ECEN474/704: (Analog) VLSI Circuit Design
Spring 2018

Lecture 11: Noise

1876

Sam Palermo
Analog & Mixed-Signal Center
Texas A&M University

Announcements

- HW3 is due Mar. 27

- Exam dates have changed
  - Exam 2 is on Apr. 10
  - Exam 3 is on May 3 (3PM-5PM)

Agenda & References

- Noise Types
- Noise Properties
- Resistor Noise Model
- Diode Noise Model
- MOSFET Noise
- Filtered Noise
- OTA Noise Example

- Reading
  - Razavi Chapter 7

Noise Significance

- Why is noise important?
  - Sets minimum signal level for a given performance parameter
  - Directly trades with power dissipation and bandwidth
- Reduced supply voltages in modern technologies degrades noise performance

$$\textbf{Signal Power} \propto (\alpha Vdd)^2 \Rightarrow \textbf{SNR} = P_{sig} / P_{noise} \propto \left(\frac{\alpha Vdd}{V_{noise}}\right)^2$$

- Noise is often proportional to kT/C
  - Increasing capacitance to improve noise performance has a cost in increase power consumption for a given bandwidth

Interference Noise

- Interference "Man-Made" Noise
  - Deterministic signal, i.e. not truly "random"
    - Could potentially be modeled and predicted, but practically this may be hard to do
  - Examples
    - Power supply noise
    - Electromagnetic interference (EMI)
    - Substrate coupling
  - Solutions
    - Fully differential circuits
    - Layout techniques

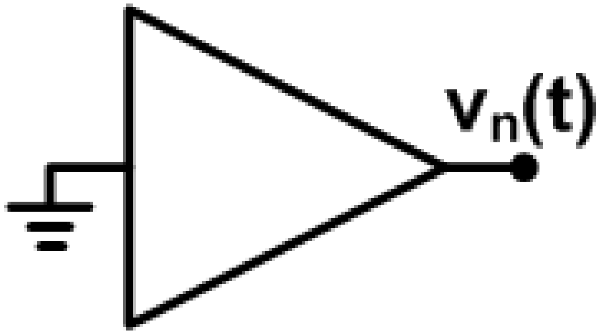
- Not the focus of this lecture
  - Unless the deterministic noise is approximated as a random process

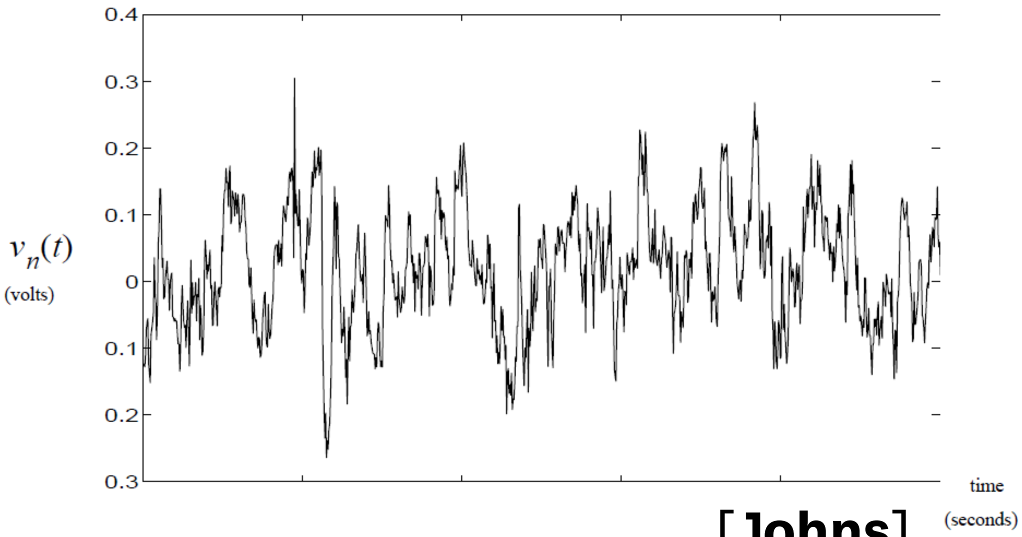
Inherent Noise

- “Electronic” or “Device” Noise
  - Random signal
  - Fundamental property of the circuits
  - Examples
    - Thermal noise caused by thermally-excited random motion of carriers
    - Flicker (1/f) noise caused by material defects
    - Shot noise caused by pulses of current from individual carriers in semiconductor junctions
  - Solutions
    - Proper circuit topology
    - More power!!!

- Is the focus of this lecture

Noise Properties





[Johns]

- Noise is **random**
  - Instantaneous noise value is unpredictable and the noise must be treated statistically
  - Can only predict the average noise power
  - Model with a Gaussian amplitude distribution
  - Important properties: mean (average), variance, power spectral density (noise frequency spectrum)

RMS Value

- If we assume that the noise has zero mean (generally valid)
- RMS or “sigma” value is the square-root of the noise variance over a suitable averaging time interval, T

$$\boxed{V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)\,dt\right]^{1/2}}$$

- Indicates the **normalized noise power**, i.e. if $v_n(t)$ is applied to a 1Ω resistor the average power would be

$$P_n = \frac{V_{n(rms)}^2}{1\Omega} = V_{n(rms)}^2$$

Signal-to-Noise Ratio (SNR)

$$\boxed{SNR \equiv 10 \log \left[\frac{\textbf{signal power}}{\textbf{noise power}} \right]}$$

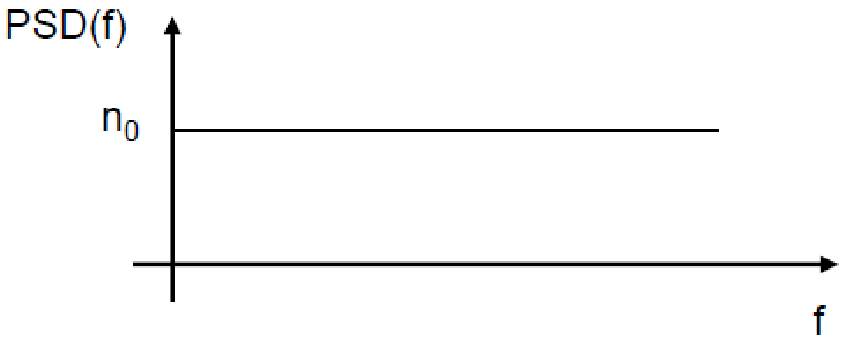
For a signal with normalized power of $\mathbf{V}^2_{\mathbf{x(rms)}}$

$$SNR \equiv 10 \log \left[\frac{V^2_{x(rms)}}{V^2_{n(rms)}} \right] = 20 \log \left[\frac{V_{x(rms)}}{V_{n(rms)}} \right]$$

- Quantified in units of dB

Thermal Noise Spectrum

- The power spectral density (PSD) quantifies how much power a signal carries at a given frequency
- Thermal noise has a uniform or "white" PSD



