Announcements & Agenda

- HW2 Due Mar 6
- Reading
  - Razavi Chapter 4
- Single-ended and differential signals
- Differential pair
- Differential amplifiers
Single-Ended & Differential Signals

- A single-ended signal is measured with respect to a fixed potential (ground)
- A differential signal is measured between two equal and opposite signals which swing around a fixed potential (common-mode level)
- You can decompose differential signals into a differential mode (difference) and a common-mode (average)

\[
V_{DM} = V_{out}^+ - V_{out}^- \quad V_{CM} = \frac{V_{out}^+ + V_{out}^-}{2}
\]
Single-Ended & Differential Amplifiers

- Differential signaling advantages
  - Common-mode noise rejection
  - Higher (ideally double) potential output swing
  - Simpler biasing
  - Improved linearity
- Main disadvantage is area, which is roughly double
  - Although, to get the same performance in single-ended designs, we often have to increase the area dramatically

Max Output Swing

\[ V_{DD} - (V_{GS} - V_{Tn}) \]

Max Output Swing

\[ 2(V_{DD} - (V_{GS} - V_{Tn})) \]
Common-Mode Level Sensitivity

- A design which uses two single-ended amplifiers to realize a differential amplifier is very sensitive to the common-mode input level.
- The transistors’ bias current and transconductance can vary dramatically with the common-mode input.
  - Impacts small-signal gain
  - Changes the output common-mode, which impacts the maximum output swing.
An improved differential amplifier topology utilizes a “tail” current source to keep the transistor bias current ideally constant over the common-mode input range.

- Allows for a constant small-signal gain and output common-mode level
  - Note, you still have to have keep the input pair and tail current source transistors in saturation

\[
A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = -g_m R_D
\]

where \( g_m = g_{m1} = g_{m2} \) and \( R_D = R_{D1} = R_{D2} \)

\[
V_{O,CM} = V_{DD} - \left( \frac{I_{SS}}{2} \right) R_D
\]
Differential Pair Input-Output Characteristics

- For large-signal differential inputs, the maximum output levels are well defined and ideally independent of the input common-mode.
- For small-signal differential inputs, the small-signal gain is maximum at low-input signal levels.
  - As the differential input level increases, the circuit becomes more nonlinear and the gain decreases.
**Objective:** Find the differential transistor current \((I_{D1}-I_{D2})\) as a function of the input differential voltage \((V_{in1}-V_{in2})\)

**Input Voltage Difference:**

\[
V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = (V_{GS1} - V_T) - (V_{GS2} - V_T)
\]

\[
V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} W}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} W}}
\]

Squaring both sides and using \(I_{D1} + I_{D2} = I_{SS}\)

\[
(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox}} \left(I_{SS} - 2\sqrt{I_{D1}I_{D2}}\right)
\]

\[
\frac{1}{2} \mu_n C_{ox} \left(V_{in1} - V_{in2}\right)^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}
\]

Squaring both sides and using \(4I_{D1}I_{D2} = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2 = I_{SS}^2 - (I_{D1} - I_{D2})^2\)

\[
(I_{D1} - I_{D2})^2 = -\frac{1}{4} \left(\mu_n C_{ox} \frac{W}{L}\right)^2 (V_{in1} - V_{in2})^4 + I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2
\]

\[
I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W} - (V_{in1} - V_{in2})^2}
\]

*Note, this equation is only valid for a certain maximum input differential voltage \((V_{in1}-V_{in2})\)*
The differential current is an odd function of the differential input voltage which increases linearly for small inputs.

For large differential input voltages, the output differential current compresses due to the sqrt term.

