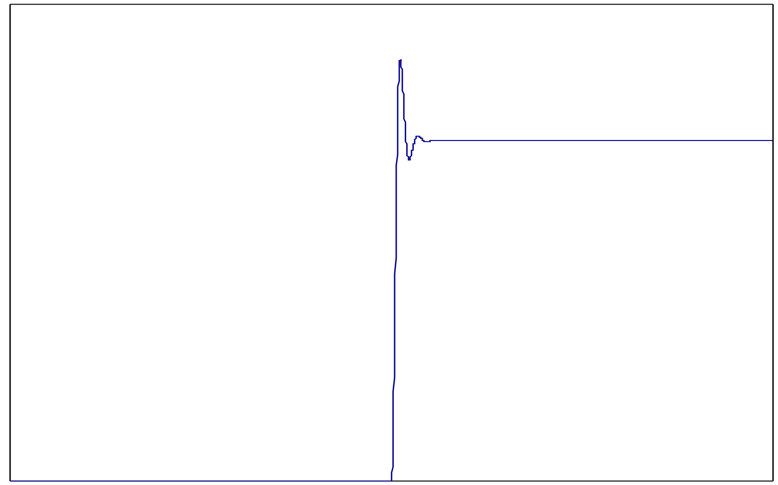


Closed-Loop Step Response

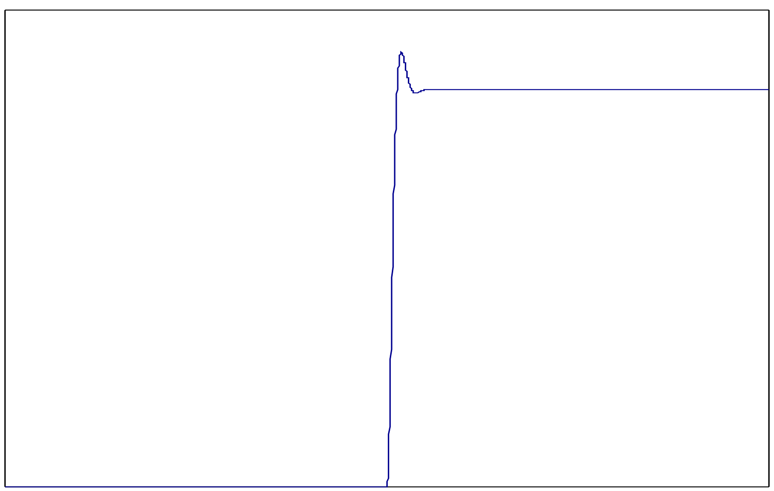
PM $= 30^\circ$



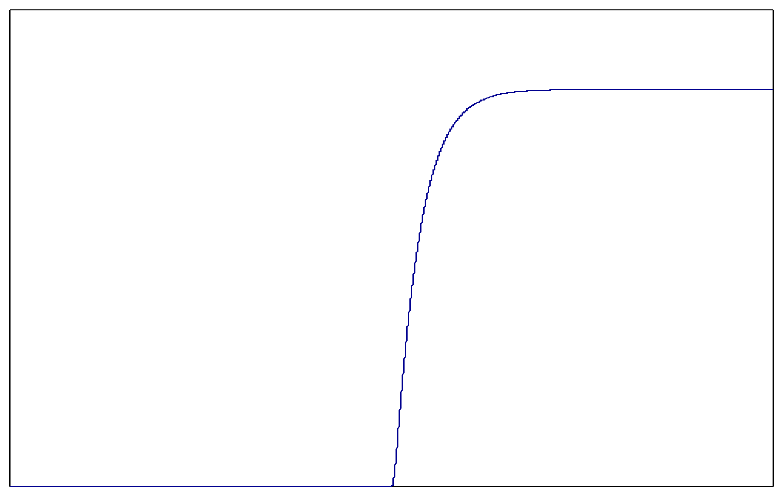
PM $= 45^\circ$



PM $= 60^\circ$

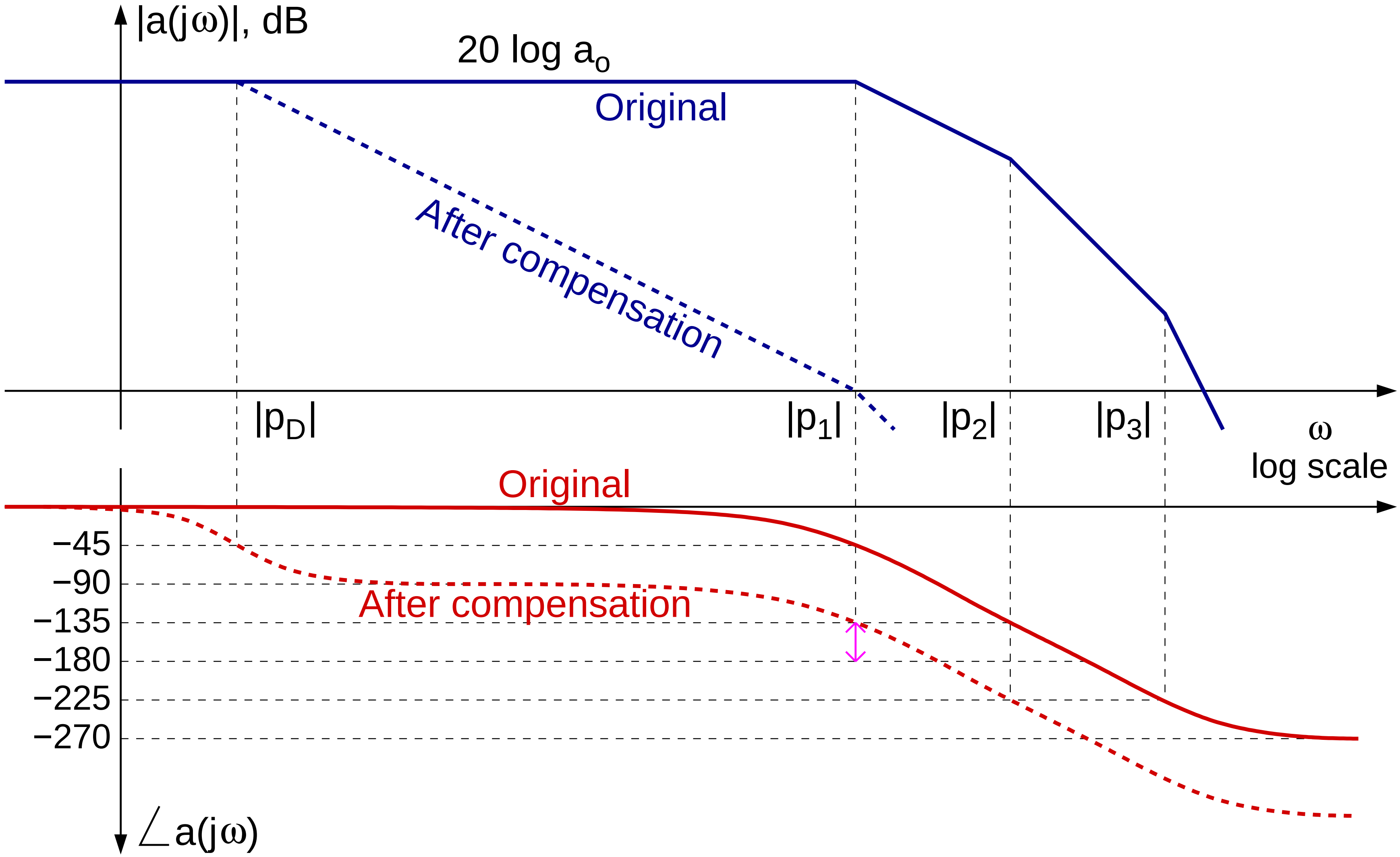


PM $= 90^\circ$



Compensation

