Announcements & Agenda

• HW2 Due Oct 20

• Reading
  • Razavi Chapters 3 & 6
Announcements & Agenda

- Common-Source Amp Frequency Response
- Open-Circuit Time Constants (OC\(\tau\)) Bandwidth Estimation Technique
- Common-Drain Amp Frequency Response
- Common-Gate Amp Frequency Response
- Cascode Amp Frequency Response
Common-Source Amplifier: Low Frequency Response

\[ v_o = -\frac{g_{m1}}{g_{o1} + g_{o2}} \]

\[ \frac{v_o}{v_i} = g_{m1} \]

\[ v_i = g_{m1} V_{gs1} \]
Common-Source Amplifier: High Frequency Response

Small-Signal Model (Assuming $V_{G2}$ is AC gnd)

KCL @ Node $v_1 : (v_1 - v_i) G_{in} + v_1 s C_{gs1} + (v_1 - v_o) s C_{gd1} = 0$

KCL @ Node $v_o : (v_o - v_1) s C_{gd1} + g_{m1} v_1 + v_o (g_o + s C_o) = 0$

where $g_o = g_{o1} + g_{o2}$

After some algebra, we get the exact transfer function:

$$\frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

where

$$a = R_{in} \left[C_{gs1} + C_{gd1} (1 + g_m r_o)\right] + r_o \left(C_{gd1} + C_o\right)$$

and

$$b = R_{in} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o\right)$$
Common-Source Amp Frequency Response

Exact Transfer Function:

$$\frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s\frac{C_{gd1}}{g_m1}\right)}{1 + sa + s^2b}$$

For the common case when the two poles are real and far apart

Denominator

$$D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

Thus,

$$\omega_{p1} = -\frac{1}{a} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}(1 + g_m r_o)\right] + r_o\left(C_{gd1} + C_o\right)}$$

and the transfer function can be approximated as a single pole system

$$A(s) = \frac{v_o}{v_i} \cong \frac{-g_m r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}(1 + g_m r_o)\right] + r_o\left(C_{gd1} + C_o\right)\right)}$$
Open-Circuit Time Constants (OC$\tau$)

- Open-circuit time constants technique can be used to estimate bandwidth
  - Much easier than deriving transfer function
  - Accurate for systems with one dominant pole

All-Pole Transfer Function: \[ \frac{v_o(s)}{v_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1)\ldots(\tau_n s + 1)} \]

Denominator: \[ b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1 \]

Here \( b_n = \prod_{i=1}^{n} \tau_i \) and \( b_1 = \sum_{i=1}^{n} \tau_i \)

A Dominant-Pole System can be approximated as

\[ \frac{v_o(s)}{v_i(s)} \approx \frac{a_0}{b_1 s + 1} = \frac{a_0}{(\sum_{i=1}^{n} \tau_i) s + 1} \]

Bandwidth \( \omega_h \approx \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \omega_{h,est} \)
Open-Circuit Time Constants (OC\(\tau\))

- To compute time-constants
  1. Compute effective resistance \(R_{ko}\) facing each kth capacitor with all other caps open-circuited
  2. Form the product \(\tau_{ko} = R_{ko} C_k\)
  3. Sum all n “open-circuit” time constants

\[
\omega_{h,est} = \frac{1}{\sum_{k=1}^{n} R_{ko} C_k}
\]
Common-Source Amp w/ OC$\tau$

- For $C_{gs1}$

\[
R_{io} = \frac{v_{io}}{i_{io}} = \frac{v_{io}}{v_{io}} = R_{in}
\]

\[
\tau_{io} = R_{in} C_{gs1}
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming V_{G2} is AC gnd)

- For C_{gd1}

(1) \( i_{2o} = g_m v_{gs1} + \frac{(v_{2o} + v_{gs1})}{r_o} \)

(2) \( v_{gs1} = -i_{2o} R_{in} \)

