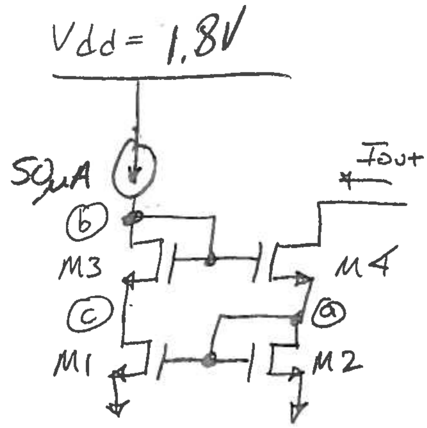
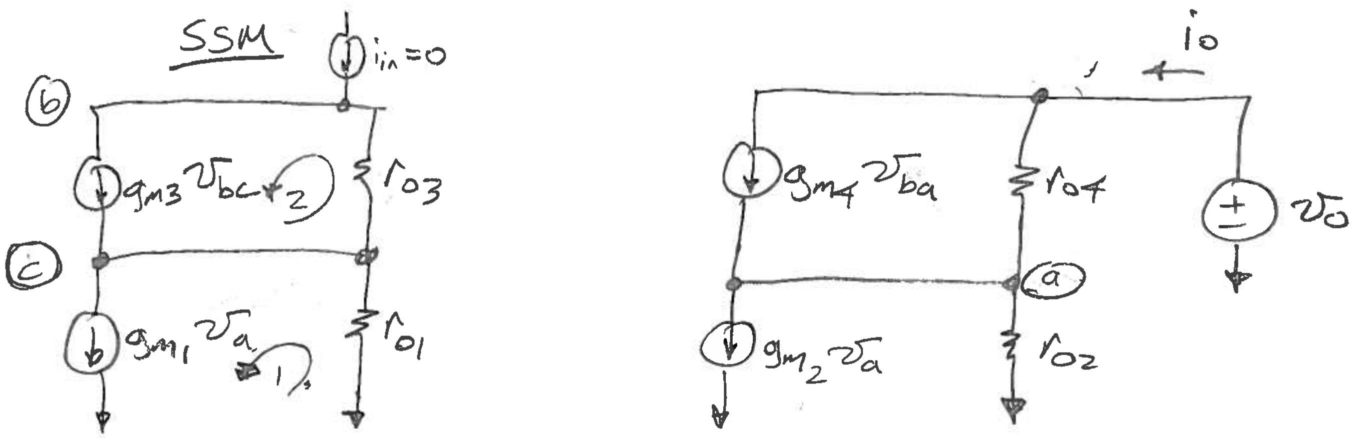
ECEN 474 HW #3 Solutions

1. Wilson Current Mirror



a. Low Frequency Output Impedance Derivation



Since $i_{in}=0 \overset{KVL\,①}{\Longrightarrow} v_c = -g_{m1}v_a r_{o1}$ ①

KVL ② $v_b = v_c - g_{m3}(v_b - v_c) r_{o3}$

$$(v_b - v_c)\left[1 + g_{m3}r_{o3}\right] = 0$$

$$\boxed{v_b = v_c}$$

③ KCL at v_o $\quad -i_o + g_{m4}(v_b - v_a) + (v_o - v_a)g_{o4} = 0$

$$-i_o + g_{m4}\left[-g_{m1}v_a r_{o1} - v_a\right] + (v_o - v_a)g_{o4} = 0$$

④ KCL at v_a: $-i_o + g_{m2} v_a + v_a g_{o2} = 0$

$$v_a = \frac{i_o}{g_{m2} + g_{o2}}$$

Plugging this into the results from ③

$$-i_o + g_{m4}\left[\frac{-g_{m1} r_{o1} i_o}{g_{m2}+g_{o2}} - \frac{i_o}{g_{m2}+g_{o2}}\right] + \left(v_o - \frac{i_o}{g_{m2}+g_{o2}}\right) g_{o4} = 0$$

$$i_o\left[1 + \frac{g_{m4} g_{m1} r_{o1}}{g_{m2}+g_{o2}} + \frac{g_{m4}}{g_{m2}+g_{o2}} + \frac{g_{o4}}{g_{m2}+g_{o2}}\right] = v_o g_{o4}$$

$$r_o = \frac{v_o}{i_o} = r_{o4}\left[1 + \frac{g_{m4} g_{m1} r_{o1}}{g_{m2}+g_{o2}} + \frac{g_{m4}}{g_{m2}+g_{o2}} + \frac{g_{o4}}{g_{m2}+g_{o2}}\right]$$

$$\simeq r_{o4} + \frac{g_{m4} g_{m1} r_{o1} r_{o4}}{g_{m2}} + \frac{g_{m4} r_{o4}}{g_{m2}} + \frac{1}{g_{m2}}$$