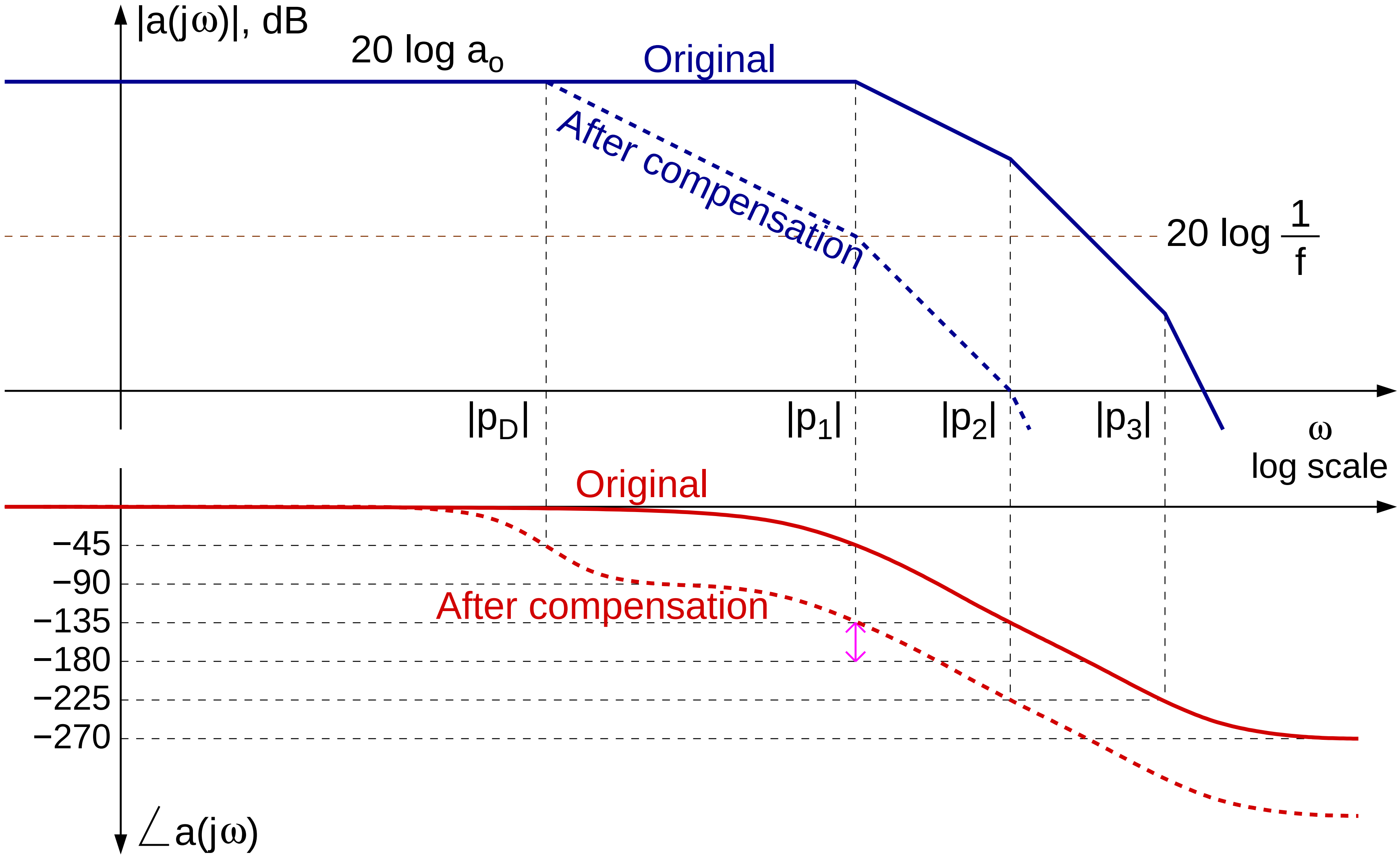
Dominant pole $(\mathbf{p_D})$ added, $\mathbf{f = 1}$



$$|\mathbf{p_D}| \approx |\mathbf{p_1}|/\mathbf{a_o}$$

Compensation

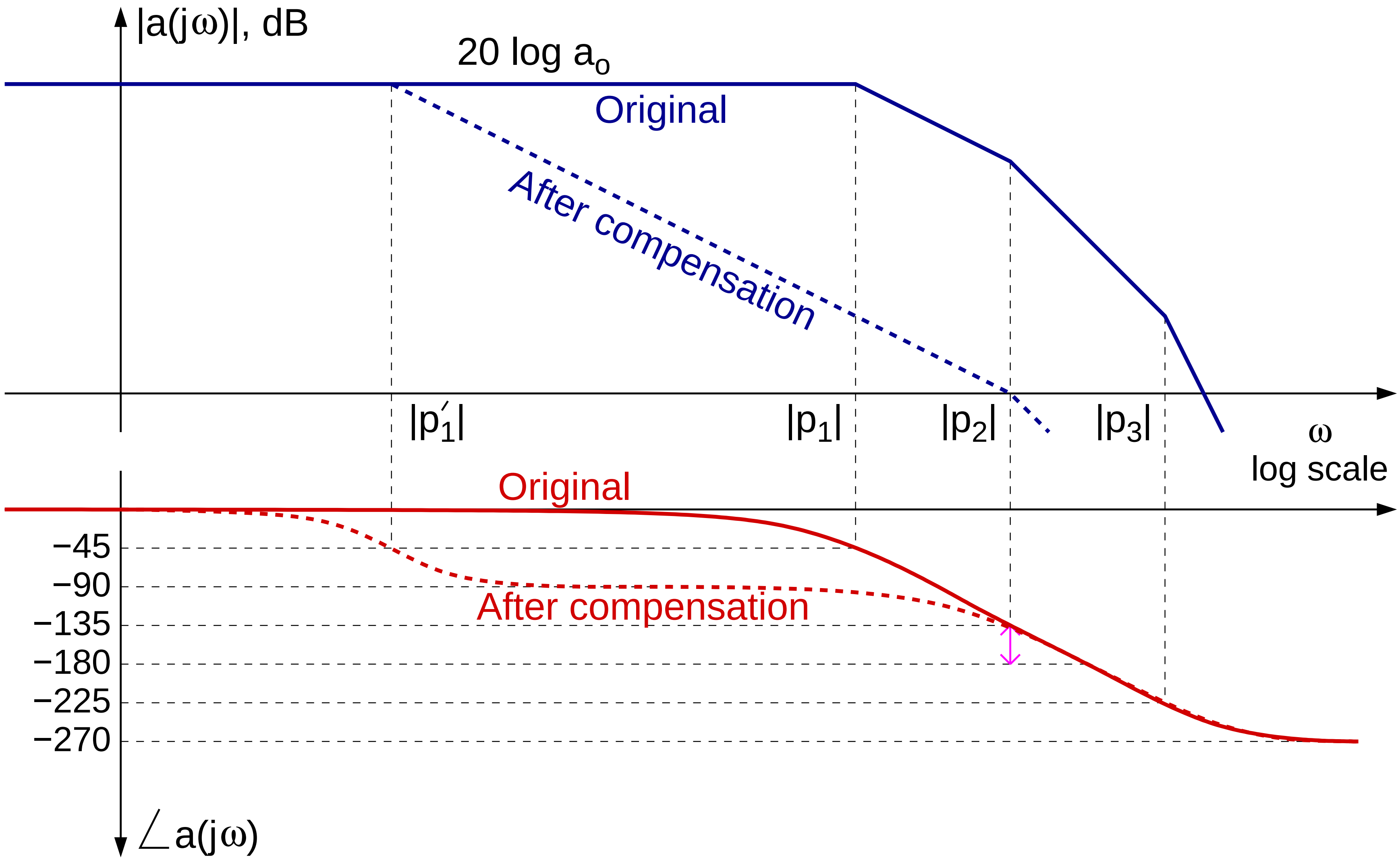
Dominant pole $(\mathbf{p_D})$ added, $\mathbf{f} < \mathbf{1}$