Plugging (2) into (1) and solving for \( \frac{v_{2o}}{i_{2o}} \)

\[ R_{2o} = \frac{v_{2o}}{i_{2o}} = R_{in} (1 + g_{m1} r_o) + r_o \]

\[ r_{2o} = \left( R_{in} (1 + g_{m1} r_o) + r_o \right) C_{gd1} \]
Common-Source Amp w/ OC$\tau$

- For $C_o$

$$R_{3o} = \frac{v_{3o}}{i_{3o}} = \frac{v_{3o}}{v_{3o}} = r_o$$

$$\tau_{3o} = r_o C_o$$
Common-Source Amp w/ OCᵣ

Small-Signal Model (Assuming \( V_{G2} \) is AC gnd)

3 Time Constants: \( \tau_{1o} = R_{in} C_{gs1} \), \( \tau_{2o} = (R_{in}(1 + g_m r_o) + r_o)C_{gd1} \), \( \tau_{3o} = r_o C_o \)

\[
b_1 = \sum_{i=1}^{n} \tau_i = R_{in} C_{gs1} + (R_{in}(1 + g_m r_o) + r_o)C_{gd1} + r_o C_o
\]

\[
\omega_{h,est} = \frac{1}{b_1} = \frac{1}{R_{in} C_{gs1} + (R_{in}(1 + g_m r_o) + r_o)C_{gd1} + r_o C_o}
\]

Exactly the same as what we derived in Slide 6!

\[
A(s) = \frac{v_o}{v_i} \approx \frac{-g_m r_o}{1 + s\left[R_{in}\left[C_{gs1} + C_{gd1}(1 + g_m r_o)\right] + r_o\left(C_{gd1} + C_o\right)\right]}
\]
Common-Source Amp w/ Large $R_{in}$

- Example: Using common-source output stage in a 2-stage OpAmp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}\left(C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right) + r_o\left(C_{gd1} + C_o\right)}$$

with $R_{in} \gg r_o$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}\left(C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right)}$$

$$\omega_{p1} = -\frac{1}{R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right]}$$

- Dominant pole is formed by input resistance times transistor $C_{gs}$ and $C_{gd}$ which has been multiplied by $1-A_{dc}$
  - $C_{gd}(1-A_{dc})$ is called the Miller capacitance
Miller’s Theorem

If $A_v$ is the gain from node 1 to 2, then a floating impedance $Z_F$ can be converted to two grounded impedances $Z_1$ and $Z_2$.

$I_1$ should be the same in both circuits

$$ I_1 = \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} $$

$$ Z_1 = \left( \frac{V_1}{V_1 - V_2} \right) Z_F = \left( \frac{1}{1 - \frac{V_2}{V_1}} \right) Z_F = \frac{Z_F}{1 - A_v} $$

where $A_v = \frac{V_2}{V_1}$

$I_2$ should be the same in both circuits

$$ I_2 = \frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2} $$

$$ Z_2 = \left( \frac{V_2}{V_2 - V_1} \right) Z_F = \left( \frac{1}{1 - \frac{V_1}{V_2}} \right) Z_F = \frac{Z_F}{1 - \frac{1}{A_v}} $$
With Miller’s theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

\[
Z_{in} = \frac{1}{j \omega C_F (1 - A_v)} = \frac{1}{j \omega C_F (1 - (-A_o))} = \frac{1}{j \omega C_F (1 + A_o)}
\]

Equivalent to an input cap that is the original \( C_F \) multiplied by \( 1 + A_o \)

Following a similar procedure, the output cap is the original \( C_F \) multiplied by \( 1 + \frac{1}{A_o} \)
• What about the second pole?