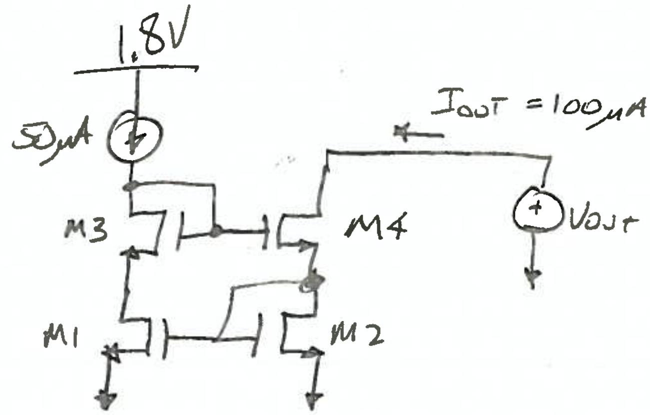
$$= r_{o4} + r_{o4} \frac{g_{m4}}{g_{m2}}\left(g_{m1} r_{o1} + 1\right) + \frac{1}{g_{m2}}$$

$$r_o \simeq g_{m4}(1 + A_v)\left(\frac{1}{g_{m2}}\right) r_{o4}$$

where $A_v = g_{m1} r_{o1}$

b. Design 1:2 current Mirror w/ $I_{in} = 50\mu A$

$R_{OUT} \geq 2M\Omega$ for $V_{out} \geq 0.7V$



From Problem 1a, we derived that

$$R_{OUT} \simeq g_{m4}(1 + g_{m1} r_{o1})\left(\frac{1}{g_{m2}}\right) r_{o4}$$

If $\frac{W}{L_2} = \frac{W}{L_4}$, because they have the same I_D and neglecting body effect, $g_{m4} \doteq g_{m2}$

Thus, $R_{out} \simeq (1 + g_{m1} r_{o1}) r_{o4} \simeq g_{m1} r_{o1} r_{o4}$

In order to achieve the $2M\Omega$ r_{out} spec, we need to insure that all output transistors are saturated.

$$\Longrightarrow V_{TH2} + V_{DSAT2} + V_{DSAT4} \leq 0.7V$$

Here I would use $L = 0.72\mu m$ for lower V_{TH} and higher r_{out}
Assuming $V_{TH} \simeq 0.4V \Rightarrow V_{DSAT2,4} \leq 150mV$ is appropriate

* The following design plots are from Shunlong Xiao who used

$$\left(\frac{W}{L}\right)_{2,4} = 40 \text{ fingers of } \frac{3.6\mu}{0.72\mu}$$

$$\left(\frac{W}{L}\right)_{1,2} = 20 \text{ fingers of } \frac{3.6\mu}{0.72\mu}$$