$$|\mathbf{p_D}| \approx |\mathbf{p_1}|/(\mathbf{a_o f})$$

Compensation

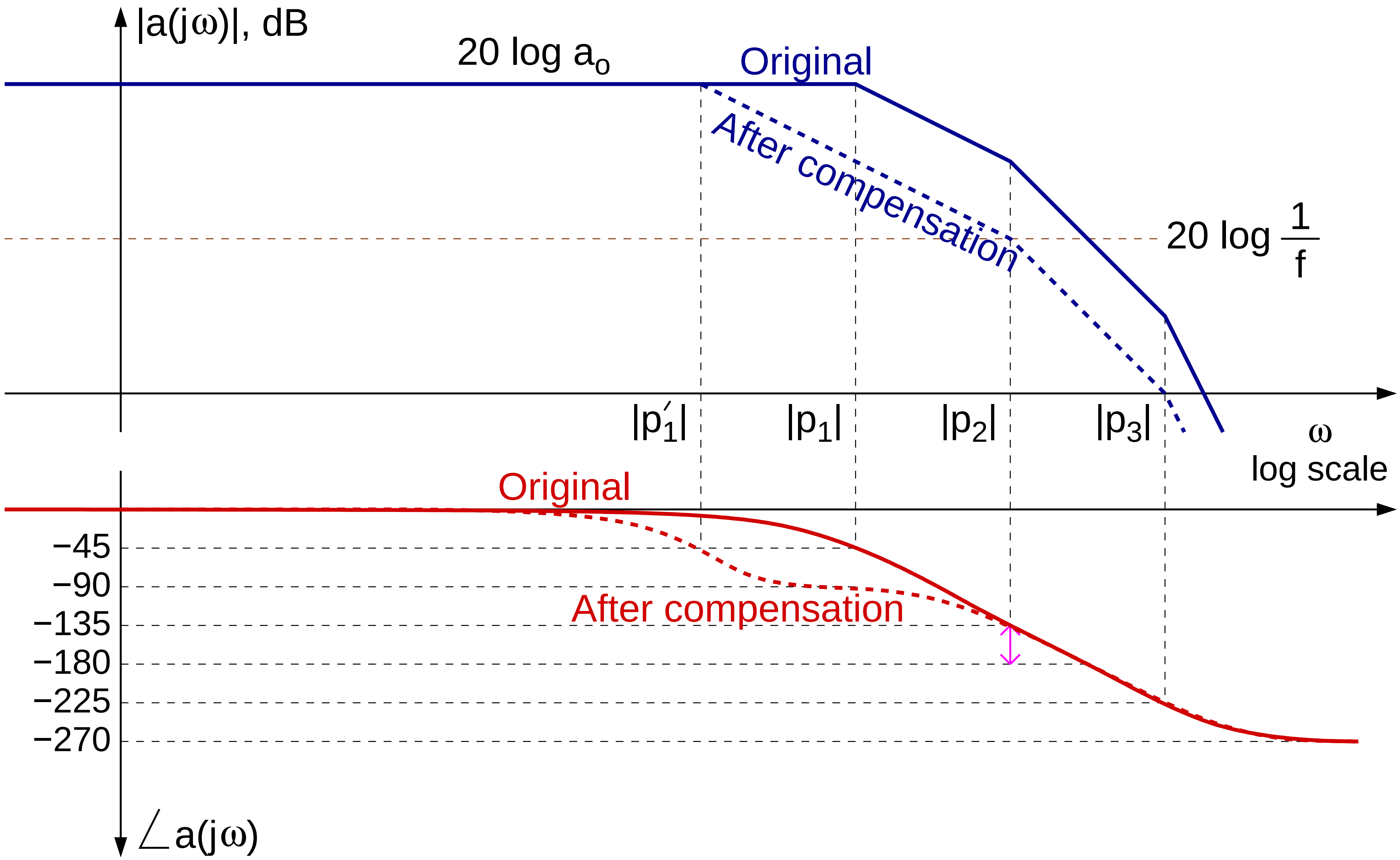
$\mathbf{p_1}$ shifted to $\mathbf{p_1'}$, $\mathbf{f = 1}$



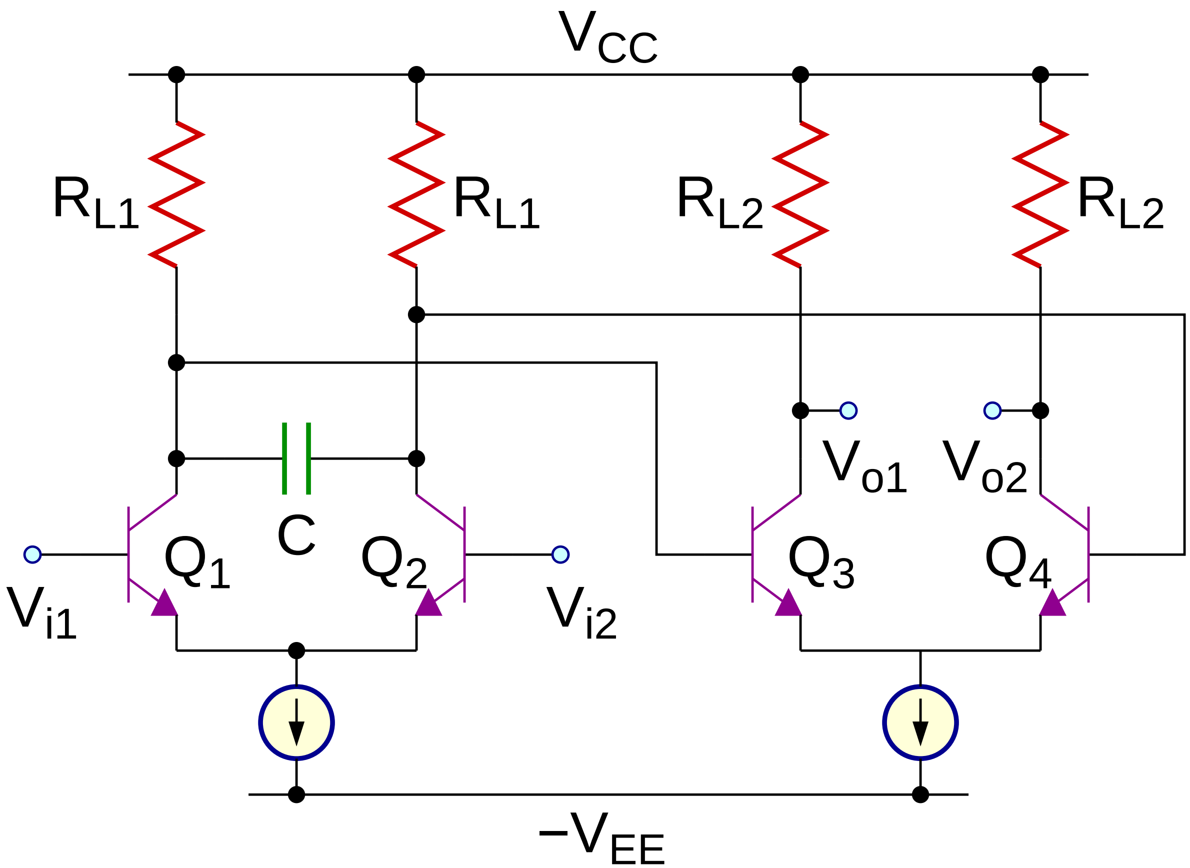
$$|\mathbf{p_1'}| \approx |\mathbf{p_2}|/\mathbf{a_o}$$

