Announcements & Agenda

• HW2 Due Mar 6

• Reading
  • Razavi Chapters 3 & 6
Announcements & Agenda

- Common-Source Amp Frequency Response
- Open-Circuit Time Constants (OC$\tau$)
  Bandwidth Estimation Technique
- Common-Drain Amp Frequency Response
- Common-Gate Amp Frequency Response
- Cascode Amp Frequency Response
Common-Source Amplifier: Low Frequency Response

\[ \frac{v_o}{v_i} = -\frac{g_m1}{g_{o1} + g_{o2}} \]
Common-Source Amplifier:  
High Frequency Response

\[ v_{1} = (v_{1} - v_{i})g_{in} + v_{1}sC_{gs1} + (v_{1} - v_{o})sC_{gd1} = 0 \]

\[ v_{o} = (v_{o} - v_{1})sC_{gd1} + g_{m1}v_{1} + v_{o}(g_{o} + sC_{o}) = 0 \]

where \( g_{o} = g_{o1} + g_{o2} \)

After some algebra, we get the exact transfer function:

\[
\frac{v_{o}}{v_{i}} = \frac{-g_{m}r_{o}\left(1-s\frac{C_{gd1}}{g_{m1}}\right)}{1+sa+s^{2}b}
\]

where

\[ a = R_{in}\left[C_{gs1} + C_{gd1}(1+g_{m}r_{o})\right] + r_{o}\left(C_{gd1} + C_{o}\right) \]

and

\[ b = R_{in}r_{o}\left(C_{gd1}C_{gs1} + C_{gs1}C_{o} + C_{gd1}C_{o}\right) \]
Common-Source Amp Frequency Response

[Diagram of common-source amplifier]

Exact Transfer Function: \[ \frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b} \]

For the common case when the two poles are real and far apart

Denominator \[ D(s) = \left(1 - \frac{s}{\omega_{p1}}\right) \left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1} \omega_{p2}} \approx 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1} \omega_{p2}} \]

Thus, \[ \omega_{p1} = -\frac{1}{a} = -\frac{1}{R_{in} \left[C_{gs1} + C_{gd1} \left(1 + g_{m1} r_o \right)\right] + r_o \left(C_{gd1} + C_o\right)} \]

and the transfer function can be approximated as a single pole system

\[ A(s) = \frac{v_o}{v_i} \approx \frac{-g_m r_o}{1 + s \left(R_{in} \left[C_{gs1} + C_{gd1} \left(1 + g_{m1} r_o \right)\right] + r_o \left(C_{gd1} + C_o\right)\right)} \]
Open-Circuit Time Constants (OC$\tau$)

- Open-circuit time constants technique can be used to estimate bandwidth
  - Much easier than deriving transfer function
  - Accurate for systems with one dominant pole

All-Pole Transfer Function: \[ \frac{v_o(s)}{v_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1) \ldots (\tau_n s + 1)} \]

Denominator: \( b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + 1 \)

Here \( b_n = \prod_{i=1}^{n} \tau_i \) and \( b_1 = \sum_{i=1}^{n} \tau_i \)

A Dominant-Pole System can be approximated as

\[ \frac{v_o(s)}{v_i(s)} \approx \frac{a_0}{b_1 s + 1} = \frac{a_0}{\left(\sum_{i=1}^{n} \tau_i\right) s + 1} \]

**Bandwidth**
\[ \omega_h \approx \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \omega_{h,\text{est}} \]
Open-Circuit Time Constants (OC$\tau$)

- To compute time-constants
  1. Compute effective resistance $R_{ko}$ facing each $k$th capacitor with all other caps open-circuited
  2. Form the product $\tau_{ko} = R_{ko}C_k$
  3. Sum all $n$ “open-circuit” time constants

\[
\omega_{h,est} = \frac{1}{\sum_{k=1}^{n} R_{ko}C_k}
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming \( V_{G2} \) is AC gnd)

- For \( C_{gs1} \)

\[
R_{1o} = \frac{V_{1o}}{i_{1o}} = \frac{V_{1o}}{\left( \frac{V_{1o}}{R_{in}} \right)} = R_{in} \quad \tau_{1o} = R_{in} C_{gs1}
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming VG2 is AC gnd)

