ECEN474/704: (Analog) VLSI Circuit Design
Spring 2018

Lecture 8: Frequency Response

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Announcements & Agenda

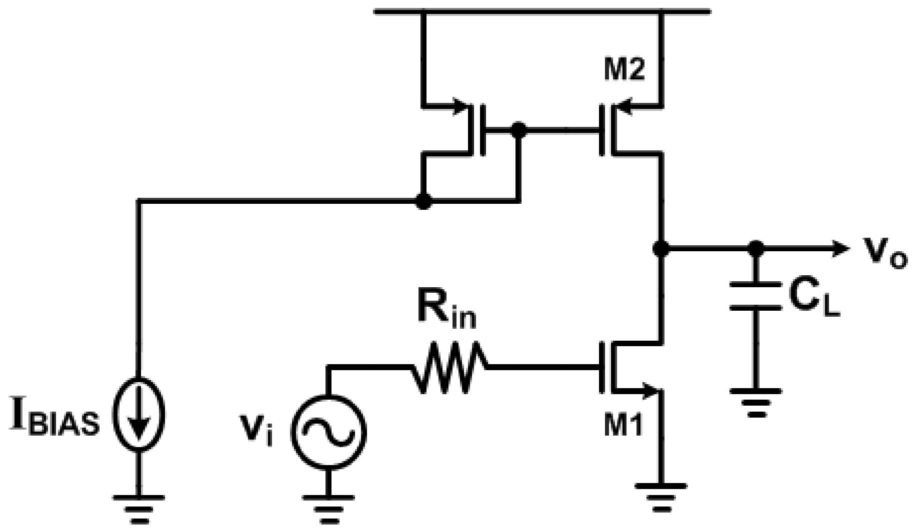
- HW2 Due Mar 6

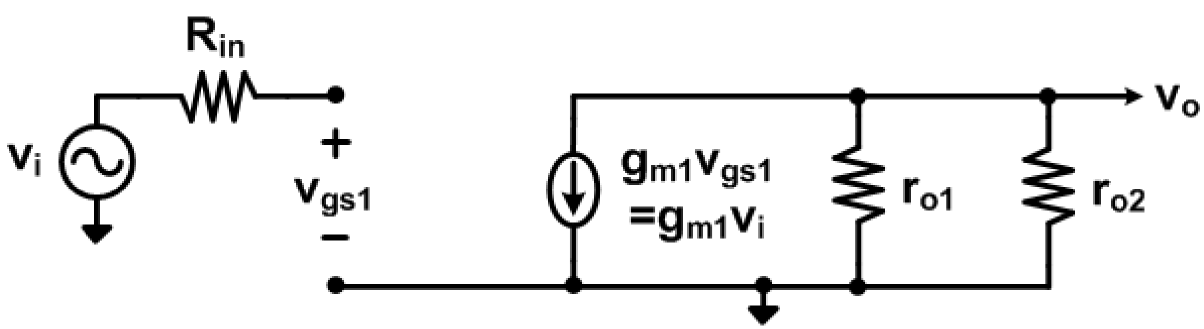
- Reading
  - Razavi Chapters 3 & 6

Announcements & Agenda

- Common-Source Amp Frequency Response
- Open-Circuit Time Constants (OCτ) Bandwidth Estimation Technique
- Common-Drain Amp Frequency Response
- Common-Gate Amp Frequency Response
- Cascode Amp Frequency Response

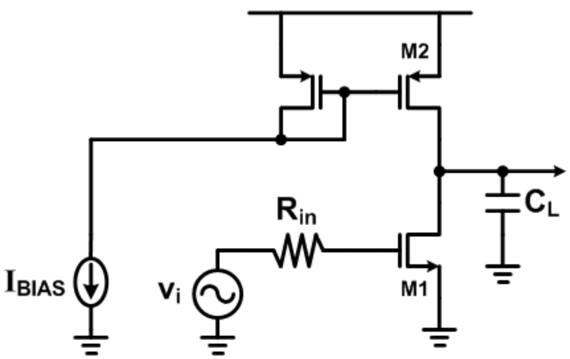
Common-Source Amplifier: Low Frequency Response



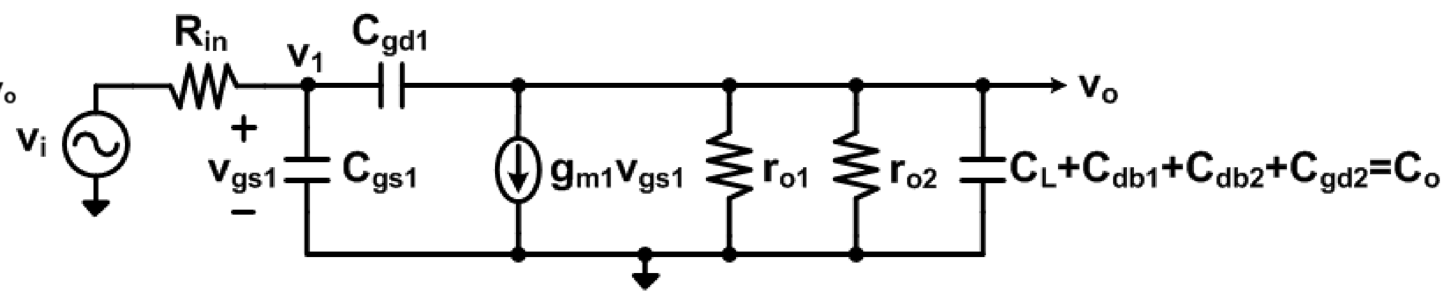


$$\frac{v_o}{v_i} = -\frac{g_{m1}}{g_{o1} + g_{o2}}$$

Common-Source Amplifier: High Frequency Response



Small-Signal Model (Assuming V_{G2} is AC gnd)



$$\textbf{KCL @ Node } \mathbf{v_1}: (v_1 - v_i)G_{in} + v_1 sC_{gs1} + (v_1 - v_o)sC_{gd1} = 0$$

$$\textbf{KCL @ Node } \mathbf{v_o}: (v_o - v_1)sC_{gd1} + g_{m1}v_1 + v_o(g_o + sC_o) = 0$$

$$\textbf{where } g_o = g_{o1} + g_{o2}$$

After some algebra, we get the exact transfer function:

$$\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\dfrac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

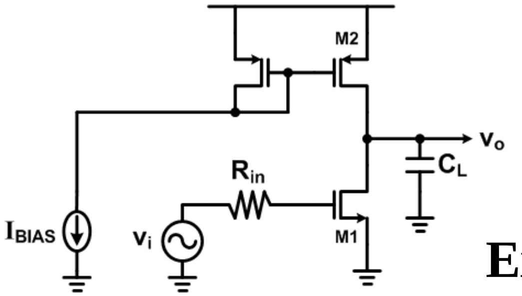
where

$$a = R_{in}\left[C_{gs1} + C_{gd1}(1 + g_m r_o)\right] + r_o\left(C_{gd1} + C_o\right)$$

and

$$b = R_{in} r_o\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

Common-Source Amp Frequency Response



Exact Transfer Function : $\dfrac{v_o}{v_i} = \dfrac{-g_m r_o\left(1 - s\dfrac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$

For the common case when the two poles are real and far apart

Denominator $D(s) = \left(1 - \dfrac{s}{\omega_{p1}}\right)\left(1 - \dfrac{s}{\omega_{p2}}\right) = 1 - s\left(\dfrac{1}{\omega_{p1}} + \dfrac{1}{\omega_{p2}}\right) + \dfrac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \dfrac{s}{\omega_{p1}} + \dfrac{s^2}{\omega_{p1}\omega_{p2}}$

Thus, $\omega_{p1} = -\dfrac{1}{a} = -\dfrac{1}{R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right] + r_o\left(C_{gd1} + C_o\right)}$

and the transfer function can be approximated as a single pole system

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1}r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right] + r_o\left(C_{gd1} + C_o\right)\right)}$$

Open-Circuit Time Constants (OCτ)