- The total average noise power P_n in a particular frequency band is found by integrating the PSD

$$\boxed{P_n = \int_{f_1}^{f_2} PSD(f)\,df}$$

For white noise spectrum : $P_n = n_0(f_2 - f_1) = n_0 \Delta f$

Thermal Noise of a Resistor

- The noise PSD of a resistor is

$$PSD(f) = n_0 = 4kT$$

where k is the Boltzmann constant and T is the absolute temperature (K)

- The total average power of a resistor in a given frequency band is

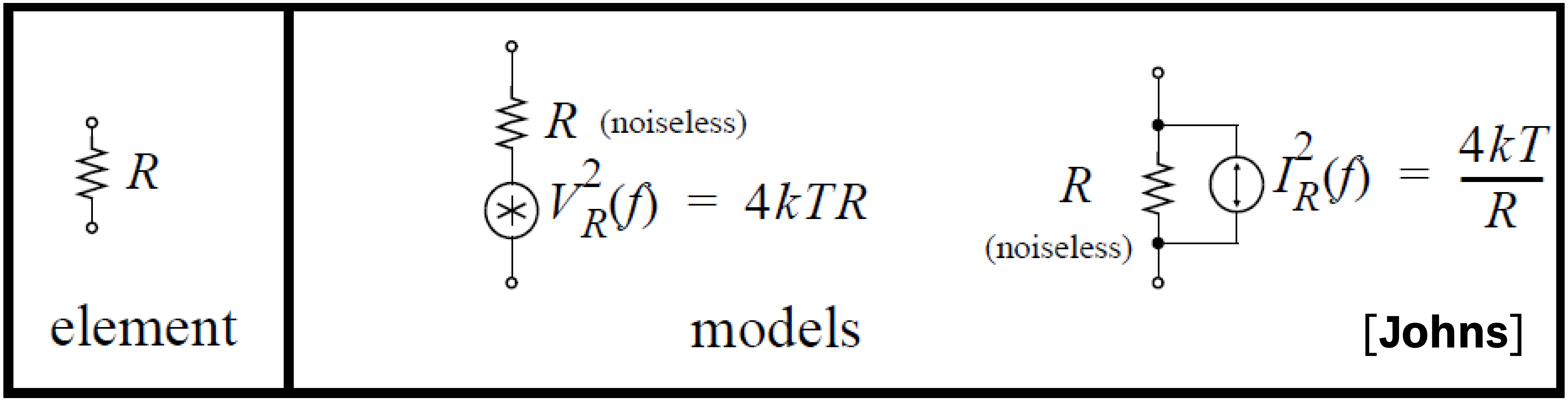
$$P_n = \int_{f_1}^{f_2} 4kTdf = 4kT(f_2 - f_1) = 4kT\Delta f$$

Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

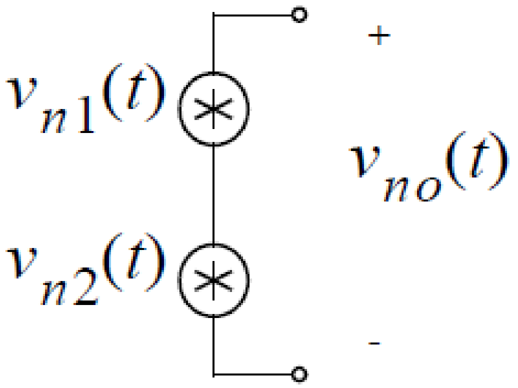
$$\boxed{V_{Rn}^2 = P_n R = 4kTR\Delta f}$$

$$\boxed{I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R}\Delta f}$$

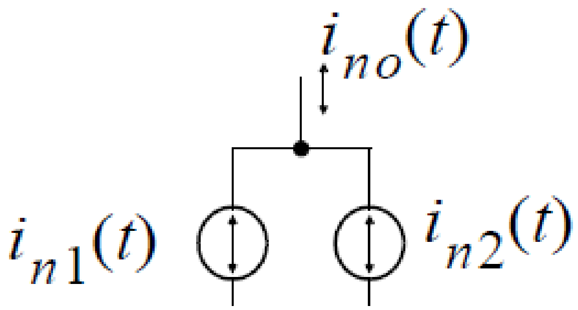


- Recall the PSD is white (uniform w/ frequency)

Noise Summation



Voltage



Current

$$v_{no}(t) = v_{n1}(t) + v_{n2}(t)$$

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\, dt$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t) v_{n2}(t)\, dt$$

- Same procedure applies to noise current summing at a node

Correlation

- Last term describes the correlation between the two signals, defined by the correlation coefficient, C

$$C = \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)dt}{V_{n1(rms)}V_{n2(rms)}}$$

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2CV_{n1(rms)}V_{n2(rms)}$$

- Correlation always satisfies -1≤C≤1
    - C=+1, fully-correlated in-phase (0°)
    - C=-1, fully-correlated out-of-phase (180°)
    - C=0, uncorrelated (90°)

Uncorrelated Signals

- For two uncorrelated signals, the mean-squared sum is given by

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2$$

Add as though they were vectors at right angles

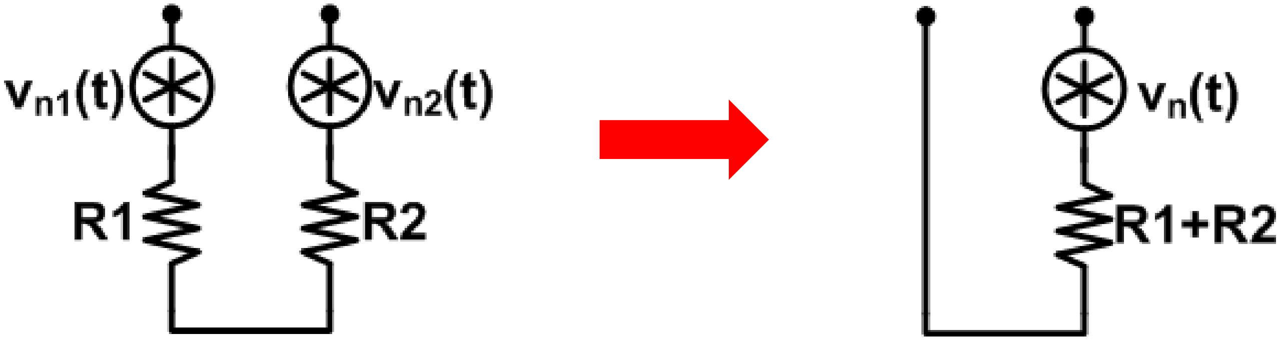
- For two fully correlated signals, the mean-squared sum is given by

$$V_{no(rms)}^2 = \left(V_{n1(rms)} \pm V_{n2(rms)}\right)^2$$

Sign is determined by phase relationship

RMS values add linearly (aligned vectors)