Compensation

p_1 shifted to p_1', $f < 1$

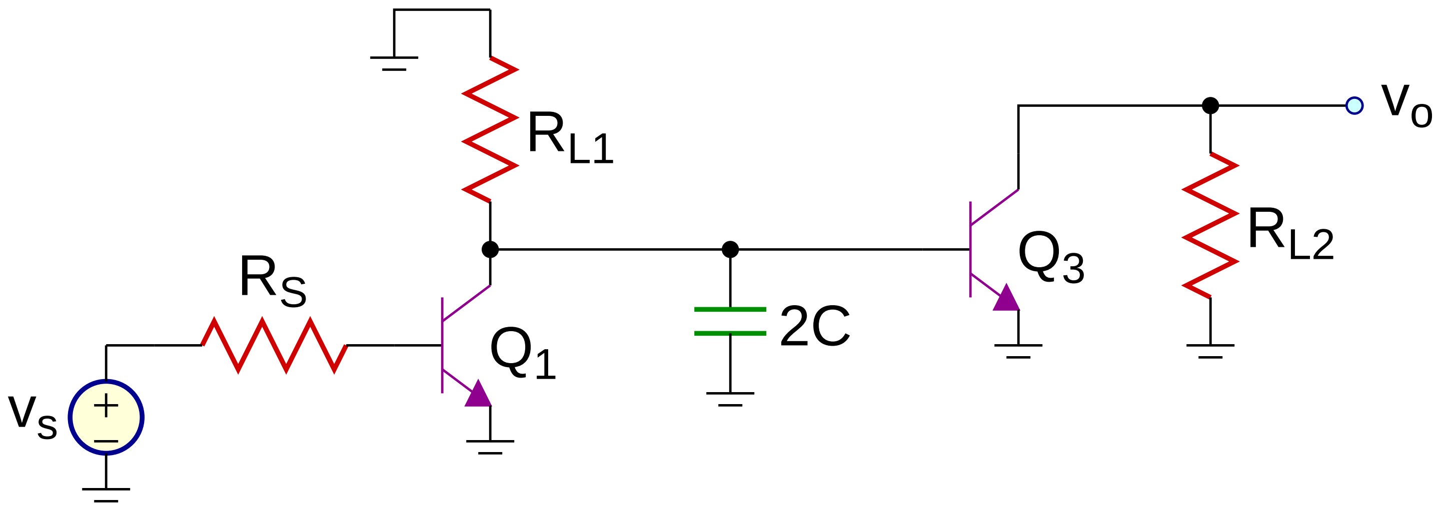


$$|p_1'| \approx |p_2|/(a_o f)$$



C : Compensation capacitor

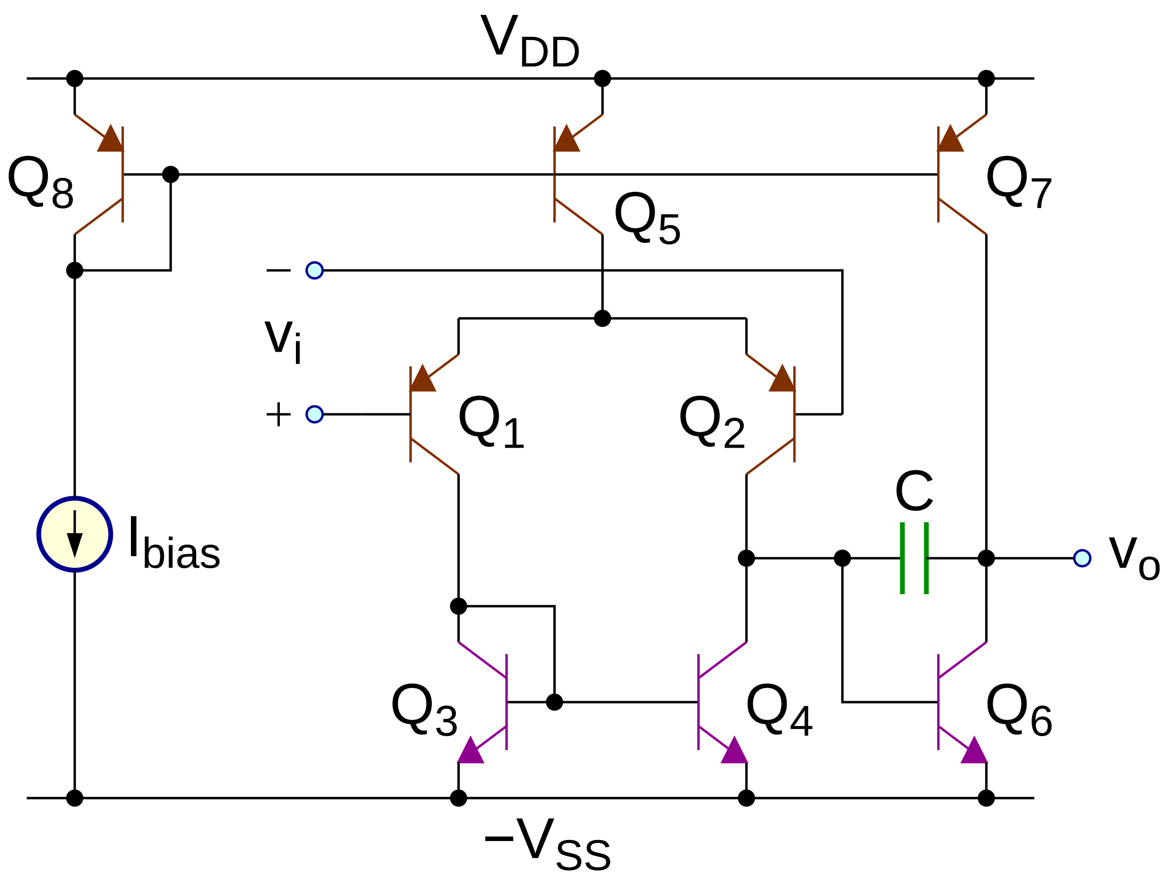
2-Stage Amplifier



$$|p_D| = \frac{1}{2RC}$$

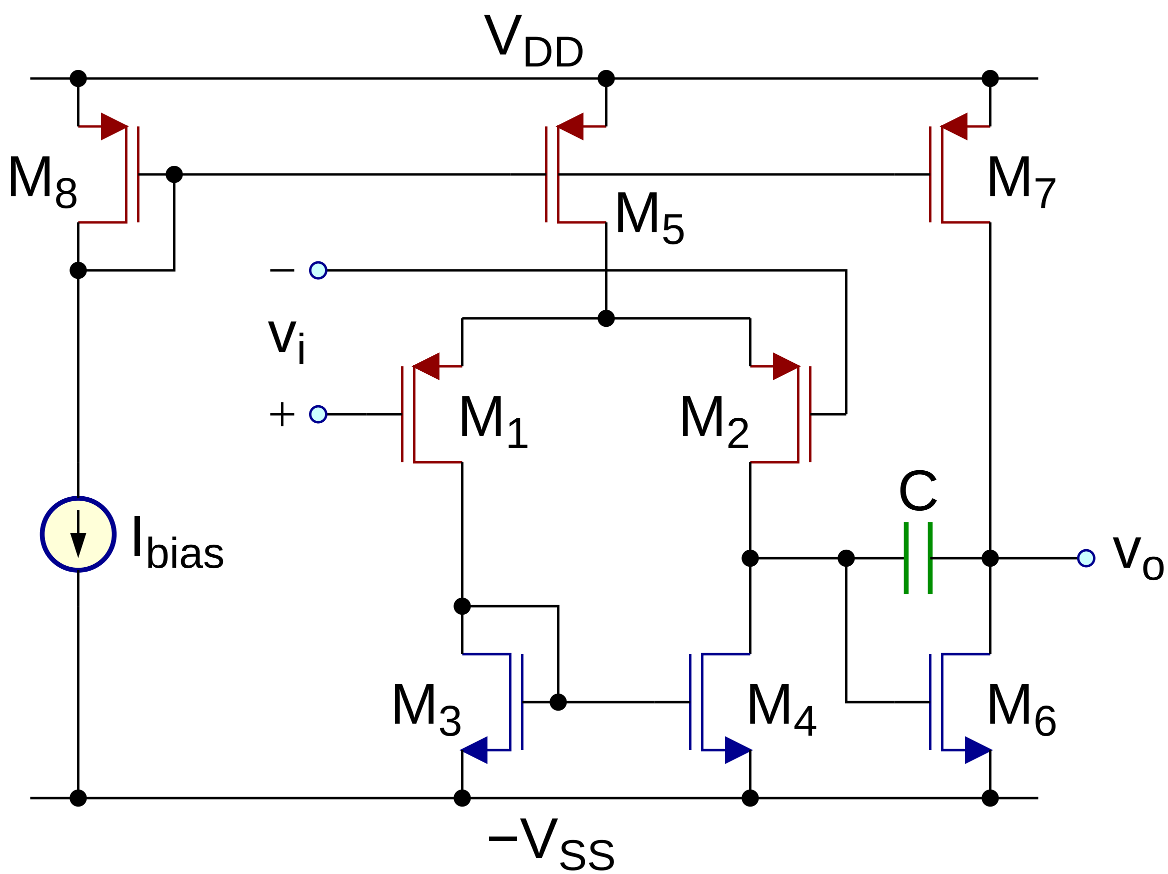
$$R = R_{L1} \parallel (r_{b3} + r_{\pi 3})$$

The value of **C** required is usually very large (typically $> 1\text{nF}$)

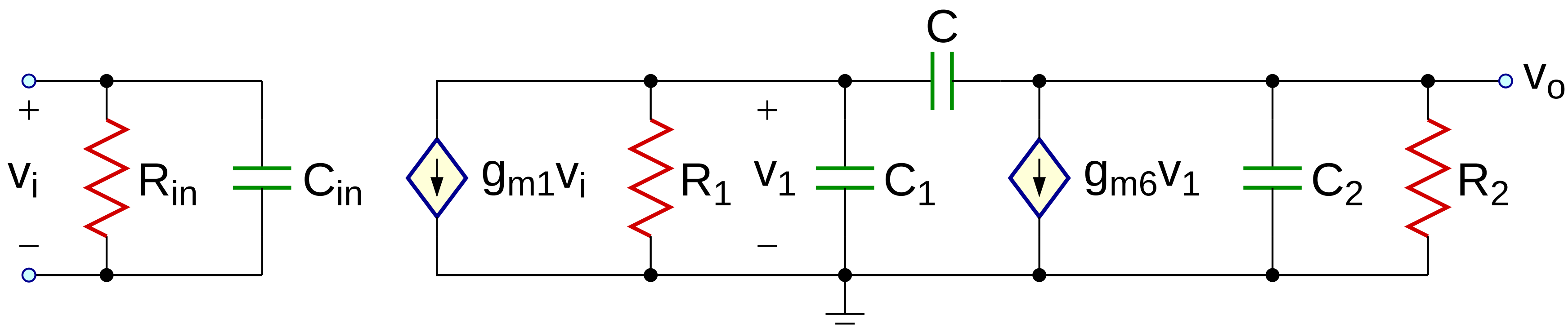


C : Compensation capacitor

Compensation of Opamps



C : Compensation capacitor



