Problem 1 (45 points). Minimize your algebra!
The array of devices shown below have been fabricated on a P-substrate and that \( VT=0.7V \) and body effects is negligible (\( \gamma=0 \)). The P-substrate is connected to ground while voltage at the gate terminal is 1.5V, T1 attached to ground, T2 voltage is 0.1V, T3 voltage is 1V and voltage at T4 is 0.2V.

- 15 points: Extract the transistors embedded in the layout and give the dimensions (W, L, source and drain area and perimeter) of each one. Find the region of operation of all transistors and identify their terminals (source, drain, gate).
- 10 points: Provide the expression for the T2 terminal-to-substrate capacitors (all of them). Provide the value of relevant dimensions needed for the computation of these capacitors.
- 10 points: Assuming that only the concentrations and dimensions (A, B and L) are known give the expression of channel-to-T2 resistance. Assume that \( R_c \) (Resistance per square) is known and that the resistance per contact is \( R_c \).
- 10 points: Provide the expression for the transconductance and conductance of the transistor associated with T2 terminal.

\[
\text{Work each transistor } = A_{\mu} \text{ for all transistors } = L_{\mu}.
\]

\[
\text{Area source } = A^2 = A_{\text{source}} \text{ Drain}
\]

\[
\text{Perimeter for Sidewall capacitor } = 2A
\]

\[
G_{\text{source}} = \frac{C_{\text{perim}}}{L_{\text{perim}}}
\]

\[
C_{\text{sidewall}} = \frac{C_{JSW}(LA)}{1 + \frac{0.1}{\phi_j}}
\]

\[
C_{\text{bottom}} = \frac{C_{JSW}(LA)}{1 + \frac{0.1}{\phi_j}}
\]
Transistor is in triode

\[ C_{\text{channel}} = \frac{2}{3} \text{Cox AL} \]

Triode \( \Rightarrow \) \[ C_{\text{D-channel}} = \frac{2}{3} \text{Cox AL} \]

\[ C_{\text{T2-total}} = C_{\text{bottom}} + C_{\text{sidewall}} + \frac{4}{3} \text{Cox AL} \]

c) \[ C_{\text{channel}} - T_2 \]

\[ R_{\text{total}} = R_0 \cdot \frac{B}{A} \]

We have 2 resistors in parallel (upstairs)

\[ R_{\text{channel}} - T_2 \approx R_0 \cdot \frac{B}{2A} + \frac{R_c}{2} \]

d) \[ T_1 \leftrightarrow T_2 \]

\[ V_{gs} = 1.5 \]
\[ V_{ps} = 0.1 \]

\[ \delta \rho = \frac{\partial \rho}{\partial V_{gs}} \bigg|_0 = \mu n \text{Cox} \frac{W}{L} (V_{gs} - V_T - V_{ps}) \]

\[ \delta \eta = \frac{\partial \eta}{\partial V_{gs}} \bigg|_0 = \mu n \text{Cox} \frac{W}{L} V_{gs} \]

\[ T_2 \leftrightarrow T_4 \]

\[ \delta \rho = \mu n \text{Cox} \frac{W}{L} (V_{gs} - V_T - V_{ps}) \]
\[ \delta \eta = \mu n \text{Cox} \frac{W}{L} V_{ps} \]

\[ V_{gs} = 1.4 \]
\[ V_{ps} = 0.1 \]
Consider the small signal model \( i_d = \frac{\mu_o C_{OX}}{2(1 + \theta(V_{gs} - V_T))} \left( \frac{W}{L} \right) (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) \): does not neglect the mobility degradation terms; assume that \( V_T, L, \theta \) and \( V_{CRIT} \) are voltage invariant parameters.