Noise Example #1: Two Series Resistors



$$v^2_{n(rms)} = v^2_{n1(rms)} + v^2_{n2(rms)} + 2Cv_{n1(rms)}v_{n2(rms)}$$

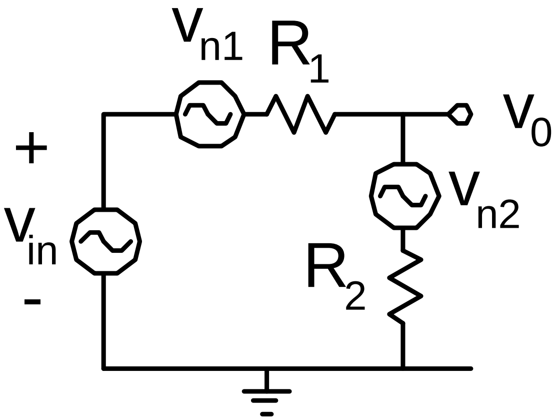
- The noise of the two resistors is uncorrelated or statistically independent, so C=0

$$v^2_{n(rms)} = v^2_{n1(rms)} + v^2_{n2(rms)} = 4kT\left(R_1 + R_2\right)\Delta f$$

- Always add independent noise sources using mean squared values
  - Never add RMS values of independent sources

Noise Example #2: Voltage Divider

- Lets compute the output voltage: **Apply superposition (noise sources are small signals, you can use small signal models)!**



$$v_0 = \left(\frac{R2}{R1+R2}\right) v_{in} + \left(\frac{R2}{R1+R2}\right) v_{n1} + \left(\frac{R1}{R1+R2}\right) v_{n2}$$

Above is what you do for deterministic signals, but we cannot do this for the resistor noise

But noise is a random variable, power noise density has to be used rather than voltage; then the output referred noise density (noise in a bandwidth of 1 Hz) becomes

$$v_{0n}^{2} = \left(\frac{R2}{R1+R2}\right)^2 v_{n1}^{2} + \left(\frac{R1}{R1+R2}\right)^2 v_{n2}^{2}$$

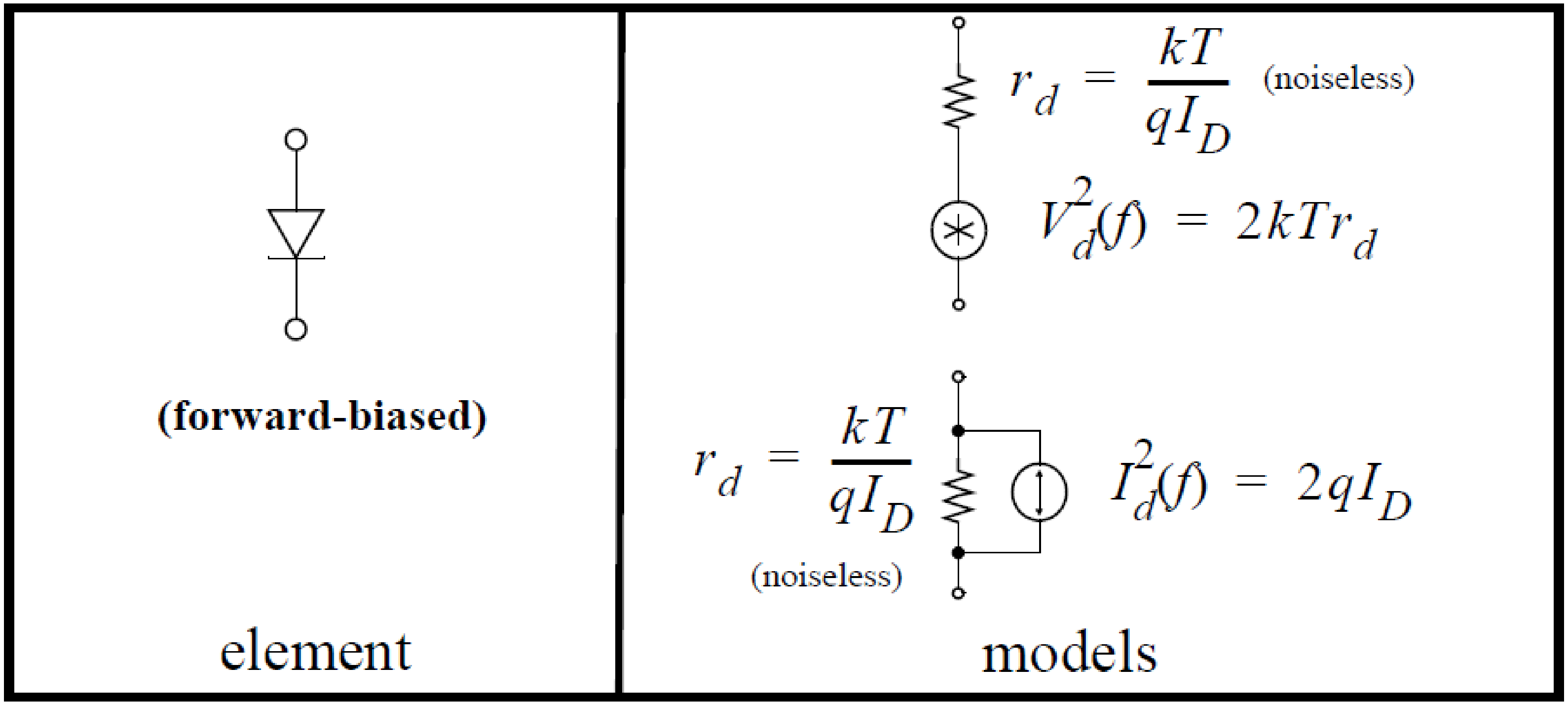
$$v_{0n}^{2} = \left(\frac{R2}{R1+R2}\right)^2 4kTR1 + \left(\frac{R1}{R1+R2}\right)^2 4kTR2$$

General Case :

$$v_{on,T}^{2}(f) = \sum_{x} \left|H_x(s)\right|^2_{s=j2\pi f} \; v_x^2(f)$$

Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions
- White spectral density

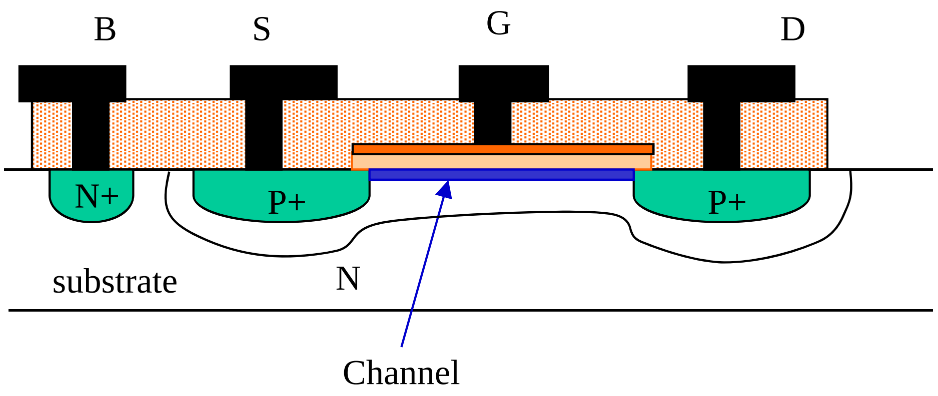


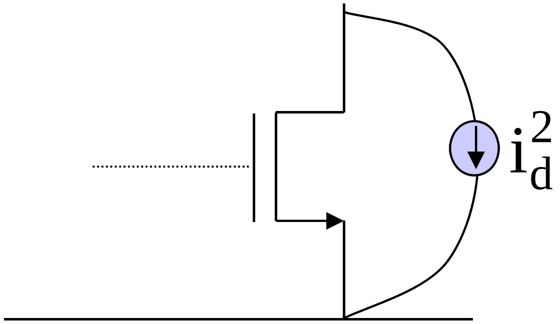
[Johns]

