ECEN474: (Analog) VLSI Circuit Design
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Lecture 16: Feedback & Stability

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Agenda

• Feedback in opamp circuits
• Stability Considerations
  • Nyquist Criteria
  • Phase & Gain Margin
If the OPAMP is not precise, how we can design accurate systems?

Answer is FEEDBACK!!!

Examples of our daily life
• Can you shave yourself closing your eyes?
• Can you drive your car closing your eyes?
• Can you adjust the supply voltages without a voltage indicator?
• Can you measure (without any equipment) the magnetic field generated by your cellular phone?
• Can you control properly your daily activities without feedback?

• If you measure the output at the time you apply the stimuli you can better control the system!!
Typical values for low-frequency opamps:
$A_V \sim 10^5$
$\omega_p \sim 100$ rad/sec
$r_0 \sim < 100$ Ohms

If $A_V \sim \infty$ then
$V_- = 0$  VIRTUAL GROUND

$\frac{v_0}{g_m} \leq CM$
If $A_V$ is finite then $v_-$ $\neq$ 0

$$i_1 = -i_2 \quad \text{if} \quad Z_{in} = \infty$$

or

$$\left( v_i - \frac{-v_0}{A_V} \right) Y_1 = \left( v_0 - \frac{-v_0}{A_V} \right) Y_2$$

$$A_V = \frac{A_{VDC}}{1 + \frac{s}{\omega_p}}$$

$R_{out} = 0$ and $Z_{in} = \infty$

Using the approximation $\frac{1}{1 + x} \approx 1 - x$ for small $x$

$$\text{Error} = -\frac{1}{A_V} \left( 1 + \frac{Z_2}{Z_1} \right) \approx -\frac{1 + \frac{s}{\omega_p}}{A_{DC}} \left( 1 + \frac{Z_2}{Z_1} \right)$$
**FEEDBACK:**

- If you measure the output at the time you apply the stimuli you can better control the system!!

**Definitions:**

- **A(s):** Amplifier gain (very large but not very well controlled)
- **B(s):** Feedback Factor (Very well controlled)
- **T(s)=A(s)B(s):** Loop Gain (Extremely important parameter!!)

**Applying Mason Rule:**

\[
\frac{v_o}{v_i} = \frac{\text{Direct trajectory}}{1 - \text{loop gain}} = \frac{A(s)}{1 - (-T(s))} = \frac{A(s)}{1 + A(s)B(s)}
\]
Feedback Configuration

Here $f = \text{feedback factor (B(s) in previous slides)}$

$$a(s) = \frac{V_o(s)}{V_\varepsilon(s)} = \frac{a_0}{1 - \frac{s}{p_1}}$$

$$A_{CL}(s) = \frac{V_o(s)}{V_i(s)} = \frac{a(s)}{1 + a(s)f} = \frac{a_0}{1 + a_0f} \frac{s}{1 - \frac{s}{(1 + a_0f)p_1}}$$
Gain-Bandwidth

Gain magnitude, dB

\[ 20 \log a_0 \]

\[ 20 \log \frac{a_0}{1+a_0f} \]

\[ |a(j\omega)| \]

\[ |A_{CL}(j\omega)| \]

\[ |p_1| \]

\[ (1+a_0f)|p_1| \]

\[ -20 \text{ dB/dec} \]

\[ \omega_t \]

\[ \omega \]

log scale
Instability and the Nyquist Criterion

Transfer function of a 3-pole amplifier:

\[ a(s) = \frac{a_0}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})(1 - \frac{s}{p_3})} \]

Nyquist criterion for stability of the amplifier:

Consider a feedback amplifier with a stable \( T(s) \). If the Nyquist plot of \( T(j\omega) \) encircles the point (-1,0), the feedback amplifier is unstable.

Recall \( T(s) \) is the loop gain

\[ T(s) = A(s)B(s) = a(s)f \]
Magnitude & Phase

3-pole amplifier

\[ |a(j\omega)|, \text{dB} \]

20 log \( a_0 \)

20 log \( a_{180} \)

0

[|p_1| |p_2| \( \omega_{180} \) |p_3|]

\( \omega \)

\( \omega \)

log scale

\( \angle a(j\omega) \)

-20 dB/dec

-40 dB/dec

-60 dB/dec

[Karsilayan]
Magnitude & Phase

\[ T(s) = a(s)f_1 \]

-20 dB/dec
-40 dB/dec
-60 dB/dec

log scale
Nyquist Plot

Frequency Sweep of Loop Gain, $T(s)$

$T(s) = a(s)f_1$

[Karsilayan]
Magnitude & Phase

\[ T(s) = a(s)f_2 \]

[Image of a graph showing magnitude and phase characteristics with labels and annotations.]
Nyquist Plot

\[ T(s) = a(s)f_2 \]

[Karsilayan]
Gain & Phase Margin

$|a(j\omega)|$, dB

$|T(j\omega)|=0$ dB

$\omega$ axis for $|T(j\omega)|$

Gain margin

$20 \log a_0$

$20 \log a_{180}$

$0$

$\angle a(j\omega), \angle T(j\omega)$

Phase margin

$20 \log \frac{1}{f}$

[Karsilayan]
Stability Criteria

Nyquist:

\[ |T(j\omega_{180})| = a_{180^\circ} f < 1 \Rightarrow \text{Stable} \]