The differential output current maxes out when all the current flows through one transistor at $\Delta V_{in1}$. 

\[ I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \left( \sqrt{\frac{4I_{ss}}{\mu_n C_{ox}} \frac{W}{L}} - (V_{in1} - V_{in2})^2 \right) \]

\[ \Delta V_{in1} = \frac{\sqrt{2I_{ss}}}{\sqrt{\mu_n C_{ox} \frac{W}{L}}} \]
Differential Pair I-V Characteristics (Large Signal)

For the maximum current range consider the case when all current flows through M1

\[ I_{D1} - I_{D2} = I_{D1} - 0 = I_{SS} \Rightarrow I_{D1} = I_{SS} \]

For \( I_{D2} = 0 \), ideally \( V_{GS2} = V_T \)

\[ \Delta V_{in} = V_{GS1} - V_{GS2} = V_{GS1} - V_T = \Delta V_{in1} \]

At this \( \Delta V_{in1} \), M1 must support all of \( I_{SS} \)

Maximum Differential Input: \( \Delta V_{in1} = V_{GS1} - V_T = \sqrt{ \frac{2I_{SS}}{W} } \sqrt{\mu_n C_{ox} \frac{W}{L}} \)

We can relate this to the zero differential input overdrive

Zero Differential Input Overdrive: \( (V_{GS} - V_T)_{1,2} = \sqrt{ \frac{I_{SS}}{W} } \sqrt{\mu_n C_{ox} \frac{W}{L}} = \frac{\Delta V_{in1}}{\sqrt{2}} \)

- The differential output current will saturate if the differential input voltage exceeds \( \sqrt{2} \) times the equilibrium input overdrive voltage.
Differential Pair Transconductance

\[ I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{ss}}{W}} - (V_{in1} - V_{in2})^2 \]

Define \( \Delta I_D = I_{D1} - I_{D2} \) and \( \Delta V_{in} = V_{in1} - V_{in2} \)

\[ G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\mu_n C_{ox}}{L} \sqrt{\frac{4I_{ss}}{W}} - \Delta V_{in}^2 \]

The small-signal transconductance at \( \Delta V_{in} = 0 \) is

\[ G_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{ss}} \]

Considering the load resistors \( R_D \), the small-signal gain is

\[ |A_v| = G_m R_D = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{ss} R_D} \]

- The differential pair transconductance and gain is maximum near zero input differential voltage
Differential Pair Small-Signal Analysis
Method 1 - Superposition

Find $V_{out}(V_{in1})$

Note that $g_{m1} = g_{m2} = g_m$ and $R_{D1} = R_{D2} = R_D$

$$\frac{V_X}{V_{in1}} = -\frac{g_{m1}R_{D1}}{1 + \frac{g_{m1}}{g_{m2}}} = -\frac{g_mR_D}{2}$$

- The X output from Vin1 is modeled as a degenerated CS amplifier
Differential Pair Small-Signal Analysis
Method 1 - Superposition

Find $V_{out}(V_{in1})$

Note that $g_{m1} = g_{m2} = g_m$ and $R_{D1} = R_{D2} = R_D$

$$
\frac{V_Y}{V_{in1}} = \frac{g_{m2}R_{D1}}{1 + \frac{g_{m2}}{g_{m1}}} = \frac{g_mR_D}{2}
$$

- The Y output from Vin1 is modeled as a Thevenin equivalent driving a CG amplifier
Differential Pair Small-Signal Analysis
Method 1 - Superposition

To find the total $V_{out}|_{V_{in1}}$

$$V_{out}|_{V_{in1}} = (V_X - V_Y)|_{V_{in1}} = \left(\frac{-g_{m1}R_{D1}}{1 + \frac{g_{m1}}{g_{m2}}} - \frac{g_{m2}R_{D1}}{1 + \frac{g_{m2}}{g_{m1}}}\right) V_{in1} = \left(\frac{-g_mR_D}{2} - \frac{g_mR_D}{2}\right) V_{in1} = -g_mR_D V_{in1}$$

From the circuit symmetry, $V_{out}|_{V_{in2}} = -V_{out}|_{V_{in1}}$

$$V_{out}|_{V_{in2}} = (V_X - V_Y)|_{V_{in2}} = g_mR_D V_{in2}$$

Differential Gain:

$$\frac{(V_X - V_Y)_{tot}}{V_{in1} - V_{in2}} = -g_mR_D \frac{(V_{in1} - V_{in2})}{V_{in1} - V_{in2}} = -g_mR_D$$
Differential Pair Small-Signal Analysis
Method 2 – Half Circuit