- Open-circuit time constants technique can be used to estimate bandwidth
  - Much easier than deriving transfer function
  - Accurate for systems with one dominant pole

$$\text{All-Pole Transfer Function: } \frac{v_o(s)}{v_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1)\ldots(\tau_n s + 1)}$$

$$\text{Denominator: } b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1$$

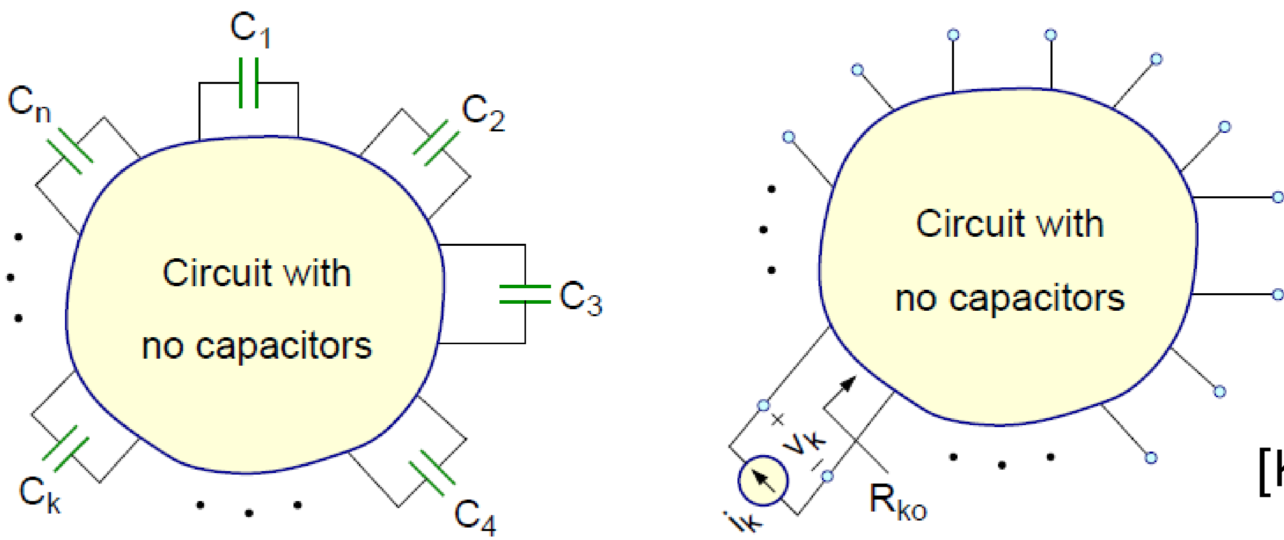
$$\text{Here } \quad b_n = \prod_{i=1}^{n} \tau_i \quad \text{and} \quad b_1 = \sum_{i=1}^{n} \tau_i$$

A Dominant-Pole System can be approximated as

$$\frac{v_o(s)}{v_i(s)} \cong \frac{a_0}{b_1 s + 1} = \frac{a_0}{\left(\sum_{i=1}^{n} \tau_i\right) s + 1}$$

$$\textbf{Bandwidth} \quad \omega_h \cong \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \omega_{h,est}$$

Open-Circuit Time Constants (OCτ)

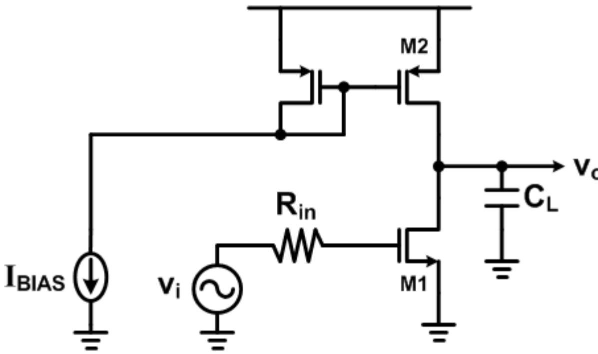


[Karsilayan]

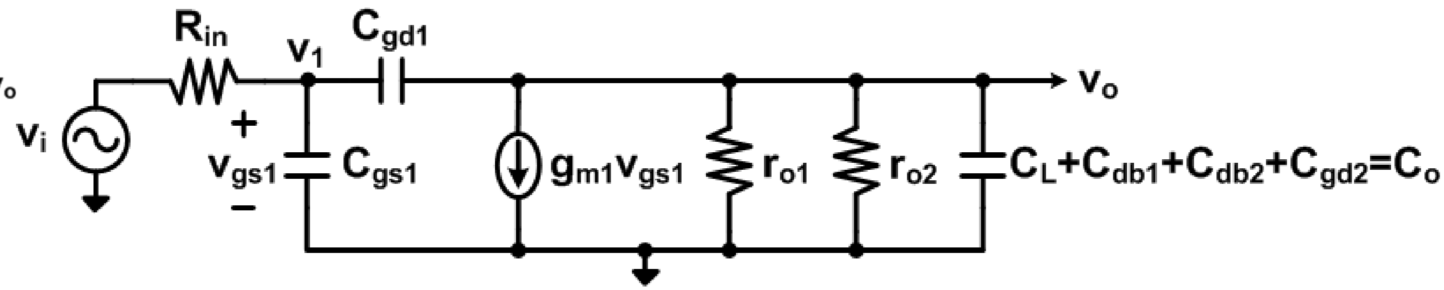
- To compute time-constants
  1. Compute effective resistance R_{ko} facing each kth capacitor with all other caps open-circuited
  2. Form the product $\tau_{ko}=R_{ko}C_k$
  3. Sum all n "open-circuit" time constants

$$\omega_{h,est} = \frac{1}{\sum_{k=1}^{n} R_{ko} C_k}$$

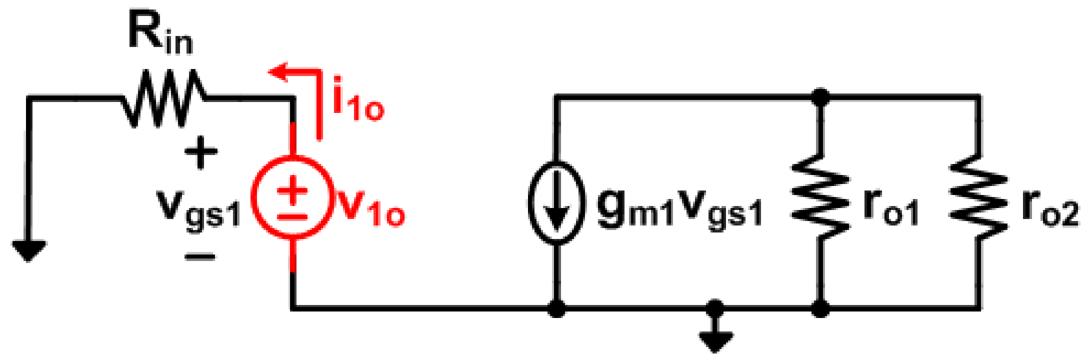
Common-Source Amp w/ OCτ



Small-Signal Model (Assuming V_{G2} is AC gnd)



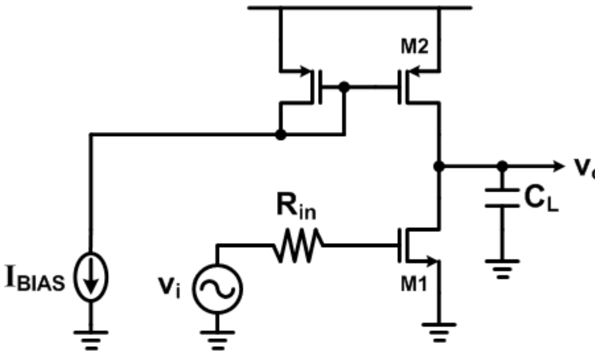
- For C_{gs1}



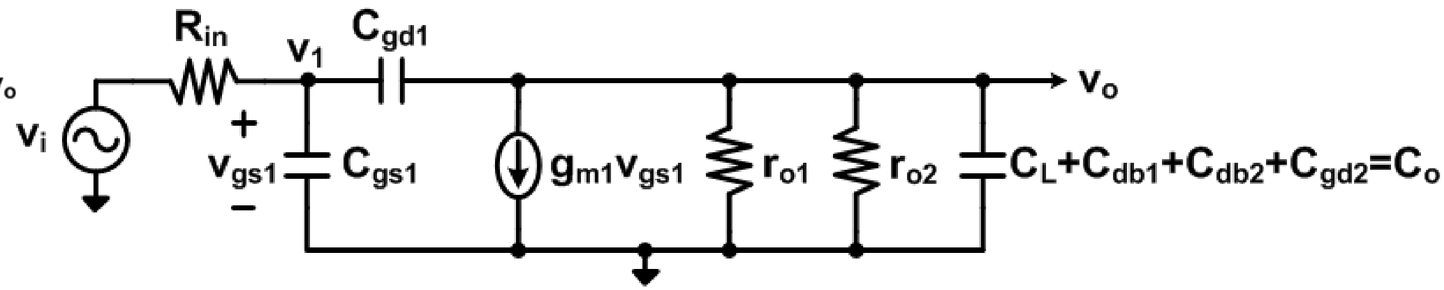
$$R_{1o} = \frac{v_{1o}}{i_{1o}} = \frac{v_{1o}}{\left(\frac{v_{1o}}{R_{in}}\right)} = R_{in}$$

$$\tau_{1o} = R_{in} C_{gs1}$$

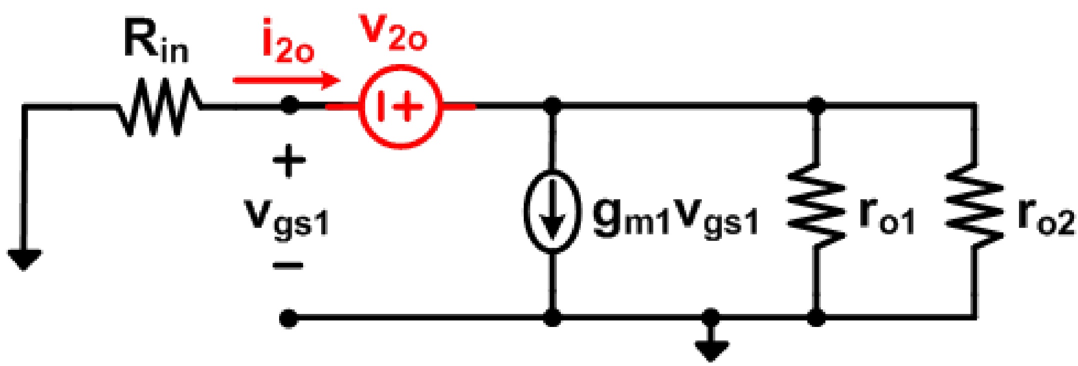
Common-Source Amp w/ OCτ



Small-Signal Model (Assuming V_{G2} is AC gnd)