• For C_{gd1}

(1) \[ i_{2o} = g_m v_{gs1} + \frac{(v_{2o} + v_{gs1})}{r_o} \]

(2) \[ v_{gs1} = -i_{2o} R_{in} \]

Plugging (2) into (1) and solving for \( \frac{v_{2o}}{i_{2o}} \)

\[ R_{2o} = \frac{v_{2o}}{i_{2o}} = R_{in} (1 + g_m r_o) + r_o \]

\[ \tau_{2o} = (R_{in} (1 + g_m r_o) + r_o) C_{gd1} \]
Common-Source Amp w/ OC$\tau$

- For $C_o$

$$R_{3o} = \frac{v_{3o}}{i_{3o}} = \frac{v_{3o}}{\left(\frac{v_{3o}}{r_o}\right)} = r_o$$

$$\tau_{3o} = r_o C_o$$
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming \( V_{G2} \) is AC gnd)

3 Time Constants: \( \tau_{1o} = R_{in} C_{gs1} \), \( \tau_{2o} = \left( R_{in} \left( 1 + g_m r_o \right) + r_o \right) C_{gd1} \), \( \tau_{3o} = r_o C_o \)

\[
b_1 = \sum_{i=1}^{n} \tau_i = R_{in} C_{gs1} + \left( R_{in} \left( 1 + g_m r_o \right) + r_o \right) C_{gd1} + r_o C_o
\]

\[
\omega_{h,est} = \frac{1}{b_1} = \frac{1}{R_{in} C_{gs1} + \left( R_{in} \left( 1 + g_m r_o \right) + r_o \right) C_{gd1} + r_o C_o}
\]

Exactly the same as what we derived in Slide 6!

\[
A(s) = \frac{v_o}{v_i} \approx \frac{-g_m r_o}{1 + s \left( R_{in} \left( C_{gs1} + C_{gd1} \left( 1 + g_m r_o \right) + r_o \left( C_{gd1} + C_o \right) \right) \right)}
\]
Common-Source Amp w/ Large $R_{in}$

- Example: Using common-source output stage in a 2-stage OpAmp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1}r_o\right)\right] + r_o\left(C_{gd1} + C_o\right)\right)}$$

with $R_{in} >> r_o$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1}r_o\right)\right]}$$

$$\omega_{p1} = -\frac{1}{\frac{R_{in}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1}r_o\right)\right]}{}}$$

- Dominant pole is formed by input resistance times transistor $C_{gs}$ and $C_{gd}$ which has been multiplied by $1-A_{dc}$
  - $C_{gd}(1-A_{dc})$ is called the Miller capacitance
Miller’s Theorem

If $A_v$ is the gain from node 1 to 2, then a floating impedance $Z_F$ can be converted to two grounded impedances $Z_1$ and $Z_2$.

$I_1$ should be the same in both circuits

$$I_1 = \frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$Z_1 = \left(\frac{V_1}{V_1 - V_2}\right)Z_F = \left(\frac{1}{1 - \frac{V_2}{V_1}}\right)Z_F = \frac{Z_F}{1 - A_v}$$

where $A_v = \frac{V_2}{V_1}$

$I_2$ should be the same in both circuits

$$I_2 = \frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2}$$

$$Z_2 = \left(\frac{V_2}{V_2 - V_1}\right)Z_F = \left(\frac{1}{1 - \frac{V_1}{V_2}}\right)Z_F = \frac{Z_F}{1 - \frac{1}{A_v}}$$
With Miller’s theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

\[
Z_{in} = \frac{1}{j\omega C_F (1 - A_v)} = \frac{1}{j\omega C_F (1 - (-A_o))} = \frac{1}{j\omega C_F (1 + A_o)}
\]

Equivalent to an input cap that is the original \( C_F \) multiplied by \( (1 + A_o) \)

Following a similar procedure, the output cap is the original \( C_F \) multiplied by \( 1 + \frac{1}{A_o} \)
Common-Source Amp w/ Large $R_{in}$

- What about the second pole?