Exact Transfer Function: \[ \frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_{m1}} \right)}{1 + sa + s^2 b} \]

Denominator \[ D(s) = \left(1 - \frac{s}{\omega_{p1}}\right) \left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1} \omega_{p2}} \approx 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1} \omega_{p2}} \]

\[ \frac{1}{\omega_{p1} \omega_{p2}} = b = \frac{R_{in} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)}{1} \]

\[ \omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} = -\frac{R_{in} \left[C_{gs1} + C_{gd1} \left(1 + g_{m1} r_o \right) \right]}{R_{in} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} \]

Assuming that the Miller Cap, \( C_{gd1} \left(1 + g_{m1} r_o \right) \), dominates

\[ \omega_{p2} \approx -\frac{g_m C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o} \]
Common-Source Amp w/ Small $R_{\text{in}}$

- Example: Source-follower driving the common-source amp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_m r_o}{1 + s(R_{\text{in}}[C_{gs1} + C_{gd1}(1 + g_m r_o)] + r_o[C_{gd1} + C_o])}$$

with $r_o \gg R_{\text{in}}$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_m r_o}{1 + s r_o(C_{gd1} + C_o)}$$

$$\omega_{p1} = -\frac{1}{r_o(C_{gd1} + C_o)}$$

- Dominant pole is formed by output resistance times output capacitance plus transistor $C_{gd}$
Common-Source Amp w/ Small $R_{in}$

- What about the second pole?

**Exact Transfer Function:**

$$v_o = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_m} \right)}{1 + sa + s^2b}$$

**Denominator**

$$D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \cong 1 - s\left(\frac{1}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}\right)$$

$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in}r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1}R_{in}r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{r_o \left(C_{gd1} + C_o\right)}{R_{in}r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

$$\omega_{p2} = -\frac{C_{gd1} + C_o}{R_{in} \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} \approx -\frac{1}{R_{in} \left(C_{gs1} + C_{gd1}\right)} \quad \text{(with large $C_o$)}$$
Common-Source Amp Frequency Response

\[ A_{dc} = -g_m r_o \quad \omega_z = \frac{g_{m1}}{C_{gd1}} \]

\[ \omega_{p1} = -\frac{1}{R_{in} \left[ C_{gs1} + C_{gd1} \left( 1 + g_{m1} r_o \right) \right]} \]

\[ \omega_{p2} \approx -\frac{g_{m1} C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o} \]

\[ \omega_{p1} = \frac{1}{r_o \left( C_{gd1} + C_o \right)} \]

\[ \omega_{p2} \approx \frac{1}{R_{in} \left( C_{gs1} + C_{gd1} \right)} \]
Common-Source Amp Input Impedance

Neglecting Output Cap: \( Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]/s} \)

Input impedance is purely capacitive (\(C_{gs}\) + Miller \(C_{gd}\))

Considering Output Cap: \( \frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})/s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)} \)

Low frequency is capacitive, but then impedance experiences a zero followed by a second pole
Small signal analysis: Common-drain (source follower) amplifier

Small signal equivalent circuit

\[
\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m1} + g_{mb} + g_{01} + g_{02}}
\]
Common-Drain Amplifier: High Frequency Response

• Simplifying the schematic a bit for SSA
  • Ideal current source load and neglecting transistor $r_o$ and $g_{mb}$ (i.e. $\lambda=\gamma=0$)
  • Will result in an optimistic DC gain estimate

[Razavi]
Common-Drain Amplifier: High Frequency Response

KCL @ Node $v_a : (v_a - v_i)G_S + v_a sC_{gd} + (v_a - v_o)sC_{gs} = 0$

KCL @ Node $v_o : (v_o - v_a)sC_{gs} - g_m(v_a - v_o) + v_o sC_L = 0$

After some algebra:

$$\frac{V_{out}(s)}{V_{in}} = \frac{g_m + C_{GSS} s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$
Common-Drain Amplifier:
High Frequency Response

\[
\frac{V_{out}(s)}{V_{in}} = \frac{g_m + C_{GS}s}{R_s(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_s C_{GD} + C_L + C_{GS})s + g_m}
\]