- Where $q=1.6\text{x}10^{-19}\text{C}$ and I_D is the diode DC current

Thermal Noise

=> Spectral Density of the thermal noise drain current (CMOS transistor biased @ linear region)



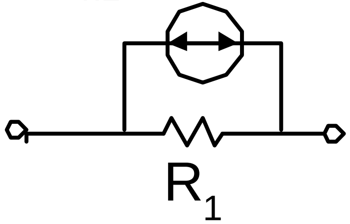


Transistor

$$\overline{i_d^2} = \frac{4kT}{R_{DS}}$$

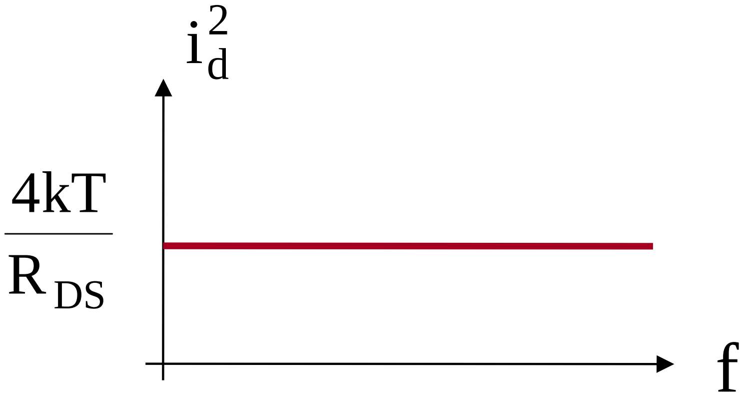
Resistor

$i_{n1}^2=4kT/R1$



$$R_{DS} \cong \frac{1}{\mu C_{ox} \frac{W}{L} \left(V_{GS} - V_T - V_{DS}\right)}$$

White Noise



@ Triode region

$$\overline{i_d^2} = \left[4kT\mu C_{ox}\right]\left[\frac{W}{L}\right]\left[V_{GS} - V_T - V_{DS}\right]$$

Low current noise => W/L ↓ => g_m or g_o ↓

@ Saturation

$$g_o = \frac{1}{R_{DS}} \rightarrow \frac{2}{3} g_m$$

$$\overline{i_d^2} = \frac{8}{3} kTg_m$$

$$\Rightarrow \quad \overline{i_d^2} = \left(\frac{8kT}{3}\right)\left(\mu C_{ox}\right)\left(\frac{W}{L}\right)\left(V_{GS} - V_T\right)$$

MOSFET 1/f (Flicker) Noise

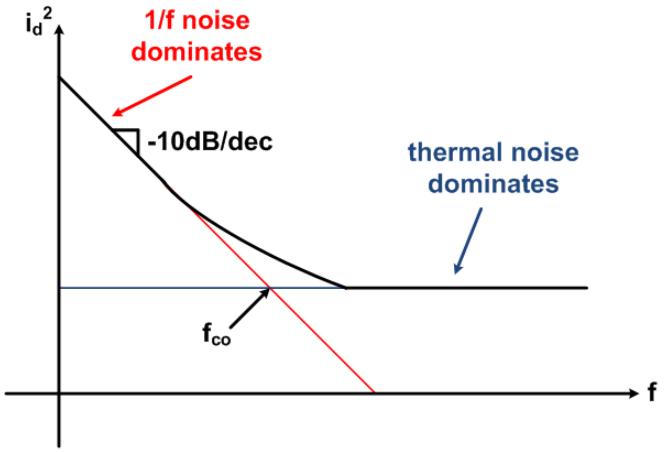
- Caused by traps near Si/SiO$_2$ interface that randomly capture and release carriers

$$\boxed{i_d^2(f) = \frac{K_F g_m^2}{W L C_{ox} f}}$$

- K$_F$ is strongly dependent on the technology

1/f Noise Corner Frequency

- This is the frequency at which the flicker noise density equals the thermal noise density



$$\frac{K_F g_m^2}{WLC_{ox} f_{co}} = 4kT\gamma g_m$$

$$f_{co} = \frac{K_F}{4kT\gamma C_{ox}} \frac{g_m}{WL} = \frac{K_F}{4kT\gamma C_{ox}} \frac{1}{L} \left(\frac{g_m}{I_D}\right)\left(\frac{I_D}{W}\right)$$

- For a given g_m/I_D (which sets I_D/W), the only way to reduce f_{co} is to use longer channel devices

Output and input referred noise

Current noise is the real one

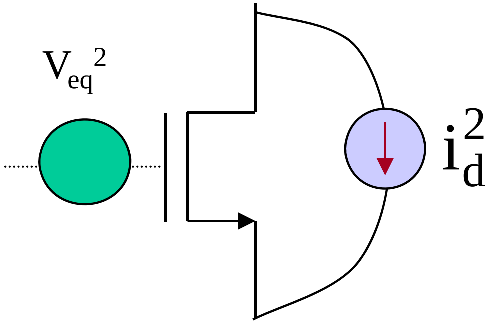
Thermal Noise

$$i_d = g_m V_{gs}$$

$$i_d^2 = g_m^2 V_{gs}^2$$

Voltage noise representation is an artifact to facilitate system analysis

$$\Rightarrow \quad V_{gseq}^2 = \frac{8}{3}\frac{kT}{g_m}$$



Flicker Noise

$$i_d^2 = \frac{K_F g_m^2}{WLC_{ox} f}$$

Referred to the input

$$v_{eq}^2 = \frac{K_F g_m^2}{WLC_{ox} f}\frac{1}{g_m^2}$$

$$v_{eq}^2 = \frac{K_F}{C_{ox}}\left(\frac{1}{WL}\right)\left(\frac{1}{f}\right)$$

Equivalent input referred voltage noise

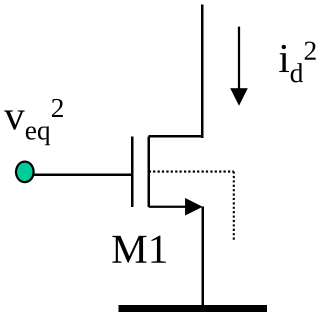
$$V_{eq}^2 = \frac{i_{dth}^2 + i_{df}^2}{g_m^2}$$