**Exact Transfer Function:**

$$\frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_m} \right)}{1 + sa + s^2 b}$$

**Denominator $D(s)$:**

$$D(s) = \left(1 - \frac{s}{\omega_{p1}}\right) \left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1} \omega_{p2}} \approx 1 - s \left(\frac{1}{\omega_{p1}} + \frac{s^2}{\omega_{p1} \omega_{p2}}\right)$$

$$\frac{1}{\omega_{p1} \omega_{p2}} = b = r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} = -\frac{R_{in} \left[C_{gs1} + C_{gd1} (1 + g_m r_o) \right]}{R_{in} r_o \left(C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)}$$

**Assuming that the Miller Cap, $C_{gd1} (1 + g_m r_o)$, dominates**

$$\omega_{p2} \approx -\frac{g_m C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o}$$
Common-Source Amp w/ Small $R_{in}$

- Example: Source-follower driving the common-source amp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + s\left(R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)\right] + r_o\left(C_{gd1} + C_o\right)\right)}$$

with $r_o \gg R_{in}$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sr_o\left(C_{gd1} + C_o\right)}$$

$$\omega_p = -\frac{1}{r_o\left(C_{gd1} + C_o\right)}$$

- Dominant pole is formed by output resistance times output capacitance plus transistor $C_{gd}$
Common-Source Amp w/ Small $R_{in}$

- What about the second pole?

**Exact Transfer Function**:

$$v_o = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_m1}\right)}{1 + sa + s^2 b}$$

**Denominator** $D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \approx 1 - s \left(\frac{1}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}\right)$

$$\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$

$$\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} = -\frac{r_o \left(C_{gd1} + C_o\right)}{R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)}$$

$$\omega_{p2} = -\frac{C_{gd1} + C_o}{R_{in} \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)} \approx -\frac{1}{R_{in} \left(C_{gs1} + C_{gd1}\right)} \quad \text{(with large } C_o)$$
Common-Source Amp Frequency Response

\[ A_{dc} = -g_m r_o \]
\[ \omega_z = \frac{g_m}{C_{gd1}} \]
\[ \omega_{p1} = -\frac{1}{R_{in} \left[ C_{gs1} + C_{gd1} \left( 1 + g_m r_o \right) \right]} \]
\[ \omega_{p2} \approx -\frac{g_m C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o} \]

\[ \omega_{p1} = -\frac{1}{r_o \left( C_{gd1} + C_o \right)} \]
\[ \omega_{p2} \approx -\frac{1}{R_{in} \left( C_{gs1} + C_{gd1} \right)} \]
Common-Source Amp Input Impedance

Neglecting Output Cap:  
\[ Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s} \]

Input impedance is purely capacitive \((C_{gs} + \text{Miller } C_{gd})\)

Considering Output Cap:  
\[ \frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)} \]

Low frequency is capacitive, but then impedance experiences a zero followed by a second pole

[Image of a common-source amplifier circuit diagram]
Small signal analysis: Common-drain (source follower) amplifier

Small signal equivalent circuit

\[
\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m1} + g_{mb} + g_{01} + g_{02}}
\]
Common-Drain Amplifier: High Frequency Response

• Simplifying the schematic a bit for SSA
  • Ideal current source load and neglecting transistor $r_o$ and $g_{mb}$ (i.e. $\lambda = \gamma = 0$)
  • Will result in an optimistic DC gain estimate

[Razavi]
Common-Drain Amplifier:
High Frequency Response

\[ V_{\text{out}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m} \]
Common-Drain Amplifier: High Frequency Response

From this simplified transfer function:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} (s) = \frac{g_m + C_{GS} s}{R_s (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) s^2 + (g_m R_s C_{GD} + C_L + C_{GS}) s + g_m}
\]

- From this simplified transfer function:

\[
A_{dc} = \frac{g_m}{g_m} = 1 \quad \text{(Optimistic)}
\]

Exact \( A_{dc} = \frac{g_m}{g_m + g_o + g_{mb}} \)

\[
\omega_z = -\frac{g_m}{C_{gs}}
\]

2 poles, If we assume that they are spaced far apart:

\[
\omega_{p1} \approx \frac{g_m}{g_m R_s C_{GD} + C_L + C_{GS}} = \frac{1}{R_s C_{GD} + \frac{C_L + C_{GS}}{g_m}}
\]
Common-Drain Amp Input Impedance

\[ Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{g_{mb}}\right) \frac{1}{g_{mb} + C_{LS}} \]