$$\frac{v_o}{v_i}=\frac{g_{m1}R_1R_2(g_{m6}-sC)}{1+s[R_1(C_1+C)+R_2(C_2+C)+g_{m6}R_1R_2C]+s^2[R_1R_2(C_1C_2+CC_1+CC_2]}$$

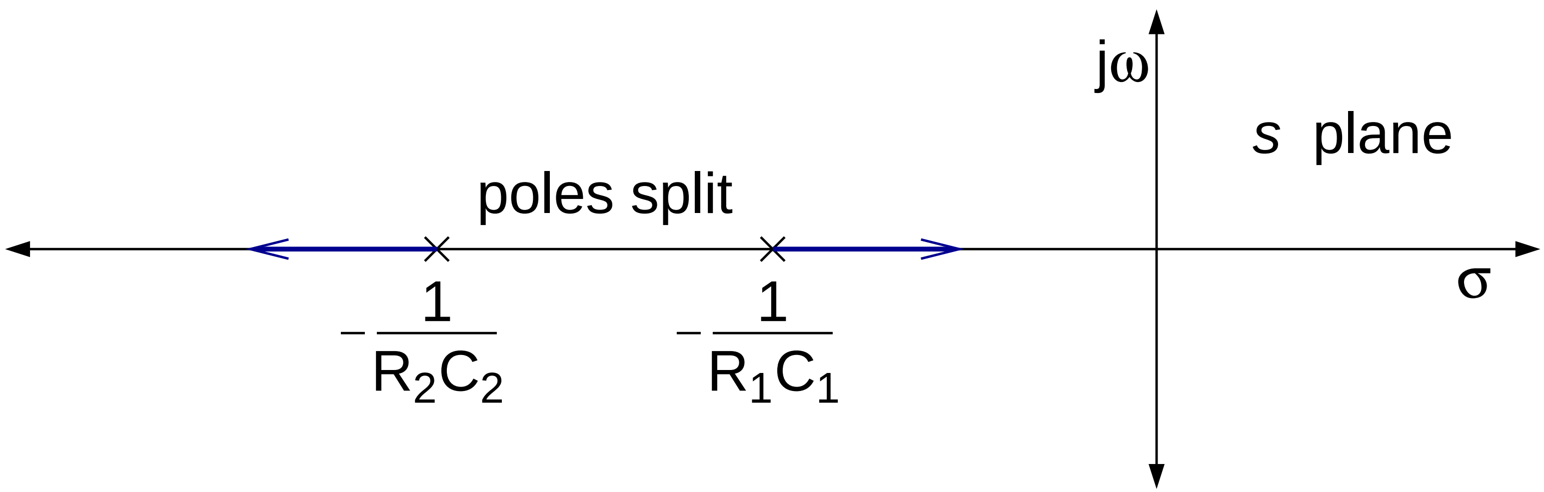
If p_1 and p_2 are real, $|p_1|\ll|p_2|$, C is large, $g_{m6}R_1, g_{m6}R_2 \gg 1$:

$$p_1 \approx -\frac{1}{R_1(C_1+C)+R_2(C_2+C)+g_{m6}R_1R_2C} \approx -\frac{1}{g_{m6}R_1R_2C}$$

$$p_2 \approx -\frac{g_{m6}C}{C_1C_2+C(C_1+C_2)} \qquad z=\frac{g_{m6}}{C}$$

$$C = 0 \;\Rightarrow\; p_1 = -\frac{1}{R_1C_1} \quad , \quad p_2 = -\frac{1}{R_2C_2}$$