- For C_{gd1}



(1) $i_{2o} = g_m v_{gs1} + \frac{(v_{2o} + v_{gs1})}{r_o}$

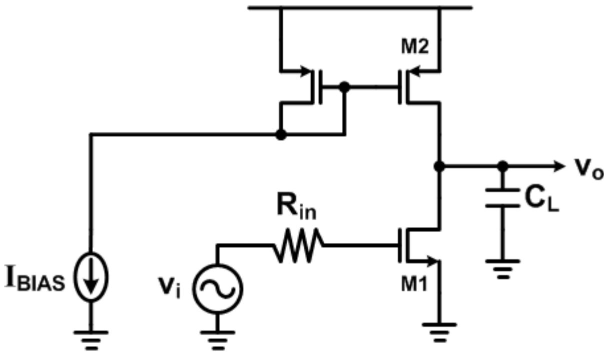
(2) $v_{gs1} = -i_{2o} R_{in}$

Plugging (2) into (1) and solving for $\frac{v_{2o}}{i_{2o}}$

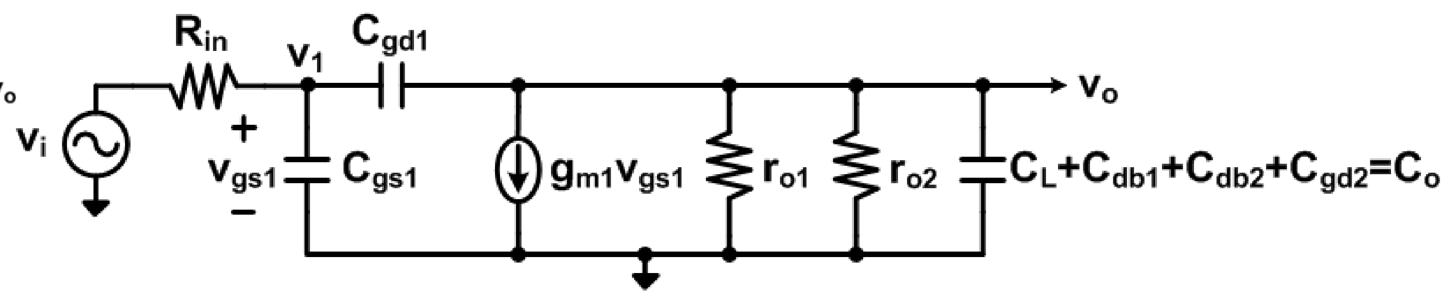
$$R_{2o} = \frac{v_{2o}}{i_{2o}} = R_{in}(1 + g_{m1} r_o) + r_o$$

$$\tau_{2o} = (R_{in}(1 + g_{m1} r_o) + r_o) C_{gd1}$$

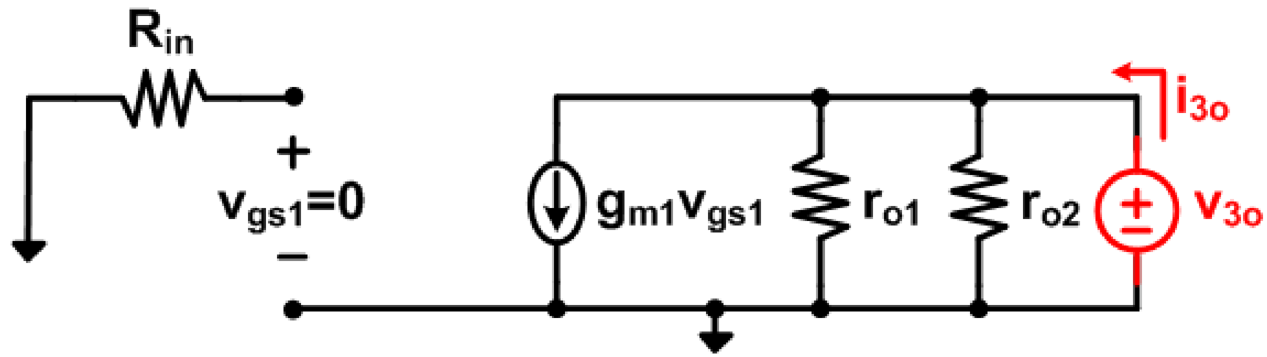
Common-Source Amp w/ OCτ



Small-Signal Model (Assuming V_{G2} is AC gnd)



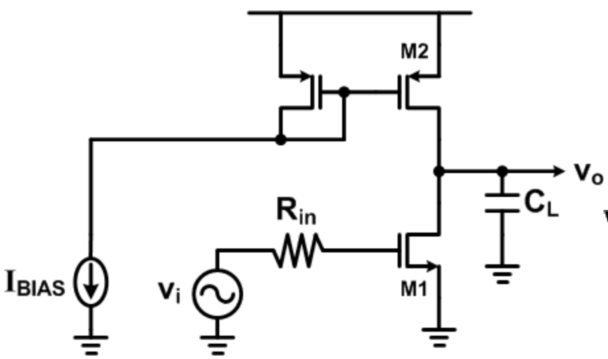
- For C_o



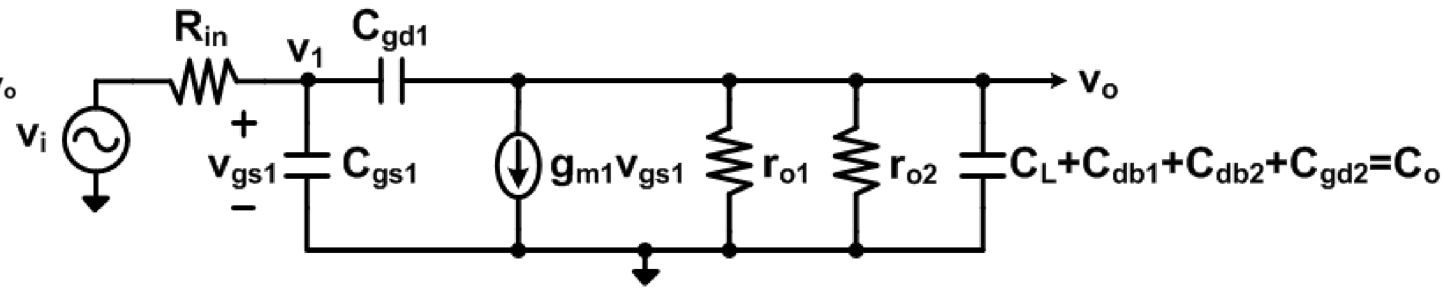
$$R_{3o} = \frac{v_{3o}}{i_{3o}} = \frac{v_{3o}}{\left(\frac{v_{3o}}{r_o}\right)} = r_o$$

$$\tau_{3o} = r_o C_o$$

Common-Source Amp w/ OCτ



Small-Signal Model (Assuming V_{G2} is AC gnd)



3 Time Constants : $\tau_{1o} = R_{in}C_{gs1},\ \tau_{2o} = \left(R_{in}\left(1+g_m r_o\right)+r_o\right)C_{gd1},\ \tau_{3o} = r_o C_o$

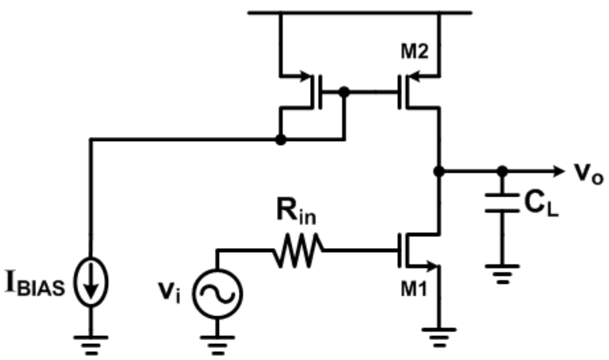
$$b_1 = \sum_{i=1}^{n} \tau_i = R_{in}C_{gs1} + \left(R_{in}\left(1+g_m r_o\right)+r_o\right)C_{gd1} + r_o C_o$$

$$\omega_{h,est} = \frac{1}{b_1} = \frac{1}{R_{in}C_{gs1} + \left(R_{in}\left(1+g_m r_o\right)+r_o\right)C_{gd1} + r_o C_o}$$

Exactly the same as what we derived in Slide 6!