Problem1 Plots

1. As Fig. 1, the transistors' sizes, bias current are clearly labeled. DC working points are present on the table 1.

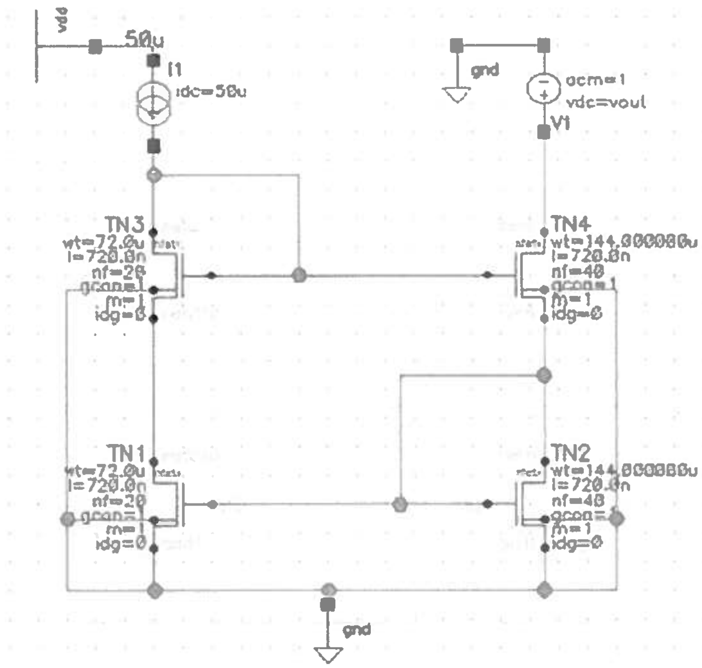


Table.1

signal	OP("/V1" "??")
i	-99.97u
pwr	-69.98u
v	700m
signal	OP("/I1" "??")
i	50u
pwr	-2.5K
v	-50M

2. As Fig. 2, the plot of output current vs sweeping output voltage is shown;

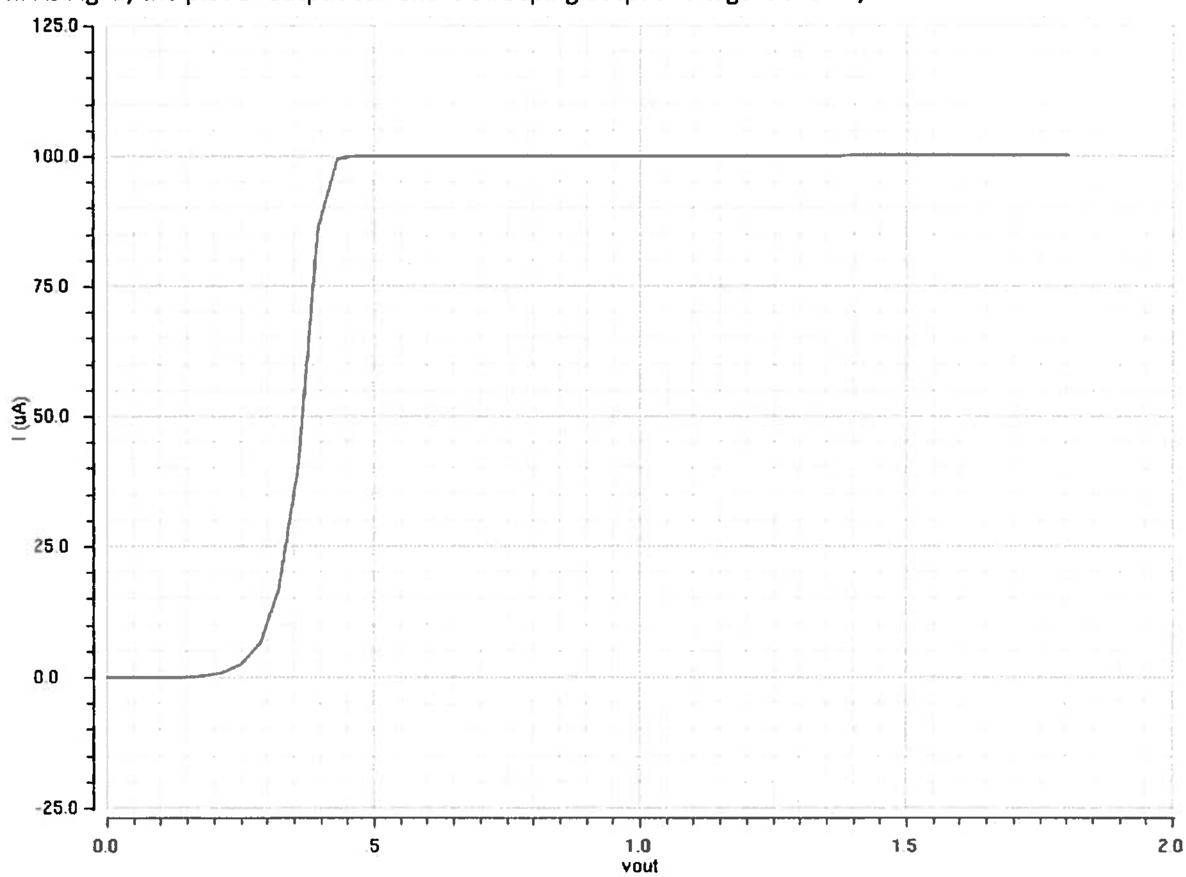


Fig. 2

3. The low frequency output impedance dependency on Vout is shown as Fig. 3. It can be seen that the output impedance is greater than 3.37M Ω for output voltage greater than 0.7V.