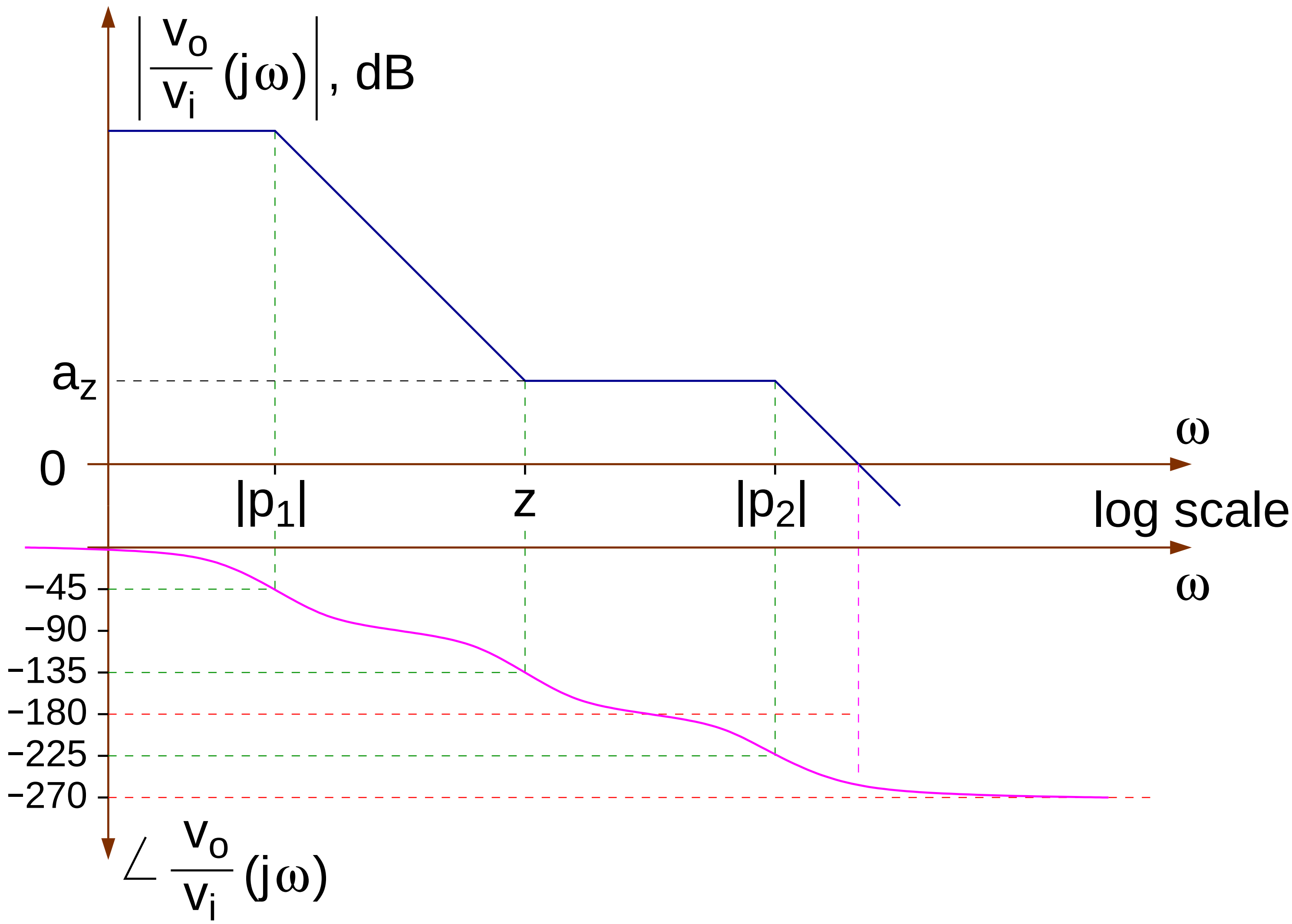
$$C \gg C_1, C_2 \;\Rightarrow\; p_1 \approx -\frac{1}{g_{m6}R_1R_2C} \quad , \quad p_2 \approx -\frac{g_{m6}}{C_1 + C_2}$$



As $\mathbf{C}$ increases, $|\mathbf{p_1}|$ decreases and $|\mathbf{p_2}|$ increases