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1}r_o}{1+s\left(R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]+r_o\left(C_{gd1}+C_o\right)\right)}$$

Common-Source Amp w/ Large R_{in}



- Example: Using common-source output stage in a 2-stage OpAmp

$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+s\left(R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]+r_o\left(C_{gd1}+C_o\right)\right)}$$

with $R_{in} >> r_o$

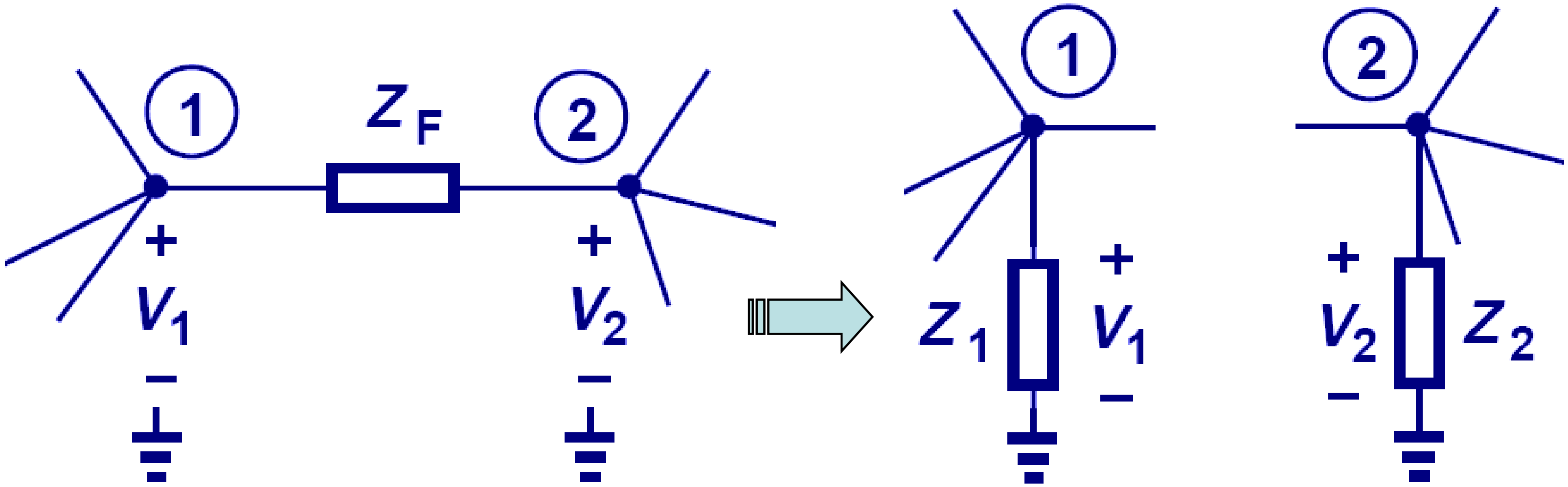
$$A(s)=\frac{v_o}{v_i}\cong\frac{-g_{m1}r_o}{1+sR_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]}$$

$$\omega_{p1}=-\frac{1}{R_{in}\left[C_{gs1}+C_{gd1}\left(1+g_{m1}r_o\right)\right]}$$

- Dominant pole is formed by input resistance times transistor C_{gs} and Cgd which has been multiplied by $1\text{-}A_{dc}$
  - $C_{gd}(1\text{-}A_{dc})$ is called the Miller capacitance

Miller's Theorem

- If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.



I_1 should be the same in both circuits

$$I_1 = \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$Z_1 = \left(\frac{V_1}{V_1 - V_2}\right) Z_F = \left(\frac{1}{1 - \frac{V_2}{V_1}}\right) Z_F = \frac{Z_F}{1 - A_v}$$

where $A_v = \frac{V_2}{V_1}$

$$Z_1 = \frac{Z_F}{1 - A_v}$$

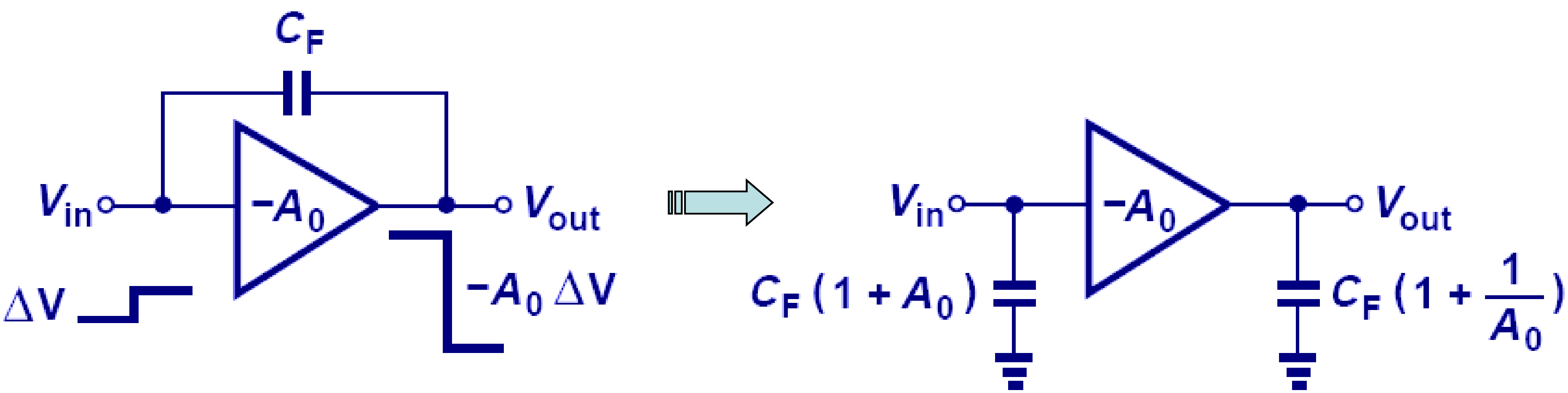
$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

I_2 should be the same in both circuits

$$I_2 = \frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2}$$

$$Z_2 = \left(\frac{V_2}{V_2 - V_1}\right) Z_F = \left(\frac{1}{1 - \frac{V_1}{V_2}}\right) Z_F = \frac{Z_F}{1 - \frac{1}{A_v}}$$

Miller Multiplication



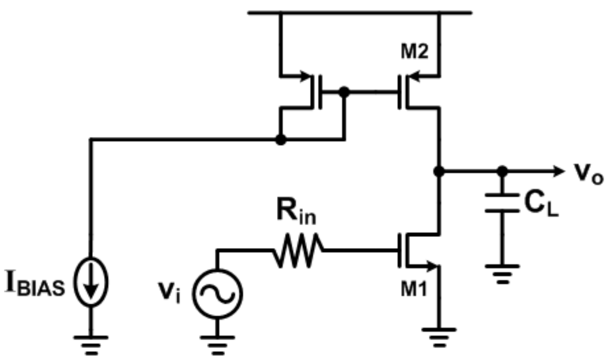
$$Z_{in} = \frac{1}{j\omega C_F(1-A_v)} = \frac{1}{j\omega C_F(1-(-A_o))} = \frac{1}{j\omega C_F(1+A_o)}$$

Equivalent to an input cap that is the original C_F multiplied by $(1+A_o)$

Following a similar procedure, the output cap is the original C_F multiplied by $\left(1+\frac{1}{A_o}\right)$

- **With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.**

Common-Source Amp w/ Large R_{in}



- What about the second pole?

Exact Transfer Function : $$\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

Denominator $$D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

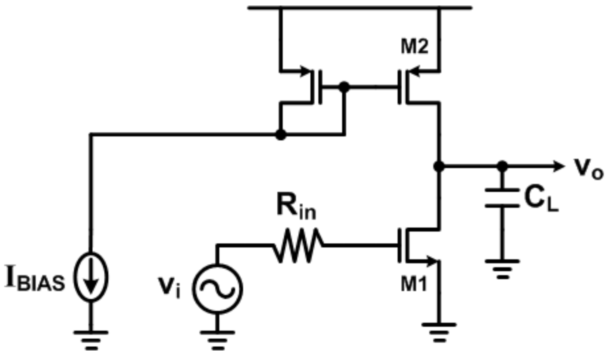
$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1} r_o)\right]}{R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

Assuming that the Miller Cap, $C_{gd1}(1 + g_{m1} r_o)$**, dominates**

$$\boxed{\omega_{p2} \cong -\frac{g_m C_{gd1}}{C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o}}$$

Common-Source Amp w/ Small R_{in}



- Example: Source-follower driving the common-source amp

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1} r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1} r_o\right)\right] + r_o\left(C_{gd1} + C_o\right)\right)}$$

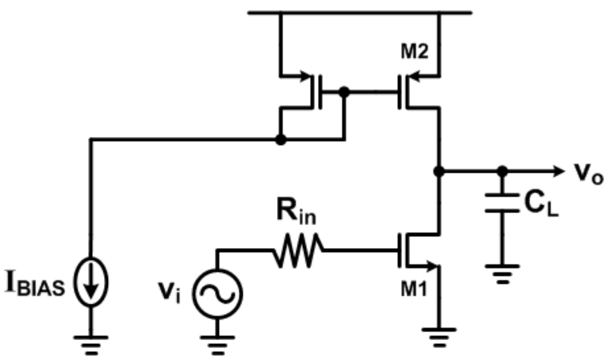
with $r_o >> R_{in}$

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_{m1} r_o}{1 + s r_o\left(C_{gd1} + C_o\right)}$$

$$\omega_{p1} = -\frac{1}{r_o\left(C_{gd1} + C_o\right)}$$

- Dominant pole is formed by output resistance times output capacitance plus transistor C_{gd}

Common-Source Amp w/ Small R_{in}



- What about the second pole?