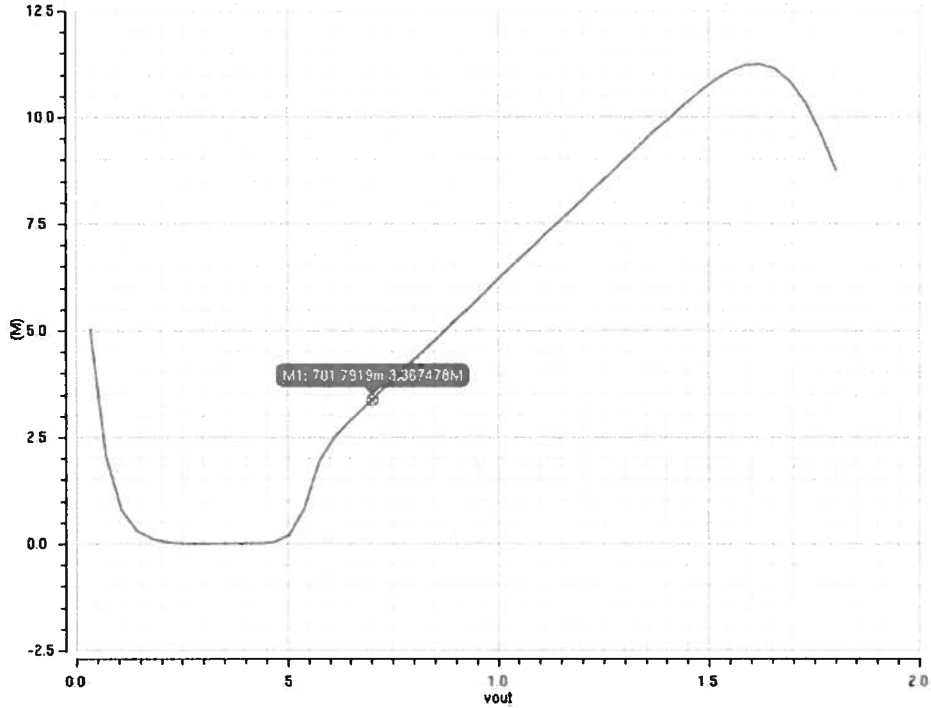


Fig. 3

4. The AC frequency response of the output impedance from 10Hz~100MHz with Vout=1V is as Fig. 4. The dB20 plot indicates pole location: 139.675kHz. The zero [redacted] are around 47MHz.