Gain Margin (GM):

\[ GM = 20 \log \frac{1}{|T(j\omega_{180})|} = -20 \log |T(j\omega_{180})| \]

\[ GM > 0 \Rightarrow \text{Stable} \]

Phase Margin (PM):

\[ PM = 180^\circ + \angle T(j\omega_0) \]

\[ PM > 0 \Rightarrow \text{Stable} \]
Phase Margin

\[ |T(j\omega_0)| = 1 \Rightarrow |a(j\omega_0)|f = 1 \Rightarrow |a(j\omega_0)| = \frac{1}{f} \]

PM = 45° \quad \Rightarrow \quad \angle T(j\omega_0) = -135°, \quad A_{cl}(j\omega_0) = \frac{a(j\omega_0)}{1 + T(j\omega_0)}

\[ A_{cl}(j\omega_0) = \frac{a(j\omega_0)}{1 + e^{-j135^\circ}} = \frac{a(j\omega_0)}{1 - 0.7 - 0.7j} \]

\[ |A_{cl}(j\omega_0)| = \frac{|a(j\omega_0)|}{|0.3 - 0.7j|} = \frac{1}{0.76f} = 1.3 \]

PM = 30° \quad \Rightarrow \quad \angle T(j\omega_0) = -150°, \quad |A_{cl}(j\omega_0)| = 1.92/f

PM = 60° \quad \Rightarrow \quad \angle T(j\omega_0) = -120°, \quad |A_{cl}(j\omega_0)| = 1/f

PM = 90° \quad \Rightarrow \quad \angle T(j\omega_0) = -90°, \quad |A_{cl}(j\omega_0)| = 0.7/f
Closed-Loop Frequency Response

Relative Gain, dB

Relative Frequency

PM=30
PM=45
PM=60
PM=90
Closed-Loop Step Response

PM = 30°

PM = 45°

PM = 60°

PM = 90°
Non-Inverting Amplifier Example

If you want to amplify your signal: B(s) must be an attenuator (voltage divider!!)

The error is determined by the overall loop gain: $T(s) = A(s)B(s)$

Key points:

If $T(s) >> 1$, then $\frac{v_o}{v_i} \approx \frac{A(s)}{1 - (-T(s))}$

For Error, can write: $\frac{v_o}{v_i} = \frac{1}{B(s)} \left[ \frac{T(s)}{1 + T(s)} \right] = \frac{1}{B(s)} \left[ \frac{1}{1 + \frac{1}{T(s)}} \right]

Error $\propto \frac{1}{T(s)}$

$B(s) = \frac{Z_1}{Z_1 + Z_2} = \frac{1}{1 + \frac{Z_2}{Z_1}}$

$\frac{v_o(s)}{v_i(s)} \approx 1 + \frac{Z_2}{Z_1}$

$A(s)$ is amplifier response only

$A(s)$

Max Error

Error

$\omega_{max}$

$v_i$ $+$

$+$

$v_o$

$v_i - v_o B(s)$

$v_i$ $-$

$v_0$
Inverting Amplifier Example

Inverting Amplifier: Apply superposition

A(s) and B(s) are sharing some elements!!
A(s) = ? B(s) = ? T(s) = ?

\[ A(s) = \frac{Z_2 A_V}{Z_1 + Z_2} \]

From \( T(s) = A(s)B(s) \)

\[ B(s) = \frac{T(s)}{A(s)} = -\frac{Z_1}{Z_2} \]

\[ \frac{v_o(s)}{v_i(s)} \approx \frac{1}{B(s)} = -\frac{Z_2}{Z_1} \]

\[ \frac{v_0}{v_i} = \frac{A(s)}{1 + T(s)} = -\frac{Z_2 A_V}{1 + \frac{Z_1 A_V}{Z_1 + Z_2}} = -\frac{Z_2}{Z_1} \left(1 + \frac{Z_2}{\frac{Z_1}{A_V}}\right) \]
Inverting Amplifier: consider the non-zero output impedance!!

\[ A'(s) = -\frac{(Z_2 + r_0)A_V}{Z_1 + Z_2 + r_0} \]

\[ \frac{V_0'}{V_i}(s) = -\frac{(Z_2 + r_0)A_V}{Z_1 + Z_2 + r_0} \]

\[ \frac{V_0'}{V_i}(s) = \left( \frac{v_o'}{v_i} \right) \frac{Z_1 + Z_2}{Z_1 + Z_2 + r_0} = -\frac{Z_2}{Z_1} \left[ 1 - \frac{r_0}{A_VZ_2} \right] \frac{1}{1 + \frac{Z_2 + r_0}{Z_1}} \]
The error can be approximated as:

\[
\text{Error} \approx -\frac{r_0}{A_V Z_2} - \frac{1 + \frac{Z_2 + r_0}{Z_1}}{A_V}
\]

Determined by the OPAMP open-loop gain

Determined by the OPAMP output impedance
Next Time

• Common-Mode Feedback Techniques