Exact Transfer Function : $$\frac{v_o}{v_i} = \frac{-g_m r_o\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

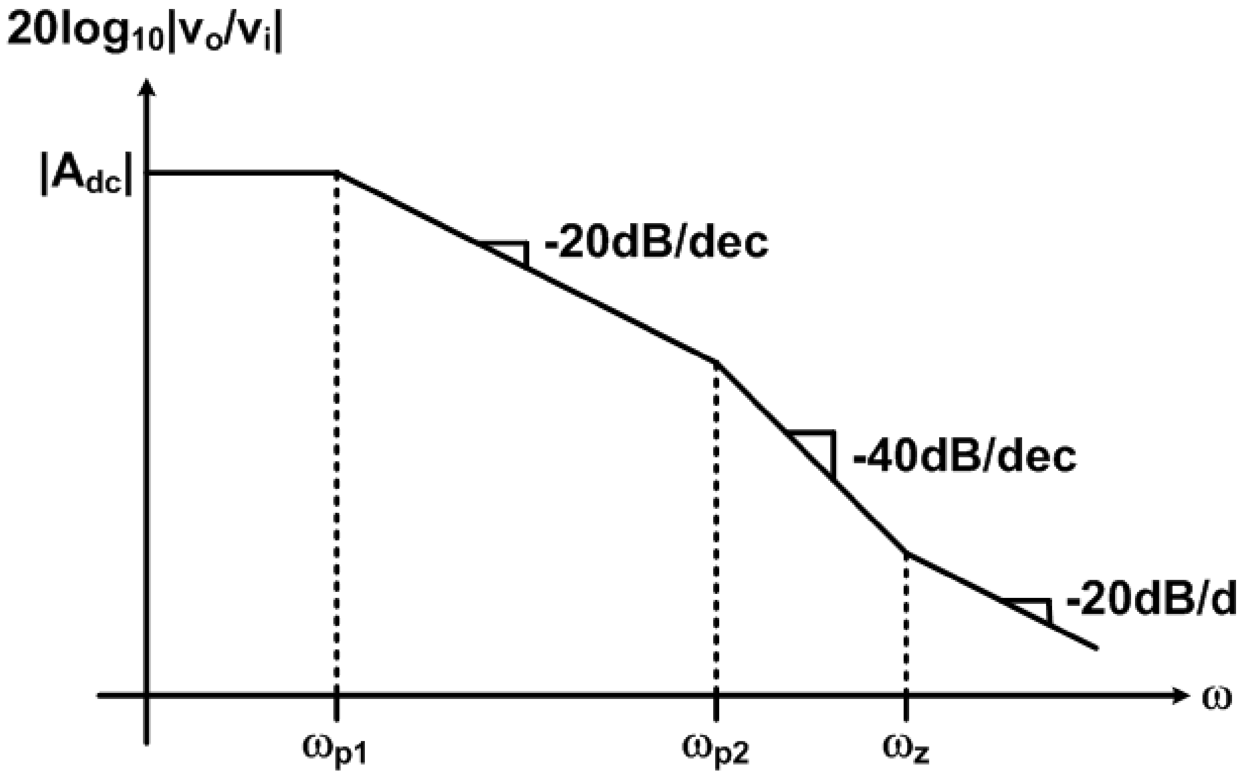
Denominator $$D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{r_o\left(C_{gd1} + C_o\right)}{R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

$$\omega_{p2} = -\frac{C_{gd1} + C_o}{R_{in}\left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} \cong -\frac{1}{R_{in}\left(C_{gs1} + C_{gd1}\right)} \quad \textbf{(with large } \mathbf{C_o}\textbf{)}$$

Common-Source Amp Frequency Response



$$A_{dc} = -g_m r_o \qquad \omega_z = \frac{g_{m1}}{C_{gd1}}$$

Large R_{in}

$$\omega_{p1} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1} r_o\right)\right]}$$

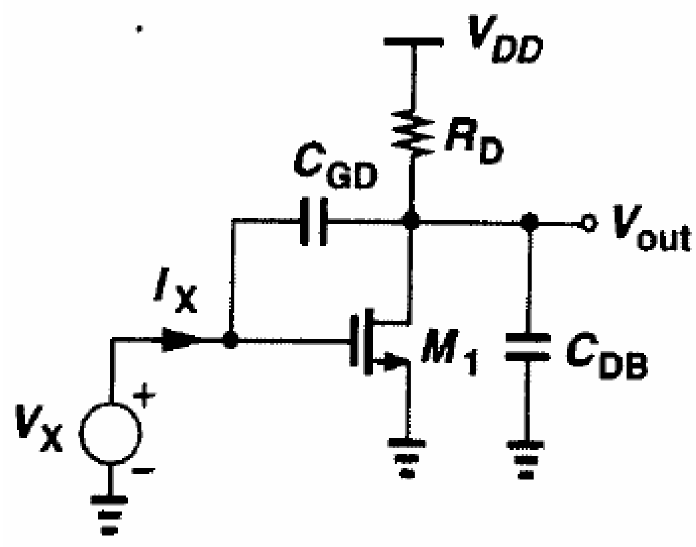
$$\omega_{p2} \cong -\frac{g_{m1} C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o}$$

Small R_{in}

$$\omega_{p1} = -\frac{1}{r_o\left(C_{gd1} + C_o\right)}$$

$$\omega_{p2} \cong -\frac{1}{R_{in}\left(C_{gs1} + C_{gd1}\right)}$$

Common-Source Amp Input Impedance



[Razavi]

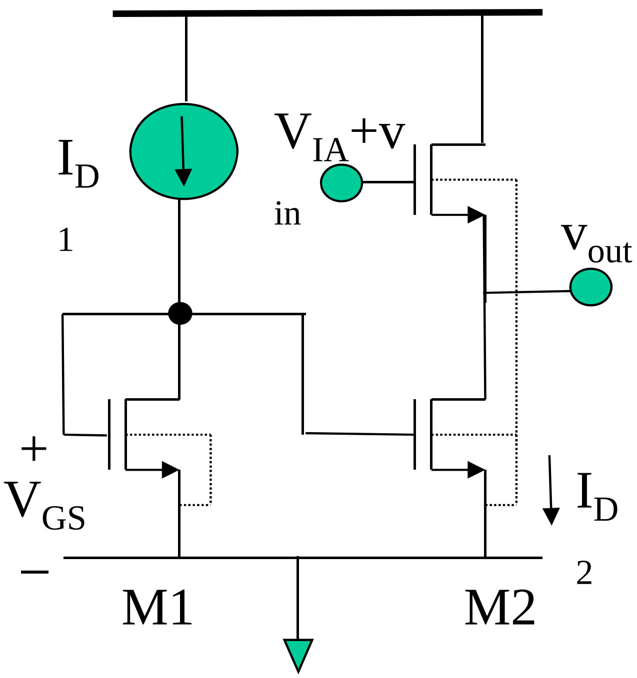
Neglecting Output Cap: $$Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

Input impedance is purely capacitive (C_{gs} + Miller C_{gd})

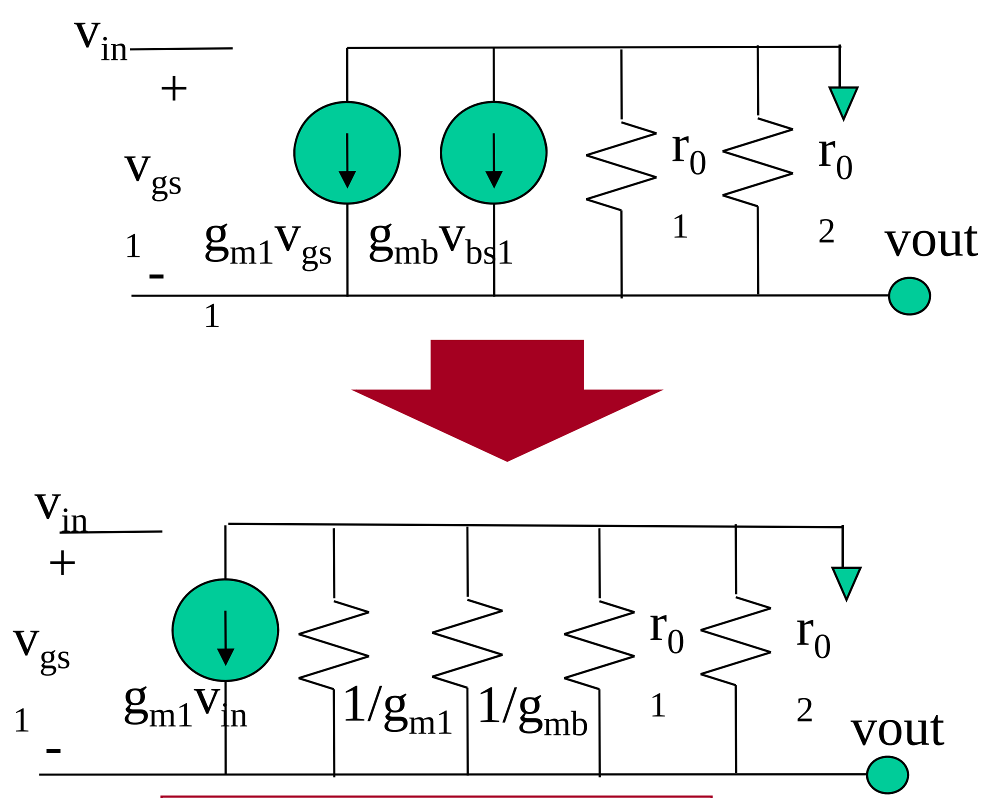
Considering Output Cap: $$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB} s)}$$