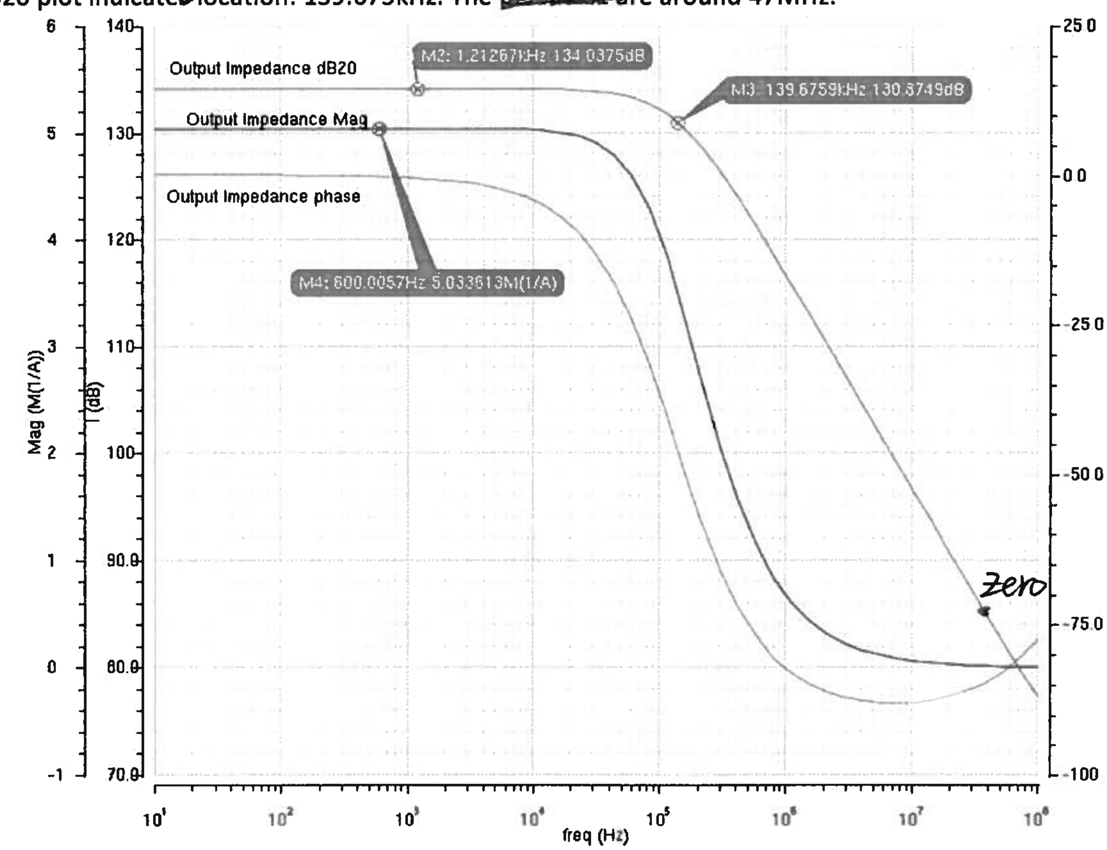


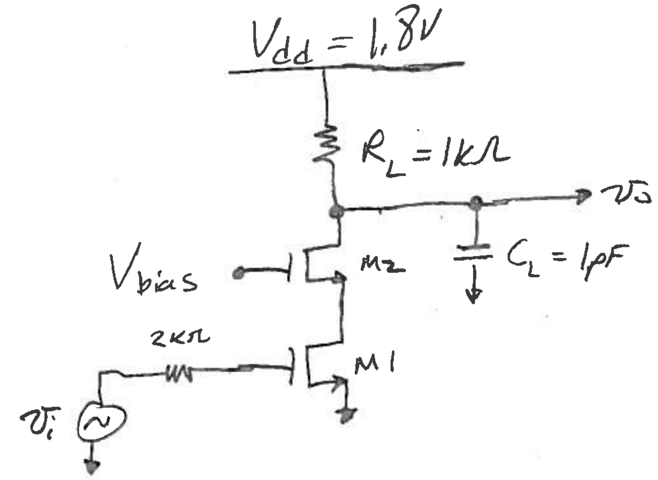
Fig. 4

6. Small-signal device parameters for N1, N2, N3, N4 at Vout=0.7V.

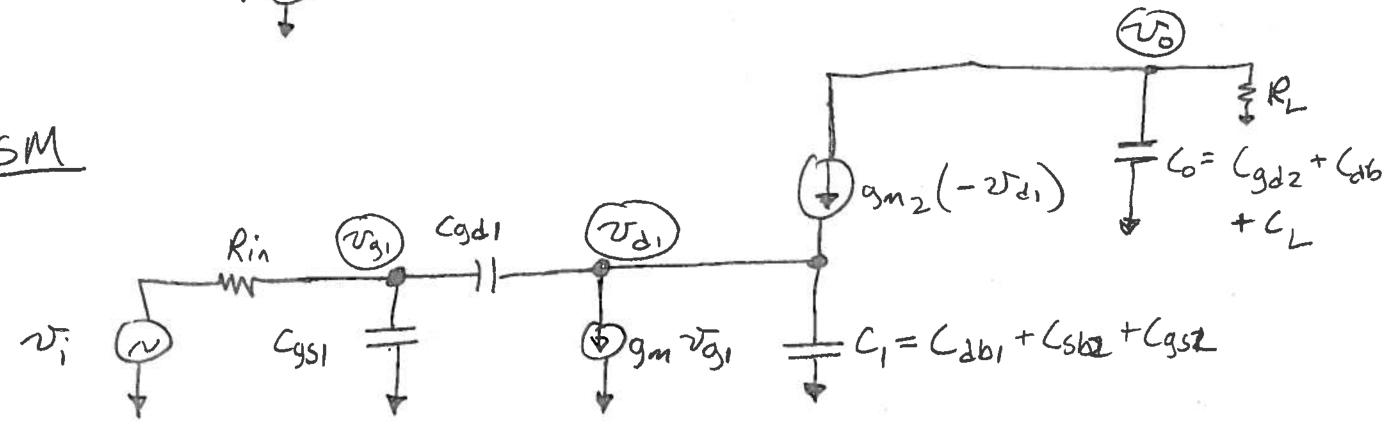
Signal/Transistor	N1	N2	N3	N4
betaeff	8.647m	17.28m	8.934m	17.87m
cbb	720.4f	1.494p	634.8f	1.27p
cbd	-198.4f	-458.1f	-226.2f	-452.9f
cbdbi	-410.6a	-15.37f	-176.8a	-355.2a
cbg	-111f	-200.3f	-139.8f	-279.6f
cbs	-411f	-835.5f	-268.8f	-537.7f
cbsbi	-185f	-383f	-268.8f	-537.7f
cdb	-257.9f	-578.4f	-318.5f	-637.7f
cdd	245.4f	577.6f	274f	548.6f
cddbi	-881.5a	38.34f	-202.9a	-402.1a
cdg	-287.9f	-632f	-284.6f	-569.2f
cds	300.4f	632.8f	329.1f	658.2f
cgb	-146.7f	-274.1f	-177.6f	-355.2f
cgbovl	0	0	0	0
cgd	-45.61f	-129.8f	-47.49f	-94.99f
cgdbi	2.648f	-33.3f	765.1a	1.527f
cgdovl	48.26f	96.52f	48.26f	96.52f
cgg	806.3f	1.674p	826.8f	1.654p
cggbi	709.8f	1.481p	730.3f	1.461p
cgs	-614f	-1.27p	-601.8f	-1.204p
cgsbi	-565.7f	-1.174p	-553.5f	-1.107p
cgsovl	48.26f	96.52f	48.26f	96.52f
cjd	198f	442.7f	226f	452.5f
cjs	226f	452.5f	0	0
csb	-315.7f	-641.5f	-138.7f	-277.4f
csd	-1.356f	10.33f	-385.3a	-769.6a
csg	-407.5f	-842f	-402.4f	-804.8f
css	724.6f	1.473p	541.5f	1.083p
cssbi	450.3f	924.1f	493.2f	986.4f
gbd	0	0	0	0
gbs	0	0	0	0
gds	4.389u	119.5u	2.618u	5.241u