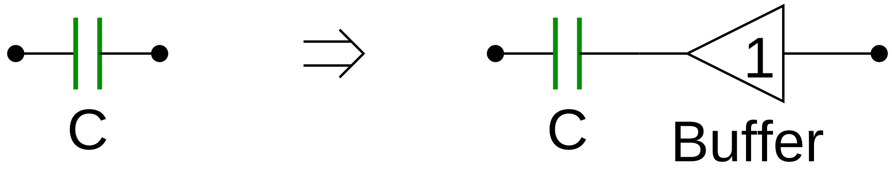
Effect of RHP Zero

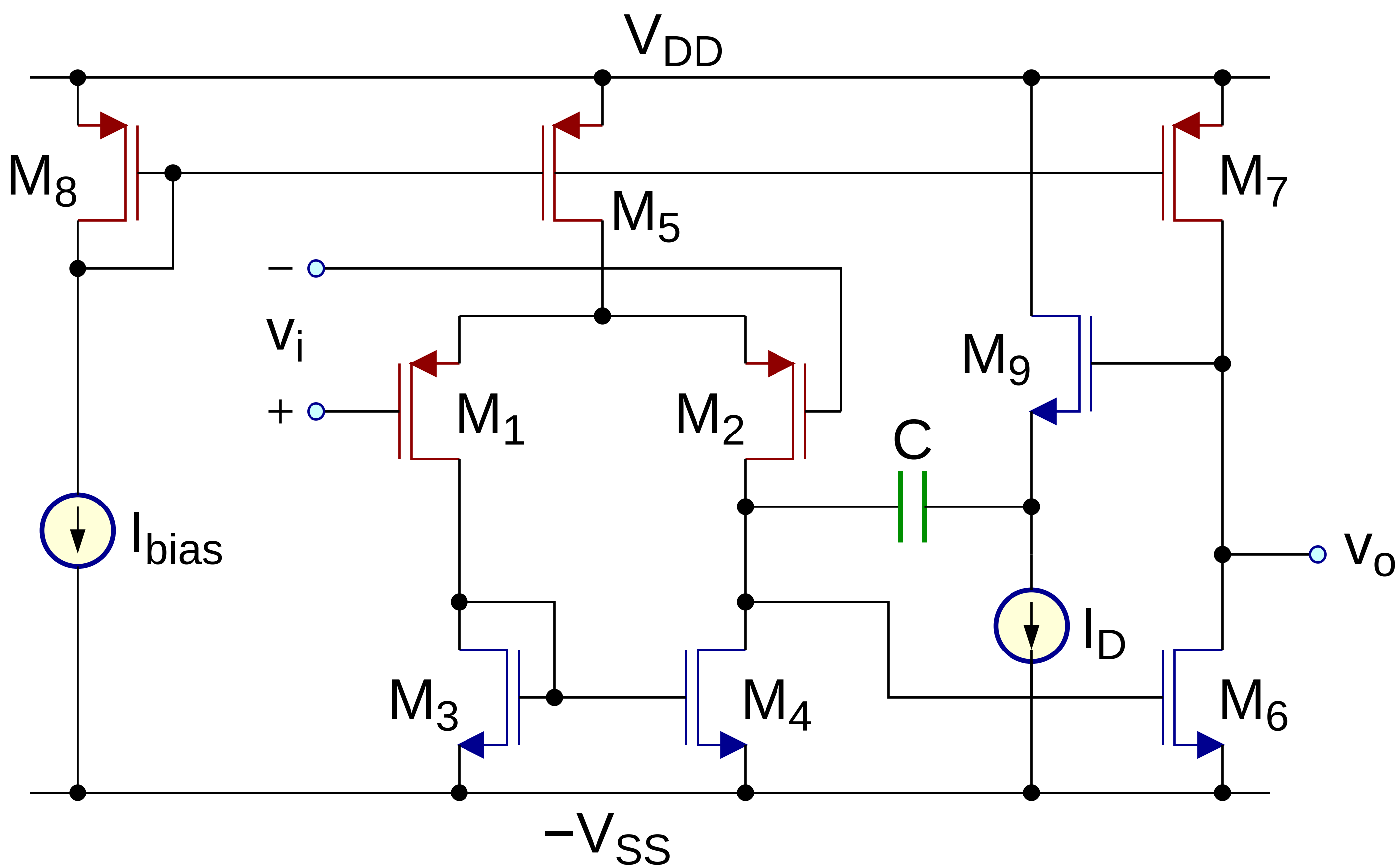
$z = g_{m6}/C$



If $a_z > 0$ dB and $z < |p_2|$, PM becomes negative

Elimination of RHP Zero Effect





Elimination of RHP Zero Effect

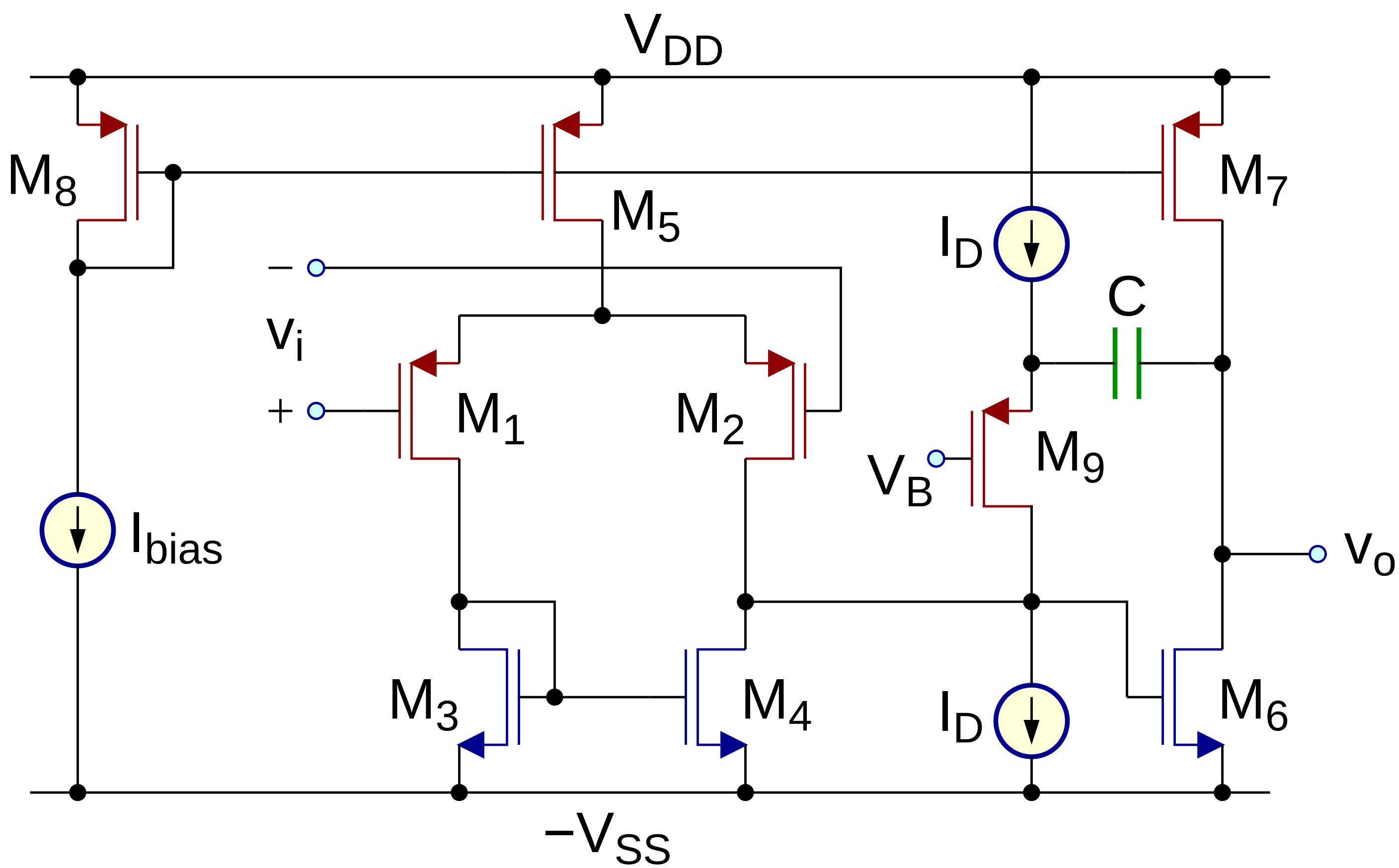
Assuming $g_{m6}R_1, g_{m6}R_2 \gg 1$ and C is large,

$$p_1 \approx -\frac{1}{g_{m6}R_1R_2C}$$

$$p_2 \approx -\frac{g_{m6}C}{(C_1 + C)C_2} \approx -\frac{g_{m6}}{C_2}$$

- The zero has been eliminated
- p_1 is unchanged
- p_2 is approximately the same as before
- Extra devices and bias current are required
- Output swing is affected