Low frequency is capacitive, but then impedance experiences a zero followed by a second pole

Small signal analysis: Common-drain (source follower) amplifier



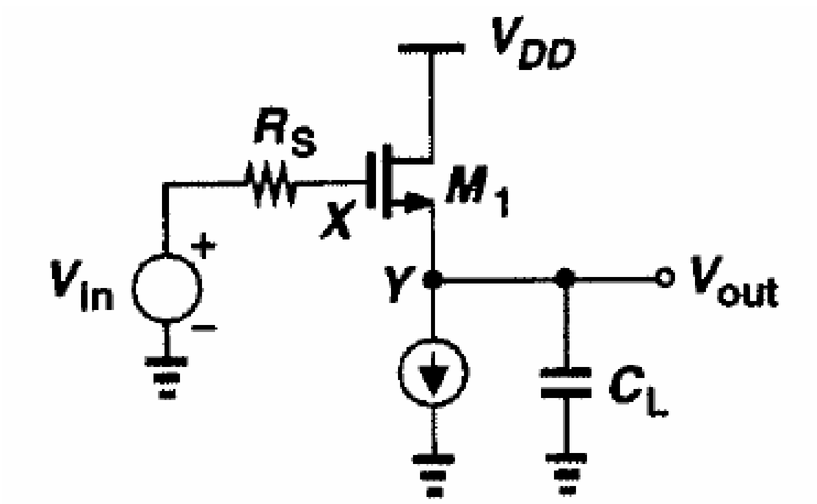
Small signal equivalent circuit

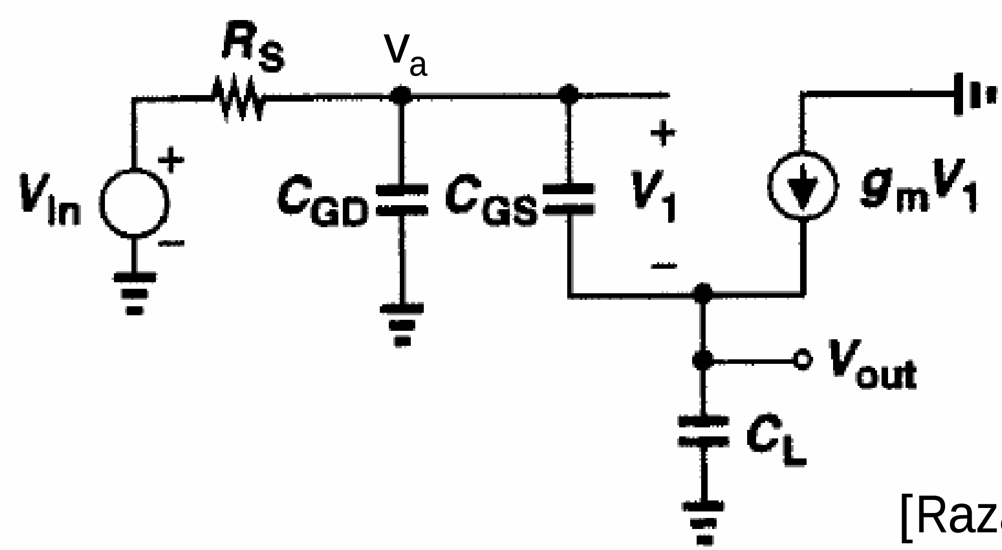


$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{mb} + g_{01} + g_{02}}$$

Common-Drain Amplifier: High Frequency Response

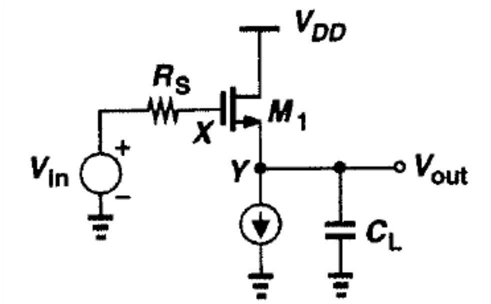
- Simplifying the schematic a bit for SSA
  - Ideal current source load and neglecting transistor r_o and g_{mb} (i.e. $\lambda=\gamma=0$)
  - Will result in an optimistic DC gain estimate

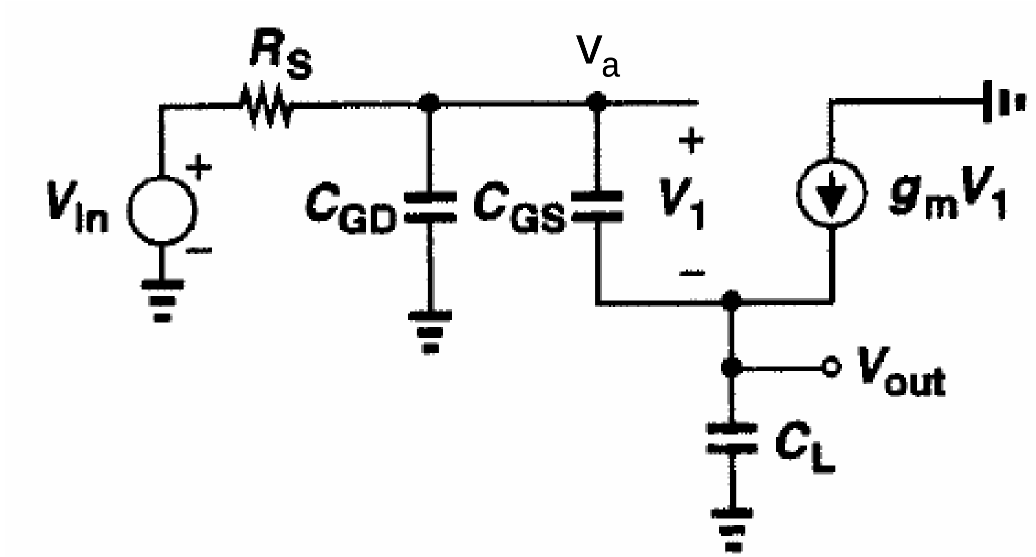




[Razavi]

Common-Drain Amplifier: High Frequency Response





[Razavi]

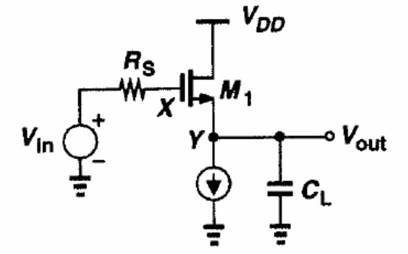
$$\textbf{KCL @ Node } \mathbf{v_a}: (v_a - v_i)G_S + v_a sC_{gd} + (v_a - v_o)sC_{gs} = 0$$

$$\textbf{KCL @ Node } \mathbf{v_o}: (v_o - v_a)sC_{gs} - g_m(v_a - v_o) + v_o sC_L = 0$$

After some algebra:

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Common-Drain Amplifier: High Frequency Response



$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

- From this simplified transfer function:

$$A_{dc} = \frac{g_m}{g_m} = 1 \quad \textbf{(Optimistic)}$$

$$\textbf{Exact} \quad A_{dc} = \frac{g_m}{g_m + g_o + g_{mb}}$$

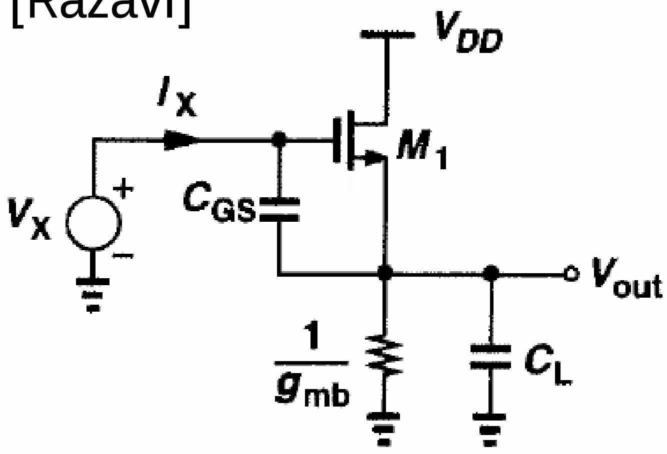
$$\omega_z = -\frac{g_m}{C_{gs}}$$

2 poles, If we assume that they are spaced far apart :

$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \frac{C_L + C_{GS}}{g_m}}$$

Common-Drain Amp Input Impedance

[Razavi]



$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

Low Frequency: $Z_{in} \approx \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$