gm	677.7u	1.275m	662.1u	1.324m
gmbs	167u	319.3u	247.4u	494.6u
gmoverid	13.55	12.75	13.24	13.24
i1	50u	99.96u	50u	99.96u
i3	-50u	-99.96u	-50u	-99.96u
i4	-520f	-1.04p	-260f	-520f
ibd	-260f	-520f	-260f	-520f
ibs	-260f	-520f	0	0
ibulk	-520f	-1.04p	-260f	-520f

id	50u	99.96u	50u	99.96u
ids	50u	99.96u	50u	99.96u
igb	0	0	0	0
igcd	0	0	0	0
igcs	0	0	0	0
igd	0	0	0	0
igidl	0	0	0	0
igisl	0	0	0	0
igs	0	0	0	0
is	-50u	-99.96u	-50u	-99.96u
isub	0	0	0	0
pwr	55.93u	14.05u	43.36u	85.91u
qb	-1.107p	-2.209p	-776.9f	-1.554p
qbd	-451.9f	-492.7f	-216.7f	-429.8f
qbi	-1.107p	-2.209p	-776.9f	-1.554p
qbs	-216.7f	-429.8f	0	0
qd	-22.55f	-145.8f	-24.04f	-48.83f
qdi	-22.55f	-50.7f	-24.42f	-48.83f
qg	1.217p	2.536p	879f	1.759p
qgi	1.163p	2.332p	837.9f	1.676p
qinv	613.4u	1.523m	655.1u	1.31m
qsi	-33.81f	-72.84f	-36.59f	-73.18f
qsrco	-87.79f	-181.6f	-78.07f	-156.1f
region	2	2	2	2
reversed	0	0	0	0
ron	22.37K	1.406K	17.35K	8.598K
type	0	0	0	0
vbs	-867.3m	-859.4m	0	0
vdb	1.986	1	867.3m	859.4m
vds	1.119	140.6m	867.3m	859.4m
vdsat	119.9m	123.8m	113.3m	113.3m
vfbeff	-900.9m	-896.6m	-843.5m	-843.5m
vgb	1.986	1.986	859.4m	859.4m
vgd	0	985.8m	-7.829m	0
vgs	1.119	1.126	859.4m	859.4m
vgsteff	92.84m	97.64m	95.67m	95.67m
vth	1.04	1.041	774.8m	774.8m