Equivalent input referred noise voltage means that all **current noise sources are accounted as drain current and represented by an “equivalent” noise voltage at transistor gate**

$$V_{eq}^2 = \frac{8}{3}\frac{kT}{g_m} + \frac{K_F}{C_{ox}}\frac{1}{WL}\frac{1}{f}$$

$$V_{eq_{total}}\left(\mathrm{RMS}\right) = \sqrt{\int_{BW} v_{eq}^2(f)\,df}$$

NOISE COMPONENTS (values provided are for a 0.8 μm technology)



Noise density (V^2/Hz)

Spice model

$$v_{eq}^2 = v_{th}^2 + v_{1/f}^2$$

$$v_{eq}^2 = \frac{8}{3}\frac{kT}{g_m}df + \frac{K_F}{WLC_{OX}f}df$$

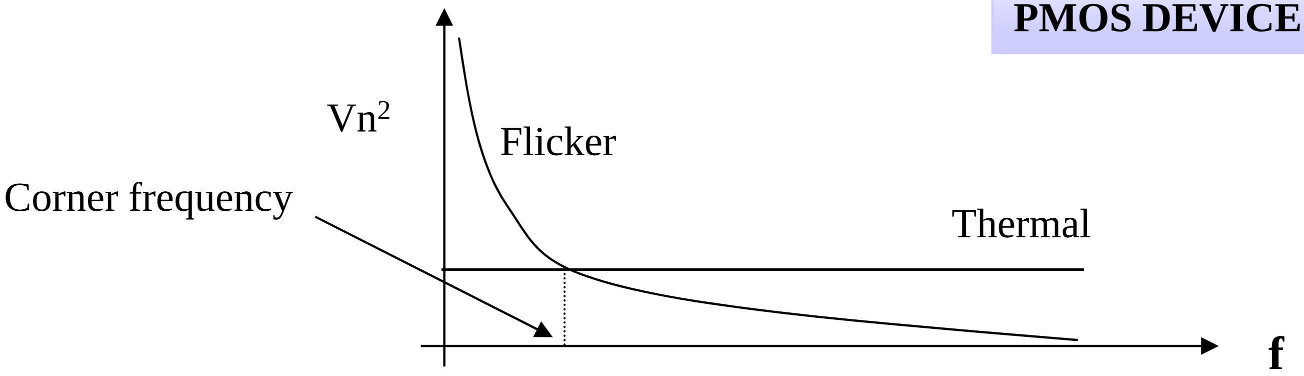
$$\frac{K_F}{C_{OX}} = 9.8x10^{-9}\,V^2/\mu m - Hz(NMOS)$$

$$= 0.5x10^{-9}\,V^2/\mu m - Hz(PMOS)$$

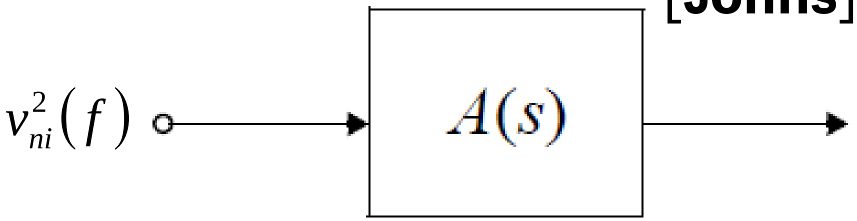
FOR LOW-FREQUENCY APPLICATIONS,

WHEREIN 1/F NOISE IS DOMINANT,

PMOS DEVICES MUST BE USED.



Filtered Noise

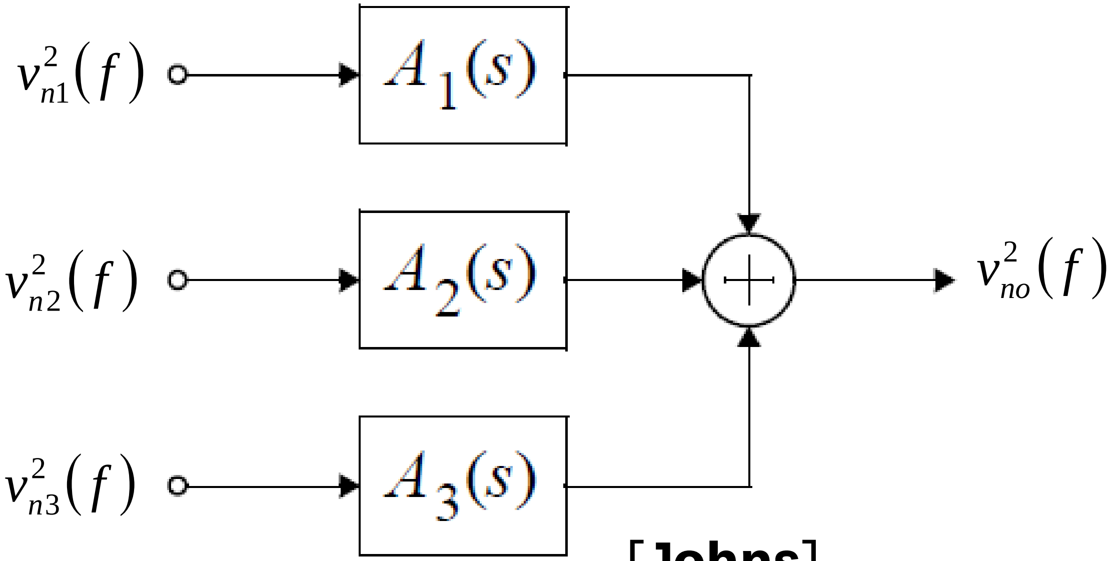


[Johns]

$$v_{no}^2(f) = |A(j2\pi f)|^2 v_{ni}^2(f)$$

$$v_{no}(f) = |A(j2\pi f)| v_{ni}(f)$$

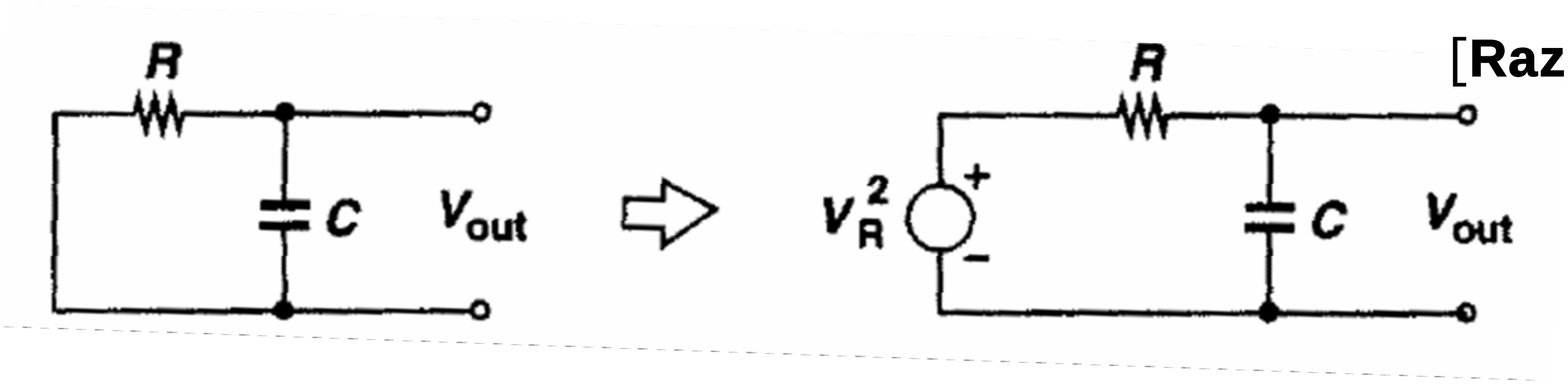
- Noise output spectral density is a function only of the magnitude of the transfer function, and not its phase
- With multiple uncorrelated noise sources, combined output is also uncorrelated