Low Frequency: \[ Z_{in} \approx \frac{1}{C_{GS}s} \left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}} \]

Equivalent to a series capacitive term \( C_{gs} \left(\frac{g_{mb}}{g_m + g_{mb}}\right) \) and resistive term \( \frac{1}{g_{mb}} \)

High Frequency: \[ Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_{LS}s} + \frac{g_m}{C_{GS}C_{LS}s^2} \]

Series combination of \( C_{gs} \) and \( C_{L} \) and a negative resistance term \( -\frac{g_m}{C_{gs}C_{L}\omega^2} \)

The negative resistance term can be utilized in oscillator design
Common-Drain Amp Output Impedance

- Pole at very high frequency
- Zero at potentially low frequency if $R_S$ is large
  - Impedance can increase with frequency, i.e. display inductive behavior

\[ Z_{out} = \frac{V_X}{I_X} = \frac{R_SC_{GS}s + 1}{g_m + C_{GS}s} \]
Common-Drain Amp Output Impedance

\[ Z_{out} = \frac{V_x}{I_x} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s} \]

\[ R_2 = \frac{1}{g_m} \]

\[ R_1 = R_S - \frac{1}{g_m} \]

\[ L = \frac{C_{GS}}{g_m} \left( R_S - \frac{1}{g_m} \right) \approx \frac{R_S C_{GS}}{g_m} \text{ if } R_S >> \frac{1}{g_m} \]
Transient Behavior w/ Large $C_L$

- Inductive output impedance in combination with a large load capacitance can create undesired “ringing” in the transient response.
- If we have a large $RS$ and $CL$, then the assumption that we have one dominant pole is no longer valid.
- Both poles (potentially complex) should be considered in the analysis.

\[
\frac{V_{out}(s)}{V_{in}} = \frac{g_m + C_{GS}s}{R_s(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_mR_sC_{GD} + C_L + C_{GS})s + g_m}
\]
Common-Gate Amp Low Frequency Response

- **No $R_S$**

  Neglecting transistor $r_o$

  \[
  \frac{v_{out}}{v_{in}} = (g_m + g_{mb})R_D
  \]

- **With $R_S$**

  \[
  \frac{v_{out}}{v_a} \text{ is given from left}
  \]

  How to get from $v_{in}$ to $v_a$?

  Use amplifier input impedance and voltage divider

  \[
  R_{in} = \frac{1}{g_m + g_{mb}}
  \]

  \[
  v_a = \frac{1}{g_m + g_{mb}}v_{in} = \frac{1}{1 + (g_m + g_{mb})R_S}v_{in}
  \]

  \[
  v_{out} = \frac{v_a}{v_{in}}v_{out} = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \approx \frac{R_D}{R_S} \text{ if } R_S \text{ is large}
  \]
Common-Gate Amp Frequency Response

No zero
No Miller capacitor multiplication
Low input impedance limits effectiveness as a voltage amplifier
Useful as a current-to-voltage (transimpedance) amplifier

\[
\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{1 + \left(\frac{C_S}{g_m + g_{mb} + R_S^{-1}s}\right) (1 + R_D C_{DS}s)}
\]

\[C_D = C_{db} + C_{gd}\]
Cascode Amp Frequency Response

- If we associate the poles with the nodes A, X, and Y
  - Note, this is only an approximation, as it ignores interactions caused by “feedforward” caps ($C_{gd}$) and resistors
- 3 pole system

\[
\omega_{p,A} = \frac{1}{R_s \left[ C_{GS1} + \left( 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) C_{GD1} \right]} \\
\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}} \\
\omega_{p,Y} = \frac{1}{R_d(C_{DB2} + C_L + C_{GD2})}
\]
Neglecting $C_{GD1}$ and $C_Y$

$$Z_{out} = r_{o2} + Z_X + g_{m2}r_{o2}Z_X$$

where $Z_X = r_{o1} \frac{1}{sC_X}$

**Output Impedance Pole**

$$\omega_{Zout} = \frac{1}{r_{o1}C_X}$$
Next Time

• Differential Amplifiers