Lecture 4: Channel Pulse Model & Modulation Schemes

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Announcements & Agenda

- Lab 1 Report and Prelab 2 due Feb. 6
- ISI
- Channel pulse model
- Peak distortion analysis
- Compare NRZ, PAM-4, and Duobinary modulation
- Reference material for this lecture
  - Peak distortion analysis paper by Casper (posted on web)
  - Notes from H. Song, Arizona State
  - Papers posted on PAM-4 and duobinary modulation
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI).
- ISI is caused by:
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

\[ y^{(1)}(t) = c^{(1)}(t) * h(t) \]
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated “1”s and “0”s

\[
\text{Data} = \text{“1000101”}
\]

\[
\begin{align*}
\text{“1” Symbol} & : c_k^{(1)}(t) = u(t - kT) - u(t - (k + 1)T) \\
\text{“0” Symbol} & : c_k^{(0)}(t) = -c_k^{(1)}(t)
\end{align*}
\]

where \( u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \)
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated “1”s and “0”s

\[ V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t) \]
Channel Response to NRZ Data

- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses

\[ V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H(c_k^{(d_k)}(t)) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \]
Channel Pulse Response

\[ y^{(d_k)}(t) = c^{(d_k)}(t) \ast h(t) \]

\( y^{(1)}(t) \) sampled relative to pulse peak:

\[ [... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...] \]

\( k = [... -2 1 0 1 2 3 4 5 6 ...] \)

By Linearity: \( y^{(0)}(t) = -1 \times y^{(1)}(t) \)
Channel Data Stream Response

Input Data Stream

Pulse Responses

Channel Response
Channel “FIR” Model

\[ c_0^{(1)}(t) \xrightarrow{H} H(c_0^{(1)}(t)) = y_0^{(1)}(t) \]

\[ c_0^{(1)}(t) \xrightarrow{Z^{(D-1)}} Z^{-1} Z^{-1} Z^{-1} Z^{-1} Z^{-1} \ldots \]

\[ a = [\ldots a_2 \ a_1 \ a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ldots] \]

\[ y^{(1)}(t) \text{ sampled relative to pulse peak:} \]
\[ \ldots 0.003 \ 0.036 \ 0.540 \ 0.165 \ 0.065 \ 0.033 \ 0.020 \ 0.012 \ 0.009 \ldots \]
Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case “1” is summation of a “1” pulse with all negative non k=0 pulse responses

\[
s_1(t) = y_0^{(1)}(t) + \sum_{k=-\infty}^{\infty} y^{(d_k)}(t-kT) \bigg|_{y(t-kT)<0}
\]

- Worst-case “0” is summation of a “0” pulse with all positive non k=0 pulse responses

\[
s_0(t) = y_0^{(0)}(t) + \sum_{k=-\infty}^{\infty} y^{(d_k)}(t-kT) \bigg|_{y(t-kT)>0}
\]
Peak Distortion Analysis

- Worst-case eye height is \( s_1(t) - s_0(t) \)

\[
s(t) = s_1(t) - s_0(t) = (y_0^{(1)}(t) - y_0^{(0)}(t)) + \left( \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \big|_{y(t-kT)<0} - \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \big|_{y(t-kT)>0} \right)
\]

Because \( y_0^{(0)}(t) = -1(y_0^{(1)}(t)) \)

\[
s(t) = 2 \left( y_0^{(1)}(t) + \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \big|_{y(t-kT)<0} - \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \big|_{y(t-kT)>0} \right)
\]

"1" pulse worst-case "1" edge

"1" pulse worst-case "0" edge

- If symmetric "1" and "0" pulses (linearity), then only positive pulse response is needed
Peak Distortion Analysis Example 1

\[ y_0^{(1)}(t) = 0.540 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \bigg|_{y(t-kT)<0} = -0.007 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \bigg|_{y(t-kT)>0} = 0.389 \]

\[ s(t) = 2(0.540 - 0.007 - 0.389) = 0.288 \]
Worst-Case Bit Pattern

- Pulse response can be used to find the worst-case bit pattern

\[ \text{Pulse } a = [\ldots a_{-2} \ a_{-1} \ a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ \ldots] \]

- Flip pulse matrix about cursor \( a_0 \) and the bits are the inverted sign of the pulse ISI

\[ [\ldots -\text{sign}(a_6) -\text{sign}(a_5) -\text{sign}(a_4) -\text{sign}(a_3) -\text{sign}(a_2) -\text{sign}(a_1) 1 -\text{sign}(a_{-1}) -\text{sign}(a_{-2}) \ldots] \]
Peak Distortion Analysis Example 2

\[ y_0^{(1)}(t) = 0.426 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT) < 0} = -0.053 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT) > 0} = 0.542 \]