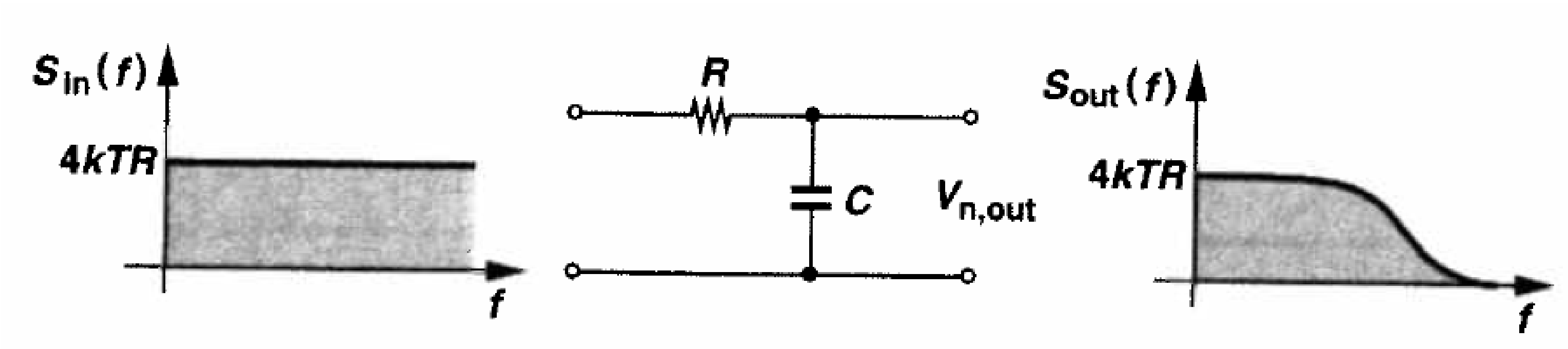
$$v_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 v_{ni}^2(f)$$

[Johns]

First-Order RC Circuit Example

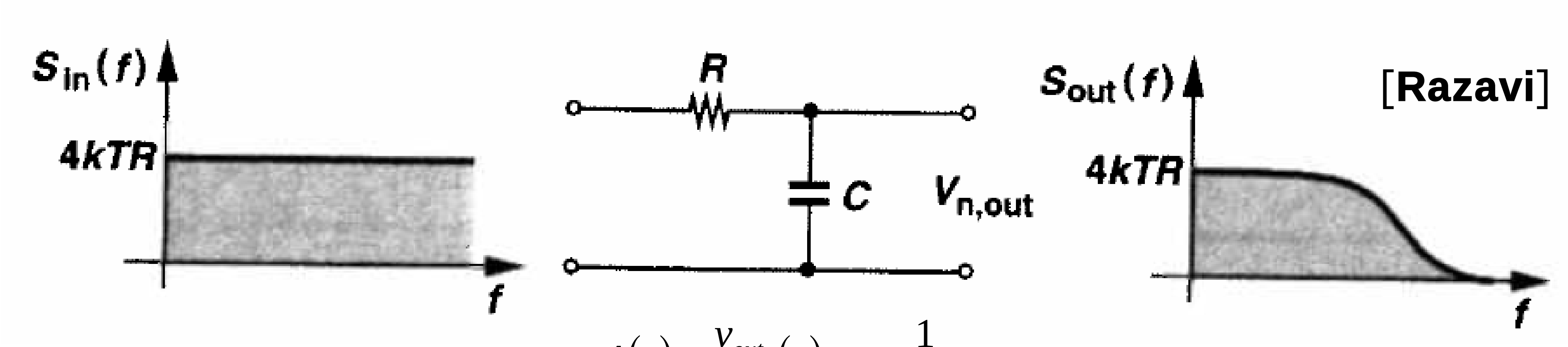


[Razavi]



What is the total output noise power?

First-Order RC Circuit Example



[Razavi]

$$A(s) = \frac{v_{out}}{v_R}(s) = \frac{1}{1+sRC}$$

$$v_{out}^2(f) = \left|A(j2\pi f)\right|^2 v_R^2(f) = \frac{1}{1+4\pi^2 f^2 R^2 C^2} 4kTR$$

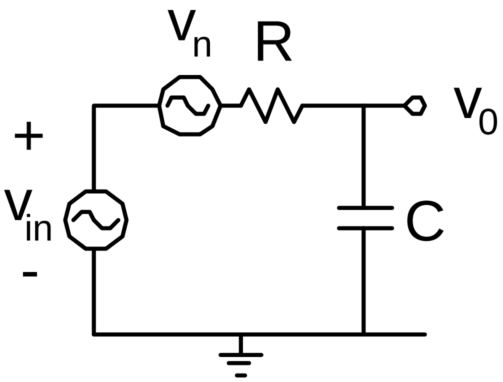
To calculate Total Noise Power integrate over all frequencies

$$v_{out}^2 = \int_0^{\infty} \frac{4kTR}{1+4\pi^2 f^2 R^2 C^2}$$

Using $\int \frac{dx}{x^2+1} = \tan^{-1} x$

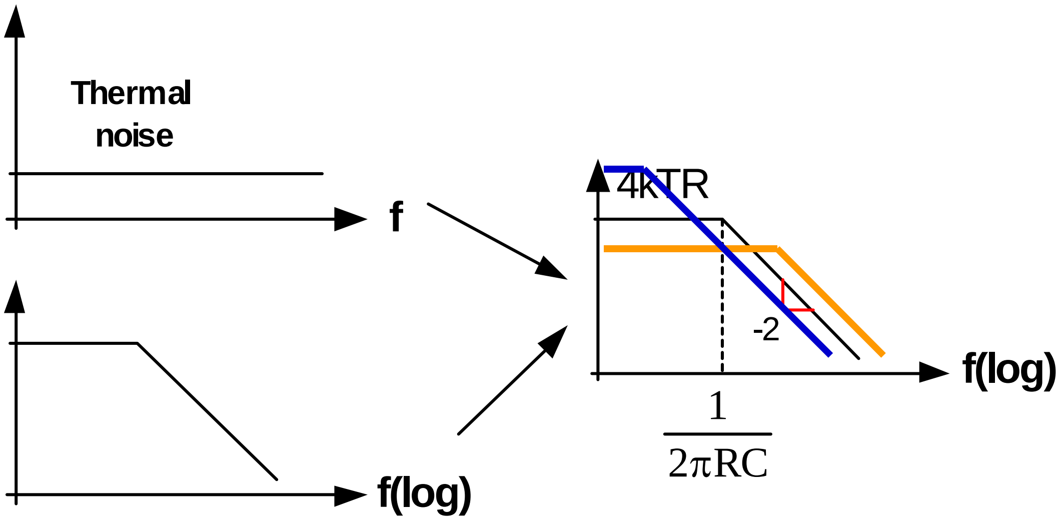
$$v_{out}^2 = \frac{2kT}{\pi C} \tan^{-1}(2\pi fRC)\Big|_{f=0}^{f=\infty} = \frac{2kT}{\pi C}\left[\frac{\pi}{2} - 0\right] = \frac{kT}{C}$$