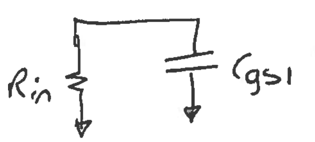
2. Open-Circuit Time Constants Bandwidth Estimation



SSM



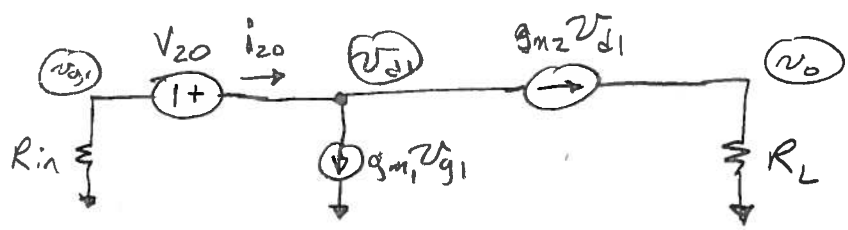
Cgs1



$$R_{10} = R_{in} = 2k\Omega$$

$$\tau_{10} = R_{in} C_{gs1} = (2k\Omega)(220fF) = 440ps$$

Cgd1



KCL at v_{d1}:
$$-i_{20} + g_{m1}(-i_{20}R_{in}) + g_{m2}\left[-i_{20}R_{in} + V_{20}\right] = 0$$

$$i_{20}\left[1 + g_{m1}R_{in} + g_{m2}R_{in}\right] = g_{m2}V_{20}$$

$$R_{20} = \frac{V_{20}}{i_{20}} = R_{in}\left(1 + \frac{g_{m1}}{g_{m2}}\right) + \frac{1}{g_{m2}}$$

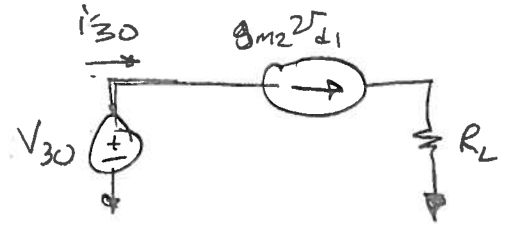
$$\tau_{20} = C_{gd1} R_{20}$$

2a

$$R_{20} = 2K\Omega\left(1 + \frac{12mA/V}{12mA/V}\right) + \frac{1}{12^{mA/V}} = 4.08K\Omega$$

$$\tau_{20} = 4.08K\Omega(45fF) = 184ps$$

C1



$$V_{d1} = V_{30}$$

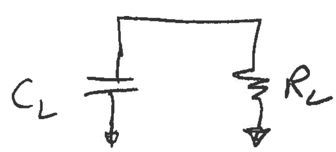
$$i_{30} = g_{m2}V_{30}$$

$$R_{30} = \frac{1}{g_{m2}} = \frac{1}{12^{mA/V}} = 83\Omega$$

$$C1 = C_{db1} + C_{sb1} + C_{gs1} = 90fF + 130fF + 220fF$$
$$= 440fF$$

$$\tau_{30} = (83\Omega)(440fF) = 37ps$$

CO



$$R_{40} = R_L = 1K\Omega$$

$$CO = C_{gd2} + C_{db2} + C_L = 45fF + 90fF + 1pF$$
$$= 1.135pF$$

$$\tau_{40} = (1K\Omega)(1.135pF) = 1.135ns$$

$$\Sigma OCT = 440ps + 184ps + 37ps + 1.135ns = 1.796ns$$

$$BW \simeq \frac{1}{\Sigma OCT} = 557 Mrad/s = 89 MHz$$