\[ s(t) = 2(0.426 - 0.053 - 0.542) = -0.338 \]
Modulation Schemes

- **Binary, NRZ, PAM-2**
  - Simplest, most common modulation format
- **PAM-4**
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run $\frac{1}{2}$ speed
- **Duobinary** $w[n] = x[n] + x[n-1]$
  - Allows for controlled ISI, symbol at RX is current bit plus preceding bit
  - Results in less channel equalization

\[
\begin{align*}
1 & \rightarrow 1 & \text{if } x[n-1] = 1 \\
0 & \rightarrow 0 & \text{if } x[n-1] = 0 \\
1 & \rightarrow 0 & \text{if } x[n-1] = 0 \text{ OR } x[n] = 1 \\
0 & \rightarrow 1 & \text{if } x[n-1] = 1 \text{ OR } x[n] = 0
\end{align*}
\]
Modulation Frequency Spectrum

Majority of signal power in 1GHz bandwidth

Majority of signal power in 0.5GHz bandwidth

Majority of signal power in 0.5GHz bandwidth
Nyquist Frequency

- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect $R_s$ (symbols/s) without ISI is $R_s/2$ (Hz)
  - Thus, a system with bandwidth $W=1/2T=R_s/2$ (Hz) can support a maximum transmission rate of $2W=1/T=R_s$ (symbols/s) without ISI
    \[
    \frac{1}{2T} = \frac{R_s}{2} \leq W \Rightarrow \frac{R_s}{W} \leq 2 \quad \text{(symbols/s/Hz)}
    \]
  - For ideal Nyquist pulses (sinc), the required bandwidth is only $R_s/2$ to support an $R_s$ symbol rate

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Bits/ Symbol</th>
<th>Nyquist Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ</td>
<td>1</td>
<td>$R_s/2 = 1/2T_b$</td>
</tr>
<tr>
<td>PAM-4</td>
<td>2</td>
<td>$R_s/2 = 1/4T_b$</td>
</tr>
</tbody>
</table>

- Duobinary is not Nyquist signaling, as it employs controlled ISI for reduced signal bandwidth
NRZ vs PAM-4

- PAM-4 should be considered when
  - Slope of channel insertion loss ($S_{21}$) exceeds reduction in PAM-4 eye height
    - Insertion loss over an octave is greater than $20 \times \log_{10}(1/3) = -9.54$ dB
  - On-chip clock speed limitations
PAM-4 Receiver

- 3x the comparators of NRZ RX

[Stojanovic J SSC 2005]
NRZ vs PAM-4 – Desktop Channel

- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is only 2.7dB
  - NRZ has better voltage margin

![Channel Frequency Responses](image)

![Desktop 10Gb/s NRZ Eye](image)

![Desktop 10Gb/s PAM-4 Eye](image)
NRZ vs PAM-4 – T20 Server Channel

- Eyes are produced with 4-tap TX FIR equalization
- Loss in the octave between 2.5 and 5GHz is 15.8dB
  - PAM-4 “might” be a better choice
Multi-Level PAM Challenges

- Receiver complexity increases considerably
  - 3x input comparators (2-bit ADC)
  - Input signal is no longer self-referenced at 0V differential
    - Need to generate reference threshold levels, which will be dependent on channel loss and TX equalization

- CDR can display extra jitter due to multiple “zero crossing” times

- Smaller eyes are more sensitive to cross-talk due to maximum transitions

- Advanced equalization (DFE) can allow NRZ signaling to have comparable (or better) performance even with >9.5dB loss per octave
Duobinary Signaling

\[ w[n] = x[n] + x[n-1] \]

Binary \((1, -1)\) \(\rightarrow\) TX EQ \(\rightarrow\) Channel \(\rightarrow\) RX EQ \(\rightarrow\) Duobinary \((2, 0, -2)\)

\[ H(z) = 1 + z^{-1} \]

duobinary response

Superposition of single bit response
Duobinary Signaling w/ Precoder

- With precoder, “middle” signal at the receiver maps to a “1” and “high” and “low” signal maps to a “0”
- Precoder allows for binary signal out of transmitter resulting in a power gain
- Channel can be leveraged to aid in duobinary pulse shaping
- Eliminates error propagation at receiver
- Similar performance to using a 1-tap loop-unrolled DFE at RX
10Gb/s Modulation Comparisons

- Channel input = 600mV_{pp}
- 2-tap TX FIR equalization
- Both duobinary and PAM-4 perform better
- With more equalization NRZ will be more competitive
Modulation Take-Away Points

- Loss-slope guidelines are a good place to start in consideration of alternate modulation schemes

- More advanced modulation trades-off receiver complexity versus equalization complexity

- Advanced modulation challenges
  - Peak TX power limitations
  - Setting RX comparator thresholds and controlling offsets
  - CDR complexity
  - Crosstalk sensitivity (PAM-4)

- Need link analysis tools that consider voltage, timing, and crosstalk noise to choose best modulation scheme for a given channel
Next Time

• Link Circuits
  • Termination structures
  • Drivers
  • Receivers