- The symmetric differential pair can be modeled as a Thevenin equivalent to observe how the tail node P changes with the differential input signal.
- If $R_{T1}=R_{T2}$ and the input is a truly differential signal, node P remains constant.
- This allows the tail node to be treated as a “virtual ground”
1. KVL around the input networks

\[ v_{in1} - v_1 = v_P = v_{in2} - v_2 \]

2. KCL at node P

\[ g_{m1} v_1 + g_{m2} v_2 = 0 \]

For small signals \( g_{m1} = g_{m2} \)

\[ v_1 = -v_2 \]

For differential operation \( v_{in1} = -v_{in2} \), and using the above KVL

\[ 2v_{in1} = 2v_1 \]

which implies that

\[ v_P = v_{in1} - v_1 = 0 \]
Differential Pair Small-Signal Analysis
Method 2 – Half Circuit

- Applying the virtual ground concept allows modeling as two “half circuits”

\[
\begin{align*}
V_X &= -g_m R_D \\
V_Y &= -g_m R_D \\
\left(\frac{V_X - V_Y}{V_{in1} - (-V_{in1})}\right) &= -\frac{2g_m R_D V_{in1}}{2V_{in1}} = -g_m R_D
\end{align*}
\]
Differential Pair Common-Mode Response

- Ideally, a differential amplifier completely rejects common-mode signals, i.e. $A_{v,CM}=0$
- In reality, the finite tail current source impedance results in a finite common-mode gain

\[
A_{v,CM} = \frac{V_{out}}{V_{in,CM}} = -\frac{2g_m \left( \frac{R_D}{2} \right)}{1 + 2g_m R_{SS}} = -\frac{g_m R_D}{1 + 2g_m R_{SS}}
\]
Differential Pair with Diode Loads

While the gain of this amplifier is relatively small, it is somewhat predictable, as it is defined by the ratio of the transistor sizes and the n/p mobility.

Assuming $\gamma = 0$

$$A_v = -\frac{g_{m1}}{g_{o1} + g_{m3} + g_{o3}} \approx -\frac{g_{m1}}{g_{m3}}$$

$$A_v \approx -\frac{g_{m1}}{g_{m3}} = -\frac{\sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{SS}}}{\sqrt{\mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{SS}}} = -\frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}$$
Differential Pair w/ Current-Source Loads

While the gain of this amplifier is higher, it is somewhat unpredictable, as it is defined by the transistor output resistance, which changes dramatically with process variations.

Assuming $\gamma = 0$

$$A_v = -\frac{g_{m1}}{g_{o1} + g_{o3}}$$
Differential Pair w/ Diode & Parallel Current-Source Loads

Adding a parallel current source to a diode connected load allows for increased gain which is still somewhat predictable.

\[
A_v = -\frac{g_{m1}}{g_{o1} + g_{m3} + g_{o3} + g_{o5}} \approx -\frac{g_{m1}}{g_{m3}} + \frac{\sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1} I_{ss}}{\sqrt{\mu_p C_{ox} \left(\frac{W}{L}\right)_3 (1-\alpha) I_{ss}}}
\]

Assuming \( \gamma = 0 \)

where \( \alpha \) is the current percentage that the current source "steals" from the diode load.
Cascode Differential Pair

Assuming $\gamma = 0$

$$A_v = -g_{m1} \left( (r_{o3} + r_{o1} + g_{m3} r_{o1} r_{o3}) \parallel (r_{o5} + r_{o7} + g_{m5} r_{o7} r_{o5}) \right)$$

$$A_v \approx -g_{m1} \left( g_{m3} r_{o1} r_{o3} \parallel g_{m5} r_{o7} r_{o5} \right)$$

- Using a cascode differential pair and cascode current-source loads allows for a considerable increase in gain
- However, a relatively large power supply may be required to supply the necessary voltage “headroom” to keep all the transistors in saturation
Next Time

- Simple OTA