- From this simplified transfer function:

\[A_{dc} = \frac{g_m}{g_m} = 1 \quad \text{(Optimistic)}\]

\[
\text{Exact} \quad A_{dc} = \frac{g_m}{g_m + g_o + g_{mb}}
\]

\[\omega_z = -\frac{g_m}{C_{gs}}\]

2 poles, If we assume that they are spaced far apart:

\[\omega_{p1} \approx \frac{g_m}{g_m R_s C_{GD} + C_L + C_{GS}} = \frac{1}{R_s C_{GD} + \frac{C_L + C_{GS}}{g_m}}\]
Common-Drain Amp Input Impedance

\[ Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right) \frac{1}{g_{mb} + C_{LS}} \]

Low Frequency:
\[ Z_{in} \approx \frac{1}{C_{GS}s} \left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}} \]

Equivalent to a series capacitive term \( C_{gs} \left(\frac{g_{mb}}{g_m + g_{mb}}\right) \) and resistive term \( \frac{1}{g_{mb}} \)

High Frequency:
\[ Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_{LS}s} + \frac{g_m}{C_{GS}C_{LS}s^2} \]

Series combination of \( C_{gs} \) and \( C_L \) and a negative resistance term \( \left(-\frac{g_m}{C_{gs}C_L\omega^2}\right) \)

The negative resistance term can be utilized in oscillator design
Common-Drain Amp Output Impedance

- Pole at very high frequency
- Zero at potentially low frequency if $R_S$ is large
  - Impedance can increase with frequency, i.e. display inductive behavior

$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GSS}s + 1}{g_m + C_{GSS}s}$$
Common-Drain Amp Output Impedance

\[ Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s} \]

\[ R_2 = \frac{1}{g_m} \]
\[ R_1 = R_S - \frac{1}{g_m} \]

\[ L = \frac{C_{GS}}{g_m} \left( R_S - \frac{1}{g_m} \right) \approx \frac{R_S C_{GS}}{g_m} \text{ if } R_S \gg \frac{1}{g_m} \]
Transient Behavior w/ Large $C_L$

- Inductive output impedance in combination with a large load capacitance can create undesired “ringing” in the transient response.
- If we have a large $R_S$ and $C_L$, then the assumption that we have one dominant pole is no longer valid.
- Both poles (potentially complex) should be considered in the analysis.

\[
\frac{V_{out}(s)}{V_{in}} = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}
\]
Common-Gate Amp Low Frequency Response

**No $R_S$**

Neglecting transistor $r_o$

$$\frac{v_{out}}{v_{in}} = (g_m + g_{mb})R_D$$

**With $R_S$**

$$\frac{v_{out}}{v_{a}} \text{ is given from left}$$

How to get from $v_{in}$ to $v_a$?

Use amplifier input impedance and voltage divider

$$R_{in} = \frac{1}{g_m + g_{mb}}$$

$$v_a = \frac{1}{g_m + g_{mb}} \frac{v_{in}}{1 + (g_m + g_{mb})R_S}$$

$$v_{out} = \frac{v_a}{v_{in}} v_{out} = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \approx \frac{R_D}{R_S} \text{ if } R_S \text{ is large}$$
Common-Gate Amp Frequency Response

- No zero
- No Miller capacitor multiplication
- Low input impedance limits effectiveness as a voltage amplifier
- Useful as a current-to-voltage (transimpedance) amplifier
Cascode Amp Frequency Response

- If we associate the poles with the nodes A, X, and Y
  - Note, this is only an approximation, as it ignores interactions caused by “feedforward” caps ($C_{gd}$) and resistors
- 3 pole system
Cascode Amp Output Impedance

- Simplified Model

Neglecting $C_{GD1}$ and $C_Y$

$$Z_{out} = r_{o2} + Z_X + g_{m2}r_{o2}Z_X$$

where $Z_X = r_{o1} \left| \frac{1}{sC_X} \right|$

Output Impedance Pole $\omega_{Z_{out}} = \frac{1}{r_{o1}C_X}$
Next Time

• Differential Pairs