- 10 points. Find the expression for the small signal transconductance.
- 10 points. Find the expression for the small signal conductance.
- 10 points. In case \( \theta \) is not provided by the silicon foundry, discuss a practical method for extracting this parameter. Assume that you can use Cadence.

\[
\begin{align*}
\mathbf{a)} & \quad g_m &= \frac{\partial i_d}{\partial V_{gs}} \bigg|_{Q} = \frac{\partial}{\partial V_{gs}} \left[ \frac{\mu_o C_{OX}}{2(L)} \left( \frac{W}{L} \right) (V_{gs} - V_T)^2 \right] \\
& &= A \left[ \frac{2(V_{gs} - V_T)}{1 + \theta(V_{gs} - V_T)} - \frac{(V_{gs} - V_T)^2}{(1 + \theta(V_{gs} - V_T))^2} \right] \\
& &= A \left[ \frac{2(V_{gs} - V_T)}{1 + \theta(V_{gs} - V_T)} \right] \theta \\
\mathbf{b)} & \quad g_o &= \frac{\partial}{\partial V_{gs}} \left[ \frac{\mu_o C_{OX}}{2(L)} \left( \frac{W}{L} \right) (V_{gs} - V_T)^2 \right] \\
& &= B \left[ \frac{1 + \lambda V_{ds}}{1 + \frac{V_{ds}}{V_{CRIT}}} - \frac{(1 + \lambda V_{ds})}{V_{CRIT}} \right] \\
& &= B \left[ \frac{1 + \lambda V_{ds}}{1 + \frac{V_{ds}}{V_{CRIT}}} \right] \theta \\
\end{align*}
\]

(a) 
(b) 

\text{Ratio of extrapolation and Cadence} = 1 + \theta (V_{gs} - V_T) 

Pick a proper \( V_{gs} - V_T \) and compute \( \theta \).
Problem 3 (30 points) **Minimize your algebra!**

Assume that the two transistors shown below are identical and are biased with same voltages except the substrate of the second transistor. Assume that $V_T=0.5\text{V}$ and $\lambda=0\text{ V}^{-1}$, $\gamma=0.1\text{V}^{1/2}$ and $\Phi_F=0.25\text{V}$.

- 10 points. Find the expression of the current difference between the two transistors in $\%$.
- 10 points. Find the expression of the transconductance difference between the two transistors in $\%$.
- 10 points. If $\lambda \neq 0$ find the expression of the conductance difference between the two transistors in $\%$.

**Hint:** Difference in percentage (normalized) of two parameters $x_1$ and $x_2$ should be defined as $(x_1-x_2)/x_1$.

\[
\frac{I_{\text{diff}}}{I_{\text{Total}}} = \frac{(V_{GS} - U_{T1})^2}{(V_{GS} - U_{T2})^2} = \frac{(V_{GS} - U_{TO}) - (\sqrt{2}\Phi_F) - (\sqrt{2}\Phi_F)}{(V_{GS} - U_{TO} - \lambda) \left( \sqrt{2}\Phi_F + 0.5 - \sqrt{2}\Phi_F \right)}
\]

\[
= \frac{1}{\left(1 - \frac{\sqrt{2}\Phi_F + 0.5 - \sqrt{2}\Phi_F}{V_{GS} - U_{TO}}\right)^2}
\]

Current in percentage $\approx -2\ \%\ \frac{\sqrt{2}\Phi_F + 0.5 - \sqrt{2}\Phi_F}{V_{GS} - U_{TO}}$

\[
\frac{\delta m_1}{m_1} = \mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{TO})
\]

\[
\frac{\delta m_2}{m_2} = \mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{TO} - \lambda \left( \sqrt{2}\Phi_F + 0.5 - \sqrt{2}\Phi_F \right))
\]

\[
\frac{\delta m_2}{m_2} = \frac{1 - \lambda \left( \sqrt{2}\Phi_F + 0.5 - \sqrt{2}\Phi_F \right)}{V_{GS} - U_{TO}}
\]

\[
\frac{\delta I_{DS2}}{\delta I_{DS1}} = \frac{\mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{T1}) \lambda_2}{\mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{TO}) \lambda_1} = \left( \frac{V_{GS} - U_{T1}}{V_{GS} - U_{TO}} \right) \frac{\lambda_2}{\lambda_1}
\]

\[
\frac{\delta I_{DS2}}{\delta I_{DS1}} = \mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{T1}) \lambda_1
\]

\[
\frac{\delta I_{DS2}}{\delta I_{DS1}} = \mu_n \cos \frac{U_{T1}}{L} (V_{GS} - U_{TO}) \lambda_1
\]