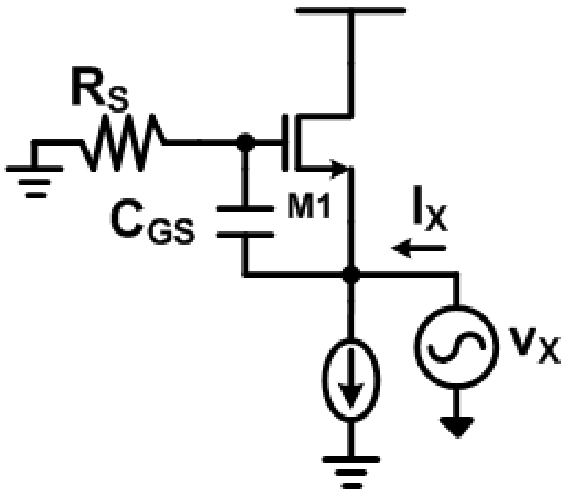
Equivalent to a series capacitve term $C_{gs}\left(\frac{g_{mb}}{g_m + g_{mb}}\right)$ **and resistive term** $\frac{1}{g_{mb}}$

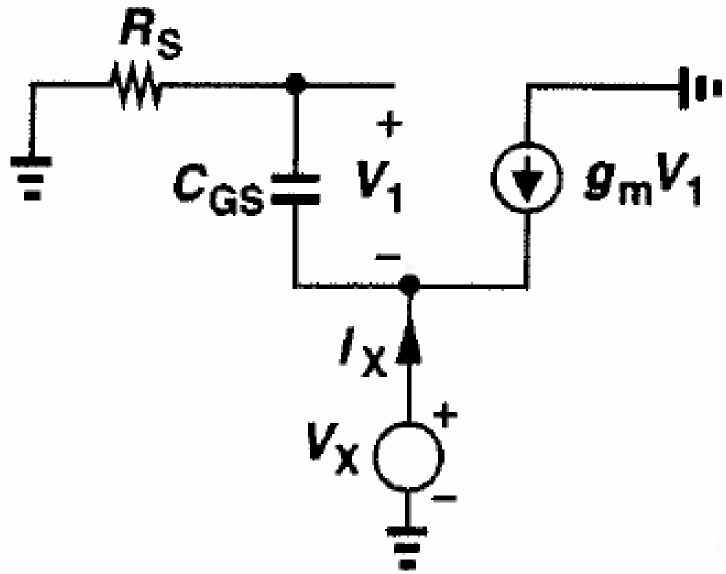
High Frequency: $Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS}C_L s^2}$

Series combination of C_{gs} **and** C_L **and a negative resistance term** $\left(-\frac{g_m}{C_{gs}C_L\omega^2}\right)$

The negative resistance term can be utilized in oscillator design

Common-Drain Amp Output Impedance

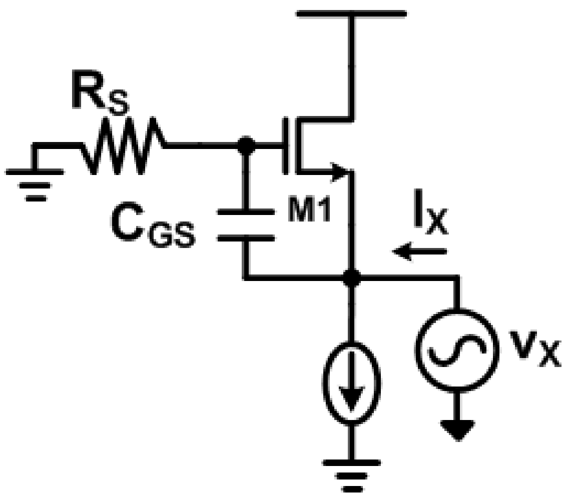




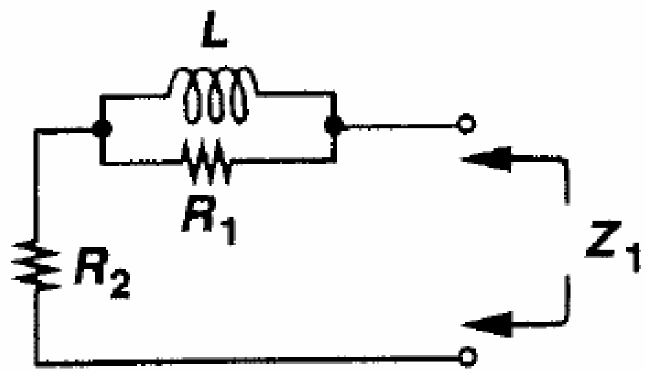
$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

- Pole at very high frequency
- Zero at potentially low frequency if R_S is large
  - Impedance can increase with frequency, i.e. display inductive behavior

Common-Drain Amp Output Impedance



$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

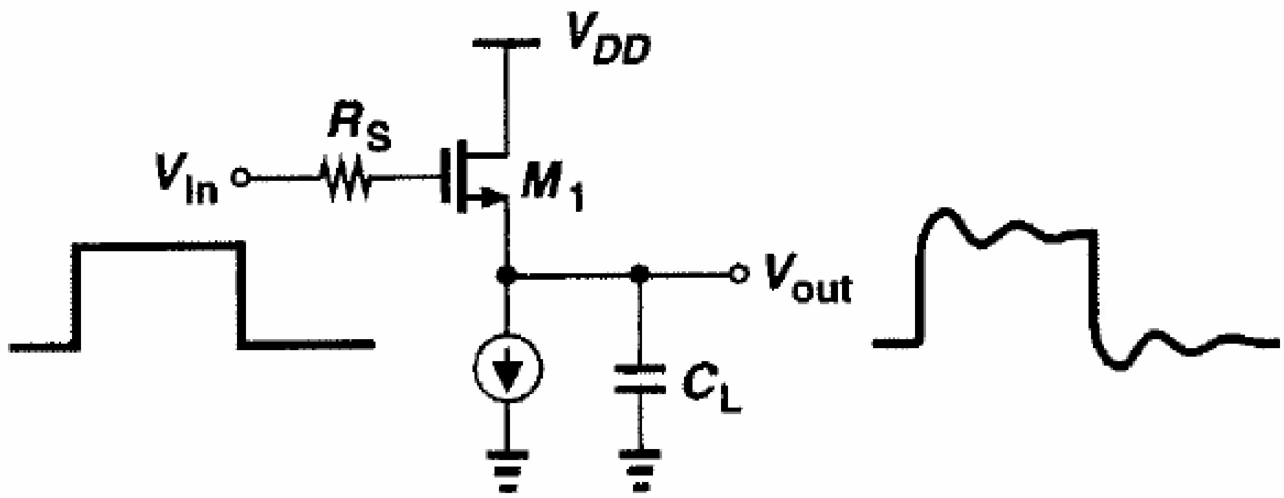


$$R_2 = \frac{1}{g_m}$$

$$R_1 = R_S - \frac{1}{g_m}$$

$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right) \approx \frac{R_S C_{GS}}{g_m} \ \textbf{if} \ R_S >> \frac{1}{g_m}$$

Transient Behavior w/ Large C_L

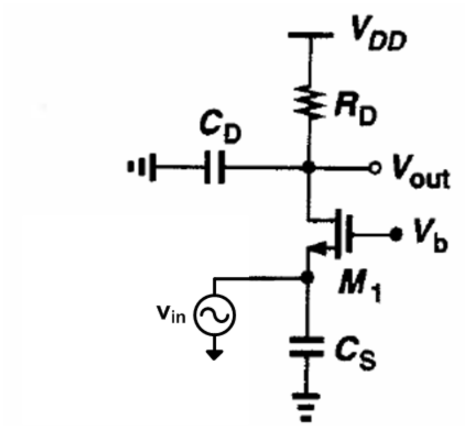


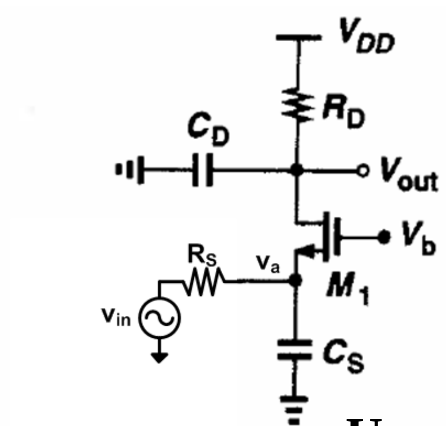
- Inductive output impedance in combination with a large load capacitance can create undesired “ringing” in the transient response
- If we have a large RS and CL, then the assumption that we have one dominant pole is no longer valid
- Both poles (potentially complex) should be considered in the analysis

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Common-Gate Amp Low Frequency Response

- No R_S
- With R_S





$$\frac{v_{out}}{v_a}$$ **is given from left**

How to get from v_{in} to v_a?