Elimination of RHP Zero Effect



Assuming $g_{m6}R_1, g_{m6}R_2 \gg 1$ and C is large,

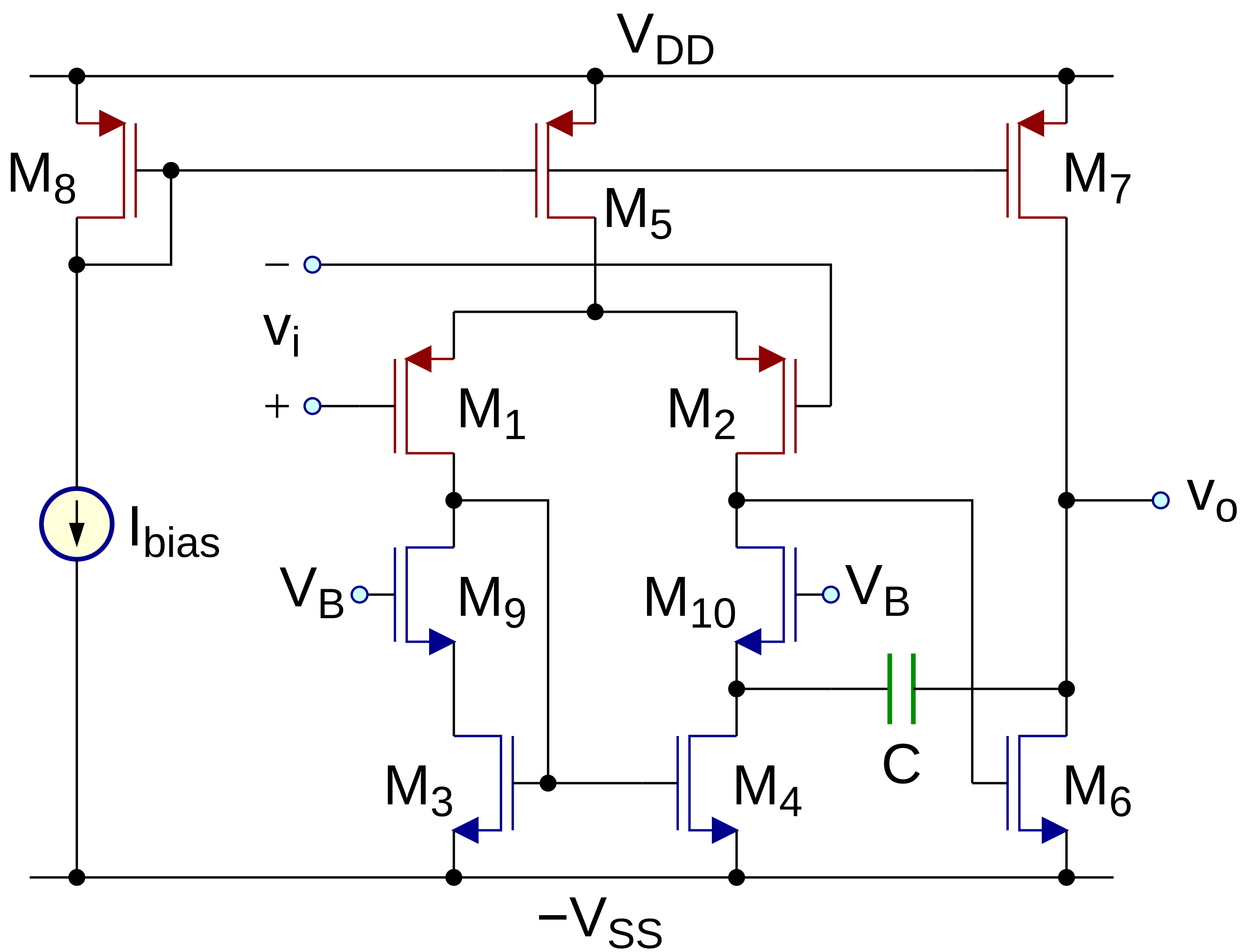
$$p_1 \approx -\frac{1}{g_{m6}R_1R_2C}$$

$$p_2 \approx -\frac{g_{m6}}{C + C_2}\frac{C}{C_1}$$

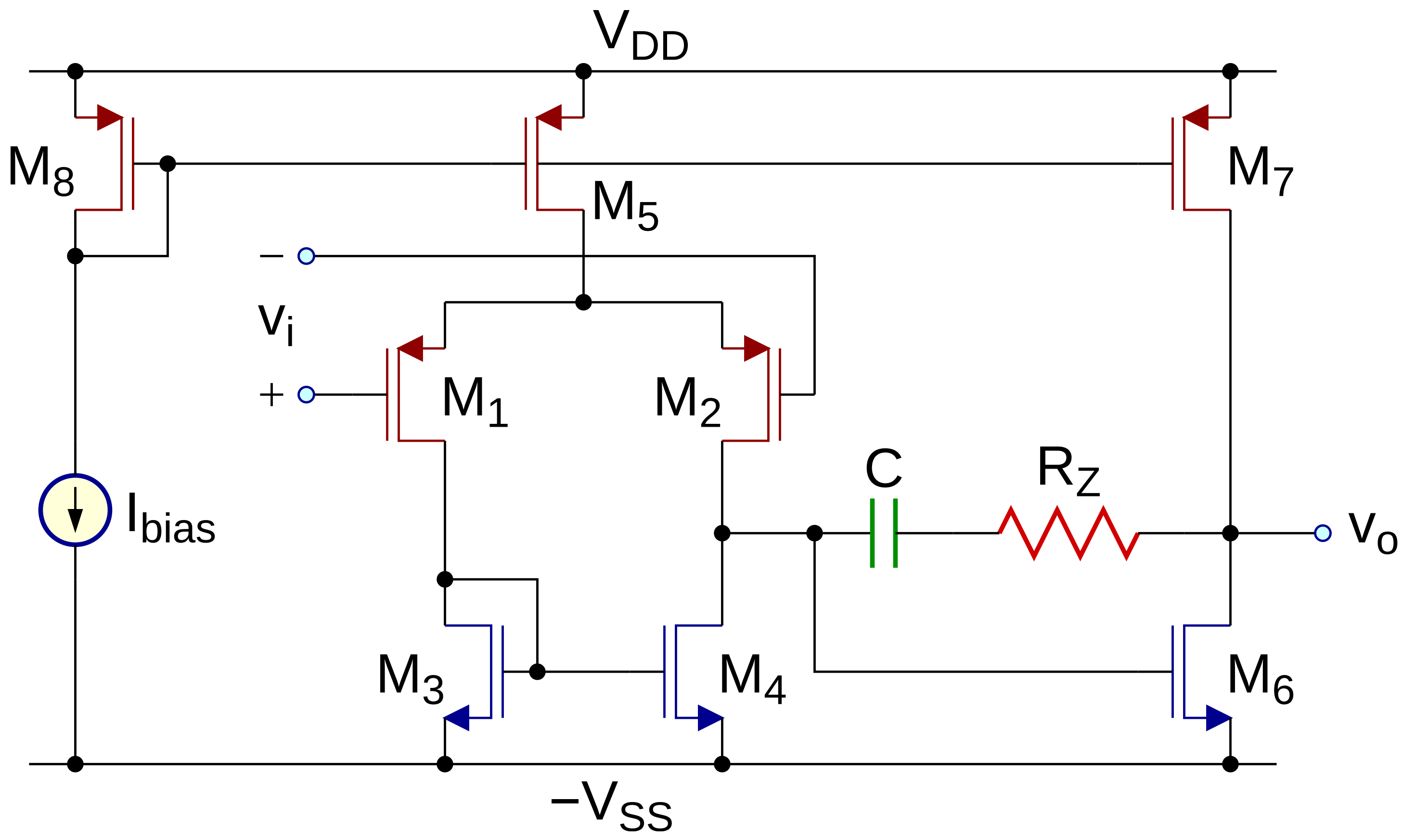
- The zero has been eliminated
- p_1 is unchanged
- p_2 is at a higher frequency
- Extra devices and bias current are required
- Input offset voltage is affected if I_Ds mismatch

Elimination of RHP Zero Effect

Solution #2 - alt.



Elimination of RHP Zero Effect



Elimination of RHP Zero Effect

Assuming $g_{m6}R_1, g_{m6}R_2 \gg 1$ and C is large,

$$p_1 \approx -\frac{1}{g_{m6}R_1R_2C}$$

$$p_2 \approx -\frac{g_{m6}C}{C_1C_2 + C(C_1+C_2)} \approx -\frac{g_{m6}}{C_1+C_2}$$

$$p_3 \approx -\frac{1}{R_ZC_1}$$

$$z = \frac{1}{\left(\frac{1}{g_{m6}} - R_Z\right)C}$$

$$R_Z = \frac{1}{g_{m6}} \quad \Rightarrow \quad z = \infty$$

$$R_Z > \frac{1}{g_{m6}} \quad \Rightarrow \quad \text{LHP zero } (z < 0)$$