Noise is generated by R but integrated noise is function of C (??)



$$v_{total}^{2} = \int_{0}^{\infty} \left(\left(\frac{1}{1+(\omega RC)^{2}} \right) (4kTR) \right) df = \frac{kT}{C}$$

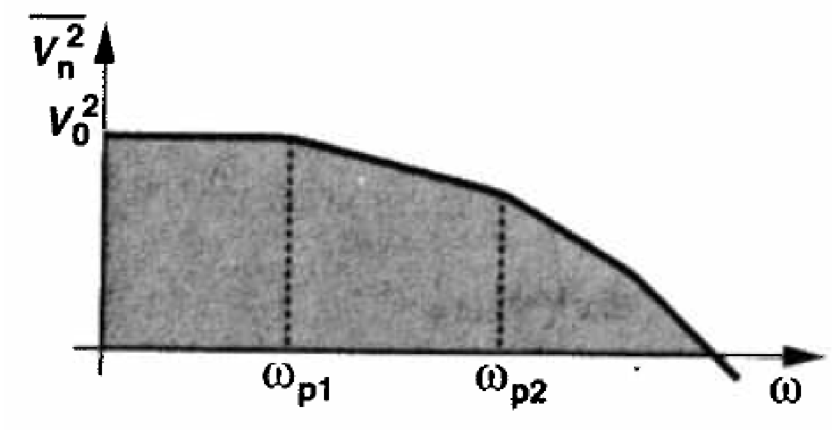
To get more insight, lets have a closer look on the operations!

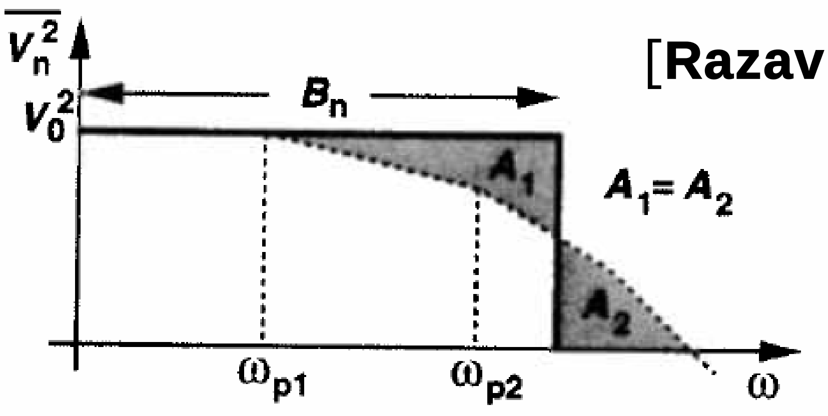


Notice that:

When R increases thermal noise increases too but the corner frequency decreases, leading to a constant area under the curves!

Noise Bandwidth





[Razavi]

- The noise bandwidth is equal to the frequency span of a brickwall filter having the same output noise rms value

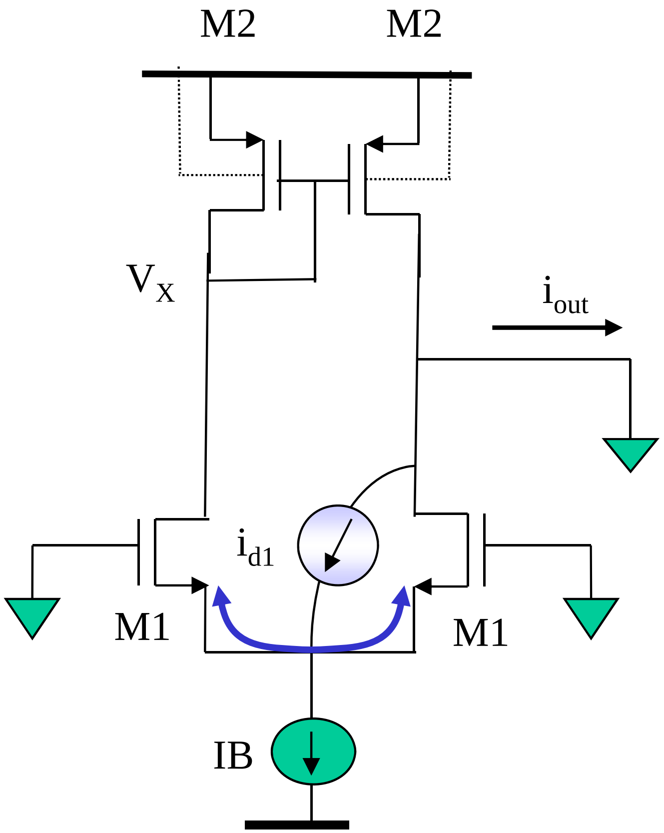
$$v_0^2 B_n = \int_0^\infty v_{no}^2 \, df$$

$$\textbf{For a first-order filter } B_n = \frac{\pi}{2}\omega_p$$

Validating with previous slides derivation :

$$\textbf{Total Noise Output} = v_0^2 B_n = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right) = \frac{kT}{C}$$

Output referred noise: Take advantage of SYMMETRIES!



Noise injected into the common-source node equally splits into the two branches

Output referred current noise density

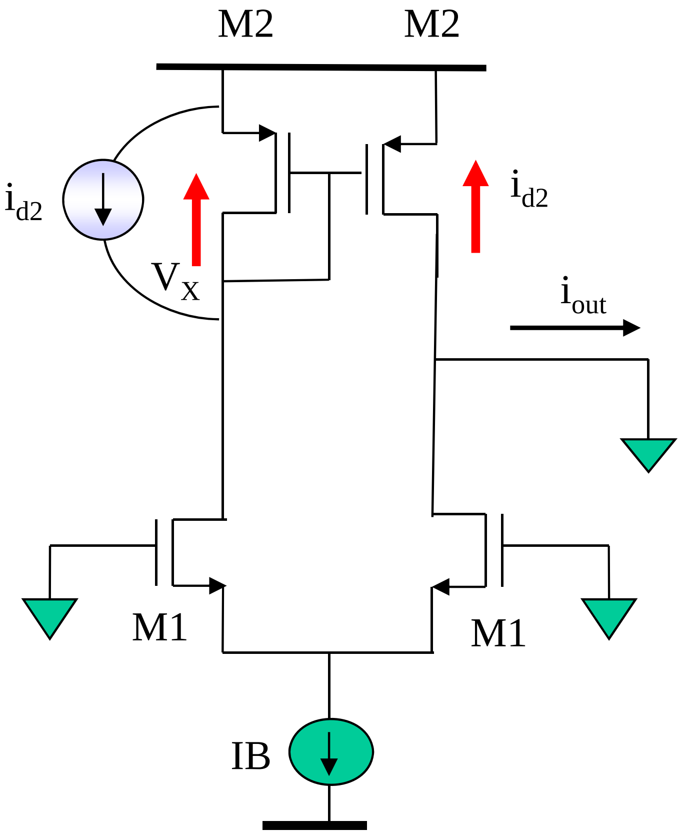
Superposition: Every transistor contributes; consider one at the time.