Use amplifier input impedance and voltage divider

Neglecting transistor r_o

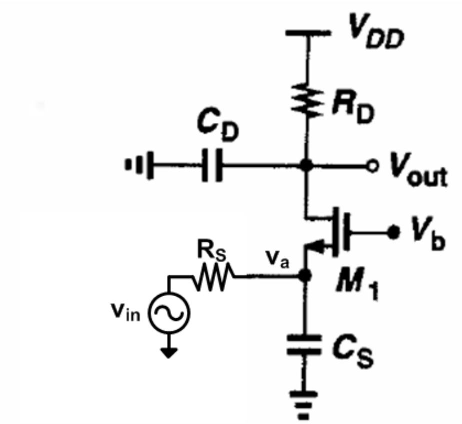
$$\frac{v_{out}}{v_{in}} = (g_m + g_{mb})R_D$$

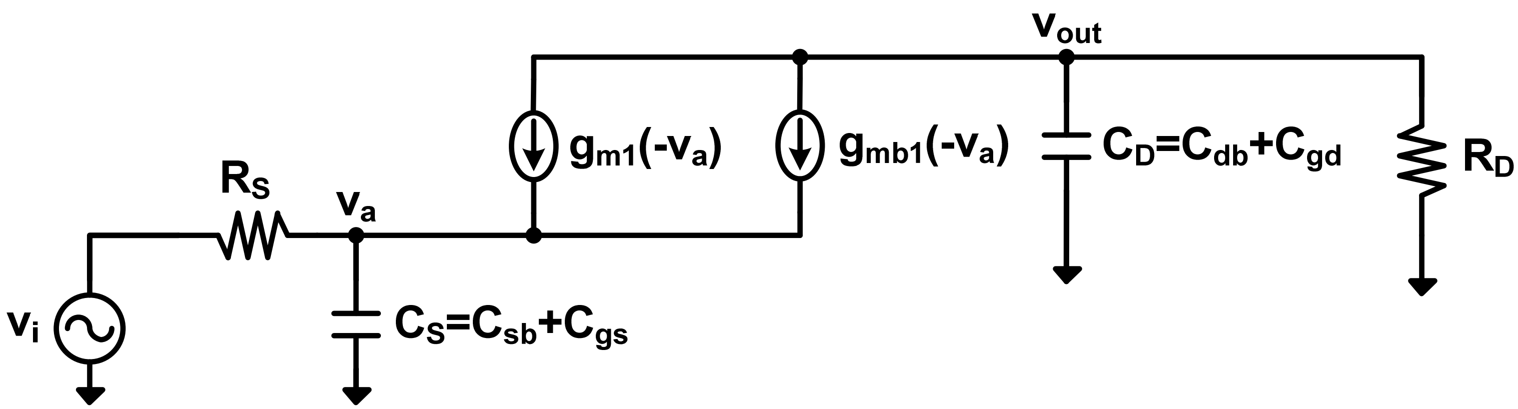
$$R_{in} = \frac{1}{g_m + g_{mb}}$$

$$v_a = \frac{\frac{1}{g_m + g_{mb}}}{R_S + \frac{1}{g_m + g_{mb}}} v_{in} = \frac{1}{1 + (g_m + g_{mb})R_S} v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{v_a}{v_{in}} \frac{v_{out}}{v_a} = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \approx \frac{R_D}{R_S}$$ **if R_S is large**

Common-Gate Amp Frequency Response



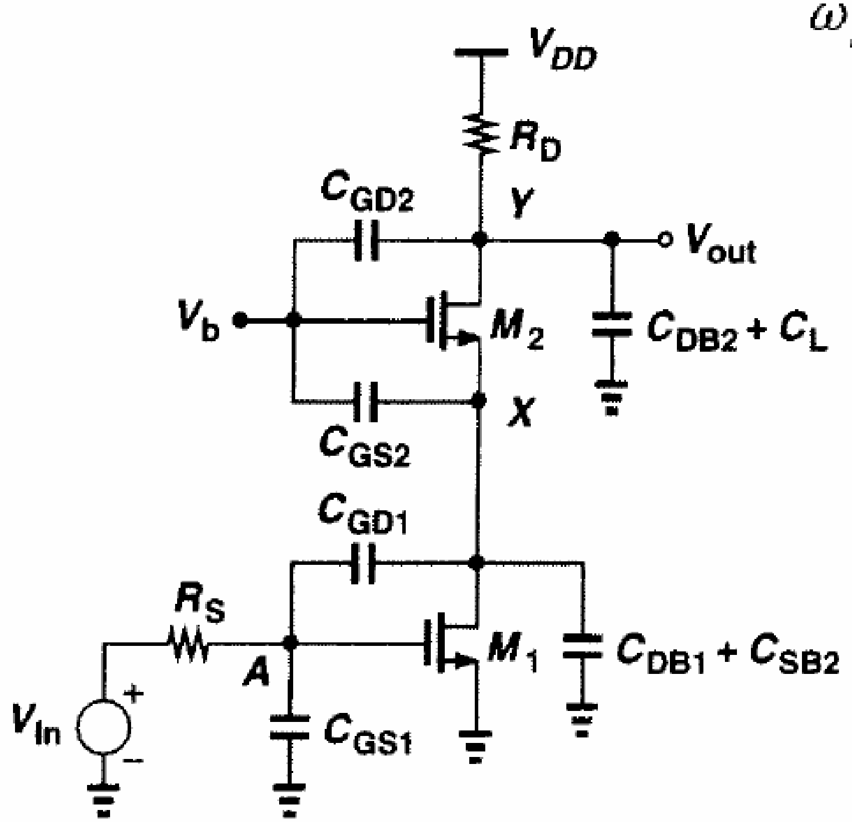


$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$

- No zero
- No Miller capacitor multiplication
- Low input impedance limits effectiveness as a voltage amplifier
- Useful as a current-to-voltage (transimpedance) amplifier

Cascode Amp Frequency Response

- If we associate the poles with the nodes A, X, and Y
  - Note, this is only an approximation, as it ignores interactions caused by “feedforward” caps (C_{gd}) and resistors
- 3 pole system



Input Pole

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right)C_{GD1}\right]}$$

Internal Pole – High Frequency

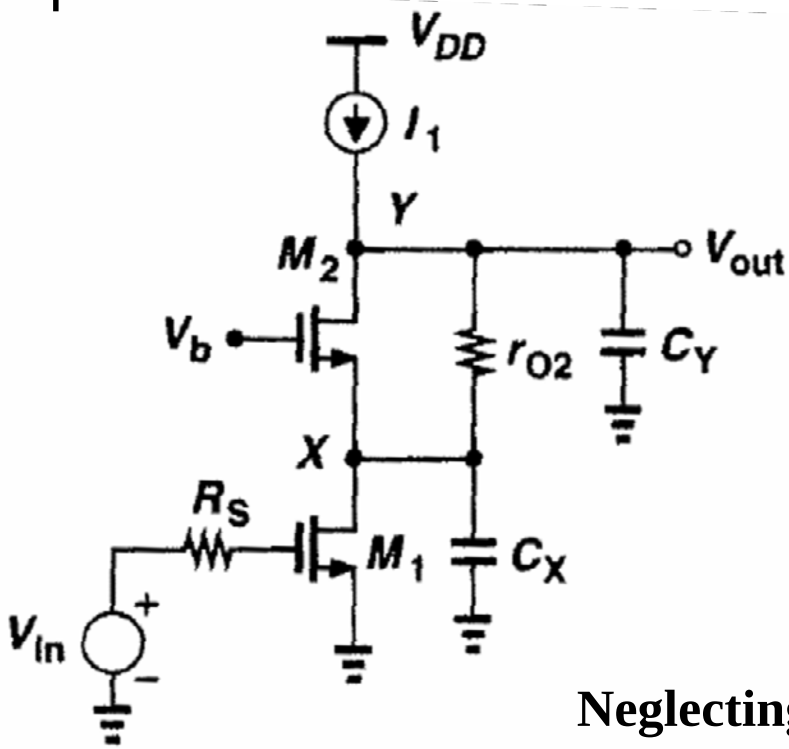
$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

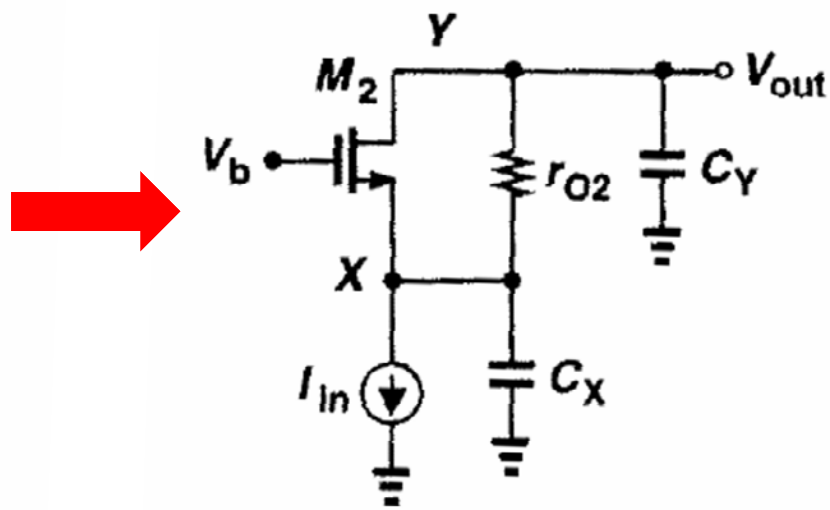
Output Pole

$$\omega_{p,Y} = \frac{1}{R_D(C_{DB2} + C_L + C_{GD2})}$$

Cascode Amp Output Impedance

- Simplified Model





Neglecting C_{GD1} and C_Y

$$Z_{out} = r_{o2} + Z_X + g_{m2} r_{o2} Z_X$$

where $$Z_X = r_{o1} \left\| \frac{1}{sC_X} \right.$$

Output Impedance Pole $$\omega_{\mathbf{Zout}} = \frac{1}{r_{o1} C_X}$$

Next Time

- Differential Amplifiers