Analysis: You can use standard circuit analysis techniques but at the end of the day you have to consider POWER.

Output noise density: Each noise component represent the RMS value of random uncorrelated noise! Then add the power noise components

$$i_{out1}^{\;2} = \frac{8}{3} kT g_{m1}$$

Output referred noise: Take advantage of SYMMETRIES!



Noise injected into the common-source node equally splits into the two branches

This noise can usually be ignored, as ideally zero net current flows to the output

Output referred current noise density due to the P-type devices:

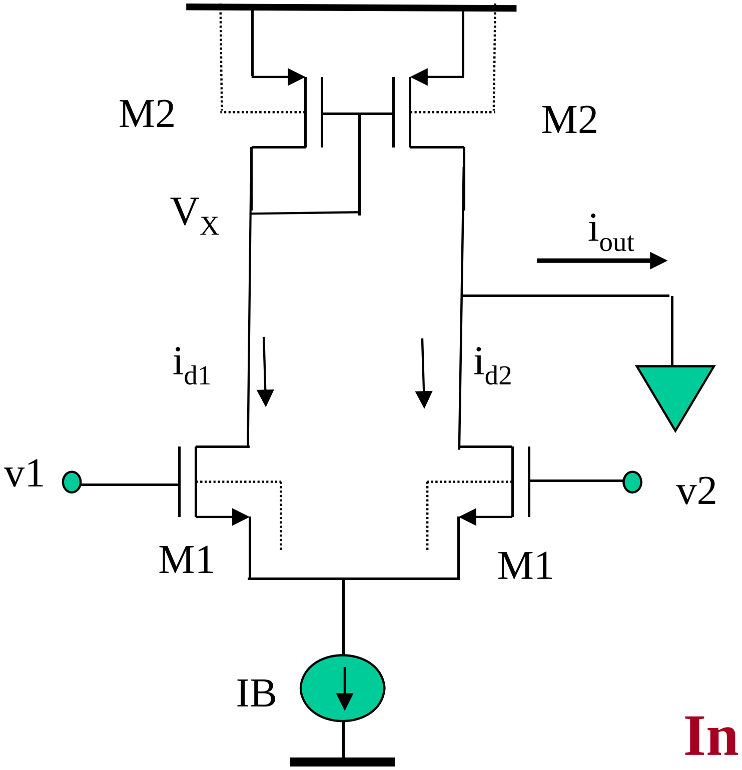
Left hand side transistor:

$$\overline{i_{out2}}^2 \cong \overline{i_{d2}}^2 = \frac{8}{3}kTg_{m2}$$

Right hand side transistor

$$\overline{i_{out2}}^2 = \frac{8}{3}kTg_{m2}$$

Output and input referred noise



Output referred current noise density

$$i_{out}^2 = 2\left(\frac{8}{3}kTg_{m1}\right) + 2\left(\frac{8}{3}kTg_{m2}\right)$$

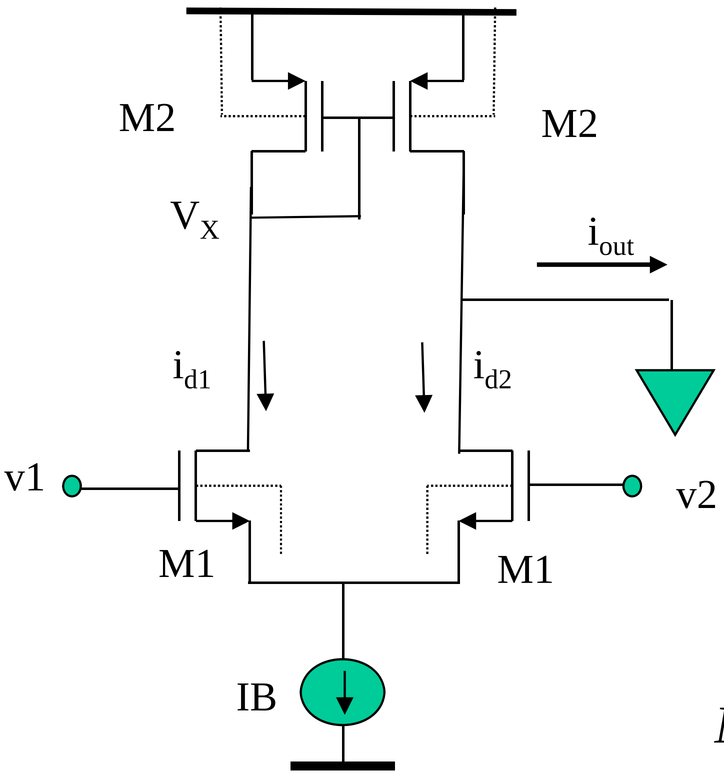
Input referred noise density (V^2/H_Z)

$$v_{in,eq}^2 = 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\right) + 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\frac{g_{m2}}{g_{m1}}\right)$$

In this case, noise due to the current source produces no net output noise

Be careful because this is not always the case!

Integrated Input referred noise



Input referred thermal noise density (V^2/Hz)

$$v_{in,eq}^{2} = 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\right) + 2\left(\frac{8}{3}\frac{kT}{g_{m1}}\frac{g_{m2}}{g_{m1}}\right)$$

Input referred noise level (volts)

$$\mathrm{Noise}(V_{RMS}) = \sqrt{\int_{BW} V_{in,eq}^{2}\,df}$$

Example: for thermal noise, the noise level becomes (assuming a single-pole system)

$$Noise(V_{RMS}) = \sqrt{\frac{16kT}{3}}\sqrt{\frac{1}{g_{m1}}}\sqrt{1+\frac{g_{m2}}{g_{m1}}}\left(\sqrt{\frac{\pi}{2}BW}\right)$$

I should advise you to use:

$$\mathrm{Noise}(V_{RMS}) \cong \sqrt{\frac{8kT}{g_{m1}}}\sqrt{1+\frac{g_{m2}}{g_{m1}}}\left(\sqrt{BW}\right)$$

$4kT \approx 16\times10^{-21}$ coul.V

Next Time

- Three Current Mirror OTA